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The Properties of the Equity Premium and the Risk-Free Rate: An Investigation Across Time and Countries

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We examine the relationship between the equity premium and the risk-free rate over time for Group of Seven countries. We show the existence of subsample instabilities, cross-country differences, and examine whether a consumption-based CAPM model is able to explain the heterogeneity of the data when cross-country and time-series differences in technology parameters are accounted for. We demonstrate that the basic features of the equity premium and risk-free puzzles remain regardless of the sample period and the country considered. Modifications of the basic setup also fall short of providing an explanation for the puzzles. [JEL C15, E43, G12]

The historical magnitude of the equity premium and of the average real risk-free rate in the United States has been the object of intense study in the past 15 years and continues to fascinate investors and academics. It is well known, for example, that the average real return on stocks is much higher than the average short-term real interest rate and that the volatility of the former is much higher than the volatility of the latter. In the past five years the excess return of equity over short-term risk-free interest rates has been even larger than in the previous decade and stock volatility has also substantially increased. These two facts together have prompted some commentators to mention the possibility that a bubble may have artificially inflated stock (and other asset) prices.

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Economists have struggled to try to understand these patterns and to identify the fundamentals that have caused movements in stock returns (and the equity premium) over time. Since the seminal work of Mehra and Prescott (1985) many authors, including Reitz (1988), Weil (1989), Labadie (1989), Epstein and Zin (1990), Constantinides (1990), Mankiw and Zeldes (1991), Cecchetti, Lam, and Mark (1993), and Campbell and Cochrane (1999), have modified a basic consumption-based CAPM model to account for the wide discrepancy in the properties of data simulated by the model and the actual U.S. data. The existing literature, thoroughly surveyed by Kocherlakota (1996) and Campbell (1999), has however overlooked several empirical issues that may be useful in understanding the extent and the dimensions of the "puzzle" and in formulating suitable models capable of explaining the relationship between returns on equities and the risk-free rate.

First, apart from Campbell (1999), the relationship between the equity premium and the risk-free rate has been documented almost exclusively for the United States, and evidence from other countries has been disregarded. Second, the historical features of the equity premium and of the risk-free rate have changed over time (see, for example, Mehra and Prescott, 1985, p. 147). One might wonder whether the choice of sample period would influence our perception of the economic relevance of the phenomenon and whether considering data that is more homogeneous—say, only post-WWII or post-1970 data—might change the features of the relationship. Indeed, Jagannathan, McGrattan, and Scherbina (2000) provide evidence of a significant decline of the U.S. equity premium in the past three decades.

The task of this paper is twofold. First, we characterize the relationship between equity premium (EP) and the risk-free rate (R) in the Group of Seven (G-7) countries, using quarterly data from 1970 onward. The G-7 countries as a block are large in world financial markets (the total value of their stock and T-bill markets exceeds 70 percent of the world market) and their economies are sufficiently homogeneous and developed to make the comparison meaningful. Second, we want to know whether a standard consumption-based CAPM model can account for the time variations and the cross-country differences present in the data. In this respect, examining cross-country and cross-time variations in the distribution of the (EP, R) pair offers a much more challenging term of comparison for judging the validity of existing theories.

The remainder of the paper is organized in four sections. Section I documents the properties of the two variables for the United States, Canada, United Kingdom, Japan, Italy, Germany, and France for four samples (1971:1–1999:3, 1971:1–1980:4, 1981:1–1990:1, and 1991:1–1999:3). For each country we present sample estimates of the first two moments and of the autoregressive coefficient of (EP, R), and we examine whether estimates are stable across time and similar across countries. We show that there are important differences in the mean and in the standard deviation of the two variables both across time within countries and across countries for a given sample period. Rolling estimates of the moments of the two variables indicate the presence of somewhat similar patterns of time variations across countries. Heterogeneities in the mean of the risk-free rate appear to be related to time changes and cross-country differences in the distributional properties of the inflation rate. Variations across time in the equity premium, on the other hand, are due equally to changes in the properties of equity returns and of the risk-free rate.

In Section II we briefly summarize the features of the model we will use to explain the cross-country and the cross-sample evidence. We highlight the relationship between the parameter of the model and the moments of the equity premium and the risk-free rate. Following Mehra and Prescott (1985), we are interested in knowing what kind of preference parameters are necessary to match the moments of the mean equity premium and the mean risk rate across countries and across subsamples. Moreover, following Hansen and Jagannathan (1991), we are interested in estimating the variability of the implicit discount factor needed to match Sharpe ratios in each country and each sample period.

Section III describes the results. We find that the risk aversion parameter and the implicit rate of time preference needed to match the mean of the actual (EP, R) pair are within economically unreasonable ranges both across countries and across time. We also show that in all countries and all sample periods the variability of the implicit discount factor substantially exceeds the variability of the discount factor used by the model (a power function of consumption growth) and this is the reason for the poor match in the dimensions examined.

In Section IV we discuss modifications of the basic model that aim to increase the variability of the discount factor of the model. We consider allowing for heteroscedasticity in the driving forces (as in Kandel and Stambaugh, 1990; or Canova and Marrinan, 1993), for the presence of inflation effects (as in Labadie, 1989), for leverage (as in Benninga and Protopapadakis 1990; or Kandel and Stambaugh, 1991), for non-expected utility functions (as in Epstein and Zin, 1990; or Weil, 1989), and argue that none is able to improve the performance of the model in the dimensions examined. Finally, we examine whether time variations in risk are able to improve the performance of the model (as in Campbell and Cochrane, 1999). We find that the implicit "stock of habit," which we derive from our rolling estimates of the risk aversion coefficient, has time-series properties that are inconsistent with those of aggregate per capita consumption, therefore implicitly denying the usefulness of this route to solve the puzzle.

Overall, our results suggest that the (EP, R) puzzle is worse than commonly thought: it exists regardless of the country and may worsen with the sample period used. Moreover, none of the modifications that the literature has suggested for increasing the variability of the discount factor in the asset pricing equation appear to deliver time-series properties of the model close to those of the existing data.

I. The Properties of the Equity Premium and the Risk-Free Rate

This section documents the time-series properties of the (EP, R) pair for the United States, Canada, United Kingdom, Japan, Italy, Germany, and France for the period 1971:1–1999:3 and for three subperiods (1971:1–1980:4; 1981:1–1990:4; 1991:1–1999:3). The subperiod division is somewhat arbitrary but the break point is chosen keeping in mind the behavior of inflation during the three subperiods. Garcia and Perron (1991) show that the real risk-free rate in the United States reached a breaking point in 1981 due to changes in Fed policies. Because after that date real rates moved to a higher mean level all over the world, it is likely that this date is also important for the other G-7 countries. Moreover, in the 1990s inflation declined substantially in all countries and nominal interest rates dropped around the industrialized world. In computing both the real equity premium and the real risk-free rate we use the domestic CPI and calculate returns in local currencies. Substitution of the GNP deflator for the CPI leaves the results qualitatively unchanged. Later on, we will discuss results obtained converting foreign returns to U.S. dollars. The definition of the variables used to construct the equity premium, as well as the other relevant data for our analysis, are essentially those used by Campbell (1999). Morgan Stanley Capital International National Price and Gross Return indices in local currency are used to compute stock returns, whereas inflation, consumption data, and short-term interest rates are taken from International Financial Statistics and Organization for Economic Cooperation and Development databases.

Table 1 presents estimates of the mean, standard deviation, and AR(1) coefficient for the (EP, R) pair for the four samples under consideration and for each of the seven countries. Returns on both stocks and risk-free securities are computed for a three-month holding period, and means and standard deviations are annualized. Two aspects of the table deserve attention. First, the moments of both variables display large variations across countries and this is true even for more financially integrated countries like the United States, Canada, Germany, France, and Italy. The mean of the equity premium for the full sample ranges from 2.26 (for Canada) to 5.68 (for the United States). The mean of the risk-free rate ranges from 1.38 (for Japan) to 3.13 (for Germany). In all cases, means are significantly different from zero. Note that the means of the two variables in the United States are approximately of the same magnitude as those produced using annual data by Mehra and Prescott (1985) for the 1880–1978 period and by Bonomo and Garcia (1993) for the sample 1889–1987 period. The standard deviation of the equity premium is large, ranging from 16.76 in the United States to 27.08 in Italy, while standard deviations of the risk-free rate are smaller but there are significant variations across countries, with the United States at the lower end (1.61) and the United Kingdom at the upper end (3.05). There is no significant evidence of serial correlation in the equity premium series in any of the countries. However, The AR(1) for the risk-free rate ranges from 0.28 for Germany to 0.72 for France and in five of the seven countries is significantly different from zero.

