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Socioeconomic Institute
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Working Paper No. 0811

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Publisher Sozialökonomisches Institut
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A welfare analysis of “junk” information and spam filters *

Josef Falkinger[†]

9 October 2008

Abstract

This paper analyses the equilibrium effects of individual information filters. Information is modelled as advertisements which are distributed across a population of consumers with heterogeneous preferences. An advertisement that provides knowledge about a product with little or no utility for a consumer is considered junk. Filters are characterised by their level of tolerance. The quality of the filter is measured in terms of the share of useful items in the total set of items passing the filter. It is shown that in conditions of decentralised competition, multiple equilibria arise. A social optimum can be achieved by demanding each consumer to reject a certain percentage of advertisements, leaving the choice of what is rejected up to the consumer him/herself.

Keywords: global information society, advertising, junk information, spam filter, Internet regulation.
JEL classification: D83, L86, M38, D18.

*I wish to thank Timo Boppart and Armin Schmutzler for very helpful comments on earlier versions of this paper. I am also grateful to seminar participants at the Economic University of Vienna for their feedback.

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1 Introduction

The Internet permits distribution of information to a wide range of users. This information will be always more or less valuable to the user who receives it, however. For example, a consumer opens his or her mail box. Besides personal letters, the box will also contain advertisements. Some of the advertisements will describe useful products, whereas others will be less informative or indeed constitute pure junk for our particular consumer. In a similar way, more or less useful information is brought to the attention of consumers as they surf the World Wide Web. The quality of the service provided by the Internet can be improved through the use of a filter that focuses on valuable pieces of information and eliminates what is unwanted. This paper shows that in an information-rich economy, individual filter use generates an externality that affects other users. The reason is that the use of filters leads to a change in the equilibrium set of information distributed via the Internet.

In order to analyse the equilibrium effects of filters, I use a model of monopolistic competition under conditions of limited attention that has already been described in Falkinger [2008]. In this model, first an infinite mass of firms competes for attention by advertising different variants of a product to consumers with identical preferences. Then, those firms that succeed in attracting the consumers' attention sell their products in conditions of monopolistic price competition. In this paper, instead of assuming that all products enter the consumer's utility function in a symmetric way, I allow for heterogeneity in products and tastes. A consumer values information because his or her choices are restricted by the set of products (s)he is aware of. As Ozga [1960] and Stigler [1961] have pointed out, besides the consumers' own search, advertising is the modern method of providing potential buyers with knowledge of consumption opportunities.[1] Of course, informative material providing knowledge about a highly valued product is more useful to the consumer than an advertisement for a less valuable product. "Junk" is a subjective notion. It is modelled as advertisements for less valuable (or useless) products. Since different consumers may value the same product differently, it is possible that a particular advertisement will be useless for one consumer and at the same time very useful for another. And even when consumers have identical preferences, they will still be exposed to some amount of junk – simply because firms are ignorant of the preferences of consumers in general. This paper shows how a consumer's utility in equilibrium depends on the heterogeneity of consumer tastes and the mix of information supplied.[2]

Traditionally, economists assume that consumers are imperfectly informed because information distribution is costly and advertisements reach only a fraction of po-

[1]Obviously, there are other views on this point. For instance, Kaldor [1950-51], emphasises the persuasive character of advertising (see Bagwell [2007] for a survey of the different views in the economic literature on advertising). From this point of view, the answer as to why a consumer obtains little information from advertisements is more or less trivial. Therefore, this paper is based on the premise that advertising is informative in the sense that knowledge about the identity of a product is provided. It may still be uninformative in the sense that the advertised product is useless in the eyes of consumers.

