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# RESTORING A FISH STOCK: A DYNAMIC BANKRUPTCY PROBLEM 

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# Restoring a Fish Stock: A Dynamic Bankruptcy Problem 

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#### Abstract

Total Allowable Catch (TAC) regulating schemes have been introduced in most fisheries. TAC distribution following the Proportional Rule, based on historical catches, implies that harvesters or vessel groups which have captured more in the past and contributed to overfishing are getting larger quotas than groups that have contributed less to overfishing. In contrast to this rule a more egalitarian rule, the Constrained Equal Award Rule, is proposed for distributing the TAC. Contingent upon the fishing techniques used by the harvesters, it is demonstrated how the fish stock recovery period, harvest and profitability may vary according to these two rules. (JEL Q22 and C70)


## I. INTRODUCTION

Over the last two or three decades, various measures have been taken to regulate and build up previously overexploited fish stocks. Pivotal elements in this have included institutional changes such as the 200 miles national jurisdiction for coastal states, as well as other institutional changes within these states. More recently, through the so-called Total Allowable Catch (TAC) regulating schemes, quotas have been introduced in most fisheries (see, e.g., Brown 2000 for a general discussion). A typical TAC regulating scheme implies that in stage one the regulating authority sets a TAC for the actual fish stock for a given fishing period, often a year, based on ecological and economic considerations. In the next stage the TAC is distributed among, say, different vessel groups or fishermen who claim to have fishing rights inherited in historical catches. How the total quota should be distributed among the various harvesters involved is, however, far from clear as the sum of the claims generally exceeds what is available, i.e. the TAC. Therefore, a bankruptcy problem exists. ${ }^{1}$

Informally, a bankruptcy problem is a distribution problem that involves the allocation of a single resource, the estate, in a situation where the amount available is insufficient to satisfy the claims of all agents simultaneously. A solution to such a problem is provided by a rule prescribing how the resource should be allocated among claimants. Any overexploited fishery subject to a TAC regulating scheme in which claims are based on historical catches can be modelled as a bankruptcy problem. A fishery bankruptcy problem may be present at different levels. Harvesters may be considered as single vessels (or fishermen), various vessel groups, or even various countries. The first two situations are typically dealt with within coastal states while the distribution of the TAC among nations is frequently dealt with within the European Union (EU). The EU anglerfish fishery has been analyzed as a bankruptcy problem in Gallastegui et al. (2003), where the so-called Proportional Rule was compared with some solutions from cooperative games in a static, or one period, framework.

However, usually a TAC regulating scheme is designed for several periods, and once an initial $T A C$ is established a decision process is required to update the TAC yearly according to new information coming up about the actual fishery. Therefore, this policy obviously needs to be analyzed in a dynamic setting. The approach we develop in this paper, called the dynamic fishery bankruptcy problem, expands the one-period bankruptcy problem over time. Its basic elements are the following: i) The dynamic bankruptcy model assumes that fishing claims are based on historical catches, that the TAC is insufficient to satisfy these claims and that a rule
to distribute the TAC is needed. ii) To build up a previously overexploited stock, it is well accepted that the TAC for the first years should be quite low and well below the natural growth of the fish stock. The TAC should then increase. iii) To distribute the TAC two classical rules are considered: The Proportional Rule (as in Gallastegui et al. 2003) and the Constrained Equal Award Rule. The Proportional Rule prescribes that the share of TAC should be in proportion to the claims while the Constrained Equal Award Rule prescribes that every harvester should obtain the same quota subject to the restriction that no one should receive more than his claim (more details below). As the selectivity level of harvesters differs, meaning that the natural growth of the fish stock is affected differently by harvesters (see below), the recovery of the fish stock as well as the profitability of the fishery will also depend on the actual distribution rule.

The dynamic bankruptcy model is illustrated numerically with data from the North East Atlantic Norwegian cod fishery, where the trawler fleet and the coastal fleet are treated as two separate agents. As a stylized real-life situation in this particularly fishery, it is assumed that the coastal fleet does not influence the natural growth of the fish stock while the harvest of the trawler fleet has a negative effect as the trawlers use non-selective gear typically exploiting immature fish. Using numerical simulations, we find that the application of the Constrained Equal Award Rule, which is the more egalitarian rule (see below), yields a higher long-term ‘sustainable’ stock size. It may also shorten the length of time needed to reach the long-term stock size. For the given cost structure, the long term profitability of the fishery is also increased.

It is a large literature on fishery distribution problems, but most of these problems are not analysed (implicitly or explicitly) as bankruptcy problems. Armstrong (1999), for example, analyses the problem of distribution among different vessel groups (trawlers and the coastal fleet of a Norwegian cod fishery) utilising a pragmatic rule, the so-called 'trawl ladder' rule while many distribution problems among nations exploiting a common stock are viewed in light of a cooperative game framework pioneered in the work of Munro (1979). The distribution problem among different harvesters within a vessel group is frequently solved as an Individual Transferable Quota (ITQ) problem (see e.g. Libecap 2006 for an overview). However, as we are not considering the distribution within a vessel group, ITQ distribution is not a topic considered here.

The rest of the paper is organized as follows. In section II we briefly study a standard situation of a fishery that is unregulated, but in which harvesting selectivity is introduced. The fish population is described by a biomass model meaning that the age, or demographic, structure of the fish stock is not explicitly taken into account. Section III contains some preliminaries concerning bankruptcy problems and introduces the two rules for solving the fishery bankruptcy problem. Section IV formulates the dynamics of the fishery bankruptcy problem and describes the problem of restoring the long-run fish stock. The numerical illustrations follow in section V .