Second, large time variations of moments are evident across the subsamples. For example, the mean of the equity premium is steadily increasing across subsamples in the United States, Germany, United Kingdom, France, and Italy; in Canada it decreases in the second and increases in the third subsample, while in Japan the opposite occurs. The time-series behavior of the mean of the risk-free rate is more similar across countries: it increases everywhere in the 1980s and then declines in the 1990s. The standard deviations of both variables also change over time but variations appear to be smaller in magnitude than those observed in the mean of the variables. Notice that the variability of the equity premium steadily decreases over time in the United States and the United Kingdom, while in the other five countries it increases in the second subsample and decreases afterwards. The variability of the risk-free rate is first decreasing in the 1980s and increasing in

Table 1. Cross-Country Statistics: Equity Premium–Risk-Free Rate
(Three-month holding period)

		1971:1–1999:3		1971:1–1980:4		1981:1–1990:4		1991:1–1999:3	
Country		EP	R	EP	R	EP	R	EP	R
United States	Mean	5.68	1.61	–0.03	–0.78	4.69	3.89	13.84	1.95
	SD	16.76	1.49	19.14	1.23	17.48	1.26	11.66	0.77
	AR(1)	0.05	0.57	0.06	0.28	0.03	0.17	–0.17	0.17
Japan	Mean	3.63	1.38	4.19	–0.81	8.27	3.80	–2.33	1.33
	SD	21.57	2.18	19.82	2.79	24.61	1.25	20.12	1.34
	AR(1)	0.01	0.43	0.15	0.34	–0.21	0.00	0.18	0.31
Canada	Mean	2.26	2.95	4.22	–0.16	–3.67	5.67	6.64	3.70
	SD	17.46	1.82	17.31	1.65	19.90	1.03	14.43	1.21
	AR(1)	0.08	0.67	0.12	0.45	0.15	0.37	–0.22	0.20
Germany	Mean	3.19	3.13	–4.10	2.14	5.95	4.15	9.08	3.17
	SD	20.08	1.51	16.66	1.84	24.36	0.99	18.33	1.40
	AR(1)	0.02	0.28	–0.06	0.24	0.08	0.31	–0.09	0.23
France	Mean	4.12	2.91	–1.86	–0.30	5.37	5.06	10.03	4.40
	SD	22.95	1.66	22.66	1.15	26.64	1.03	18.32	1.21
	AR(1)	0.03	0.72	0.09	0.25	0.04	0.22	–0.16	0.71
Italy	Mean	0.17	2.89	–4.91	–2.49	–0.38	6.23	6.62	5.22
	SD	27.08	2.61	27.42	2.17	29.66	1.38	23.85	1.41
	AR(1)	0.03	0.66	0.01	0.04	0.08	0.32	–0.01	0.58
United Kingdom	Mean	5.15	1.58	1.75	–3.29	6.21	4.03	8.13	3.85
	SD	21.42	3.05	27.82	3.50	19.34	1.75	13.70	1.20
	AR(1)	0.08	0.47	0.19	0.19	–0.05	–0.01	–0.17	0.47

Notes: EP is the equity premium and R is the risk-free rate. SD is the standard deviation of the series; AR(1) is the first-order autoregressive coefficient. Means and standard deviations are annualized.

the 1990s in Germany, Canada, Japan, France, and Italy while for the United States and the United Kingdom the opposite occurs. The AR(1) coefficient of both variables displays very little variation in the first two subsamples and it increases in absolute value in the third subsample. Consistent with the characterization offered by some authors (e.g., Blanchard, 1993), we find that real equity returns and the real risk-free rate do not significantly move together over subsamples. The correlations of these two variables range from –0.43 to 0.22 in the first subsample and only the (negative) values for Germany and the United Kingdom are significant—from –0.24 to 0.10 in the second subsample and from –0.48 to 0.29 in the third subsample—and the (negative) values of Canada, France, and Italy are significant. Apparently, real equity returns and the real risk-free rate have amplitude movements across subsamples that are, in many instances, very similar.

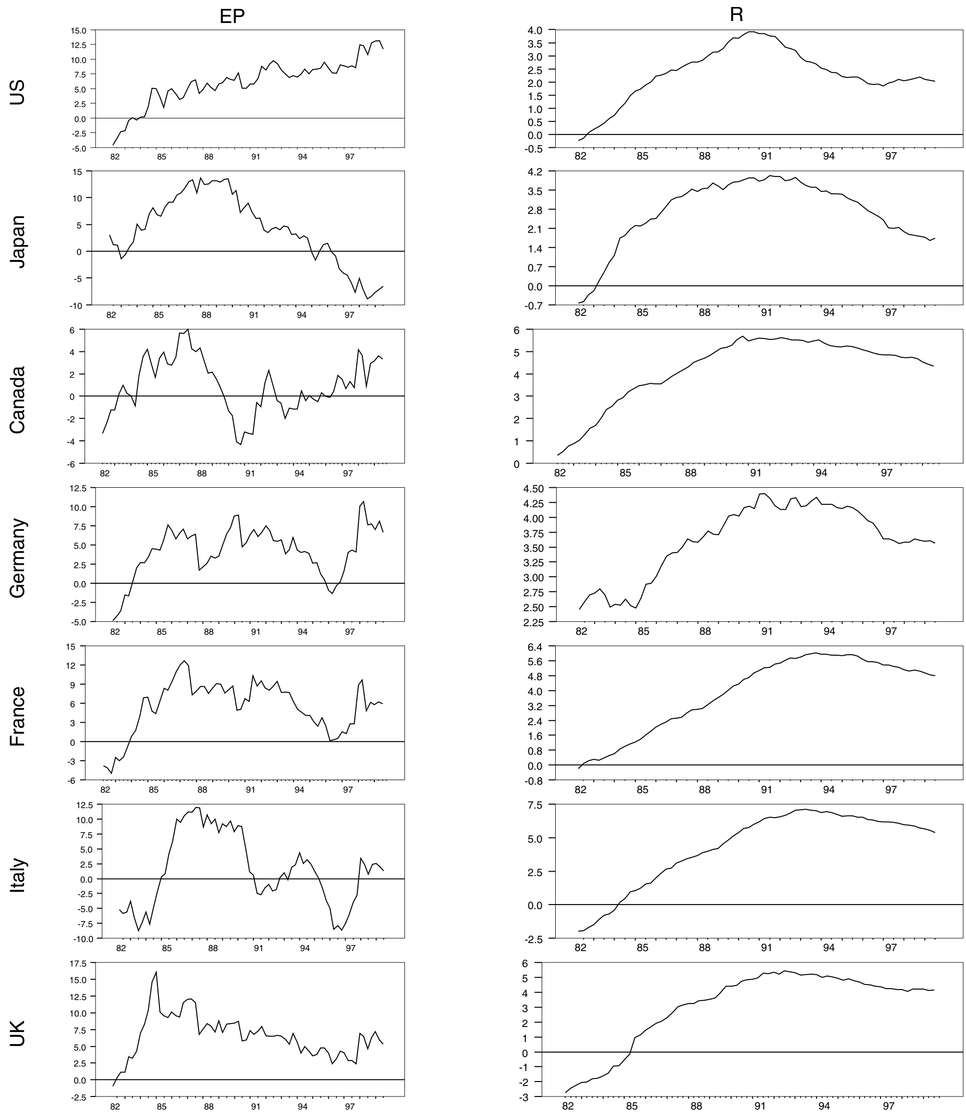
To obtain a deeper understanding of the time variations in the distribution of the two variables, we compute rolling moments using a ten-year window. We present the results graphically in Figures 1–3, which plot, respectively, the means, the standard deviations, and the AR(1) coefficients for the seven countries. The dates in each graph refer to the end of the sample period used (i.e., 1985:4 indicates that moments are computed using the sample 1976:1–1985:4). The three figures confirm the initial impression, or pervasive time variations. The mean of the equity premium steadily increases in the United States, Germany, and France; in the United Kingdom it declines after an initial jump, while in Italy and Canada there are periods (1973–1985 for the former and 1980–1990 for the latter) when the mean of the equity premium was negative. Also, consistent with a priori expectations, the mean of the equity premium in Japan is negative throughout the 1990s. The mean of the risk-free rate increases up to the beginning of the 1990s and then declines, and this pattern is approximately the same in every country.

The standard deviation of the equity premium displays a trend decline in the United States, Canada, and Italy; a sharper decline around 1985 in the United Kingdom; and large variations with no trend change in France. For Germany and Japan, two countries with severe economic problems in the past ten years, the standard deviations of the equity premium increased in the 1990s. The standard deviation of the risk-free rate displays similar features across countries: there is a trend decline in the United States, Japan, and Canada; a hump followed by a decline in the United Kingdom, France, and Italy; and in Germany the variability increased in the 1990s approximately to the level existing in the 1970s and its trend mirrors the one in the standard deviation of the equity premium.