[2]Here, the analysis is carried out on a model of information about consumption goods. However, it is worth noting that the structure analysed here can apply to any information about items that are more or less valuable for a user: for instance, intermediate inputs for manufacturers or sources of scientific knowledge for researchers, as opposed to consumption goods for households.

tential consumers (see, e.g., the models by Ozga [1960] and Butters [1977]). This suggests that advances in information technologies that reduce the cost of information and allow distribution of advertisements across a wider range of consumers should solve the problem. But the effect seems to be quite the opposite. The annoyance of being exposed to junk is closely related to the richness of information distributed to consumers. As Simon [1971] has pointed out, in an information-rich society, the resource that is scarce is attention. The reason is that a consumer's capacity to process information is limited. As long as the consumer has free capacity to evaluate any piece of information supplied, less useful pieces of information or junk will not distract his or her attention from more useful information. In an information-rich world, by contrast, the different pieces of information all compete for the consumer's attention, and less useful pieces may crowd out the more useful ones. In this case, the fit of advertised products with consumer preferences – the quality of information – becomes important. By employing a filter, the consumer focuses attention on a subset of supplied information. This paper shows that the quality of this subset depends on the filter used by the individual, but also on the quality of the pool from which the filtering takes place, that is, on the quality of the aggregate supply of information. This quality, in turn, cannot be influenced by the individual consumer's own filter decisions, rather depends on the average filter tightness adopted in the population. If many consumers protect themselves against junk, the incentives to distribute junk decrease. Thus, the quality of the information provided increases - to the benefit of all the users of the Internet. This collective aspect implies that, in general, a decentralised equilibrium is inefficient. The paper shows that the social-planner solution can be implemented by means of a simple regulation of filter tolerance.

The paper is organised as follows. The next section presents the model. Section 3 analyses consumer and producer behaviour. Section 4 presents the equilibrium analysis and discusses welfare. Section 5 deals with optimal policy. Section 6 summarises the results. Proofs are provided in the Appendix.

2 Model

The economy consists of two types of agents: monopolistic firms and price-taking consumers. The firms decide on market entry and product price. The consumers set a filter parameter and decide how much to buy of a perceived product variant. Individual agents have zero mass and take aggregate values as given.

Let $\mathbf{\Omega}$ be an exogenous set of potential variants of goods. Out of this set, firms create product innovations and advertise them to consumers. Let $\mathbf{T}$ of measure T denote the set of firms. Each firm $t \in \mathbf{T}$ draws one variant $\omega(t) \in \mathbf{\Omega}$, where $\omega(t) \neq \omega(t')$ for $t \neq t'$. Denote $\mathbf{\Omega_T} \equiv \{\omega(t)\,|t \in \mathbf{T}\}$. The measure of $\mathbf{\Omega_T}$, equal to T, describes the aggregate diversity of supply (of information and goods). The equilibrium value of T is determined by free entry. For entering, a firm has to incur a fixed cost f, which is exogenous to the individual firm but endogenously determined in the competition for attention.

Let $\mathbf{N} = [0, 1]$ be the set of consumers. Each consumer is endowed with budget y and information-processing capacity τ_0. This means that a consumer is able to perceive information about a mass of τ_0 products. For instance, τ_0 units of time are available; studying material about the characteristics of a set of products of mass 1 requires $1/\tau_0$ units of time.[3] Budget y and information-processing capacity τ_0 are exogenous.

Consumer $i \in \mathbf{N}$ is exposed to information about set $\mathbf{S}_i \subset \mathbf{\Omega_T}$ of items.[4] This information supply may be more than the consumer is actually able to perceive. Let $\mathbf{M}_i \subset \mathbf{S}_i$ be the set of products (s)he actually perceives. Denote by S_i and M_i the measures of $\mathbf{S}_i$ and $\mathbf{M}_i$, respectively. Whereas S_i describes the diversity of supply, M_i is the perceived diversity. S_i and M_i are endogenously determined. They depend on the way in which producers distribute information and consumers filter this information.

2.1 Consumer preferences

Consumers have a preference for variety. So, in principle, they like to have access to advertising about a large set of items.[5] Nonetheless, they may like some items more than others. I capture these preferences by means of the following CES utility

[3] For a more general discussion as to why human behaviour is subject to a limited capacity for perception, see the literature on attention psychology, for instance, Kahneman [1973] or the survey by Pashler [1998]. For a theoretical foundation of competitive equilibrium analysis under scarcity of attention, see Falkinger [2007].

