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Portfolio Constraints and Contagion in Emerging Markets

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The objective of this paper is twofold: (1) to analyze an optimal portfolio rebalancing by a fund manager in response to a "volatility shock" in one of the asset markets, under sufficiently realistic assumptions about the fund manager's performance criteria and portfolio restrictions; and (2) to analyze how the composition of the investor base determines the sensitivity of equilibrium asset prices to a shock originating in one of the fundamentally unrelated asset markets. The analysis confirms that certain combinations of portfolio constraints (notably short-sale constraints and benchmark-based performance criteria) can create an additional transmission mechanism for propagating shocks across fundamentally unrelated asset markets. The paper also discusses potential implications of recent and ongoing changes in the investor base for asset price volatility in emerging markets. [JEL G11, G12]

A recurrent theme in the fast-growing literature on financial contagion is contagion through portfolio reallocations unrelated to "fundamental" factors, that is, factors that determine the value of an asset. In this literature, the behavior of investors who choose to adjust their exposure to a particular asset in response to new information unrelated to asset fundamentals may or may not be fully rational.[1] The environments where such response may be rational include the ones where investors act strategically, taking into account the actions of other market participants, or where investors act as price takers, but are subject to portfolio constraints that create links

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[1]The not-fully-rational behavior is often referred to as "herding" or "following the market." See, for example, Bikhchandani and Sharma (2000) and Kaminsky and Reinhart (2000) for a review of different approaches toward modeling herding in financial markets.

between their positions in the fundamentally unrelated markets. The latter will be the focus of this paper.[2]

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(2) To analyze how the composition of the investor base determines the sensitivity of equilibrium asset prices to a shock originating in one of the fundamentally unrelated asset markets.

Why are these issues important? First, when regulators design investment guidelines for mutual funds, pension funds, and insurance companies, their first priority is investor protection; the potential impact of these guidelines on the portfolio allocation decisions of institutional investors and, in turn, on asset price dynamics are rarely taken into account. Second, in less liquid markets (including most emerging markets) and, particularly, in those where foreign institutional investors account for a large share of asset holdings and turnover, portfolio rebalancing by large investors in response to local and external shocks may have significant implications for asset price movements. Thus, a better understanding of the role played by different types of institutional investors in propagating shocks across asset markets is critical to understanding the extent to which assets prices, particularly in emerging markets, may be driven by factors unrelated to asset fundamentals.

Over the past decade, the participation of foreign institutional investors in emerging debt and equity markets increased dramatically, driven by the capital account liberalization and improved credit fundamentals in many emerging market (EM) countries, as well as by the relaxation of investment restrictions for institutional investors in mature market (MM) countries. Figures 1 and 2 show the shares of EM equity and debt instruments held by foreign investors in selected EM countries.

Virtually all types of foreign institutional investors (mutual funds, pension funds, hedge funds, and proprietary trading desks of major investment banks) are present in emerging securities markets. Although it is difficult to find a comprehensive data source on foreign holdings of EM securities, it is possible to gauge the proportions of the main types of foreign institutional investors in EMs by looking at the client base of major investment banks. For instance, based on JPMorgan's client survey of foreign investors in emerging debt markets (EDMs),[3] the proportion of crossover investors and trading accounts has declined in the past five years from about 50 percent to 30 percent, whereas the share of dedicated EM investors has increased (as local investors in EM domestic debt securities were added to the survey at the end of 2003) (see Figure 3).[4]

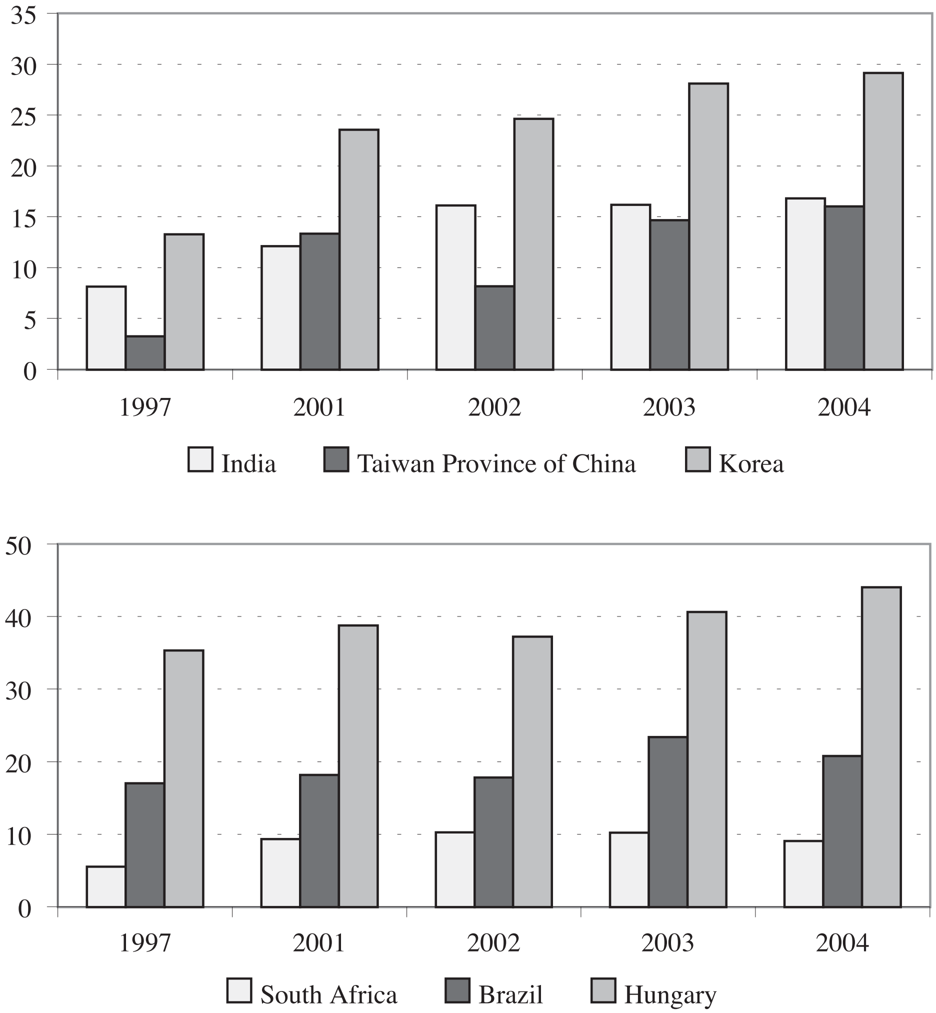
[2]The framework used here is similar to that in Chakravorti, Ilyina, and Lall (2003).

[3]EDM is the market for the dollar- or euro-denominated Eurobonds issued by the EM sovereigns and corporates.

[4]The term "dedicated EM investor" typically refers to an asset manager who has a mandate to invest exclusively in EM securities. Such an investor is usually benchmarked against an EM index (that is, investor performance is measured relative to the performance of an EM benchmark portfolio). The term "crossover investor" typically refers to an asset manager who does *not* have an EM-specific mandate, but can invest in the EM securities that are part of the asset class that is specified in his or her investment mandate (equity or fixed income). Such an investor may or may not be benchmarked against an index that includes EM assets. (For instance, for the U.S. high-grade or U.S. high-yield bond fund manager, the EM dollar-denominated bond exposure represents an "out-of-index" bet; such investors would typically cross over into EMs to pick up yield.)