## II. THE UNREGULATED FISHERY

We consider a fishery exploited by a fixed number of $N$ harvesters. When stochastic variations in ecology and environment are ignored, the biomass ('a fish is a fish') growth $X_{t}$ at time $t$ may be written as:

$$
\begin{equation*}
X_{t+1}=X_{t}+F\left(X_{t}, h_{1, t}, \ldots, h_{N, t}\right)-\sum_{i=1}^{N} h_{i, t} \tag{1}
\end{equation*}
$$

where $h_{i, t}$ is the harvest of agent $i$ and $F\left(X_{t}, h_{1, t}, \ldots, h_{N, t}\right)$ is the natural growth function, given in a standard humped, density-dependent way (more details below). In addition, the selective level of harvesters generally influences natural growth; that is, $\partial F / \partial h_{i, t} \neq 0$. Typically, when an agent uses non-selective gear natural growth will be negatively affected, $\partial F / \partial h_{i, t}<0$. Just to fix ideas, we may think of non-selective technology as gears that take a high proportion of young, or sexually immature (recruited), fish. In our biomass framework this translates into lower natural growth. On the other hand, when only mature fish are caught the fishing gear is selective and harvesting leaves the stock growth less affected (Section IV provides more details).

The current profit of harvester $i$ may be written as $\pi_{i, t}=\pi_{i}\left(h_{i, t}, X_{t}\right)$. Following the logic of the Schäfer harvesting function (see also the numerical section below), the profit is concave in the harvest while a higher stock improves profitability as the unit harvesting cost becomes lower. Due to unclear and insecure property rights (see, e.g., the classical Gordon 1954), it is assumed that the fishery is myopically exploited, meaning that each agent maximises current
profit while taking the stock as given. The own stock effect through harvesting and any possible negative selection effect are therefore not taken into account. For this reason, any strategic interaction between agents is also omitted. ${ }^{2}$ Myopic profit maximising yields harvest at time $t$ as a function of the stock at the same time $h_{i, t}=h_{i}\left(X_{t}\right)$, where higher fish abundance means more harvest as lower cost induces more harvesting effort.

The stock dynamics are completed when the (myopic) optimal harvest is substituted into the ecological growth equation (1):

$$
\begin{equation*}
X_{t+1}=X_{t}+F\left(X_{t}, h_{1}\left(X_{t}\right), \ldots, h_{N}\left(X_{t}\right)\right)-\sum_{i=1}^{N} h_{i}\left(X_{t}\right) . \tag{2}
\end{equation*}
$$

The fish stock therefore grows according to a non-linear first order difference equation, and due to the unregulated nature of the fishery we typically find that the stock becomes lower (possibly interrupted by oscillations, see e.g. May 1976) over time. The myopic nature of the fishery and the selective technology effect govern the stock depletion. In addition, standard reciprocal cost externalities are present so that the harvest of each agent reduces the stock while it increases the cost and reduces the profit of the remaining harvesters. At a given point in time the profitability of fishermen becomes low, the "sustainability" of the stock is in danger and TAC-regulation of the previously unregulated fishery is introduced. Aggregate demand exceeds supply, and we then typically find that and the fishermen claim their historical catches out of a fishing stock which is no longer sufficient to satisfy their aspirations. Consequently, the fishery bankruptcy problem emerges.

## III. THE STATIC FISHERY BANKRUPTCY PROBLEM

There is a significant tradition of analyzing bankruptcy problems in economics. The works by O'Neill (1982), and Aumann and Maschler (1985) inspired a great many papers characterizing the solutions of these problems. Thompson (2003) surveys most of these papers. Formally, the static fishery bankruptcy problem, where the evolvements of harvest and natural growth over time are of no concern, can be formulated as follows (see Gallastegui et al. 2003): let the $N$ number of claimants (fishermen, vessel groups, nations, etc.) be involved in the distribution of a given TAC. Each harvester claims a share of the TAC equivalent to its historical fishing rights, or catch, denoted by the vector $h=\left(h_{1}, \ldots, h_{N}\right)$ with each $h_{i}>0$ and the total claim as
$\sum_{i=1}^{N} h_{i}$. A fishery bankruptcy problem is present when the TAC cannot satisfy the total claim; i.e., $\sum_{i=1}^{N} h_{i}>T A C$. The pair (TAC, $h$ ) is called a fishery bankruptcy problem.

A rule for solution of a fishery bankruptcy problem is a function $f$ which assigns to each (TAC, $h$ ) a vector $q=f(T A C, h)$ representing a distribution among agents of the TAC. Hence, $q_{i}=f_{i}(T A C, h)$ is the quota assigned to harvesteri. Only division rules with the properties i) $0 \leq f_{i}(T A C, h) \leq h_{i}$ and ii) $\sum_{i=1}^{N} f_{i}(T A C, h)=T A C$ are considered. Condition i) means that no agent should receive a negative quota or more than his claim, while condition ii) states that the entire $T A C$ is to be allocated among the claimants.

As mentioned, the Proportional Rule and the Constraint Equal Award Rule will be applied to the fishery bankruptcy problem. These two rules have a long tradition in game theory literature, and are the ones most often applied to real-life problems. The Proportional Rule is deeply rooted in law and custom as a division rule, and is frequently used in environmental agreements. For example, when the industrialized countries signed the agreement in 1987 to reduce the amount of ozone-damaging chemicals that they released into the atmosphere, they cut their emissions in proportion to their historical emissions (see, e.g., Perman et al. 2003). This rule asserts that the shares of the TAC should be in proportion to the claims. To put it another way, it equalizes the ratios between claims and awards. The Proportional Rule for a fishery bankruptcy problem is denoted by $\operatorname{PROP}(T A C, h)$ and is simply defined as:

$$
\begin{equation*}
q_{i}=P R O P_{i}(T A C, h)=\frac{h_{i}}{\sum_{i=1}^{N} h_{i}} T A C ; i=1, \ldots, N . \tag{3}
\end{equation*}
$$

Pure egalitarianism is a common practice in certain socioeconomic situations, but quite often limitations are established which give rise to 'restricted egalitarian rules'. The Constrained Equal Award is one of the best known of these restricted egalitarian rules and is commonly used in some financial practices. For instance, oversubscriptions for Initial Public Offerings are often solved via a procedure in which every investor gets the same number of shares with the exception of those investors who have subscribed (and obtain) a smaller number of shares.