[4] $\subset$ denotes weak inclusion.

[5] Attitudes of the kind "leave me alone" are not considered. The problem of preferring a quiet life has a trivial solution as far as information exposure is concerned: Simply disconnect your computer and communication device. The issue of being exposed to a flood of more or less useful information only arises because we actually want an abundant variety of information.

function for consumer i:

$$U^i = \left[\int\limits_{\mathbf{M}_i} \delta_i(\omega) x_i(\omega)^\rho d\omega \right]^{1/\rho}, \quad 0 < \rho < 1, \tag{1}$$

where $x_i(\omega)$ denotes the quantity consumed of variant ω, and $\delta_i(\omega) \in [0,1]$ is a weight reflecting i's valuation of the different product variants.[6] To model junk, I assume that for each consumer i there is a set $\mathbf{I_i} \subset \mathbf{\Omega}$ of potential goods that are "ideal" for i, whereas $\omega \notin \mathbf{I_i}$ is less valuable or useless. Formally,

$$\delta_i(\omega) = \begin{cases} 1 & \text{if } \omega \in \mathbf{I_i}, \\ \delta & \text{otherwise}, \end{cases} \tag{2}$$

with $\delta \in [0,1]$. The assumption that the weight function $\delta_i(\omega)$ assumes only two values substantially simplifies the analysis. Advertisements that provide information about elements of $\mathbf{I_i}$ are welcome to the consumer. Other advertisements are considered - in a loose sense - to be junk. (In a strict sense, $\omega \notin \mathbf{I_i}$ is junk information if $\delta = 0$.) Moreover, $\delta \geq 0$ excludes harassment. Junk as modelled here has no direct negative effect on utility. It is only harmful to the extent that more useful information might be crowded out. Different consumers can have different ideal sets $\mathbf{I_i}$. However, the sets may overlap; that is, several consumers may share their valuation of advertised items. For tractability reasons, the following symmetry assumption is imposed on consumer heterogeneity.

Assumption 1. *Let* $\mathbf{I} \equiv \bigcup\limits_{i \in \mathbf{N}} \mathbf{I_i}$. *There exists* $h \in (0,1]$ *so that for all* $i \in \mathbf{N}$: $prob\{\omega \in \mathbf{I}_i | \omega \in \mathbf{I}\} = h$.

[6]This weights allow for asymmetries between goods in the Dixit and Stiglitz [1977] model of monopolistic competition while keeping the elasticity of substitution constant.

The assumption restricts the heterogeneity of tastes. The conditional probability that an item is ideal for a particular consumer, assuming that the item is ideal for anybody at all, is the same for all consumers. That is, each $\mathbf{I_i}$ has an equal share in $\mathbf{I}$. Denote by I_i and I the measures of $\mathbf{I_i}$ and $\mathbf{I}$, respectively. According to Assumption 1,

$$I_i = hI. \tag{3}$$

Thus, parameter h describes the *homogeneity* ($1/h$ the *heterogeneity*) of tastes in the population.

2.2 Innovation and distribution of information

A firm $t \in \mathbf{T}$ that wants to enter the market makes a random draw $\omega(t) \in \mathbf{\Omega}$. The drawn variant may match consumer tastes to a greater or lesser extent. The following assumption is imposed on the innovation process.

Assumption 2. *There exists* $\chi \in (0,1]$ *so that for all* $t \in \mathbf{T}$: *prob* $\{\omega(t) \in \mathbf{I}\} = \chi$.

The assumption states that the probability of an innovation being ideal for some consumers is given by an exogenous success rate (χ). Together with Assumption 1, this implies that the probability that an innovation is ideal for consumer i is $h\chi$. Whether or not a particular variant is liked by some consumers is revealed only after it has been advertised. This requires fixed costs $f \geq f_0$, where f_0 is exogenous. The variable unit costs of production are given by a constant c. The fixed costs depend on the intensity of competition for attention. If there is no such competition – because consumers have free capacity to process any piece of information they are exposed to (that is, if $S_i < \tau_0$) – then the cost of advertising a variant equals f_0. However, if information exposure exceeds the consumers' information-processing capacity, then the required intensity of advertising increases. For instance, if consumers skip every second piece of information, a firm has to approach each of them at least twice in order to obtain their attention. The determination of f through competition for attention will be described more precisely in Section 3.2, which analyses the profitability of entry.