Figure 1. Cross-Border Portfolio Investment in EM Equity Securities

(As a percent share of equity market capitalization)

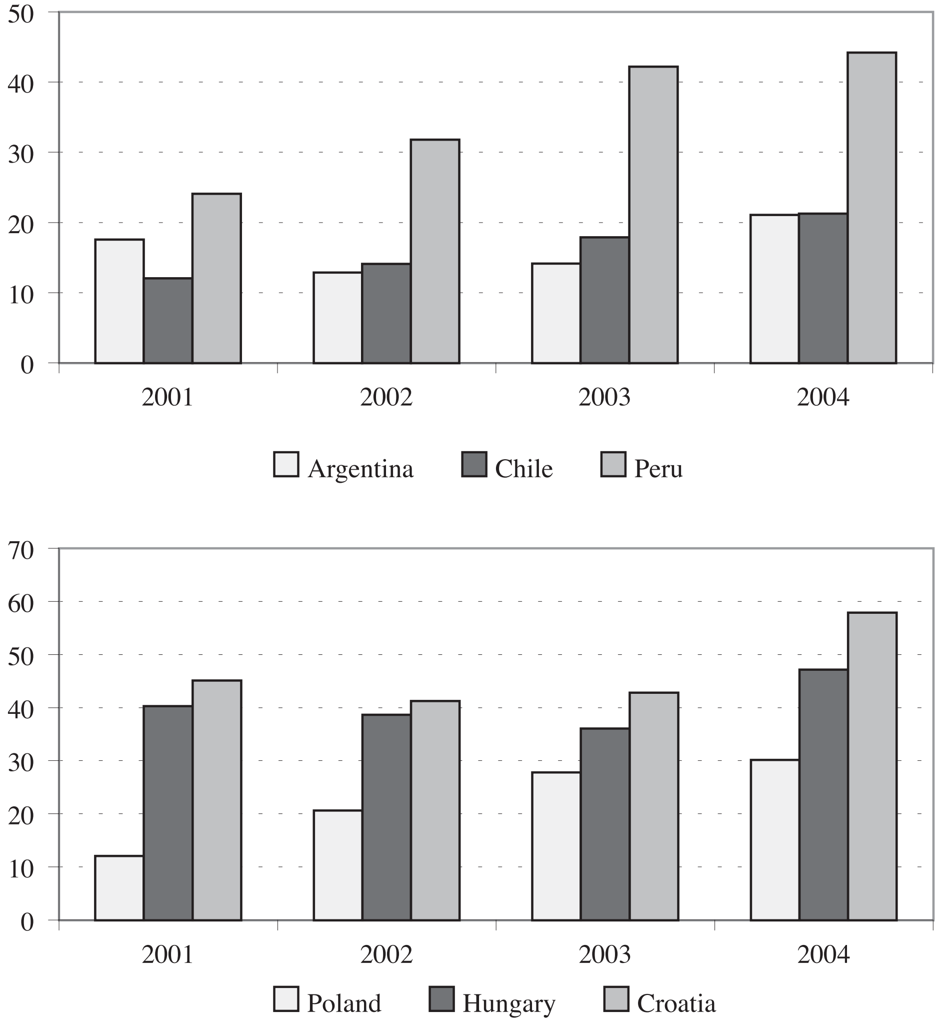


Sources: IFC; and IMF's Coordinated Portfolio Investment Survey.

Dedicated investors' allocations to the EM asset class are generally viewed as more stable ("sticky") than those of crossover investors and trading accounts. This is because the latter are typically not benchmarked against an EM index, and their investment decisions tend to be more sensitive to the developments in competing asset classes. For example, the U.S. high-grade/high-yield funds may change their EM allocations in response to developments in the U.S. corporate bond market. However, a larger share of dedicated investors in the foreign investor base for a particular EM does not necessarily imply that foreign holdings of securities in that particular EM would be more stable. This paper, for example, shows that excessive

Figure 2. Cross-Border Portfolio Investment in EM Debt Securities

(As a percent share of debt market capitalization)

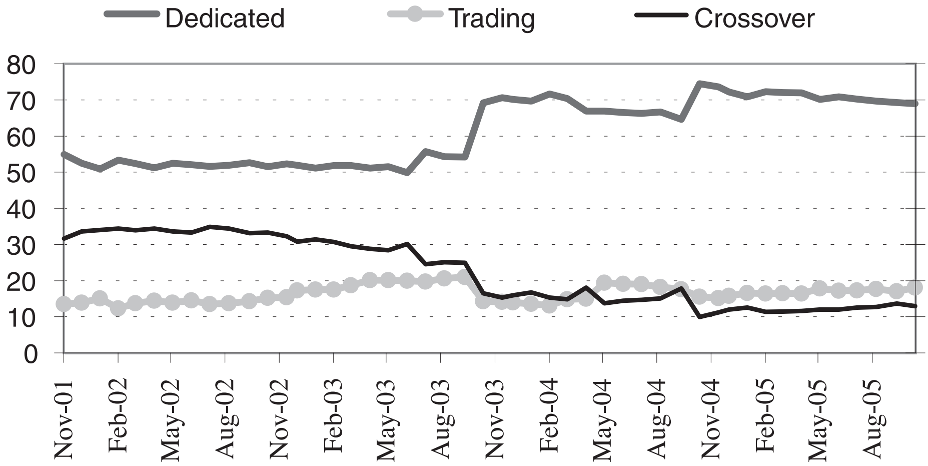


Sources: BIS; and IMF's Coordinated Portfolio Investment Survey.

price volatility unrelated to fundamentals in a particular asset market can be generated by the portfolio reallocations of dedicated fund managers that are subject to multiple portfolio constraints.

Various strands of the contagion literature study the implications of institutional differences between investors for the "transmission" of shocks across asset markets. Papers by Schinasi and Smith (2000) and Calvo and Mendoza (2000) analyze contagion in the context of a single-investor decision problem. On the other hand, Kodres and Pritsker (2002); Kyle and Xiong (2001); and Danielsson, Shin, and Zigrand

Figure 3. Assets Under Management of EM Debt Investors
(In percent)



Source: JPMorgan EM Investor Survey.

(2004) study the impact of portfolio reallocations by different types of investors on price dynamics in a general equilibrium framework. Instead of presenting a comprehensive overview of the literature on financial contagion, the rest of this section will focus on the discussion of the papers that are most closely related to the exercise presented here.

Schinasi and Smith (2000) study the optimal portfolio rebalancing in response to two types of shocks—an increase in volatility in one of the asset markets ("volatility event") and a capital loss ("capital event"). They consider different portfolio management rules within a partial equilibrium mean-variance framework, allowing portfolio managers to take both long and short positions (no short-sale constraints). They find that only in the case of a positive covariance between asset returns does a volatility event in one asset market lead to an adjustment of positions in other assets.[5] Furthermore, a leveraged investor always reduces risky asset positions when the return on the leveraged portfolio falls below the cost of funding. In this paper, we consider a broader class of portfolio management rules, including rules where the portfolio managers' compensation is explicitly linked to the performance of a benchmark index and where fund managers may be subject to short-sale constraints. In contrast with Schinasi and Smith (2000), in this paper asset values are assumed to be uncorrelated, so that any possible contagion effects cannot be attributed to "fundamental" links between asset markets.

Calvo and Mendoza (2000) study the implications of institutional restrictions (in particular, the short-sale constraint) on investors' incentives to gather costly

[5]Schinasi and Smith argue that this is the most relevant case because asset returns are generally positively correlated across countries.

information and take positions based on their private information, as opposed to imitating arbitrary market portfolios. They find that, in the presence of short-sale constraints, the gains from acquiring information at a fixed cost may diminish as markets grow (that is, as the number of assets increases). In this paper, the analysis is focused on what happens when the short-sale constraint is combined with other institutional restrictions, such as the benchmark-linked performance criterion, and whether this can create an additional transmission mechanism for contagion through portfolio rebalancing.

Danielsson, Shin, and Zigrand (2004) investigate the implications of the widespread adoption of value-at-risk (VaR) risk management techniques on asset price dynamics in a general equilibrium framework. A comparison of the simulated dynamics of asset prices with the use of VaR techniques against the asset price dynamics without VaR reveals that (1) prices are lower with VaR constraints, (2) troughs in the price paths with VaR constraints following a negative shock are deeper and longer, and (3) the variance of returns is larger with VaR constraints than without them.[6] These results raise more general concerns that the widespread use of certain combinations of portfolio constraints (including those that restrict the fund manager's portfolio choices in the interests of investor protection) may have a systematic negative impact on asset price dynamics.