The Constrained Equal Award Rule focuses on the amount that each claimant obtains and prescribes that everyone should gain equally subject to the restriction that no one should get more than he is owed. That is, this rule equally divides the $T A C$ among all claimants with the proviso that no claimant receives more than his claim. The Constrained Equal Award Rule for a fishery bankruptcy problem is denoted by $C E A(T A C, h)$ and defined as:

$$
\begin{equation*}
q_{i}=C E A_{i}(T A C, h)=\min \left\{h_{i}, \lambda\right\} ; i=1,,, N \text { such that } \lambda \text { solves } \sum_{i=1}^{n} \min \left\{h_{i}, \lambda\right\}=T A C \tag{4}
\end{equation*}
$$

The computation of this rule is the result of the following sequential process: i) Initially every agent involved in the bankruptcy problem gets the smallest claim. If this is not possible then the estate is simply divided by the total number of agents (TAC / $N$ in this setting) and the process ends. Otherwise the smallest claimant(s) receive(s) his/their claim and leave(s). ii) Starting from the smallest claim, each of the remaining agents gets an additional amount until the second smallest claim is reached. If this is not possible then the (so far) non distributed estate is equally divided and the process ends. Otherwise the second smallest claimant(s) receive(s) his/their claim and leave(s). iii) The process continues in this way until the entire estate is distributed. ${ }^{3}$

Notice that the idea of equality underlies both these rules; that is, the Proportional Rule ensures equality of ratios among claimants while the Constrained Equal Award Rule emphasizes equality in their awards. In general, each rule represents an application of certain ethical principles (defined as properties) for solving bankruptcy problems. Apart from the socalled Constrained Equal Losses Rule, these two are the only ones that satisfy the properties of equal treatment, scale invariance and consistency. Moreover, they rule out path independence. Equal treatment requires that agents with identical claims should obtain identical quotas. Scale invariance implies no influence of the units in which the estate and the
claims are measured on the shares assigned. Composition means that a when a bankruptcy problem is equivalent to the sum of two partial bankruptcy problems, then its solution should coincide with the sum of the solutions to those two partial bankruptcy problems. Path independence is a property that applies to situations where after solving a bankruptcy problem it turns out that the amount of the estate falls short of what was expected. When this happens, the property of path independence requires that if the claims were substituted by the shares initially assigned and the rule were reapplied to this problem, then the solution reached should be equal to the solution derived from a new bankruptcy problem with the initial claims and the reduced estate. Consistency implies that once a solution has been agreed upon, no group of agents should be willing to reply the rule to the reduced problem (Herrero and Villar 2001 gives more details). ${ }^{4}$

## IV. BUILDING UP THE FISH STOCK: THE DYNAMIC PROBLEM

As already indicated, the stock dynamics depend not only on the size of the TAC established in each period $t$, henceforth $T A C_{t}$, but also on the particular distribution rule through the selective level of the harvesters. These two aspects become apparent from the stock growth equation (1) rewritten as:

$$
\begin{equation*}
X_{t+1}=X_{t}+F\left(X_{t}, q_{1, t}, \ldots, q_{N, t}\right)-T A C_{t} \tag{5}
\end{equation*}
$$

and where the individual quota $q_{i, t}$, with $\sum_{i=1}^{N} q_{i, t}=T A C_{t}$, generally differs according to the distributional rules of the $T A C_{t}$.

To build up a previously overexploited stock, the resource manager typically determines that the $T A C_{t}$ should be quite low in the beginning so that natural growth strongly dominates harvest. It should then increase successively until the stock reaches the long-term socially desirable level, based on economic and ecological considerations. ${ }^{5}$ Therefore, the harvesting strategy is formulated so that the time sequence of the $T A C_{t}$ follows the pattern $T A C_{t-1} \leq T A C_{t}$ for $t=1, \ldots, T^{*}$, where $T^{*}$ is number of years before the long-term socially desirable steady-state stock level $X^{*}$ and $T A C^{*}$ are reached. In what follows, the so-called proportional threshold harvesting strategy is used to determine the sequence of $T A C_{t}$. In a
generalized framework with uncertainty and uncertain stock estimates, Lande et al. (1997) argue that this harvesting strategy has some desirable properties. ${ }^{6}$ The resource managers in Hilborn and Walters (1992) and Homans and Wilen (1997), among others, also apply this harvesting strategy.

According to the proportional threshold harvesting strategy, a fixed fraction of the surplus stock over a certain minimum level, the threshold, should be harvested every year:

$$
\begin{equation*}
T A C_{t}=\max \left[0, b\left(X_{t}-X_{\min }\right)\right], \tag{6}
\end{equation*}
$$

where $X_{\text {min }}>0$ is the threshold level, while $b>0$ is the proportional factor. It is assumed that the values of $b$ and $X_{\min }$ are scaled such that i) $\left(X_{t}-X_{\min }\right)>0$ holds, and ii) $X_{t}$ increases over time $t=1, \ldots, T^{*}$. Therefore, $T A C_{t}$ also increases over time. ${ }^{7}$

To analyse how the different distribution rules affect the stock growth (5) and the steady-state, the technology, or the selective level, of the harvesters, must be known. As already mentioned, non-selective technology may be thought of in terms of gears that take a high proportion of sexually immature fish, while the fishing that catches basically mature fish can be seen as more selective. In some fisheries, we find that small-scale coastal vessels using passive catch gear like trolls take mature fish while trawlers often take a high proportion of young fish. Recent evidence from the North Atlantic cod fishery is an example on this (Norges Naturvernforbund 2006, and see numerical illustration below). However, as pointed out by one of the referees, the situation may be different in temperate and tropical waters where juveniles tends to be inshore, while there tends to be more mature fish and less biodiversity offshore.