Finally, the AR(1) coefficient for the equity premium starts positive but becomes negative in the United States, Canada, Japan, and the United Kingdom—in these latter two countries persistently so—while for the other three countries there are large variations but no discernible trend. The AR(1) coefficient for the risk-free rate has similar time variations across countries—a decline in the 1980s after a peak in the 1970s. However, while in the United States and Canada the pattern continues in the 1990s, in the other five countries the AR(1) coefficient shows a positive trend with sharp increases noticeable in Italy and France.

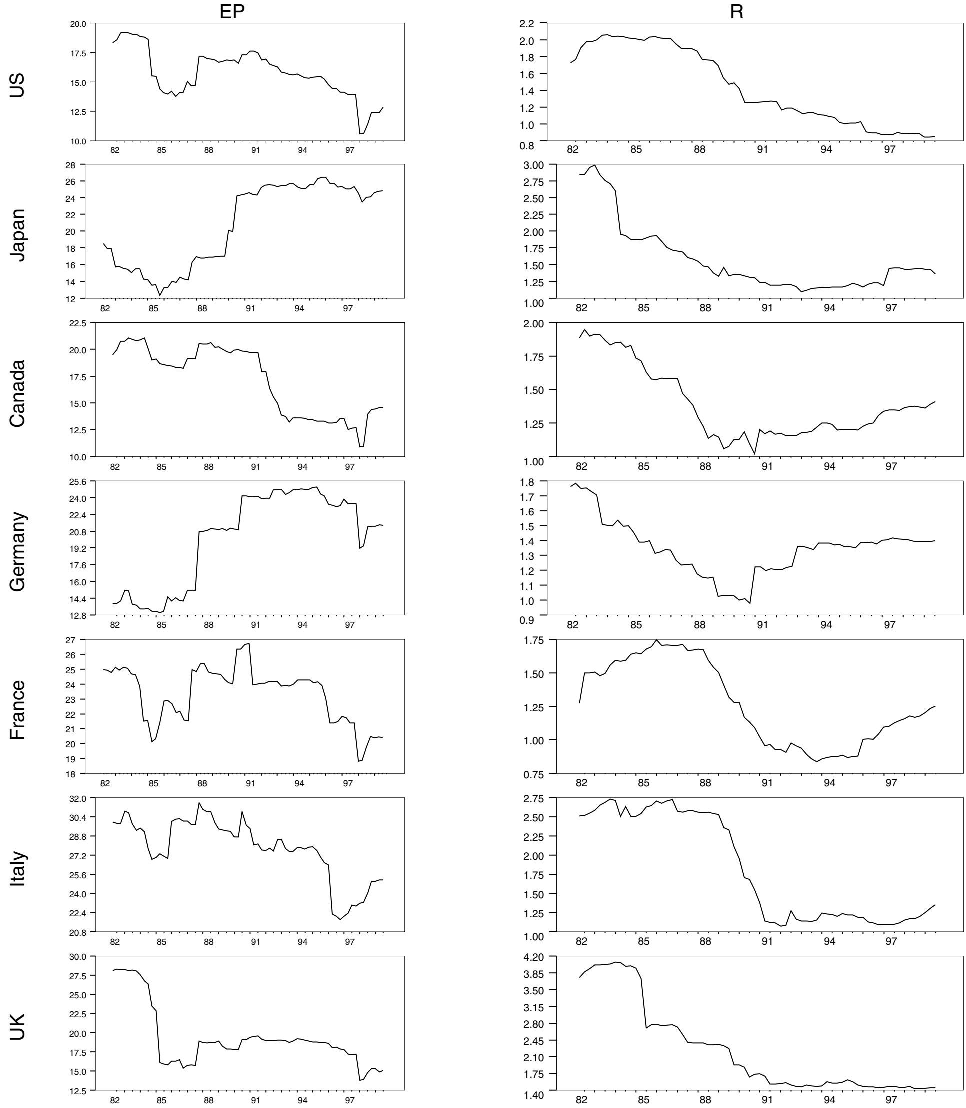
In sum, it appears that the mean of the equity premium is increasing over time in at least four of the seven countries. Its variability is decreasing (except in Japan

Figure 1. Means



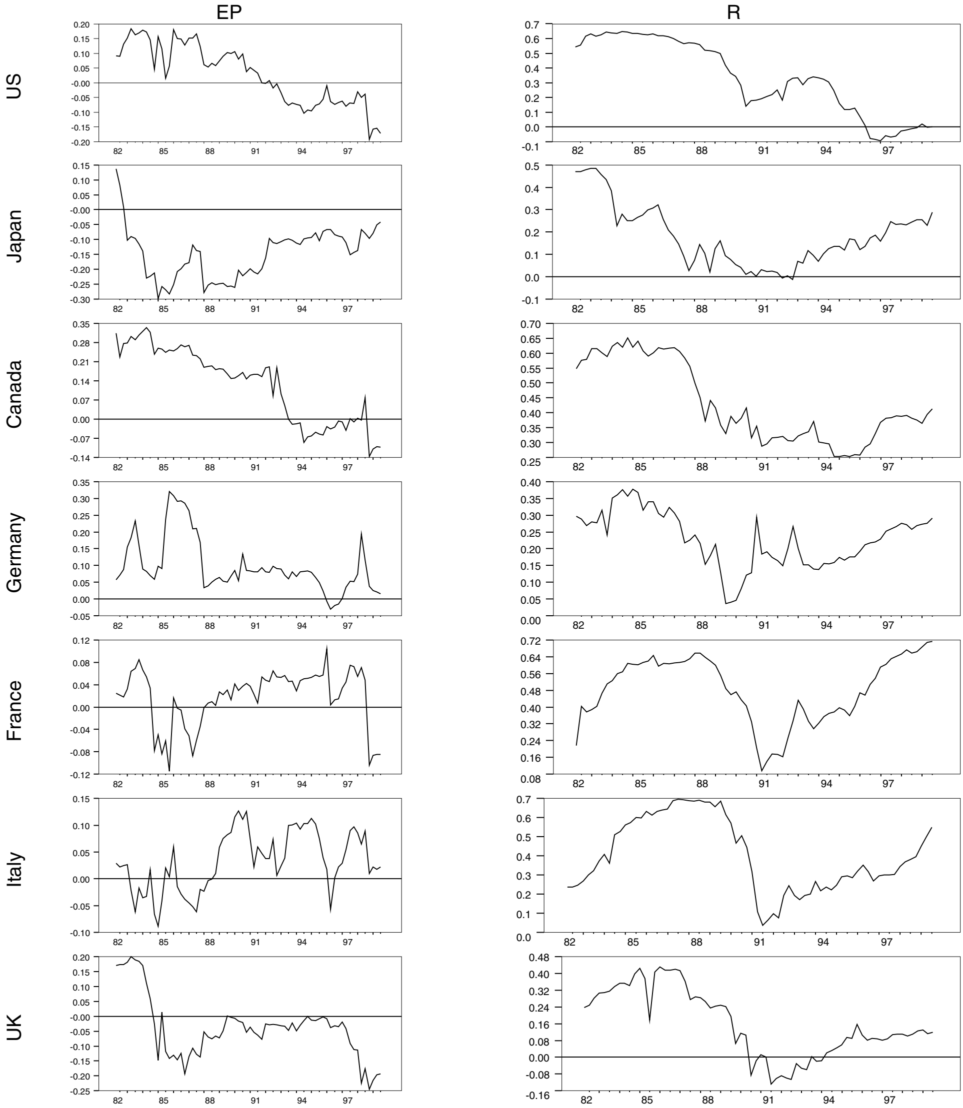
and Germany) and some mean reversion appears in the latter part of the sample in several countries. The distribution of the real risk-free rate, on the other hand, is strongly and significantly negatively linked to the behavior of inflation. In fact, the correlation between inflation and the real risk-free rate is on average equal to –0.65 for the full sample, –0.73 in the first subsample, –0.62 in the second subsample, and –0.52 in the third subsample. Overall, the mean has a hump-shaped time profile with a peak in the beginning of the 1990s, the standard deviation declines over time as inflation variability drops, and, in the latter part of the sample, the real risk-free rate acquires some of the persistence displayed by short-term nominal rates, primarily in European countries.

Figure 2. Standard Deviations



Are the differences across countries and across time statistically significant? Our interest in this question stems from two different points of view. First, we would like to ascertain the reliability of a description of the (EP-R) relationship that uses data from the entire sample. Mehra and Prescott report that the mean of the (EP-R) pair in the United States changed substantially over the first 70 years of the 1900s (see Table 1). Siegel (1992) and Blanchard (1993) present similar evidence up to the 1980s. The large time variations in Figure 1 suggest that this may be the case also for our more recent data set. Since most of the work attempting to replicate the properties of the equity premium and the risk-free rate using general equilibrium models assumes that their

Figure 3. AR1 Coefficients



distributions are time invariant, our analysis may allow us to evaluate the appropriateness of these exercises. Furthermore, many commentators have argued that the behavior of stock and bond markets in the 1990s appears to be unusual from a historical perspective. Hence, we would like to examine whether the time-series properties of the equity premium and the risk-free rate in this period are typical or not. In addition, we would like to know whether the cross-country differences we have noticed are accidental, in which case restricting the analysis to one country is sufficient, or whether there is additional information in the international cross section of data that is neglected when one focuses attention on the U.S. evidence.