I. The Model

Asset Markets

Consider a simple discrete time environment with *three risky assets*—two EM assets (or portfolios of assets) A and B, and one MM asset (or portfolio of assets) Z—and *one riskless asset,* cash. The EM assets are assumed to have higher return and higher volatility than the MM assets. There are two periods: the current period ($t = 0$) and the terminal period ($t = 1$). Investors make their portfolio decisions in period 0, based on their expectations about asset values in the terminal period, and liquidate their positions in period 1. The gross return on asset i, for $i \in \{A, B, Z\}$, is $R^i \equiv \frac{P_1^i}{P_0^i}$, where P_1^i denotes the value of asset i in the terminal period and P_0^i denotes the equilibrium price determined by the supply and demand conditions in asset market i in the current period.

The terminal values of all assets are normally distributed with means μ_i and variances σ_i^2, which are commonly known among investors. These probability distributions can be interpreted as representing uncertainty about the fundamental values of assets. To isolate the impact of portfolio allocation decisions by different types of investors on equilibrium prices in period 0 from any effect that may be due to fundamental links between asset markets, the terminal period values of assets

[6]The VaR-based rules may not be "worse" than other portfolio rules in terms of their impact on the asset price dynamics. In the partial equilibrium framework, VaR rules do not seem to produce portfolio rebalancing dynamics that are very different from a variety of other portfolio management rules (see Schinasi and Smith, 2000).

traded in different markets are assumed to be uncorrelated, that is, $Cov(P_1^i, P_1^k) = 0$ for any $i,k \in \{A, B, Z\}$.

The equilibrium analysis presented in Section III focuses on the derivation of the market clearing prices for emerging market assets *A* and *B* for a given mix of investors participating in both mature and emerging markets. Assuming that both assets *A* and *B* are infinitely divisible and available to investors in fixed (inelastic) supply, let S_A and S_B denote the supplies of assets *A* and *B,* respectively, and let $D_{J,A}$ and $D_{J,B}$ denote the aggregate demands by investor group *J* for assets *A* and *B,* respectively. Then, the market clearing conditions are $\sum_J D_{J,A} = S_A$ and $\sum_J D_{J,B} = S_B$. Each investor group is assumed to consist of a large number of investment funds, each with an initial capital of unity. Because each fund's capital is small relative to the supply of assets *A* and *B,* fund managers act as price takers in both markets.

Types of Investors

The fund manager's portfolio optimization problem is defined by (1) investment mandate, (2) performance criteria, and (3) portfolio management rule. A particular combination of 1–3 is usually chosen to align the incentives of the fund manager with the risk-return preferences of end investors. Thus, the manager's compensation is typically linked to the performance of his or her investment portfolio, while investment restrictions are chosen to minimize "excessive" risk taking by the portfolio manager. For instance, a risk-averse end investor may want to make sure that the fund manager's compensation is increasing in the expected portfolio return and decreasing in the variability of portfolio return (as in the risk-return tradeoff rule, described below).[7]

Investment mandate

The fund manager's investment mandate typically specifies the *asset class* that he or she can invest in (that is, equity or fixed income instruments in a particular country or region) as well as restrictions on the *use of leverage* (short-sale constraints). For example, dedicated EM equity funds are allowed to invest only in the locally traded shares or American (Global) Depositary Receipts issued by the EM companies.

Performance criteria

The fund manager's performance criterion can be either *absolute*—measured as the return on capital under management—or *relative*—measured as the return on the fund's investment portfolio in excess of the benchmark portfolio return.

[7]An extensive literature on delegated portfolio management explores various ways in which the incentives of fund managers can be aligned with the preferences of end investors (under different assumptions about risk aversion and information asymmetries). The detailed discussion of these issues is beyond the scope of this paper. In what follows, the analysis will focus on the rules and restrictions that are standard in the finance literature, and no distinction will be made between "investors" and "fund managers."

Table 1. Asset Managers' Investment Mandates and Performance Criteria

	Investment Mandate			
Performance Criterion	Global/ real money	EM/real money	Global/leveraged	EM/leveraged
Relative return	Pension funds	EM mutual funds	…	…
Absolute return	…	…	Global hedge funds Proprietary trading desks	EM hedge funds

Source: Prepared by the author.
Note: EM = emerging market.

The majority of institutional investors, including pension funds, endowments, and various collective investment schemes, are allowed to use only minimal leverage and only temporarily (short-term bank credit or implicit in the derivatives positions) or no leverage at all. In contrast, hedge funds and the proprietary trading desks of investment banks may establish long or short positions in any asset market. Investment funds that are not allowed to use leverage are often referred to as "real money" funds.

The institutional investors shown in Table 1 represent the main types of portfolio managers that currently operate in the international capital markets. Most real money funds also tend to be benchmarked against a particular index, whereas those fund managers that are allowed to use "unlimited" leverage (hedge funds, etc., where leverage is limited only by the internal risk management guidelines) tend to be absolute-return driven.[8]

Because the EM fund managers are often benchmarked against a particular EM index, let R^I denote the gross return (per unit of capital) on the *benchmark portfolio* consisting of assets A and B: $R^I \equiv \alpha R^A + (1 - \alpha)R^B$, where $\alpha \in [0,1]$ is exogenously given, as the weights are typically determined by the proprietor of the index and are only modified periodically.[9]

Portfolio management rules

Portfolio management rules are designed to ensure that the incentives of portfolio managers are aligned with the preferences of end investors in terms of the risk-return properties of the investment portfolio. In what follows, we will consider two portfolio

[8]However, with the recent regulatory changes in the asset management industry, the boundaries between mutual funds and hedge funds are beginning to blur. For example, in the United States, the decision by Congress to repeal the short-sale restriction for mutual funds in 1997 and the Securities and Exchange Commission's decision to expand the allowable securities list for mutual funds led to the appearance of the first long-short mutual funds in 1998. Some of these "next generation" mutual funds are also reportedly using limited leverage and limited incentive fees, and are commonly referred to as "hedged mutual funds" (HMFs). According to industry experts, assets under the management of HMFs grew from \$2.4 billion in 1998 to about \$6 billion in 2002, which was still a very small fraction of U.S. mutual fund industry assets (around \$4 trillion, as of end-2002).

[9]The commonly used EM benchmark indices are the JPMorgan Emerging Market Bond Indices for EM bonds and the Morgan Stanley Capital International (MSCI) Emerging Market Indices for EM equities.

management rules that are commonly used in the fund management industry: the *risk-return trade-off* rule and the *tracking-error variance minimization* rule.

The risk-return trade-off rule gives a portfolio manager the flexibility to select both the return and the risk of the portfolio. For instance, for an absolute-return-driven investor, this rule is an outcome of the following optimization problem:

$$\text{maximize } E(R^P) - \frac{1}{2} aVar(R^P),$$

where a denotes the coefficient of risk aversion.

For fund managers whose performance is measured relative to the EM benchmark portfolio, the risk-return trade-off rule would be an outcome of the following optimization problem:

$$\text{maximize } E\left(R^P - R^I\right) - \frac{1}{2} aVar\left(R^P - R^I\right).$$

Another rule that is commonly used by relative-return funds is the tracking-error variance minimization (TEV) rule. This rule requires that fund managers achieve a certain target level of outperformance over the benchmark index, while minimizing the volatility of the "tracking error," that is, the variability of the difference between the manager's portfolio return and the benchmark portfolio return. The manager solves

$$\begin{aligned}&\text{minimize } Var\left(R^P - R^I\right)\\&\text{subject to } E\left(R^P - R^I\right) \geq k,\end{aligned}$$

where k is the minimum (or target) level of relative outperformance. In the extreme case, the fund manager tries to "shadow" the benchmark index, which is often referred to as passive investing.[10]

The Definition of Contagion

Although there are many types of shocks that could affect asset fundamentals and be transmitted to other asset markets via portfolio rebalancing by common investors, this paper focuses on the demand/price response induced by one specific type of shock—a volatility event. Following Schinasi and Smith (2000), we define a *volatility event* at time t as an increase in the (conditional) variance of an asset's return at time $t + 1$.