The degree of selectivity may, however, also be viewed from another angle. Over the last two decades or so there has been concern about the impact of towed gears such as trawls and dredges on benthic habitats and organisms. The reasons are that benthic habitats provide refuge for juvenile fish, and the associated fauna comprise important food sources for demersal fish (see, e.g., Pilskaln et al. 1998; Engel and Kvitek 1998; Jackson et al. 2001, but also the overview in FAO 2004 and the references therein). Therefore, based on possible habitat destruction, trawlers may be considered as non-selective. ${ }^{8}$

In our biomass framework, the degree of selectivity may be modelled as if fishing activity affects the natural growth of the fish stock either through the growth rate or through the carrying capacity, where habitat degradation is typically channelled through carrying capacity. In what follows, we have chosen to model it through the growth rate and we use the functional form applied by Escapa and Prellezo (2003). Within this framework, the difference between the `net' and `gross' intrinsic growth rates become crucial and is contingent upon two factors: the degree of selectivity and the distribution of the harvest $q_{i, t}$. More specifically, in our biomass model the net intrinsic natural growth rate is defined as:

$$
\begin{equation*}
\tilde{r}_{t}=r+1-\sum_{i=1}^{N} \alpha_{i, \gamma^{\prime}} \tag{7}
\end{equation*}
$$

where $r$ is the gross intrinsic rate, $\alpha_{i, t}=q_{i, t} / T A C_{t}$ is the catch proportion of harvester $i$ so that $\sum_{i=1}^{N} \alpha_{i, t}=1$ for all $t=1, \ldots, T$ (and for both distributional rules), and $\gamma_{i} \geq 1$ measures the degree of gear selectivity among the harvesters. If the fishing technology of all harvesters hypothetically is selective and $\gamma_{i}=1$, the net and gross growth rates coincide. Otherwise, the net rate will be below the gross rate.

When the standard logistic natural growth function with $K$ as the carrying capacity is used, the natural growth becomes:

$$
\begin{equation*}
F\left(X_{t}, q_{1, t}, \ldots, q_{N, t}\right)=\tilde{r}_{t} X_{t}\left(1-X_{t} / K\right), \tag{8}
\end{equation*}
$$

while the population growth follows as:

$$
\begin{equation*}
X_{t+1}=X_{t}+\left[r+1-\sum_{i=1}^{N} \alpha_{i, t} \gamma_{i}\right] X_{t}\left(1-X_{t} / K\right)-b\left(X_{t}-X_{\min }\right) \tag{9}
\end{equation*}
$$

when inserting equations (6) (with $X_{t}-X_{\min }>0$ ), (7) and (8) into (5). From (9) it is apparent that a $T A C$-distribution that allocates more of the catch to harvesters with more selective gear, ceteris paribus, will yield a higher natural growth, and hence, in subsequent steps, a higher
stock and $T A C_{t}$. Consequently if, say, harvesters with the largest historical claims also utilize a highly non-selective technology, the Constrained Equal Awards Rule will lead to a faster recovery of the fish stock than the Proportional Rule, at least in the very beginning.

The selectivity levels of the harvesters influence the steady-state natural growth through $\tilde{r}^{*}=r+1-\sum_{i=1}^{N} \alpha_{i}^{*} \gamma_{i}$. Therefore, not surprisingly, we find that the rule that distributes the highest proportion of the catch to harvesters with more selective gear also yields a higher natural growth in the steady-state. ${ }^{9}$ As the threshold harvesting strategy implies a fixed harvesting schedule for both distributional rules, the steady-state stock $X^{*}$ and harvest $T A C^{*}$ are also higher with the rule that distributes the highest proportion of the catch to harvesters with more selective gear. ${ }^{10}$ However, whether or not the rule that produces the largest equilibrium stock yields a higher equilibrium profit (or rent) of the fishery depends crucially on the cost and price structure of the fishing fleet. Hence, we may have a situation where the Constrained Equal Award Rule restores the fish stock faster than the Proportional Rule while at the same time the profitability is lower. On the other hand, if the harvesters with the largest historical catch utilise the least selective gear and at the same time operate less efficiently, the application of the Constrained Equal Awards Rule may be in the interest of all harvesters (more details below).

## V. NUMERICAL ILLUSTRATION

## Data and specific functional forms

The model will now be illustrated numerically with data that fit the North-East Atlantic Norwegian cod fishery in a stylized way. The North-East Atlantic cod stock is managed jointly by Norway and Russia, and the division of the catch has been roughly fifty-fifty in recent years. The entire Russian quota is fished by trawlers, while the coastal fleet plays an important role on the Norwegian side (see Armstrong 1999). In what follows, however, we are not concerned with the catch taken by the Russian fleet; only the landings of the Norwegian vessels and the division between trawlers and the coastal fleet are considered. Over the last few years, trawlers have been the largest harvesters, accounting for just under $70 \%$ of the total Norwegian catch. Since 1989 the total catch of these two vessel groups has been regulated because of previous stock depletion and low profitability: precisely the mechanism described above (Section Two).