To examine the two hypotheses we used a distance-type test of the form:

$$Q = (x_1 - x_2)\sum^{-1}(x_1 - x_2)', \tag{1}$$

where x_1 and x_2 are vectors of estimated moments (either means, variances, or AR(1)) across subsamples or across countries, and Σ is the covariance matrix of $x_1 - x_2$. Under the assumption that EP and R are general linear processes with innovations having bounded fourth moments, the sample mean, the sample variance, and the AR(1) coefficient are asymptotically normally distributed (see Fuller, 1976). Hence, asymptotically, $Q : \chi^2(m)$, where $m = \dim(x_1) = \dim(x_2)$. To account for the presence of serial correlation and heteroscedasticity, we estimated Σ using Newey and West's (1987) approach. Since the size of each subsample is small and there may be doubts about the appropriateness of the asymptotic approximation, we complement these tests with a Kolmogorov-Smirnov (KS) test. The KS statistic is useful because it gives a compact indication of the significance of the differences in the distribution (as opposed to single moments) and because the small sample distribution of this statistic is tabulated in most statistical textbooks. When testing similarities across countries we present results using the United States as a benchmark for all four samples, but we also use Germany or Japan as a benchmark with qualitatively similar results. When testing similarities across subsamples for each country we examine all possible pairs for a total of six combinations.

Are Time Variations Significant?

Table 2 reports the p-values of the tests for the equality of the means across time. Despite the large differences we observed in Table 1, there are relatively few instances when statistically significant variations in the mean of the equity premium can be measured. This occurs primarily in the United States, Germany, France, and the United Kingdom, while in Japan and Canada there are significant differences across some of the subsamples. Our test confirms that in the United States the mean of EP in the 1990s is substantially different from the historical experience of the 1970s and 1980s and the mean of the equity premium in the third subsample in Japan and Germany is different from either of the previous subsamples at 5 percent significance. For the risk-free rate, rejections of the null hypothesis of equality of the means over samples are generalized: it is only when comparing the full sample and the third subperiod in Japan and the United States, and the full sample and the second subperiod in Germany, that the means of the risk-free rate are not statistically different.

As shown in Table 3, which reports the p-value of the tests of equality of the variances across time, the picture is similar. There are several samples in which the second moments of the equity premium are statistically different in the United States, Germany, and the United Kingdom. In the other countries the large variability of the second moments prevents the rejection of the null hypothesis. The second moment of the risk-free rate displays significant time variations and, in all countries but Germany, we reject the null hypothesis at least once. In the United States, the variance of the risk-free rate in the third subsample is substantially

Table 2. Tests of Equalities of Means Across Periods
(Three-month holding period)

Country	Equity Premium						Risk-Free Rate					
	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3
United States	0.11	0.75	0.00	0.26	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00
Japan	0.88	0.29	0.11	0.42	0.16	0.03	0.00	0.00	0.85	0.00	0.00	0.00
Canada	0.55	0.09	0.12	0.06	0.51	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Germany	0.01	0.38	0.08	0.01	0.00	0.65	0.00	0.16	0.00	0.00	0.00	0.03
France	0.17	0.79	0.10	0.20	0.01	0.36	0.00	0.00	0.00	0.00	0.00	0.01
Italy	0.33	0.91	0.16	0.49	0.05	0.25	0.00	0.00	0.00	0.00	0.00	0.00
United Kingdom	0.50	0.77	0.32	0.72	0.21	0.65	0.00	0.00	0.00	0.00	0.00	0.00

Notes: The table reports the significance level of a $\chi^2(1)$ test. F indicates the sample 1971:1–1999:3; S1 the sample 1971:1–1980:4; S2 the sample 1981:1–1990:4; S3 the sample 1991:1–1999:3.

different from the estimate obtained in the full sample and in the two previous subsamples. We did not report tests for the AR(1) coefficients since they were consistent with the graphical analysis; only those of the risk-free rate for France and Italy are significantly different over time.

Table 4 reports the p-values of the KS tests across time for each of the seven countries. These tests provide additional useful information: in Japan, Germany, and Italy the distribution of the equity premium (EP) in the full sample is different from the one in each of the three subsamples; in Germany this is true even when comparing the first two to the last subsample, and in the United States the distribution of EP in the third subsample is substantially different from that of previous subsamples. For the other three countries the test is unable to reject the hypothesis that the distribution of EP is unchanged across time. Two explanations are possible to reconcile these results with those obtained with the previous test. First, it is possible that variations in the first two moments are compensated by variations in higher moments, therefore resulting in changes in the shape of the distribution that are hard to detect. Second, since the KS test is based on an exact small sample distribution, it is more reliable than the asymptotic approximation based on the χ^2 statistics. These difficulties in interpreting the results do not emerge when we consider the distribution of the risk-free rate: the KS test in fact rejects the null hypothesis of stability of the distribution in almost all cases.

To summarize, there are significant changes in the moments of the EP across time, at least in the United States, Japan, Germany, and the United Kingdom, while the entire distribution has changed in Japan, Germany, and Italy. In the United States there is a significant upward trend in the mean and a significant downward trend in the variance, both of which are unprecedented from a historical point of view; in Germany the mean is significantly increasing over time, while in Japan it has been significantly declining since the middle of the 1980s; finally, in the United Kingdom the variance has significantly declined over time. The distribution of the risk-free rate appears to be unstable across time in all countries and both moments appear to be affected.

Are Cross-Country Variations Significant?

We report the results of our investigation in Table 5, where we present the p-value of the χ^2 test for testing the equality of the means across countries, and in Table 6 where we present the p-value of the KS test for testing the equality of the distributions across countries. From Table 5 we see that the first moment of the equity premium is somewhat unstable across countries: in Italy in the full sample, in Canada in the second subsample, and in Japan, Canada, Italy, and the United Kingdom in the third subsample the mean of the equity premium is significantly different from that of the United States at the 10 percent level. A similar test applied to the variances of the equity premium across countries in different subsamples indicates that no significant difference exists. These results should be contrasted with those of Table 6. Here the *distribution* of the EP in the United States is different from that of Japan, Germany, and Italy in the full sample, that of Canada in the second subsample, and that of Japan and Italy in the third subsample. Hence, it must be true that in some countries there are significant differences in higher moments of the distribution (in particular, the fourth one).

Table 3. Tests of Equalities of Variances Across Periods
(Three-month holding period)

Country	Equity Premium						Risk-Free Rate					
	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3
United States	0.39	0.76	0.00	0.60	0.01	0.02	0.28	0.31	0.00	0.90	0.06	0.02
Japan	0.56	0.26	0.63	0.16	0.93	0.19	0.25	0.00	0.00	0.01	0.01	0.71
Canada	0.95	0.41	0.09	0.46	0.30	0.08	0.62	0.00	0.02	0.09	0.22	0.44
Germany	0.06	0.18	0.42	0.02	0.40	0.08	0.71	0.79	0.24	0.61	0.24	0.56
France	0.94	0.30	0.04	0.36	0.23	0.03	0.05	0.01	0.13	0.59	0.82	0.52
Italy	0.92	0.53	0.31	0.64	0.40	0.21	0.27	0.00	0.00	0.03	0.04	0.93
United Kingdom	0.11	0.41	0.00	0.08	0.00	0.03	0.45	0.00	0.00	0.00	0.00	0.03

Notes: See Table 2.

Table 4. Tests of Equalities of Distributions Across Periods
(Three-month holding period)

Country	Equity Premium						Risk-Free Rate					
	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3	F–S1	F–S2	F–S3	S1–S2	S1–S3	S2–S3
United States	0.63	0.99	0.24	0.57	0.04	0.04	0.00	0.00	0.27	0.00	0.00	0.00
Japan	0.00	0.00	0.00	0.40	0.47	0.18	0.00	0.00	0.00	0.00	0.00	0.70
Canada	0.64	0.42	0.93	0.16	0.86	0.15	0.00	0.00	0.00	0.00	0.00	0.00
Germany	0.00	0.00	0.00	0.01	0.02	0.84	0.00	0.00	0.00	0.00	0.06	0.10
France	0.56	0.80	0.84	0.16	0.39	0.94	0.07	0.00	0.03	0.00	0.00	0.44
Italy	0.00	0.00	0.00	0.57	0.37	0.93	0.00	0.00	0.00	0.00	0.00	0.26
United Kingdom	0.62	0.99	0.95	0.57	0.24	0.93	0.00	0.00	0.03	0.00	0.00	0.06

Notes: The table reports the significance level of a Kolmogorov-Smirnov test. See Table 2.