Because the main concern is a potential sell-off in one of the emerging markets on the back of portfolio rebalancing by common investors in response to a shock originating in a fundamentally unrelated market, the following definition of contagion will be used throughout the paper. In the equilibrium context, *contagion* is defined as

[10]According to the International Organization of Securities Commissions report (IOSCO, 2004), "Passive management encompasses benchmark funds, which follow some indices with a very tight tracking error."

a simultaneous decline of the current period prices of asset *i* (P_0^i) and asset *j* (P_0^j) in response to an increase in the (conditional) variance of the value of asset *j* (P_1^j).

Thus, to transmit a volatility event from market *j* to market *i,* the investor's optimal demand function for asset *i* has to have a nonzero sensitivity to an increase in the (conditional) variance of asset *j*. For example, in the case of a negative sensitivity, the investor would prefer to reduce his or her demand for asset *i* when the (conditional) variance of return on asset *j* goes up.

II. Optimal Investment Rules

Overview of the Main Results

Table 2 gives an overview of the key results of the partial equilibrium analysis. More specifically, it shows three different types of the EM real money funds subject to the short-sale constraints and the sensitivities of their optimal demand functions for asset *A* to a higher (conditional) return variance of asset *B*. For those investors whose performance is measured relative to the EM benchmark index, the term "overweight" (O.W.) refers to the investment position in an asset that exceeds the benchmark portfolio weight of this asset, and the term "underweight" (U.W.) refers to the investment position in an asset that is below its benchmark portfolio weight. For certain configurations of parameter values, it may be optimal for the EM fund manager to be fully invested in assets *A* and *B,* in which case the "no-borrowing constraint" is binding and cash holdings are zero.

The main results presented in Table 2 are

- **Relative-return EM funds that follow the risk-return trade-off rule** react to higher volatility in market *B* by reducing their demand for asset *A* only when they are fully invested in EM assets and asset *A* is expected to outperform asset *B* (that is, they have an O.W. position in asset *A*).
- **Relative-return EM funds that follow the TEV rule** reduce their demand for asset *A* in response to higher volatility in market *B* only if they have positive cash

Table 2. Sensitivity of Demand for Asset A to Higher (Conditional) Variance of the Return on Asset B

	Optimal Allocation Across EM Assets			
Performance Criterion/ Portfolio Management Rule	Cash = 0 Asset A = O.W.	Cash = 0 Asset A = U.W.	Cash > 0 Asset A = O.W.	Cash > 0 Asset A = U.W.
Relative return (EM index) Risk-return trade-off rule	Negative	Positive	0	0
Absolute return Risk-return trade-off rule	Positive	Positive	0	0
Relative return (EM index) TEV rule	0	0	Positive	Negative

Source: Prepared by the author.

Notes: O.W. = overweight; U.W. = underweight; TEV = tracking error variance. All fund managers are subject to the short-sale constraints.

holdings, their expected relative performance is at the target level, and asset A is expected to underperform asset B (that is, they have a U.W. position in asset A).

- **Absolute-return funds that follow the risk-return trade-off rule** never react to higher volatility in market B by reducing their demand for asset A. On the contrary, they prefer to increase their exposure to an asset if the variance of an alternative asset goes up (other things being equal), but do so only when they are fully invested in EM assets.

Thus, the relative-return EM funds that are subject to short-sale constraints can potentially transmit (negative) volatility shocks across fundamentally unrelated markets, whereas absolute-return-driven funds do not transmit negative shocks.

How "special" are the circumstances under which the no-borrowing constraint is binding and the optimal cash holdings are equal to zero? First, zero cash holdings are optimal for the configurations of parameter values where EM assets are expected to outperform the risk-free asset, which does not seem implausible, given that the risk premiums on EM assets are generally positive. Second, the dedicated EM mutual funds tend to be fully invested most of the time and typically hold small positive amounts of cash (around 5 percent of total capital) for liquidity management purposes, that is, to meet redemptions.

Opportunistic Investor

Consider the portfolio optimization problem of an opportunistic global fund manager who invests in both mature and emerging market assets (A, B, and Z), whose performance is *not* measured relative to any benchmark index, and who is *not* subject to short-sale constraints.

Let R^O denote the opportunistic fund manager's portfolio return, where ϕ is the proportion allocated to asset A, δ is the proportion allocated to asset B, and η is the proportion allocated to asset Z:

$$R^O = \phi R^A + \delta R^B + \eta R^Z + (1-\phi-\delta-\eta) R^M.$$

The portfolio optimization problem is

$$\max_{\phi,\delta,\eta} \left\{ \begin{array}{l} \phi E(R^A) + \delta E(R^B) + \eta E(R^Z) + (1-\phi-\delta-\eta) R^M \\ -\dfrac{a}{2}\left(\phi^2 Var(R^A) + \delta^2 Var(R^B) + \eta^2 Var(R^Z)\right) \end{array} \right\},$$

which yields the following well-known solution:

$$\phi^* = \frac{E(R^A) - R^M}{aVar(R^A)},\ \delta^* = \frac{E(R^B) - R^M}{aVar(R^B)},\ \text{and}\ \eta^* = \frac{E(R^Z) - R^M}{aVar(R^Z)}.$$

The main properties of the opportunistic fund manager's optimal portfolio allocation are as follows:

- The fund manager has a long (short) position in a risky asset if its expected risk-adjusted excess return is positive (negative).
- Higher risk aversion induces the fund manager to scale back risky asset positions.

- Higher return variance of any risky asset induces the fund manager to scale back exposure to that asset, but does not affect other asset positions in the portfolio.

Thus, when fundamental values of assets are uncorrelated, portfolio rebalancing by opportunistic investors does not transmit volatility shocks across unrelated markets.[11] In what follows, the optimal behavior of opportunistic investors will be compared with the optimal behavior of other types of investors who face portfolio constraints.

The Relative-Return Investor with the Risk-Return Trade-Off Rule

Consider the portfolio optimization problem of an EM fund manager who is benchmarked against the EM index, follows the risk-return trade-off rule, and is not allowed to short-sell assets or borrow cash.

Let R^D denote the gross return on the portfolio of an EM fund manager, where λ is the proportion of capital invested in asset A and τ is the proportion of capital invested in asset B, with $(1 - \lambda - \tau)$ being the proportion held in cash:

$$R^D = \lambda(R^A) + \tau(R^B) + (1 - \lambda - \tau)(R^M).$$

Let $R^D - R^I$ denote the total *excess* return on the EM fund's portfolio over the benchmark portfolio return:

$$R^D - R^I = (\lambda - \alpha)R^A + (\tau - 1 + \alpha)R^B + (1 - \lambda - \tau)R^M.$$

The expected excess return and variance are

$$E\left(R^D - R^I\right) = (\lambda - \alpha)E(R^A) + (\tau - 1 + \alpha)E(R^B) + (1 - \lambda - \tau)R^M$$

and

$$Var\left(R^D - R^I\right) = \left((\lambda - \alpha)^2 Var(R^A) + (\tau - 1 + \alpha)^2 Var(R^B)\right).$$

Given that the fund manager follows the risk-return trade-off rule and is subject to the short-sale constraints, the optimization problem is as follows:

$$\max_{\lambda,\tau} \begin{Bmatrix} (\lambda - \alpha)\left(E(R^A)\right) + (\tau - 1 + \alpha)\left(E(R^B)\right) + (1 - \lambda - \tau)R^M \\ -\dfrac{a}{2}\left[(\lambda - \alpha)^2 Var(R^A) + (\tau - 1 + \alpha)^2 Var(R^B)\right] \end{Bmatrix} \tag{1}$$

$$\text{subject to} \begin{cases} \lambda \geq 0, \\ \tau \geq 0, \\ \lambda + \tau \leq 1. \end{cases}$$

[11]This may not be the case for other types of shocks. For instance, Schinasi and Smith (2000) show that leveraged risk-averse investors tend to scale back their risky asset exposures in response to a "capital event" (capital loss).