We consider the trawlers and the coastal fleet as two separate agents, and hence we analyse the distributional rules with only two claimants, $N=2 .{ }^{11}$ The fishing techniques of the trawlers and the coastal vessels are different: the coastal fleet utilises passive catch gear while the trawlers utilise active gear (see also above). The areas of fishing activity are also different as the trawlers operate in areas well away from the coast. For these two reasons, the trawlers catch fish of smaller size than the coastal fleet, and while the trawlers harvest mainly immature individuals, the coastal fleet harvests a much higher proportion mature fish. In 2004, for example, the harvest proportion of cod above 2.5 kg was about 50 percent for the coastal fleet and only 15 percent for the trawlers. ${ }^{12}$ As a stylised situation, it is assumed that the fishing technology of the coastal fleet does not influence the natural growth of the cod stock at all; that is, $\gamma_{1}=1$ for the coastal fleet while $\gamma_{2}>1$ for the trawler fleet (Table 1). ${ }^{13}$

The harvest of both groups is governed by the Schäfer function $q_{i, t}=\theta_{i} E_{i, t} X_{t}$ with $E_{i, t}$ as the effort used (number of vessels), and $\theta_{i}$ as a productivity parameter (catchability coefficient). The current profit reads accordingly $\pi_{i, t}=\left(p_{i}-c_{i} / \theta_{i} X_{t}\right) q_{i, t}$ where $p_{i}$ is the catch (ex vessel) price, which generally differs from one group of vessels to another but is assumed here to be given and fixed over time, and $c_{i}$ is the unit vessel operating cost, also assumed fixed. ${ }^{14}$

The baseline parameter values are given in Table 1. Following Armstrong (1999), the economic and technological data applied are for average trawl and coastal vessels. The catch price is higher for the coastal fleet than the trawlers due to the different size of the fish, but also due to different quality and degree of refinement of the products delivered. The effort cost and productivity parameter also differ, and as a consequence, the unit harvesting cost differs. Under the baseline assumptions, the unit cost ratio is $\left(c_{1} / \theta_{1} X_{t}\right) /\left(c_{2} / \theta_{2} X_{t}\right)=0.53$. Hence, the unit cost of the trawler fleet is close to double that of the coastal fleet. However, it should be noted that part of this difference is due to the fact that the trawler fleet is only allowed to operate in areas far from the coast. The income and cost structure imply always that the regulation, or rationing, of the harvesters is effective in the short term; that is, a higher quota yields a higher profit and induces more fishing effort among both groups. The biological data follow Eide (1993) and Armstrong (1999) while the selectivity parameters are based on Escapa and Prellezo (2003). We find that the net intrinsic growth rate is
$\tilde{r}_{t}=\tilde{r}^{*}=r+1-1=0.50$ if the whole $T A C_{t}$ is (hypothetically) allocated to the coastal fleet and $\tilde{r}=\tilde{r}^{*}=r+1-1.30=0.20$ if the whole quota is (hypothetically) allocated to the trawler fleet.

As some of these parameter values (especially the selectivity parameters) are highly uncertain, the numerical analysis is also carried out under different assumptions. Different values for the parameter governing the $T A C_{t}$ evolvement are also used. In the baseline scenario we apply $b=0.5$, meaning that $50 \%$ of the surplus threshold level stock is harvested every year. The threshold stock level is $X_{\min }=900$ (tonnes) while the initial stock size is set to $X_{0}=1000$ (tonnes). We assume that the claims based on the historical catch are $h_{1}=210$ for the coastal fleet and $h_{2}=390$ for the trawlers. Therefore, the historical fishing proportions are 0.35 and 0.65 for the coastal fleet and the trawler fleet, respectively. Finally, to calculate the present-value profit, the discount rent is assumed to be fixed and $5 \%, \delta=0.05$.

Table 1 about here

## Results

Table 2 reports the results for the Proportional Rule, where the harvesting proportion is 0.35 for the coastal fleet and 0.65 for the trawler fleet all the time. Therefore, under this rule, the net intrinsic natural growth rate is fixed over time, and is $\tilde{r}_{t}=\tilde{r}^{*}=0.305$. For the baseline parameter values, the steady-state stock $X^{*}$ will be well below that of $X^{\text {msy }}=2,500$ while the steady-state $T A C^{*}$ never reaches the previous level of 600 . The bankruptcy problem is therefore present also after the restoration period and once the steady-state is reached. It takes 15 years to reach the steady-state, which is approached smoothly. The steady-state profit becomes 2,105 (million NOK). The table also shows the present-value profit, which is calculated for a 40-year time horizon; that is, the 15 -year restoration period and an additional 25 years of equilibrium fishing. Figure 1 demonstrates in more detail the profitability expansion path during the restoration period together with profit shares. The profit share of the coastal fleet is around 0.4 and decreases modestly over time as the cost-price ratios of the two fleets change unevenly during the period of stock expansion.

Table 2 about here

Table 2 also reports results under alternative assumptions for the TAC -setting process ( $b$ shifts) as well as for more and less selective harvesting technology assumptions for the trawler fleet ( $\gamma_{2}$ shifts). Not surprisingly, the steady-state stock size and the $T A C^{*}$ become lower and the restoration process takes a shorter time when $b$ shifts up. Profitability also falls. A less selective technology for the trawler fleet yields much the same results except that the restoration process is more modestly affected. Notice also the strong profitability effect and the negative spill-over effect to the coastal fleet, i.e. a less selective technology not only hurts the trawler fleet but also badly hurts the coastal fleet and a significant social loss becomes apparent. Therefore, the external cost of the less selective trawler fleet technology is of great importance.