Table 5. Tests of Equalities of Means: Individual Countries Versus the United States
(Three-month holding period)

Country	Equity Premium				Risk-Free Rate				Risk-Free Rate in U.S. Dollars			
	1971–99	1971–80	1981–90	1991–99	1971–99	1971–80	1981–90	1991–99	1971–99	1971–80	1981–90	1991–99
Japan	0.42	0.36	0.45	0.00	0.36	0.96	0.74	0.01	0.00	0.00	0.08	0.33
Canada	0.13	0.32	0.04	0.01	0.00	0.07	0.00	0.00	0.48	0.75	0.00	0.13
Germany	0.81	0.62	0.26	0.64	0.00	0.00	0.00	0.00	0.42	0.00	0.71	0.00
France	0.55	0.71	0.89	0.28	0.00	0.08	0.00	0.00	0.40	0.19	0.99	0.88
Italy	0.06	0.38	0.35	0.09	0.00	0.00	0.00	0.00	0.02	0.00	0.83	0.26
United Kingdom	0.83	0.75	0.71	0.05	0.92	0.00	0.00	0.00	0.20	0.12	0.56	0.95

Note: The table reports the significance level of a $\chi^2(1)$ test.

Table 6. Tests of Equalities of Distributions: Individual Countries Versus the United States
(Three-month holding period)

Country	Equity Premium				Risk-Free Rate			
	1971–99	1971–80	1981–90	1991–99	1971–99	1971–80	1981–90	1991–99
Japan	0.00	0.75	0.75	0.02	0.00	0.56	0.99	0.02
Canada	0.44	0.57	0.05	0.30	0.00	0.57	0.00	0.00
Germany	0.00	0.58	0.91	0.46	0.00	0.00	0.05	0.00
France	0.55	0.91	0.74	0.30	0.00	0.91	0.05	0.00
Italy	0.00	0.26	0.57	0.05	0.00	0.16	0.00	0.00
United Kingdom	0.66	0.75	0.90	0.46	0.00	0.05	0.00	0.00

Note: The table reports the significance level of the Kolmogorov-Smirnov statistics.

For the risk-free rate, results are fairly consistent across tests: the distribution is different across countries in the full sample and in the third subsample, while in the other two subsamples there are few instances when we do not reject the null hypothesis. These changes appear to be due, to a large extent, to variation across countries in the mean and, in Germany, also to differences in the variance.

It is worth noting that the statistically insignificant differences in the mean equity premium we have found in some countries are due to the large variability of the equity premium series that masks differences that are economically significant. To see why this is the case, consider two investors, one living in the United States and one living in Canada, and suppose they both use a naive trading rule that requires zero net investment (sell short risk-free assets and buy equity each quarter). Suppose they start with one unit of currency in 1971:1 and reinvest the gross proceeds from the investment each quarter. In 1999:3 the U.S. investor would have gained approximately one-third more than the Canadian investor, and the U.S. investor's utility would be about 50 percent higher if an exponential specification for utility is used. Clearly, the investment strategy we have described is autarkic and does not allow the Canadian investor, for example, to take advantage of the higher U.S. average equity premium in exchange for an increase in risk due to fluctuations of the Canadian dollar exchange rate. Would the picture we have so far presented change if we measure returns across countries in a common currency (say, the U.S. dollar)? More generally, are our results consistent with the large body of international financial literature dealing with the issues of (i) interest rate parity, (ii) the existence of unexploited arbitrage opportunities, and (iii) capital market integration? We tackle these questions next.

Discussion

In the case of interest rate parity, the important question is whether taking into account changes in real exchange rates alters the significance of the difference in the mean of real risk-free rates across countries. There are two reasons a priori to doubt that this will be the case. First, real interest rate parity has typically been rejected due to the existence of default risk or risk premium (see Jorion, 1996). That is, U.S. dollar-denominated real returns on risk-free assets are different from the return a U.S. investor would obtain by investing abroad, either covering the investment with a forward contract or incurring exchange risk. Second, Baxter (1994) showed that the link between real interest rate differentials and exchange rate changes is empirically fairly weak even under assumptions less restrictive than uncovered interest parity and ex ante purchasing power parity. To verify the hypothesis of real uncovered interest parity, we added real exchange rate changes to foreign real risk-free rates and tested once more the hypothesis of equality of the mean return across countries (see Table 5). Although there are still instances when we reject the null hypothesis, there are cases when the mean return adjusted for real exchange rate risk is no longer different across countries. For example, if we exclude Germany, in the 1990s real risk-free returns in dollars were very similar across the G-7 countries.

Does the rejection of uncovered real interest parity imply the existence of unexploited arbitrage opportunities across countries in some of the samples? The answer is no. For example, we have seen that the real risk-free rate is negatively correlated with inflation across countries but large differences exist in these correlations in different countries. Furthermore, the inflation process in some countries, for example, Italy, is very different from that of the United States as is the credibility of the commitment to repay its debt. Overall, it appears that the nominal risk-free rate has different risk characteristics in different periods so that some differences in the real risk-free rate across countries are to be expected.

Finally, it has been suggested that since the middle of the 1980s capital markets have become more integrated, and international factors may have become dominant in determining real returns across countries. This trend may be the reason why real uncovered interest parity in the 1990s is not rejected. Is this trend toward more integrated capital markets also responsible for the time variations in equity returns that make the EP somewhat unstable over time? We believe that, although potentially interesting, this explanation does not appear to completely explain the evidence. In fact, we find only weak evidence that the mean equity premium became more similar across countries in the 1990s, and in Japan and Italy, it seems to drift away from that of the United States in this subsample.

In conclusion, the distribution of both EP and R is unstable over time in many countries and somewhat different across countries in selected time periods. While the significance of the difference varies with the sample and the country considered, we do find that the mean equity premium in the United States, Japan, and Germany in the 1990s is different from what was observed in the previous two decades and that in the 1990s real risk-free returns denominated in U.S. dollars were somewhat more similar across countries. While both the distribution of the real risk-free rate and of real equity returns are changing across time, we find only weak evidence supporting the idea that integration of capital markets is responsible for these patterns. Whether changes in the technological frontier or in the structure of monetary policy shocks are also responsible for these alterations we cannot tell, but the topic is important and worth further investigation. Overall, it seems that by separating historical episodes with different time-series characteristics in the various countries we may have a better chance to understand whether existing theoretical models fail to reproduce the evidence.

II. Comparing a Model to the Data

To try to understand whether the facts we have described pose a puzzle from the point of view of the theory and to highlight the dependence of the moments of the equity premium and the risk-free rate on certain features of the model, we consider a standard consumption-based CAPM model and make two assumptions. First, countries may differ in the parameters governing preferences and technologies, but not in institutional setups or market arrangements. This is a simplifying but reasonable assumption since we are considering the seven most developed economies of the world. Moreover, it provides a useful benchmark to compare versions of the model where institutional constraints are introduced. Second, we assume that countries' financial markets are autarkic. This is consistent with our empirical approach, which considered the seven economies in isolation, and with the observation that portfolios of agents in the real world are far from being internationally diversified (see Baxter and Jermann, 1997). Under the opposite extreme assumption of perfect capital market integration the model would predict that investors would hold a world market portfolio of risky assets independently of their country of residence, an implication empirically rejected for most countries (see French and Poterba, 1991).

The model is a frictionless pure exchange economy featuring a single representative agent, one perishable consumption good produced by a single productive unit or "tree," and two assets, an equity share and a risk-free asset. The tree yields a random dividend each period and the equity share entitles its owner to that dividend. The risk-free asset entitles its owner to one unit of the consumption good at maturity. The representative agent in each country maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\alpha} - 1}{1-\alpha} \right) \tag{2}$$

subject to

$$c_t = y_t e_{t-1} + p_t^e \left(e_{t-1} - e_t \right) + f_{t-1,1} - p_t^{f,1} f_{t,1}, \tag{3}$$

where c_t is consumption, y_t is the tree's dividend, p_t^e and $p_t^{f,1}$ are the prices of the equity and of the one-period risk-free asset, e_t and $f_{t,1}$ are the agent's holdings of equity and of the risk-free asset, E_0 is the mathematical expectation operator conditional on information at time zero, β is the discount factor, and α is the relative risk aversion parameter. Dividends evolve according to

$$y_{t+1} = x_{t+1} y_t, \tag{4}$$

where x_{t+1} denotes the gross growth rate of dividends. Because all agents are identical, no trade will occur in equilibrium. Thus, $c_t = y_t$, $e_t = 1$, and $f_{t,1} = 0$ for all dates. The returns for the riskless security and on equity investments in each country satisfy the following optimality conditions:

$$1 = \beta E_t x_{t+1}^{-\alpha} \left(1 + R_{t,1} \right) \tag{5}$$

$$1 = E_t \beta x_{t+1}^{-\alpha} \left(1 + R_{t,1}^e \right), \tag{6}$$

where $(1 + R_{t,1}) = 1/p_{t,1}^f$ is the risk-free gross return and $(1 + R_{t,1}^e) = (p_{t+1}^e + y_{t+1})/p_t^e$ is the gross return on equities. In the literature $\beta x_{t+1}^{-\alpha}$ is typically termed "stochastic discount factor" and when markets are complete such a stochastic discount factor is unique.