The set of consumers to whom information about a variety is distributed depends on the available IT possibilities and media. Moreover, this set may be influenced by attempts to target advertisements so as to match consumer tastes. For $\omega \in \mathbf{\Omega_T}$, let $\varphi(\omega, i)$ denote the probability that a firm's information about ω will reach consumer i. Then the number of items advertised to a consumer will be

$$S_i = \int_{\mathbf{\Omega_T}} \varphi(\omega, i) d\omega. \tag{4}$$

This describes the diversity of the information exposure of i. In order to separate the technical possibilities for distributing information from information-targeting measures or general junk control, I make the following simplifying assumption about information distribution.

Assumption 3. *There exists $\varphi_0 \in (0, 1], \varphi_1 \in [0, 1]$ and $\varphi_2 \in [0, 1]$ so that for all i:*

$$\varphi(\omega, i) = \begin{cases} \varphi_0 \ \ \textit{if} \ \ \omega \in \mathbf{I_i}, \\ \varphi_1\varphi_0 \ \textit{if} \ \ \omega \in \mathbf{I} - \mathbf{I_i}, \\ \varphi_2\varphi_0 \ \textit{otherwise}. \end{cases}$$

This assumption allows for differentiated distribution of information. For instance, $\varphi_2 < 1$ means that items that nobody considers as ideal ("general junk") are distributed less widely than ideals. For instance, the use of collective blacklists by providers of Internet services reduces the distribution of general junk. If, in addition, $\varphi_1 < 1$, then a consumer is predominantly exposed to targeted information about his/her ideals. Parameter φ_0 characterises the technical possibilities for distributing information. $\varphi_0 = 1$ means that the information techniques available to a firm allow the firm's advertisement to be distributed to all consumers. The following lemma characterises the diversity and quality of information exposure of consumers resulting under Assumptions 2 and 3. Moreover, for each variant ω, the lemma determines the expected range $R(\omega)$, defined as the mass of consumers a firm can expect to reach by advertising ω.

Lemma 1. *(a) For all* ω, i*:* $R(\omega) = \varphi_0\tilde{\varphi}(h,\chi)$ *and* $S_i = \varphi_0\tilde{\varphi}(h,\chi)T$*, where* $\tilde{\varphi}(h,\chi) \equiv h\chi + \varphi_1(1-h)\chi + \varphi_2(1-\chi)$*. (b) Let* q_i^S *denote the share of items in* $\mathbf{S_i}$ *that belongs to* $\mathbf{I_i}$*. For all i,* $q_i^S = \frac{h\chi}{\tilde{\varphi}(h,\chi)}$*.*

The lemma shows how homogeneity of tastes (h), success rate of innovations (χ), technical possibilities for information distribution (φ_0), and central filters and targeting measures (φ_1, φ_2) determine the expected firm range (R), the diversity of advertisements to which a consumer is exposed (S) and the quality of information received by a consumer, defined as the share of advertisements about useful products in total advertisement exposure (q^S).[7] The following examples illustrate the role of

[7]Since $R(\omega)$ is the same for all ω and S_i, q_i^S are identical for all i, the arguments of R, S, q^S are dropped in the further analysis.

the various determinants.

Example 1. *(Random distribution of information). For* $\varphi_1 = \varphi_2 = 1$*, we have* $\tilde{\varphi}(h, \chi) = 1$*, so that* $R = \varphi_0$*,* $S = \varphi_0 T$ *and* $q^S = h\chi$*.*

This example includes the case of global information distribution with $\varphi_0 = 1$. In this case, the firm range equals population size, and the size of the information exposure of consumers is equal to the number of firms. The quality of the received information is a product of the quality of innovations and the homogeneity of tastes. This explains why we are exposed to a mix of valuable information and junk when we use the Internet. The reason is that under global information technology, any piece of information reaches all of us, and what is useful for me may be useless for you. Moreover, producers are not sure what pleases consumers. The next example assumes that firms are able to distribute only useful information or that providers can suppress general junk.