Figure 1 about here

Table 3 reports the results for the Constraint Equal Award Rule. Under the baseline TACsetting and technology assumptions, the steady-state settles down after $T^{*}=13$ years without reaching the historical harvesting claim of the coastal fleet of 210 (1000 tonnes). The steadystate harvesting proportions are hence 0.50 of both fleets. As a consequence, the net intrinsic natural growth rate of the fish stock is always higher under this rule than under the Proportional Rule, and the steady-state net intrinsic growth rate is $\tilde{r}^{*}=0.350$. Compared to the baseline solution of the Proportional Rule, the steady-state stock size $X^{*}$ and the $T A C^{*}$ are thus also higher. Moreover, the time pattern of the restoration process yields a higher yearly total profit than that of the Proportional Rule (Figure 1). Both more fish at a lower unit harvesting cost and a lower external cost due to the different fleet composition work in this direction. However, while the profit of the coastal fleet becomes significantly higher, trawler fleet profitability decreases slightly. Also under this rule, the profit share of the coastal fleet decreases during the restoration period (Figure 1).

Table 3 about here

When changing the TAC -setting by shifting $b$, we find that the patterns are very much as under the Proportional Rule. The time for reaching the steady-state is seven years when $b$ shifts up and the profitability of both fleets decreases, while it is nineteen years accompanied by more profit for the downward shift. A less selectivite technology of the
trawler fleet also results here in a substantial fall in the profitability of both fishing groups. However, the profitability of the trawler fleet is now observed to be higher than under the Proportional Rule as the present value profit is 11,268 (million NOK) compared to 9,117 . While the trawler fleet obtains a lower harvesting proportion than under the Proportional Rule, better natural growth conditions under the more egalitarian Constraint Equal Award harvesting Rule hence more than compensate for the loss of harvesting rights. Therefore, this example produces a result where the application of the Constrained Equal Award Rule is in the interest of all claimants.

## VI. CONCLUDING REMARKS

FAO statistics demonstrate that many of the world's fish stocks are depleted, many are overexploited and only a small part of fishery resources may said to be in a healthy condition. Various types of quota systems have been established to remedy this sad picture. This paper studies the dynamics of restoring a fish stock through TAC regulation, modelling the problem as a dynamic bankruptcy problem. The bankruptcy problem is studied as a distribution problem among different vessel groups, and ITQ is not considered as a topic. Traditionally, $T A C$-distribution has been based on the Proportional Rule where the TAC is divided among the claimants according to their historical harvest fractions. However, this position may be challenged because TAC distribution based on the proportional historic catches implies that those fishermen, or vessel groups, that have captured the most in the past obtain the largest quotas; that is, historical overexploitation of fish stocks is rewarded. Therefore, if overfishing is not to be rewarded, a more egalitarian distribution of the TAC should be applied. The Constrained Equal Award Rule is such a rule, and is proposed in this paper. Allocation of the TAC according to the historical catches is a method of establishing property rights to fish (not for the stock). It is a first-possession rule, and as Libecap (2006) points out, this rule recognizes incumbent harvesters who have experience in exploiting fishing resources. Needless to say, this rule discriminates against new entrants. On the other hand, egalitarian rules avoid to some extent the distributional concerns associated with historical catches and reflect pure and simply egalitarian goals. This rule may be considered as a lottery (again, see Libecap 2006 for an elaboration) since each claimant is given an equal, opportunity in the assignment of rights to fish.

The dynamic bankruptcy model is illustrated numerically with data from the North East Atlantic Norwegian cod fishery, where the trawler fleet and the coastal fleet are treated as two
separate agents. For this particular fishery where the coastal fleet vessels use more selective gear and takes a high proportion of mature fish while the trawlers fleet basically takes small immature fish, we find that the Constrained Equal Award Rule yields a higher sustainable TAC and may also approach the steady-state faster than the Proportional Rule. For the given cost and price structure, the short term and long term profitability of the fishery are also improved under the Constrained Equal Award Rule. Under certain conditions, we also find that the Constrained Equal Award yields a higher profitability of the trawler fleet even if this group obtains a lower quota than under the Proportional Rule. Therefore, the application of the Constrained Equal Award Rule may be in the interest of both claimants. Under a different cost and price structure, however, the difference in profitability between the two distribution rules may change while the result for the sustainable TAC prevails. Our results are based on a biomass model where harvest selectivity is modelled in an indirect way through its effect on the intrinsic growth rate of the biomass. It is left for further research to account more explicitly for the demography of the fish population by using an age structured model and where different exploiters may exploit different stages of the population.

The results found in this paper may be different from other fisheries (e.g., in tropical waters) where the selectivity level of the coastal fleet and trawlers can be different and where the ecosystem effects may different as well. It should also be emphasised that neither the TAC setting process nor the TAC distribution processes considered represent a socially efficient solution in the traditional economic sense. If we imagine a social planner that maximises the discounted profit from the above cod fishery (cf. also note 5), it can be shown that a necessary condition for steady-state fishing of both fleets is that an uneven cost-price ratio among the harvesters should be matched by a selectivity level in the opposite direction (Escapa and Prellezo 2003). That is, when the harvesting technology of the trawler fleet is less selective than that of the coastal fleet, the cost-price ratio of the coastal fleet must be higher if fishing of both fleets is to be socially efficient. As this is not the case in the present example, present-value profit maximisation implies coastal fleet steady-state fishing only.