Proposition 1:[12] Consider the solutions of the optimization problem (1) such that the optimal holdings of both risky assets are positive.

(A) **Zero cash holdings:** For the region of parameter values where

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}>0 \text{ and } -\alpha<\frac{E(R^A)-E(R^B)}{a\left(Var(R^A)+Var(R^B)\right)}<(1-\alpha),$$

the no-borrowing constraint is binding and the optimal portfolio weights are

$$\lambda^*=\frac{E(R^A)-E(R^B)}{a\left(Var(R^A)+Var(R^B)\right)}+\alpha \text{ and } \tau^*=\frac{E(R^B)-E(R^A)}{a\left(Var(R^A)+Var(R^B)\right)}+(1-\alpha).$$

(B) **Positive cash holdings:** For the region of parameter values where

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}<0, \frac{E(R^A)-R^M}{aVar(R^A)}+\alpha>0, \text{and } \frac{E(R^B)-R^M}{aVar(R^B)}+\left(1-\alpha\right)>0,$$

the optimal portfolio weights are

$$\lambda^{**}=\frac{E(R^A)-R^M}{aVar(R^A)}+\alpha \text{ and } \tau^{**}=\frac{E(R^B)-R^M}{aVar(R^B)}+(1-\alpha).$$

Interestingly, the optimal deviation from the benchmark portfolio allocation does not depend on the value of α. This implies that two fund managers who start out with portfolios in which the weights are determined by different benchmark indices (different α's) would make the same trades, given that they have the same beliefs about asset fundamentals.[13]

Thus, Proposition 1 implies that relative-return fund managers tend to move closer to the benchmark index when either the return volatility or risk aversion increase. Corollaries 1.1 and 1.2 describe the sensitivities of the EM fund manager's optimal portfolio allocation to the changes in the level of risk aversion and conditional volatilities of the asset returns.

Corollary 1.1: Consider an optimal portfolio allocation with *positive cash holdings* (Proposition 1B):

(a) The fund manager has an O.W. (U.W.) position in a risky asset whenever its expected return is above (below) the return on cash. When $E(R^A) < R^M$ and $E(R^B) < R^M$, the fund manager has U.W. positions in both assets and holds cash.
(b) Higher risk aversion reduces the demand for a risky asset if the fund manager holds an O.W. position, and increases the demand for a risky asset if the fund manager holds an U.W. position.
(c) Higher variance of the return on a risky asset reduces the demand for this asset when the fund manager has an O.W. position and increases the demand for this asset when the fund manager has an U.W. (but positive) exposure.

[12]All proofs are in the Appendix.
[13]This is similar to the result obtained by Roll (1992).

(d) Higher variance of the return on a risky asset has no impact on the demand for an alternative risky asset; that is, there are no volatility spillovers.

Corollary 1.2: Consider an optimal portfolio allocation with *zero cash holdings* (Proposition 1A):

(a) The fund manager has an O.W. (U.W.) position in asset A whenever $E(R^A) > E(R^B)$ ($E(R^A) < E(R^B)$). If the expected returns on two risky assets are equal, the optimal portfolio weights are equal to the benchmark portfolio weights.
(b) Same as (b) in Corollary 1.1.
(c) Same as (c) in Corollary 1.1.
(d) For O.W. positions, the demand for the risky asset is decreasing in the variance of the return on an alternative risky asset. For U.W. positions, the demand for the risky asset is increasing in the variance of the return on an alternative risky asset.

Thus, for a certain range of parameter values (Proposition 1A), the tendency of the fund manager to move closer to the benchmark allocation in response to higher uncertainty about asset returns may cause volatility spillovers across unrelated markets. Consider a portfolio allocation in which optimal holdings of both EM assets are positive, with an O.W. position in asset A and an U.W. position in asset B. Suppose there is a shock to market B that causes an increase in $Var(R^B)$. Then, in response to higher $Var(R^B)$, the fund manager would prefer to reduce exposure to asset A and increase exposure to asset B. Alternatively, suppose that there is a shock to market A, which causes an increase in $Var(R^A)$. Then the optimal reallocation would still involve reducing exposure to asset A and increasing exposure to asset B, because this would bring the fund manager's portfolio allocation closer to the benchmark allocation.

Thus, the relative-return EM fund manager who follows the risk-return trade-off rule and is fully invested in assets A and B would be forced to scale back the O.W. (but not U.W.) positions in response to a volatility shock in a fundamentally unrelated asset market. This means that only outperforming (but not underperforming) asset positions would be negatively affected by a volatility event in an unrelated market.

The Absolute-Return Investor with the Risk-Return Trade-Off Rule

Consider the portfolio allocation problem of an absolute-return EM fund manager who is not benchmarked against any index, follows the risk-return trade-off rule, and cannot take short positions. The optimization problem is

$$\max_{\lambda,\tau}\left\{\lambda E(R^A)+\tau E(R^B)+(1-\lambda-\tau)R^M-\frac{a}{2}\left[\lambda^2 Var(R^A)+\tau^2 Var(R^B)\right]\right\} \qquad (2)$$

$$\text{subject to}\begin{cases}\lambda\geq 0,\\ \tau\geq 0,\\ \lambda+\tau\leq 1.\end{cases}$$

Proposition 2:[14] Consider the solutions of the optimization problem (2), such that the optimal holding of at least one risky asset is positive.

(A) **Zero cash holdings:** For the region of parameter values, where

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}>0 \text{ and } -\alpha<\frac{E(R^A)-E(R^B)}{a\left(Var(R^A)+Var(R^B)\right)}<(1-\alpha),$$

the no-borrowing constraint is binding and the optimal portfolio weights are

$$\lambda^*=\frac{E(R^A)-E(R^B)}{a\left(Var(R^A)+Var(R^B)\right)}+\frac{Var(R^B)}{Var(R^A)+Var(R^B)}$$

and

$$\tau^*=\frac{E(R^B)-E(R^A)}{a\left(Var(R^A)+Var(R^B)\right)}+\frac{Var(R^A)}{Var(R^A)+Var(R^B)}.$$

(B) **Positive cash holdings:** For the region of parameter values where

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}<0, \text{the optimal portfolio weights are}$$

$$\lambda^{**}=\begin{cases}\dfrac{E(R^A)-R^M}{aVar(R^A)}, & if\ E(R^A)>R^M\\ 0, & otherwise\end{cases}$$

and

$$\tau^{**}=\begin{cases}\dfrac{E(R^B)-R^M}{aVar(R^B)}, & if\ E(R^B)>R^M\\ 0, & otherwise\end{cases}.$$

Whenever cash holdings are positive (Proposition 2B) the absolute-return real money EM fund managers behave in the same way as opportunistic investors, except that they cannot short-sell assets. When cash holdings are zero (Proposition 2A), λ^* is increasing in $Var(R^B)$ and τ^* is increasing in $Var(R^A)$. Thus, when the optimal portfolio allocation is such that the EM fund manager has positive holdings of both EM assets and the "no-borrowing" constraint is binding, he would prefer to increase his exposure to a risky asset in response to a "volatility shock" in an alternative asset market.

Thus, EM fund managers who face short-sale constraints but are not benchmarked against any index would not transmit negative volatility shocks across fundamentally unrelated markets.

[14]The proof of Proposition 2 is similar to that of Proposition 1.

The Relative-Return Investor with the TEV Rule

Consider the portfolio optimization problem of an EM fund manager following the tracking error variance minimization (TEV) rule, that is, the fund manager has to minimize the variance of tracking error conditional on a given level of expected outperformance relative to the EM benchmark index. The dedicated EM-TEV fund manager's optimization problem is as follows:

$$\max_{\lambda,\tau}\left\{-\left[(\lambda-\alpha)^2 Var(R^A)+(\tau-1+\alpha)^2 Var(R^B)\right]\right\} \tag{3}$$

$$\text{subject to } \begin{cases} \lambda \geq 0 \\ \tau \geq 0 \\ \lambda+\tau \leq 1 \\ (\lambda-\alpha)ER^A+(\tau-1+\alpha)ER^B+(1-\lambda-\tau)R^M \geq k \end{cases},$$

where parameter k is the target level of outperformance, which is assumed to be positive and strictly greater than the return on cash.