Closed form expressions for the average risk-free rate and the average equity premium can be obtained using some distributional assumptions. Following Aiyagary (1993) and Campbell (1999), we assume that stock returns and consumption are conditionally homoscedastic log normally distributed random variables.

In particular, we let $x_t = \exp(\mu + \varepsilon_t)$ and $(1 + R_{t,1}^e) = (1 + E(R_{t,1}^e))\exp(u_t)$ where ε_t and u_t are i.i.d. normal random variables with 0 mean and variances δ^2 and σ^2, respectively, and μ is the mean of x_t. Using (5) and (6) and the fact that if X is conditionally log normally and homoscedastically distributed, $\ln(E_t X) = E_t \ln(X) + 0.5\mathrm{var}_t(\ln(X))$, where $\mathrm{var}_t(\ln(X)) = \mathrm{var}(\ln(X)) - E_t \ln(X)$. The average risk-free rate, denoted by R, and the average equity premium, denoted by $EP = R^e - R$, are given by:

$$R = -\ln(\beta) + \alpha\mu - 0.5\alpha^2\delta^2 \tag{7}$$

$$EP = \alpha\mathrm{cov}\left(\ln\left(1 + R^e_{t,1}\right), \ln\left(x_{t+1}\right)\right) - 0.5\sigma^2. \tag{8}$$

The average riskless real rate is a linear function of dividend (consumption) growth, with a slope coefficient equal to the coefficient of relative risk aversion and the variance of consumption growth. This last term in equation (7) is typically interpreted as reflecting a precautionary saving effect. Equation (8) represents a version of the relationships derived by Abel (1988) and Black (1990). It states that the log risk premium depends on the variance of equity returns and on the coefficient of relative risk aversion times the covariance of equity returns with consumption growth. If an asset has a negative covariance with consumption, it will give high returns when consumption is low and the marginal utility is high. Hence this asset has low risk and it should have a low-risk premium. Note that the variance of equity returns enters equation (8) only because of Jensen's inequality. Equations (7) and (8) explicitly show the dependence of the average risk-free rate and the average equity premium on the technology parameters (μ, δ, and the unconditional covariance between the equity return and consumption growth) and on the preference parameters α and β. Given technological parameters, variations in β affect the mean risk-free rate only, while variations in α have a monotonic effect on the mean of the equity premium and a nonmonotonic effect on the mean of the risk-free rate.

In the spirit of Mehra and Prescott (1985) and Campbell (1999), and given technological parameters, we will employ a method of moment estimation to obtain α from equation (8) for each country and each time period. Then, plugging the estimates of α into equation (7), we will obtain an estimate of β for each country and each time period. To judge the consistency of the theory with the data we will compare these estimates with explicit or implicit estimates of these two parameters available in the literature.

Since the covariance between equity returns and consumption growth can be written as $\sigma_{e,c} = \rho_{e,c}{}^*\delta^*\sigma$, where $\rho_{e,c}$ is the correlation coefficient between consumption and equity returns, equation (8) can also be written as

$$\alpha\delta \geq \frac{EP + 0.5\sigma^2}{\sigma}. \tag{9}$$

This inequality, typically referred to as the Hansen-Jagannathan (1991) (HJ) bound, states, in the context of our consumption-based CAPM model, that a linear function of the variability of consumption growth must exceed the logarithmic Sharpe ratio for equity (i.e., the excess return of equity on the riskless asset, adjusted for the Jensen's inequality term, divided by its standard deviation). Hence, an alternative test of the model would be to compute the Sharpe ratio for every country and every time period and compare it with the variability of consumption growth.

In general, the expressions for the standard deviations and the AR(1) coefficients for the two variables depend in a nonlinear way on the differences between the conditional and unconditional distributions of the exogenous forces of the model (see Canova and Marrinan, 1996). For example, the standard deviations depend on the differences between conditional and unconditional moments of the dividend growth process and on the differences between the conditional and the unconditional covariance between risk returns and dividend growth. For the simple homoscedastic case considered here, conditional and unconditional moments are identical. Therefore, second moments are degenerate, and it is not possible to examine under what conditions the model matches the second moments of the data. Canova and De Nicoló (1995) have examined a version of the model that produces nondegenerate second moments and found that it is not possible to replicate second moments under reasonable parameter configurations.

III. Results

Table 7 provides a thorough examination of the ability of the model to replicate the cross-section time-series heterogeneity in the means of the equity premium and of the risk-free rate we found in the data. In the second and third columns we report the annualized mean and standard deviation of consumption growth, in the fourth column the annualized standard deviation of equity returns, and in column five the covariance between consumption growth and the log of equity returns. In column six we present the value of α that satisfies equation (8), given the actual equity premium (provided in Table 1), the standard deviation of equity returns, and the covariance of the log equity return with consumption growth. In column seven we report the HJ bound on the variability of the discount factor, in percentage terms. Finally, in column eight we present the implicit rate of time preferences, reported in percentage points per year, which we obtain from equation (7) once we have plugged in the actual mean of the risk-free rate, the moments of consumption growth, and the estimate of the coefficient of risk aversion obtained from equation (8). These numbers should be interpreted as the value of the real risk-free rate of interest that would prevail if consumption growth were known to be constant forever at its current level.

Overall, the table shows that the model is unable to match the mean of the equity premium and of the risk-free rate observed both across countries and across time. For example, in the full sample, the coefficient of relative risk aversion needed to match the mean equity premium in the United States, Japan, Canada, and Germany exceeds 48, while in the other three countries a negative estimate of this coefficient is obtained. Similarly, the implicit real interest rate needed to match the mean of the risk-free rate is above 82 percent in five countries while in Japan and the United States it is negative, implying an estimate of β in excess of one (see also Kocherlakota, 1996). These estimates are the result of small covariances between log equity returns and consumption growth on one hand and low variability of consumption growth, on the other. Clearly, these are only point estimates for the parameters, and Geweke (1999) has shown it is possible that once the (large)

Table 7. Test of Equalities of Distributions: Individual Countries Versus the United States

(Three-month holding period)

Country	Mu	Delta	Sigma	Sigma(e, c)	Alpha	HJ bound	Beta
Full sample							
United States	2.79	1.82	16.71	5.69	115.00	39.35	–100.00
Japan	3.23	2.74	21.90	10.02	48.00	22.02	–66.00
Canada	2.91	2.16	16.87	1.71	165.00	16.78	159.00
Germany	2.86	4.17	19.83	1.72	302.00	26.39	7146.00
France	2.15	2.55	23.23	–1.13	< 0	24.10	9105.00
Italy	3.01	1.99	27.29	–9.67	< 0	8.12	82.00
United Kingdom	2.52	2.86	21.58	–0.11	< 0	31.91	1541.00
Sample 1971–80							
United States	2.46	2.34	18.36	9.79	6.00	3.55	–16.00
Japan	4.37	3.26	20.62	20.63	37.00	37.68	–90.00
Canada	4.12	2.30	15.83	–3.07	< 0	35.12	1620.00
Germany	2.95	2.51	15.06	–2.07	57.00	< 0	–62.00
France	3.08	2.62	22.93	14.13	3.00	2.13	–10.00
Italy	4.54	2.00	27.01	–2.91	106.00	< 0	–258.00
United Kingdom	1.68	3.94	27.48	–14.97	< 0	16.46	118.00
Sample 1981–90							
United States	3.11	1.71	17.64	7.05	77.00	31.00	–148.00
Japan	3.63	1.99	24.69	12.15	80.00	39.68	–160.00
Canada	2.50	2.17	19.69	10.73	< 0	< 0	83.00
Germany	2.06	2.14	24.45	–2.16	< 0	30.45	3431.00
France	2.50	1.11	26.50	–1.45	< 0	26.90	2711.00
Italy	2.37	2.20	29.61	–10.44	< 0	6.90	54.00
United Kingdom	3.15	2.47	19.57	6.63	108.00	36.66	20.00
Sample 1991–99							
United States	2.97	1.46	15.32	2.90	338.00	64.12	222.00
Japan	2.59	2.37	22.71	4.17	76.00	14.10	–30.00
Canada	2.24	2.01	17.53	4.53	29.00	7.55	–43.00
Germany	2.82	4.87	21.88	3.96	221.00	40.14	5208.00
France	1.64	2.50	23.21	–7.78	< 0	36.27	548.00
Italy	2.16	1.87	27.17	–9.93	< 0	18.87	164.00
United Kingdom	2.98	2.02	17.29	6.55	124.00	47.34	–48.00

Notes: Mu: mean of real consumption growth; Delta: standard deviation of real consumption growth; Sigma: standard deviation of equity return; Sigma (e, c): covariance of equity return with real consumption growth; Alpha: risk aversion coefficient; HJ bound: Hansen-Jagannathan Bound; Beta: rate of time preference.

standard errors of the estimates are taken into account, estimates may be much smaller and even economically reasonable.