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${ }^{1}$ A bankruptcy problem is therefore a particular case of a rationing problem where aggregate demand exceeds aggregate supply (here the TAC). A bankruptcy problem is a well defined problem in the game theory (see below) and where the term claim is used to describe individual demand. This term is used throughout this paper, and was also used by Gallastegui et al. (2003) (see below).
${ }^{2}$ In harvesting models strategic interaction is usually channelled through the resource stock, resulting in reciprocal cost externalities. Under myopic harvesting, where the stock is treated as exogenous by the exploiters, this type of strategic interaction is therefore ruled out. However, there may also be strategic interaction through the market for fish, but this possibility is not explored in this paper as the fish price is exogenously given (see the numerical section).
${ }^{3}$ The Constrained Equal Award rule computation may be illustrated with an example where we consider four harvesters with the claims $h_{1}=100, h_{2}=200, h_{3}=300$ and $h_{4}=400$, and $T A C=800$. In step i) each agent gets 100 and harvester 1 leaves. In step ii) agents 2,3 and 4 complete their payoff up to 200 and agent 2 leaves. In step iii) agents 3 and 4 cannot complete the third smallest claim, hence the non distributed estate, i.e. 100 , is divided equally between them, completing the amount they are already given. Therefore, the CEA allocation will be $q_{1}=100, q_{2}=200$ and $q_{3}=q_{4}=250$.
${ }^{4}$ While the two rules to be applied satisfy the same set of properties they are, however, distinguished by one property, called exemption, which is of interest for the fishery bankruptcy problem. This property establishes that whenever the claim of an agent is smaller than the equal division of the estate, the rule should guarantee the full claim to that agent. Formally this property is defined as follows: If $q_{i} \leq T A C / N$ then $f_{i}(T A C / N)=q_{i}$. As indicated by Herrero and Villar (2001) this property is consistent with the principle of 'progressivism' according to which those agents with a small claim should actually be exempted from being rationed. It is immediately apparent that exemption is not satisfied by the Proportional Rule but is satisfied by the Constrained Equal Award Rule. The rationale of the exemption principle reflects the idea that small claimants are not so responsible for the overexploiting of the fish stock as big claimants since if all harvesters had small claims then there would not be a fishery bankruptcy problem.
${ }^{5}$ The social planner solution is in most instances guided by the goal of maximizing present value harvesting profit. Clark (1990) analyzes factors affecting the long-term steady-state as well as the dynamics leading to the steady-state. The dynamics will typically be of the saddle-point type, which will generally be different from the proportional threshold harvesting strategy (see main text below).
${ }^{6}$ They find, for example, that for populations of species with moderately stochastic and moderate, or even high, uncertainty in population estimates, proportional threshold harvesting results in higher average and cumulative harvests than other strategies. The threshold harvesting strategy also produces a lower risk of population extinction, or collapse, compared to the other classical harvesting strategies considered. These classical strategies are (i) constant harvesting, (ii) proportional harvesting and (iii) threshold harvesting. It should be noted that none of these strategies, or indeed the proportional threshold harvesting strategy, is `efficient' in a traditional economic sense (cf. the above footnote). However, the proportional threshold strategy may, depending on the parameter values (see the main text below), mimic a saddle point path quite closely.
${ }^{7}$ The time evolvement of $T A C_{t}$ and the time $T^{*}$ for approaching the steady-state will for obvious reasons depend on the parameter values $b$ and $X_{\text {min }}$. A high $b$-value, ceteris paribus, may lead to a fast approach to the steadystate accompanied by a low stock size and a low long-run $T A C^{*}$, while a high $X_{\min }$-value typically works in the direction of a high steady-state stock size. The value of these parameters may therefore also govern a tradeoff between short term and long-term profitability of the regulated fishery in the restoration process (see numerical section).
${ }^{8}$ Several environmental groups have demanded an international moratorium on bottom trawling because of seabed ecosystem destruction. But the proposal did not win consensus amongst the delegates taking part in a UN conference on fish stocks spring 2006 (http://www.illegal-fishing.info/).
${ }^{9}$ In the two agents case where $\alpha_{1}{ }^{*}+\alpha_{2}^{*}=1$ we find $d \tilde{r}=\left(\gamma_{2}-\gamma_{1}\right) d \alpha_{1}^{*}$ (or $\left.d \tilde{r}=-\left(\gamma_{2}-\gamma_{1}\right) d \alpha_{2}{ }^{*}\right)$. In the three (or more) agents case, we have to be more specific. If, say, the proportion to agent 1 increases at the expense of an equal reduction of the other two, it turns out that the change may be written as $d \tilde{r}=\left[0.5\left(\gamma_{2}+\gamma_{3}\right)-\gamma_{1}\right] d \alpha_{1}^{*}$. Therefore, in this case the 'net' intrinsic growth rate and the natural growth increase if the selectivity level of agent 1 is below that of the weighted average of the two other agents.
${ }^{10}$ Analytically, the steady-state stock size is determined jointly by equations (7) and (9) (when $X_{t+1}=X_{t}$ ) such that $\quad X^{*}=\frac{K\left(\tilde{r}^{*}-b\right)+\sqrt{K^{2}\left(\tilde{r}^{*}-b\right)^{2}+4 \tilde{r}^{*} b K X_{\min }}}{2 \tilde{r}^{*}} \quad$ and $\quad \tilde{r}^{*}=r+1-\sum_{i=1}^{N} \frac{q_{i}^{*} \gamma_{i}}{b\left(X^{*}-X_{\min }\right)} \quad$ with $q_{i}^{*}$ determined by the distribution rule and $\sum_{i=1}^{N} q_{i}^{*}=b\left(X^{*}-X_{\min }\right)$.
${ }^{11}$ This fits the reality of the Norwegian cod fishery as the cod TAC, in a first step, is distributed between these two fleet groups. In the next step the total trawler quota is distributed among the trawler vessels (partly based on $I T Q$ ) while the total coastal quota is distributed among the different coastal vessels (partly based on ITQ, but also historical catches, Armstrong 1999). See also the introductory section.
${ }^{12}$ For the trawler fleet, the large cod proportion landed over the last years has been $8 \%$ (1999), $8 \%$ (2000), $9 \%$ (2001), $11 \%$ (2002), $14 \%$ (2003) and, as mentioned in the main text, $15 \%$ in 2004. The large cod proportion has hence increased somewhat during this period. For the coastal fleet, these proportions have been $58 \%$ (1999), 62\% (2000), 61\% (2001), 60\% (2002), 54\% (2003) and $51 \%$ (2004) (Norges Naturvernforbund 2006, 15).
${ }^{13}$ That the technology of the coastal fleet does not affect the natural population growth is obviously somewhat unrealistic (cf. also the above mentioned catch data). However, it is the variation in the selective parameters that basically influences the results.
${ }^{14}$ However, notice that this specification of the harvesting function yields no meaningful solution under myopic profit maximising exploitation as the solution demands a decreasing effort use effect (Section Two).