Proposition 3: Consider the solutions of the optimization problem (3), such that the optimal holdings of both risky assets are positive.

(A) **Zero cash holdings:** For the region of parameter values where

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}>0 \text{ and } -\alpha<\frac{k}{E(R^A)-E(R^B)}<(1-\alpha),$$

both the no-borrowing constraint and the performance constraint are binding and the optimal portfolio weights are

$$\lambda^*=\frac{k}{\left(E(R^A)-E(R^B)\right)}+\alpha \text{ and } \tau^*=\frac{-k}{\left(E(R^A)-E(R^B)\right)}+(1-\alpha).$$

(B) **Positive cash holdings:** For the region of parameter values, such that

$$\frac{E(R^A)-R^M}{Var(R^A)}+\frac{E(R^B)-R^M}{Var(R^B)}<0, \frac{E(R^A)-R^M}{Var(R^A)}+\frac{\alpha D}{kC}>0,$$

$$\text{and} \frac{E(R^B)-R^M}{Var(R^B)}+\frac{(1-\alpha)D}{kC}>0,$$

where $C = Var(R^A)Var(R^B)$ and $D = (E(R^A) - R^M)^2\, Var(R^B) + (E(R^B) - R^M)^2\, Var(R^A)$, the optimal portfolio weights are

$$\lambda^{**}=\frac{k\left(E(R^A)-R^M\right)Var(R^B)}{D}+\alpha \text{ and } \tau^{**}=\frac{k\left(E(R^B)-R^M\right)Var(R^A)}{D}+(1-\alpha).$$

Corollary 3.1: Consider an optimal portfolio allocation with *positive cash holdings* (Proposition 3B):

(a) The fund manager has an O.W. position in a risky asset whenever its expected return is higher than the return on cash.
(b) The size of the optimal deviation from the benchmark portfolio (O.W./U.W.) does not depend on the benchmark index weights (α), but does depend on the target level of relative outperformance (k).
(c) For O.W. positions, the demand for a risky asset is decreasing in the return variance of an alternative asset, whereas for U.W. positions, the demand for a risky asset is increasing in the return variance of an alternative asset.

In contrast with the EM fund manager who follows the risk-return trade-off rule, the optimal allocation of an EM-TEV manager is such that U.W. asset positions exhibit negative sensitivity to volatility events in alternative asset markets, whereas O.W. positions exhibit positive sensitivity to volatility events in alternative asset markets. This is because volatility spillovers are generated when the constraint on the expected level of outperformance vis-à-vis the benchmark index (but not the no-borrowing constraint) is binding. Note that when both constraints are binding and cash holdings are zero, the TEV fund manager does not react to any changes in (conditional) volatilities of EM assets.

Thus, a volatility event in one market may force the EM-TEV fund manager to scale back exposure to a fundamentally unrelated asset, when the latter is underperforming. Such behavior is likely to exacerbate the selling pressure in an underperforming market and magnify asset price volatility.

III. Equilibrium Framework

The analysis of the optimal investment rules for different types of portfolio managers, derived in the previous section, suggests that certain combinations of portfolio constraints can create a "link" between the fund manager's exposure to a risky asset and the (conditional) variance of an alternative risky asset held in the portfolio, even when asset markets are fundamentally uncorrelated. Such a link could serve as a mechanism for the transmission of negative volatility shocks across markets.

In this section, this phenomenon will be investigated in an equilibrium context. To illustrate how the transmission of negative volatility shocks through portfolio rebalancing by common investors can produce contagion, consider two types of fund managers operating in both emerging markets: (1) relative-return EM fund managers with the risk-return trade-off rule and (2) opportunistic investors. In what follows, the analysis will focus on the case when dedicated EM investors are fully invested in EM assets and hold zero cash (Proposition 1A).

Suppose that N dedicated EM funds and M opportunistic funds are present in both markets A and B. In what follows, the net return on cash is normalized to zero. It is also assumed that dedicated EM funds start out with their benchmark portfolio allocation, and therefore their net demands for asset A (that is, $\lambda^* - \alpha$) and for asset B (that is, $\tau^* - (1 - \alpha)$) are entirely driven by the need to rebalance their portfolios in response to new information about asset values.

Assuming that asset supplies in both markets are inelastic, the market clearing conditions are as follows:

$$S_A = N\left(\frac{(\mu_A) - (\mu_B)\left(P_0^A / P_0^B\right)}{a\left(\sigma_A^2 + \sigma_B^2\left(P_0^A / P_0^B\right)^2\right)}\right) + M\left(\frac{\mu_A - P_0^A}{a\sigma_A^2}\right) \tag{4}$$

and

$$S_B = N\left(\frac{(\mu_B) - (\mu_A)\left(P_0^B / P_0^A\right)}{a\left(\sigma_A^2\left(P_0^B / P_0^A\right)^2 + \sigma_B^2\right)}\right) + M\left(\frac{\mu_B - P_0^B}{a\sigma_B^2}\right), \tag{5}$$

where S_A and S_B are the quantities of assets A and B, and P_0^A and P_0^B are the market clearing prices.

Proposition 5: Suppose that the number of opportunistic investors (M) is such that $M > \max\left\{\frac{aS_A\sigma_A^2}{\mu_A}, \frac{aS_B\sigma_B^2}{\mu_B}\right\}$, then the market clearing prices of assets A and B are well defined (positive).

Intuitively this means that, in order for both markets to clear, the number of opportunistic investors must be sufficiently large.

Proposition 6: In equilibrium, dedicated EM investors prefer to have an O.W. position in asset A when $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$, and an U.W. position in asset A when $\frac{S_B\sigma_B^2}{\mu_B} > \frac{S_A\sigma_A^2}{\mu_A}$.

Note that the condition $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$, under which the EM investors would prefer to have an O.W. position in asset A, can be rewritten as follows:

$$\left(\frac{\mu_A}{\mu_B}\right) \Bigg/ \left(\frac{\mu_A - \dfrac{aS_A\sigma_A^2}{(M+N)}}{\mu_B - \dfrac{aS_B\sigma_B^2}{(M+N)}}\right) > 1. \tag{6}$$

The left-hand side of equation (6) is the ratio of the expected return on asset A to the expected return on asset B, given the market clearing prices. Thus, the EM dedicated investors prefer to have an O.W. position in asset A when this ratio is greater than 1.

Now, imagine that only opportunistic funds are present in markets A and B. Then market clearing prices would be $\overline{\overline{P}}_0^A = \mu_A - \frac{aS_A\sigma_A^2}{M}$ and $\overline{\overline{P}}_0^B = \mu_B - \frac{aS_B\sigma_B^2}{M}$, which implies that there would be no volatility spillovers.

Proposition 7: Suppose that $M > \max\left\{\frac{aS_A\sigma_A^2}{\mu_A}, \frac{aS_B\sigma_B^2}{\mu_A}\right\}$ and that $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$ (that is, the dedicated EM investors prefer to have an O.W. position in asset *A*), then the market clearing prices of both assets *A* and *B* are decreasing in σ_B^2.

The main implication of Proposition 7 is consistent with the results of the partial equilibrium analysis; that is, whenever the EM fund managers' optimal portfolio allocation is such that they have an O.W. position in asset *A,* the market clearing price of asset *A* is a decreasing function of the (conditional) variance of the value of asset *B* (σ_B^2).

Thus, when dedicated EM investors have an O.W. position in asset *A* and their no-borrowing constraint is binding, an increased uncertainty about the fundamental value of asset *B* forces them to scale back their exposure to asset *A,* putting downward pressure on the price of asset *A*. At the same time, higher (conditional) variance of the value of asset *B* (σ_B^2) induces investors to reduce their demand for asset *B* as well, which results in a simultaneous decline of both EM asset prices (contagion).