The picture is no more encouraging for subsamples. If we exclude the estimates obtained in the United States and France for the first subsample, the values of α needed to match the mean equity premium in all countries for all four subsamples exceed 29 or, even worse, are negative, while estimates of the implicit rate of time preference are either negative or, when positive, are exceedingly large, implying that agents have a very short planning horizon.

It is instructive to consider the HJ bounds as they give an alternative perspective on the extent of the failures of the model. For the full sample, estimates of this lower bound range from 8, in the case of Italy, to 39 in the case of the United States. For the first subsample the range is from 2 (for France) to 37 (for Japan); in the second subsample from 6 (for France) to 39 (for Japan); in the third from 7 (for Germany) to 64 (for the United States). These numbers are very large when compared with the variability of consumption growth (see third column of Table 7) except for France in the first subsample. Hence, the failures of the model to match the means of the equity premium and risk-free rate across countries and sample periods are due to the lack of variability of the discount factor implicitly used to price assets in the model.

Are there ways to increase the variability of the discount factor using alternative specifications of the model? In other words, can some of the assumptions of the theory be altered so as to have a better match with the data in the dimensions we examined? We consider this question next.

IV. Extensions

Many authors have modified the basic model of Section II and have claimed some success in reproducing features of the equity premium in the United States. Here we examine whether some of the proposed modifications have the potential to explain those features of the mean equity premium and of the mean risk-free rate across countries and sample periods that are left unexplained by the basic model. Conceptually, many of these alternative setups differ from the basic model because they introduce one or more parameters in the specification without altering the number of moments to be matched. Therefore, in judging their success one should also discount the additional degrees of freedom allowed in the exercise.

As Abel (1988), Black (1990), and Canova and Marrinan (1993) have pointed out, changes in the riskiness of an asset may have direct and indirect effects on asset prices. For example, an increase in the variance of dividends increases the equity premium and reduces the riskless rate of return as portfolio holders move away from riskier equities toward riskless assets. To study the implications of changes in the riskiness of equity, Kandel and Stambaugh (1990), Cecchetti, Lam, and Mark (1993), Bonomo and Garcia (1993), and Abel (1994) have adopted a Markov switching model for the dividend process and have claimed various degrees of success in replicating the first two moments of the equity premium and the autocorrelation function of equity returns at various horizons. Is the introduction of time variations in the riskiness of dividends quantitatively important in bringing the model closer to the international evidence? In principle, this feature could be crucial as it may increase both conditionally and unconditionally the variability of the discount factor and therefore allow a better match with the data. In practice, this road does not seem too promising in our context. If the lack of heteroscedasticity in dividends is the reason the model fails, we need substantial cross-country differences in the structure of the time variation of second moments to match the existing cross-country differences in the mean of the equity premium and an increase in time variation in the second moments in the latter subsamples to account for the increase in magnitude of the mean equity premium. None of these features are present in the data: simple ARCH tests for conditional heteroscedasticity do not reject the null hypothesis of no time variations in the second moments of consumption in all countries. Moreover, we do not find any evidence of additional conditional heteroscedasticity in the latter two subsamples.

Labadie (1989) argued that the lack of a riskless rate of return in the real world may help explain the equity premium puzzle. For this reason she examined a version of the basic monetary model with cash-in-advance constraints. She showed that there are two channels through which inflation affects equity returns. First, because dividends are paid in money and can be used for consumption only in the next period, random variation in the money supply leads to variations in the purchasing power of dividends and equity returns over time. Second, because the intertemporal marginal rate of substitution and inflation are correlated, the model generates an inflation risk premium that equally affects the risk-free rate and equity returns. Labadie argues that the second effect is of minor importance and that once the link between inflation and the purchasing power of dividends is taken into account, the mean equity premium generated by the model is broadly consistent with the historical U.S. experience. Once again, this alteration is potentially important as inflation risk may substantially increase the variability of the discount factor. Nevertheless, adding inflation effects to the model does not help. First, note that the majority of the differences across countries and samples in the time series for the equity premium are due to differences in the time series of the risk-free rate. Because neither of the two effects influences the risk-free rate only, it is unlikely that adding inflation effects provides a better match of the mean of the equity premium. Following Labadie, we define the inflation risk premium as the covariance between S_t and π_t^{-1}, where

$$S_t = \beta\left(\frac{c_{t+k}}{c_t}\right)^{-\sigma}$$

and π_t is the inflation rate. Given a dividend process, we can obtain estimates of this covariance term, after we have pinned down β and α. Setting $\beta = 0.98$ and α varying from 2 to 20, we find that the largest inflation risk premium generated by the model is 0.003. Hence, quantitatively, the magnitude of the effect is small.

Next, we consider the issue of leverage. Mehra and Prescott (1985) examined whether leverage was crucial in accounting for the large discrepancy between the model and the data and concluded that it was not. On the other hand, Benninga and Protopapadakis (1990) and Kandel and Stambaugh (1991) argue that leverage is an important ingredient if one wants to obtain a better match between the model and the data. However, the value of the leverage parameter used by these authors is either too high or too unconstrained. Cecchetti, Lam, and Mark (1993) claim that once the ratio of dividends to consumption is set close to the historical average, the model fails to match the data. Can leverage generate substantial variability to the discount factor for all countries and all samples? The answer is no. The countries we are considering are the most industrialized in the world and their leverage characteristics are similar except, perhaps, for Italy, where the percentage of equity financing is slightly lower than in the other countries (0.11 vs. 0.17 on average for the other G-7 countries). Similarly, the leverage characteristics of these countries did not display marked differential trends across time able to account for the time variation in the means we highlighted in Section I.

In an attempt to solve the (EP-R) puzzle with U.S. data, several researchers, including Weil (1989) and Campbell (1999), have used a more flexible specification of preferences than the one employed in Section II. The basic specification forces the elasticity of substitution and the coefficient of relative risk aversion to be reciprocals of one another, but there is no reason to expect that this is the case, because risk aversion describes the reluctance of agents to substitute across states of the world while elasticity of substitution describes the preference of consumers for smoothing across time. Can a more flexible specification of preferences correct the inherent failures of the model? Instead of equation (2), consider the following recursive utility function:

$$U_t = \left\{ (1-\beta) C_t^{\frac{1-\alpha}{\theta}} + \beta E_t \left(U_{t+1}^{1-\alpha} \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\alpha}}, \tag{10}$$

where $\theta = (1-\alpha)/(1-\phi^{-1})$, ϕ is the elasticity of substitution, and α, as before, is the coefficient of relative risk aversion. It is easy to verify that equation (2) can be obtained by setting $\alpha = \phi^{-1}$. Also, let the wealth accumulation equation be $Z_{t+1} = (1+R^z_{t+1})^*(Z_t - C_t)$, where R^z_{t+1} is the net rate of return on the portfolio of all invested wealth. Repeating the maximization problem and assuming that consumption and equity returns are homoscedastic and jointly log normally distributed, we obtain:

$$R = -\ln(\beta) + \frac{1}{\phi}\mu + \frac{\theta - 1}{\phi}\sigma_z^2 - \frac{0.5\delta^2\theta}{\phi^2} \tag{11}$$

$$EP = \frac{\theta}{\phi}\operatorname{cov}\Big(\ln\big(1+R^e_{t,1}\big), \ln\big(x_{t+1}\big)\Big) + (1-\theta)\operatorname{cov}\Big(\ln\big(1+R^e_{t,1}\big), \ln\big(1+R^z_{t+1}\big)\Big) - 0.5\sigma^2. \tag{12}$$

The equity premium is now a weighted average of the covariance between equity returns and consumption and the covariance between equity returns and wealth returns. Note that when $\theta = 0$ we obtain the standard static CAPM model. Since we have a new parameter to play with (ϕ), the tight link between α and β and the means of the equity premium and the risk-free rate are severed. For example, it is no longer true that a low ϕ is needed to solve equation (12) since the covariance between equity and wealth may be high. Furthermore, high variance of wealth or high values of ϕ may make estimates of β obtained from equation (11)

Table 8. Equity Premium and Risk-Free Rate: Nonexpected Utility Specification

(Three-month holding period)