TABLE 1
Baseline values, prices and costs, ecological parameters and other parameters

| Parameter | Parameter description | Value |
| :--- | :--- | :--- |
| $r$ | 'Gross' intrinsic growth rate | 0.50 |
| $K$ | Carrying capacity | 5,000 (1000 tonnes) |
| $\gamma_{1}$ | Coastal fleet selectivity <br> parameter | 1.0 |
| $\gamma_{2}$ | Trawler fleet selectivity <br> parameter | 1.3 |
| $X_{\text {min }}$ | TAC threshold level | 900 (1000 tonnes) |
| $b$ | TAC proportional factor <br> coastal fleet | 0.5 |
| $h_{1}$ | Historical fishing rights of <br> trawlers | 390 (1000 tonnes) |
| $h_{2}$ | Coastal fleet harvesting price | 8.6 (mill NOK/1000 tonnes) |
| $p_{1}$ | Trawler fleet harvesting <br> price | 7.6 (mill NOK/1000 tonnes) |
| $p_{2}$ | Coastal fleet harvesting cost | 1.5 (mill NOK/vessel) |
| $c_{1}$ | Trawler fleet harvesting cost | 18.8 (mill NOK/vessel) |
| $c_{2}$ | Coastal fleet catchability <br> coefficient | 0.0011 (1/coastal vessel) |
| $\theta_{1}$ | Trawler fleet catchability <br> coefficient | 0.0066 (1/trawl vessel) |
| $\theta_{2}$ | Discount rent | 0.05 |
| $\delta$ |  |  |

TABLE 2
Proportional Rule. Steady-state stock size $X^{*}$ (1000 tonne biomass), steady-state TAC ${ }^{*}$ (1000 tonne biomass), time before steady-state $T^{*}$ is reached (years), steady-state profit $\pi^{*}$ (million NOK), present-value profit coastal fleet $P V_{1}$, trawler fleet $P V_{2}$ and total present-value profit $P V$ (million NOK).

|  | Baseline assumptions; $b=0.5, \gamma_{2}=1.3$ | Alternative TACsetting; $b=0.8, \gamma_{2}=1.3$ | Alternative TACsetting; $b=0.3, \gamma_{2}=1.3$ | Alternative technology trawler fleet; $b=0.5, \gamma_{2}=1.5$ | Alternative technology trawler fleet; $b=0.5, \gamma_{2}=1.15$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $X^{*}$ | 1,553 | 1,259 | 2,145 | 1,223 | 1,835 |
| TAC* | 326 | 287 | 373 | 162 | 467 |
| $T^{*}$ | 15 | 6 | 27 | 11 | 16 |
| $\pi^{*}$ | 2,105 | 1,753 | 2,564 | 976 | 3,124 |
| $P V_{1}$ | 14,075 | 12,875 | 1,4977 | 7,003 | 20,248 |
| $P V_{2}$ | 19,394 | 16,934 | 21,663 | 9,117 | 28,755 |
| PV | 33,470 | 29,809 | 36,640 | 16,120 | 49,003 |

TABLE 3
Constrained Equal Award Rule. Steady-state stock size X ${ }^{*}$ (1000 tonne biomass), steady-state TAC* (1000 tonne biomass), time before steady-state $T^{*}$ is reached (years), steady-state profit $\pi^{*}$ (million NOK), present-value profit coastal fleet $P V_{1}$, trawler fleet $P V_{2}$ and total presentvalue profit $P V$ (million NOK).

|  | Baseline assumptions; $b=0.5, \gamma_{2}=1.3$ | Alternative TACsetting; $b=0.8, \gamma_{2}=1.3$ | Alternative TACsetting; $b=0.3, \gamma_{2}=1.3$ | Alternative technology trawler fleet; $b=0.5, \gamma_{2}=1.5$ | Alternative technology trawler fleet; $b=0.5, \gamma_{2}=1.15$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $X^{*}$ | 1,680 | 1,326 | 2,336 | 1,404 | 1,871 |
| $T A C^{*}$ | 390 | 341 | 431 | 252 | 486 |
| $T^{*}$ | 13 | 7 | 19 | 12 | 12 |
| $\pi^{*}$ | 2,670 | 2,219 | 3,092 | 1,663 | 3,327 |
| $P V_{1}$ | 24,080 | 21,846 | 24,750 | 15,535 | 27,031 |
| $P V_{2}$ | 18,130 | 15,684 | 20,049 | 11,268 | 25,846 |
| PV | 42,210 | 37,530 | 44,799 | 26,803 | 52,877 |



FIGURE 1
Time path (number of years) total profit and profit shares under the Proportional Rule (PROP) and the Constrained Equal Award Rule (CEA).