IV. Conclusions

The results presented in this paper provide further support to the notion that the design of investment guidelines for institutional investors, which often has a narrow investor-protection focus (the single fund manager's perspective), should also take into account the impact of portfolio rebalancing (induced by these rules) by a large number of fund managers on the asset price dynamics. A better understanding of these implications would involve a simulation of the dynamic interaction of different types of institutional investors facing portfolio constraints.

Some of the conclusions that emerge from the analysis presented in the paper are related to the roles of different types of institutional investors in emerging markets:

- The analysis confirms that opportunistic investors can play a stabilizing role in asset markets when other market participants that (collectively) have significant market power face tight portfolio constraints. This can happen, for instance, because opportunistic investors can take a contrarian position in the face of a sell-off by real money funds reacting to a shock originating in a fundamentally unrelated market.
- The analysis presented in the paper also suggests that the inclusion of EM securities in global equity or bond indices may not necessarily reduce the volatility of portfolio flows to the individual EMs. Although it is true that the inclusion of EM securities in global benchmark indices can broaden the investor base for these assets, it can also increase the volatility of flows owing to portfolio reallocations induced by a combination of benchmark-based performance criteria and other portfolio constraints.

APPENDIX

Proof of Proposition 1

The Lagrangian for the optimization problem (1) can be written as follows:

$$L = (\lambda - \alpha)\big(E(R^A)\big) + (\tau - 1 + \alpha)\big(E(R^B)\big) + (1 - \lambda - \tau)R^M$$
$$-\frac{a}{2}\Big[(\lambda - \alpha)^2 Var(R^A) + (\tau - 1 + \alpha)^2 Var(R^B)\Big] + \varphi(1 - \lambda - \tau),$$

where φ is the Lagrange multiplier associated with the no-borrowing constraint. Assuming that $\lambda > 0$ and differentiating L with respect to λ, we obtain

$$\lambda = \frac{ER^A - R^M - \varphi}{aVar(R^A)} + \alpha. \tag{A1}$$

Assuming that $\tau > 0$ and differentiating with respect to τ, we obtain

$$\tau = \frac{ER^B - R^M - \varphi}{aVar(R^B)} + (1 - \alpha). \tag{A2}$$

The complementary slackness condition and non-negativity constraint are

$$\varphi(1 - \lambda - \tau) = 0, \varphi \geq 0.$$

Thus, if the no-borrowing constraint is not binding, that is, $\lambda + \tau < 1$, then the multiplier must be $\varphi = 0$. Alternatively, if the multiplier is positive, that is, $\varphi > 0$, the no-borrowing constraint must be binding, that is, $\lambda + \tau = 1$.

Suppose that $\varphi > 0$ and the no-borrowing constraint is binding, that is, $\lambda + \tau = 1$. Then, the optimal value φ^* derived from equations (A1) and (A2) and $\lambda + \tau = 1$ is

$$\varphi^* = \frac{\big(ER^A - R^M\big)Var(R^B) + \big(ER^B - R^M\big)Var(R^A)}{\big(Var(R^A) + Var(R^B)\big)},$$

which is positive whenever $\dfrac{\big(ER^A - R^M\big)}{Var(R^A)} + \dfrac{\big(ER^B - R^M\big)}{Var(R^B)} > 0.$

The optimal portfolio weights λ^* and τ^* are

$$\lambda^* = \frac{ER^A - ER^B}{a\big(Var(R^A) + Var(R^B)\big)} + \alpha \text{ and } \tau^* = \frac{ER^B - ER^A}{a\big(Var(R^A) + Var(R^B)\big)} + (1 - \alpha).$$

Suppose that the no-borrowing constraint is not binding, that is, $\lambda + \tau < 1$ and $\varphi = 0$. Then, the optimal portfolio weights are

$$\lambda^{**} = \frac{ER^A - R^M}{Var(R^A)} + \alpha \text{ and } \tau^{**} = \frac{ER^B - R^M}{Var(R^B)} + (1 - \alpha).$$

Proof of Proposition 3

The Lagrangian for the optimization problem (3) can be written as follows:

$$L = \gamma\left((\lambda-\alpha)ER^A + (\tau-1+\alpha)ER^B + (1-\lambda-\tau)R^M - k\right)$$
$$-\left((\lambda-\alpha)^2 Var(R^A) + (\tau-1+\alpha)^2 Var(R^B)\right) + \varphi(1-\lambda-\tau),$$

where γ is the Lagrange multiplier associated with the constraint

$$(\lambda-\alpha)ER^A + (\tau-1+\alpha)ER^B + (1-\lambda-\tau)R^M \geq k,$$

and φ is the Lagrange multiplier associated with the constraint

$$\lambda + \tau \leq 1.$$

Assuming that $\lambda > 0$ and differentiating L with respect to λ, we obtain

$$\lambda = \frac{\gamma\left(ER^A - R^M\right) - \varphi}{2Var(R^A)} + \alpha. \tag{A3}$$

Assuming that $\tau > 0$ and differentiating with respect to τ, we obtain

$$\tau = \frac{\gamma\left(ER^B - R^M\right) - \varphi}{2Var(R^B)} + (1-\alpha). \tag{A4}$$

The complementary slackness conditions and the corresponding non-negativity constraints are as follows:

$$\varphi(1-\lambda-\tau) = 0,\ \varphi \geq 0 \tag{A5}$$

and

$$\gamma\left((\lambda-\alpha)ER^A + (\tau-1+\alpha)ER^B + (1-\lambda-\tau)R^M - k\right) = 0, \gamma \geq 0. \tag{A6}$$

Suppose that $\gamma > 0$ and $\varphi = 0$, then the optimal values λ^{**}, τ^{**}, and γ^{**} derived from equations (A3), (A4), and (A6) are

$$\gamma^{**} = \frac{2kVar(R^A)Var(R^B)}{\left(ER^A - R^M\right)^2 Var(R^B) + \left(ER^B - R^M\right)^2 Var(R^A)},$$

$$\lambda^{**} = \frac{(k)\left(ER^A - R^M\right)Var(R^B)}{\left(ER^A - R^M\right)^2 Var(R^B) + \left(ER^B - R^M\right)^2 Var(R^A)} + \alpha, \text{ and}$$

$$\tau^{**} = \frac{(k)\left(ER^B - R^M\right)Var(R^A)}{\left(ER^A - R^M\right)^2 Var(R^B) + \left(ER^B - R^M\right)^2 Var(R^A)} + (1-\alpha).$$

Note that because it was assumed that $\varphi = 0$, it must be the case that $\lambda + \tau < 1$, or

$$\frac{(k)\left(ER^A - R^M\right)Var(R^B) + (k)\left(ER^B - R^M\right)Var(R^A)}{\left(ER^A - R^M\right)^2 Var(R^B) + \left(ER^B - R^M\right)^2 Var(R^A)} < 0.$$

Because the denominator is always positive, the latter holds whenever

$$\frac{ER^A - R^M}{Var(R^A)} + \frac{ER^A - R^M}{Var(R^B)} < 0.$$

Then, for the region of parameter values where $\dfrac{ER^A - R^M}{Var(R^A)} + \dfrac{ER^A - R^M}{Var(R^B)} \geq 0$, we have to consider a possibility that the no-borrowing constraint is also binding.

Suppose that $\varphi > 0$ and $\gamma > 0$, then the optimal values α^*, τ^*, φ^*, and γ^* derived from equations (A3)–(A6) are

$$\varphi^* = \frac{2k\left[\left(ER^A - R^M\right)Var(R^B) + \left(ER^B - R^M\right)Var(R^A)\right]}{\left(ER^A - ER^B\right)^2},$$

$$\gamma^* = \frac{2k\left[Var(R^B) + Var(R^A)\right]}{\left(ER^A - ER^B\right)^2}, \text{ and}$$

$$\lambda^* = \frac{k}{\left(ER^A - ER^B\right)} + \alpha \text{ and } \tau^* = \frac{-k}{\left(ER^A - ER^B\right)} + (1 - \alpha).$$

Suppose that $\varphi > 0$ and $\gamma = 0$, then $\lambda = \dfrac{-\varphi}{2Var(R^A)} + \alpha$ and $\tau = \dfrac{-\varphi}{2Var(R^B)} + (1 - \alpha)$. Because cash holdings are zero (that is, $1 - \lambda - \tau = 0$), $\varphi(-Var(R^A) - Var(R^B)) = 0$. But the latter can hold only if $\varphi = 0$, which contradicts our assumption that $\varphi > 0$.