	Phi = 0.5		Phi = 1.5		Phi = 0.5		Phi = 1.5	
Country	Alpha	Beta	Alpha	Beta	Alpha	Beta	Alpha	Beta
Full sample								
United States	151	–3.59	28	–1.01	< 0	–2.86	1713	15.85
Japan	113	–4.03	81	–0.24	9	–0.51	594	9.33
Canada	170	–2.62	266	5.29	214	3.86	204	4.02
Germany	319	–3.44	233	6.02	253	5.42	261	5.47
France	41	0.70	128	3.95	155	4.64	179	5.17
Italy	106	–1.34	95	2.52	88	3.23	78	3.17
United Kingdom	< 0	–0.34	163	3.27	215	3.98	255	4.51

Notes: Phi: intertemporal elasticity of substitution; Alpha: risk aversion coefficient; Beta: implicit rate of time preference.

more reasonable. But is this the case in our data? Table 8 reports estimates of α and of the implicit rate of time preferences when we set the intertemporal elasticity of substitution ϕ to 0.5, 1.5, 2.5, and 5. Since estimates of the elasticity of substitution are typically small, we regard the first two values as more probable than the others. In the context of the expected utility framework these values would imply a coefficient of relative risk aversion of 2, 0.66, 0.4, and 0.2. To conserve space, we present the results for the full sample only. It is clear that estimates of the coefficients of relative risk aversion are still unreasonably large, regardless of the value of ϕ, typically 28 or larger. Estimates of the implicit rate of time discount are, however, a little more reasonable. For example, for the case of $\phi = 1.5$, estimates range from 2.5 percent to 6 percent, while for Japan and the United States estimates are negative but small. Hence, separating risk aversion and the elasticity of substitution improves the qualitative ability of the model to replicate the mean of the risk-free rate. However, the model still struggles to match the mean of the equity premium.

Finally, several authors, including Abel (1999) and Campbell and Cochrane (1999), have argued that to match both the time-series properties of the equity premium and stock return volatility in the United States it is necessary that risk aversion vary over time. From equation (9) it is clear that if risk aversion varies over time, the variability of the discount factor will increase. Since none of the models we have considered so far displays this feature, they suggest using preference specifications that display habit formation. The utility function they use is

$$U_t = \frac{\beta^j \left(C_t - X_t\right)^{1-\alpha} - 1}{1-\alpha}, \tag{13}$$

where X_t is the level of habit that, in representative agent models, corresponds to the past level of per capita consumption. Defining $S_t = (X_t - C_t)/C_t$, the coefficient of risk aversion at date t is $\zeta_t = \alpha / S_t$. In a final attempt to see if existing modifications can help to reconcile the differences between the model and the equity premium data, we first computed values of ζ_t that match the mean of the equity premium on a rolling sample of 10 years for each country. Then, given several values of α and the process for the mean of consumption for that decade, we produced a time-series estimate of the level of habit needed to explain the mean of the equity premium in the seven countries. We display this time series in Figure 4, when we select $\alpha = 2$, together with the time series for the mean of consumption (computed over the 10-year window) and the implicit rate of time preference that would be obtained from the corresponding equation for the risk-free rate. The habit persistence specification will receive support in the data if the time series for the level of habit and for real consumption have roughly the same properties for all G-7 countries.

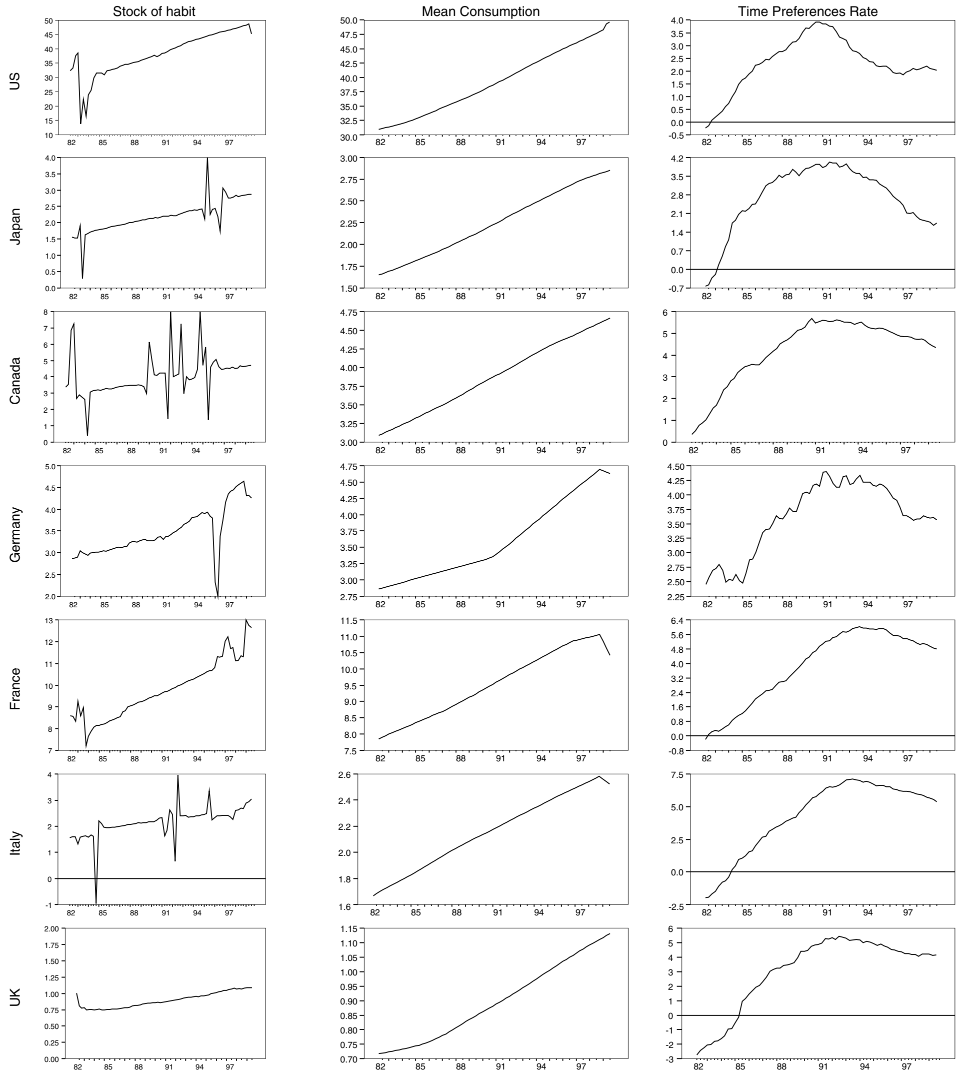
The figure shows that the time-series properties of the estimate of the level of habit are very different from those of consumption. The series are more volatile, have a somewhat different trend (more evident in the case of the United Kingdom), and present large spikes typically at times when volatility of stock returns increases. The implied rate of time preference has a hump-shaped path that mirrors, once again, the behavior of the mean of inflation in the period. Hence, we conclude that even a model with time-varying risk aversion is not able to explain the time-series properties of the mean of the equity premium across countries.

V. Conclusions

In this paper we studied the (EP-R) relationship from two different points of view. First, we characterized the relationship empirically in a number of industrialized countries for various subsamples starting in 1970. We showed that important instabilities emerge both across time and across countries. These features are neglected when we restrict the analysis to the United States alone or to the entire time period. We highlighted that both the distribution of the risk-free rate and of the equity premium display differences across countries and time periods, that the heterogeneities in the risk-free rate are linked to differences in inflation rates across time and countries, and that the differences in the equity premium are equally due to differences in the risk-free rate and in equity returns across countries and time.

We examined the performance of a consumption-based CAPM when confronted with the richness of the cross-country cross-sample evidence. We showed that the model fails to account for the heterogeneities in the data. The value of the risk aversion parameter needed to match the mean equity premium is exceedingly large, while the value of the rate of time preferences needed to match the mean of the risk-free rate is in most cases unreasonable. The Hansen-Jagannathan bounds indicate that in all cases it is the low variability of the discount factor used to price assets that is to blame for the failures. We then studied whether several modifications of the model, designed to increase the variability of this discount factor, help to reconcile the model with the data. Although all these alterations are potentially useful, and some of them help to make the rate of time preference more reasonable, in practice their role in solving the equity premium puzzle is small. Hence, the discrepancy between the theory and the data is still large and more work needs to be done to try to explain the time-series patterns that emerge from stock and bond markets.

Figure 4. Habit Formation Model



Overall, our results suggest that a truly satisfactory understanding of these patterns is more challenging than previously believed: it requires a model capable of accounting not only for the relative size of the equity premium and of the risk-free rate, but also for the cross-country and cross-time differences we have documented. We believe that either some form of heterogeneity, which may be individual-specific, as in Constantinides and Duffie (1996), or country-specific, along the lines of Mankiw and Zeldes (1991) or Marcet and Singleton (1999), or the presence of market incompleteness and transaction costs, along the lines of Telmer (1993) or Heaton and Lucas (2000), could help explain the complexity of asset return characteristics across time and countries.

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