Suppose that none of the constraints are binding, that is, $\varphi = 0$ and $\gamma = 0$, then $\lambda = \alpha$ and $\tau = (1 - \alpha)$. But then it must be the case that the no-borrowing constraint is binding, which contradicts our assumption.

Proof of Proposition 5

Let $x = P_0^A\ y = P_0^B/P_0^A$, then equations (4) and (5) can be rewritten as follows:

$$S^A = N\left(\frac{\mu_A - \mu_B\left(\frac{1}{y}\right)}{a\left(\sigma_A^2 + \sigma_B^2\left(\frac{1}{y}\right)^2\right)}\right) + M\left(\frac{\mu_A - x}{a\sigma_A^2}\right) \text{ and}$$

$$S^B = N\left(\frac{\mu_B - \mu_A y}{a\left(\sigma_A^2 y^2 + \sigma_B^2\right)}\right) + M\left(\frac{\mu_B - xy}{a\sigma_B^2}\right).$$

Solving the system of equations for x and y, we obtain the following:

$$y = \frac{(M+N)\mu_B - aS_B\sigma_B^2}{(M+N)\mu_A - aS_A\sigma_A^2}$$

and

$$x = \frac{\left((M+N)\mu_A - aS_A\sigma_A^2\right)\left(A - (2M+N)B + \left(M^2 + MN\right)C\right)}{M\left(A - 2(M+N)B + (M+N)^2 C\right)},$$

where $A = a^2\sigma_A^2(S_B\sigma_B^2)^2 + a^2\sigma_B^2(S_A\sigma_A^2)^2$, $B = aS_A\mu_A\sigma_A^2\sigma_B^2 + aS_B\mu_B\sigma_A^2\sigma_B^2$, and $C = \sigma_A^2(\mu_B)^2 + \sigma_B^2(\mu_A)^2$.

The market clearing prices can be rewritten as follows:

$$\bar{P}_0^A = \frac{\left((M+N)\mu_A - aS_A\sigma_A^2\right)F}{M\cdot G},\quad \bar{P}_0^B = \frac{\left((M+N)\mu_B - aS_B\sigma_B^2\right)F}{M\cdot G},\ \text{where}$$

$$F = \sigma_B^2\left(M\mu_A - aS_A\sigma_A^2\right)\left((M+N)\mu_A - aS_A\sigma_A^2\right) + \sigma_A^2\left(M\mu_B - aS_B\sigma_B^2\right)\left((M+N)\mu_B - aS_B\sigma_B^2\right)$$

and

$$G = \sigma_B^2\left(M\mu_A - aS_A\sigma_A^2\right)^2 + \sigma_A^2\left(M\mu_B - aS_B\sigma_B^2\right)^2.$$

Because G is always positive, the sign of P_0^A (or P_0^B) depends on the sign of the numerator. It is straightforward to verify that when $M > \max\left\{\frac{aS_A\sigma_A^2}{\mu_A}, \frac{aS_B\sigma_B^2}{\mu_B}\right\}$ F is positive for any parameter values a, μ_A, μ_B, σ_A^2, σ_B^2, S_A, S_B.

Proof of Proposition 6

The demand for asset A evaluated at the market clearing prices, that is,

$$\frac{\mu_A\left((M+N)\mu_B - aS_B\sigma_B^2\right)^2 - \mu_B\left((M+N)\mu_A - aS_A\sigma_A^2\right)\left((M+N)\mu_B - aS_B\sigma_B^2\right)}{a\left(\sigma_A^2\left((M+N)\mu_B - aS_B\sigma_B^2\right)^2 + \sigma_B^2\left((M+N)\mu_A - aS_A\sigma_A^2\right)^2\right)},$$

is positive whenever $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$.

The demand for asset B evaluated at the market clearing prices, that is,

$$\frac{\left(\mu_B\left((M+N)\mu_A - aS_A\sigma_A^2\right) - \mu_A\left((M+N)\mu_B - aS_B\sigma_B^2\right)\right)\left((M+N)\mu_A - aS_A\sigma_A^2\right)}{a\left(\sigma_A^2\left((M+N)\mu_B - aS_B\sigma_B^2\right)^2 + \sigma_B^2\left((M+N)\mu_A - aS_A\sigma_A^2\right)^2\right)},$$

is positive whenever $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$.

Proof of Proposition 7

Differentiating $\bar{P}_0^A$ with respect to σ_B^2 and simplifying, we obtain

$$\frac{\partial \bar{P}_0^A}{\partial \sigma_B^2} = \frac{\left((M+N)\mu_A - aS_A\sigma_A^2\right)}{M}\left(\frac{N\cdot Q}{G^2}\right),$$

where

$$Q = \sigma_B^2\left((M+N)\mu_A - aS_A\sigma_A^2\right)\left(\left(\frac{aS_B\sigma_B^2}{\mu_B}\right)^2\mu_A - (M+N)aS_A\sigma_A^2\right)$$
$$+ \sigma_A^2\left((M+N)\mu_B - aS_B\sigma_B^2\right)\left(\left(\frac{aS_B\sigma_B^2}{\mu_B}\right)^2\mu_B - (M+N)aS_B\sigma_B^2\right).$$

Differentiating $\bar{P}_0^B$ with respect to σ_B^2 and simplifying, we obtain

$$\frac{\partial \bar{P}_0^B}{\partial \sigma_B^2} = \frac{(-aS_B)\cdot F}{M\cdot G} + \frac{\left((M+N)\mu_B - aS_B\sigma_B^2\right)}{M}\left(\frac{N\cdot Q}{G^2}\right),$$

where F and G are shown to be positive for any well-defined market clearing prices (see Proposition 5). Then $\frac{\partial \overline{P}_0^A}{\partial \sigma_B^2}$ and $\frac{\partial \overline{P}_0^B}{\partial \sigma_B^2}$ are negative whenever $Q < 0$.

Because $M > \max\left\{\frac{aS_A\sigma_A^2}{\mu_A}, \frac{aS_B\sigma_B^2}{\mu_B}\right\}$ (by assumption), it must be the case that

$\left((M+N)\mu_A - aS_A\sigma_A^2\right) > 0$, $\left((M+N)\mu_B - aS_B\sigma_B^2\right) > 0$, and

$\left(\left(\frac{aS_B\sigma_B^2}{\mu_B}\right)^2 \mu_B - (M+N)aS_B\sigma_B^2\right) < 0.$

Then Q is negative whenever $\left(\frac{aS_B\sigma_B^2}{\mu_B(M+N)}\right)^2 < \frac{aS_A\sigma_A^2}{\mu_A(M+N)}$ and positive whenever $\left(\frac{aS_B\sigma_B^2}{\mu_B(M+N)}\right)^2 > \frac{aS_A\sigma_A^2}{\mu_A(M+N)}$.

Note that $\frac{aS_B\sigma_B^2}{\mu_B M} < 1$ and $\frac{aS_A\sigma_A^2}{\mu_A M} < 1$ (by assumption). Then $\frac{aS_B\sigma_B^2}{\mu_B(M+N)} < 1$ and $\frac{aS_A\sigma_A^2}{\mu_A(M+N)} < 1$ must be true as well. Finally, because $\frac{S_A\sigma_A^2}{\mu_A} > \frac{S_B\sigma_B^2}{\mu_B}$ (by assumption), then $\left(\frac{aS_B\sigma_B^2}{\mu_B(M+N)}\right)^2 < \frac{aS_A\sigma_A^2}{\mu_A(M+N)}$ for any a, μ_A, μ_A, σ_A^2, σ_B^2, S_A, S_B, which implies that $Q < 0$.

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