Example 2. *(Suppression of general junk). For* $\varphi_1 = 1$*,* $\varphi_2 = 0$*, we have* $\tilde{\varphi}(h, \chi) = \chi$*, so that* $R = \varphi_0 \chi$*,* $S = \varphi_0 \chi T$ *and* $q^S = h$*.*

If only those items are advertised that are ideal at least for some of the consumers, then the size of information exposure decreases and the quality of received information increases. However, this quality still remains limited by h, the homogeneity of tastes. Only perfect targeting would eliminate the role of h. If both φ_1 and φ_2 were zero, then we would get $q^S = 1$. so that the problem of junk information would vanish. The following assumption excludes this possibility.

Assumption 4. $q^S < 1$.

This assumption also excludes the case of complete homogeneity of tastes ($h = 1$) combined with a flawless process of innovation ($\chi = 1$). According to Lemma 1, this would imply $q^S = 1$ and render filters meaningless. Assumption 4 reflects the fact that $\mathbf{I_i}$ is an individual preference characteristic and thus private knowledge belonging to the consumer. Neither firms nor a central information agency can perfectly match i's ideal varieties.

2.3 Spam filter and perception of information

To repeat, a consumer is exposed to information about the set $\mathbf{S_i}$ of products advertised to him/her. (S)he focuses attention on $\mathbf{M_i} \subset \mathbf{S_i}$. The focus of attention is determined by the consumer's information-processing capacity and the filter used. The capacity constraint limits the size of the perceived set of items to

$$M_i \leq \tau_0. \tag{5}$$

If information exposure S is less than or equal to τ_0, each piece of information advertised to the consumer will be perceived by the consumer. By contrast, if $S > \tau_0$, then there is scarcity of attention and some fraction of advertisements will have to be skipped. By using a personalised filter – with individual whitelists and blacklists – a consumer can influence the quality of the information requesting his/her attention. However, perfect filtering is unavailable. Typically, there are two types of errors: First, a valuable item may be rejected; second, junk may pass the filter. This can be modelled as follows. Based on the profile provided by the consumer, a filter assigns to each $\omega \in \mathbf{S_i}$ a probability $a_i(\omega)$ that $\omega \notin \mathbf{I_i}$ ("junk probability"). $1 - a_i(\omega)$ is the probability that $\omega \in \mathbf{I_i}$. The consumer can choose the tightness of the filter by fixing the maximal junk probability (s)he tolerates. Under tolerance level A, the set of items passing the filter is then given by $\mathbf{F}^i_A \equiv \{\omega \in \mathbf{S_i} | a_i(\omega) \leq A\}$. Let

$$\psi_1(A) \equiv \text{prob } \{\omega \in \mathbf{F}^i_A | \omega \in \mathbf{I_i}\}$$
$$\psi_2(A) \equiv \text{prob } \{\omega \in \mathbf{F}^i_A | \omega \notin \mathbf{I_i}\}$$

be the probability that the filter does not reject ideals and the probability that junk passes the filter, respectively. Then,

$$z(A) \equiv \frac{\psi_1(A)}{\psi_2(A)}$$

is a measure for the precision of the filter.

A consumer may be more or less careful in personalising the filter. This clearly affects filter precision z. However, given the personal profile, it is tolerance A that determines precision. For instance, for $A = 0$, only items from the "whitelist", which are definitely considered valuable by the consumer, will pass the filter. If A increases, then any item has a higher chance of passing the filter, and in particular junk information will also be more likely to pass. This is captured by the following assumption.

Assumption 5. *(a) For $A < 1$, $\frac{d\psi_j}{dA} > 0$, $j = 1, 2$. (b) $z(A) = 1$ (random filter) or: $z(A) > 1$ and $\frac{dz}{dA} < 0$ for $A < 1$.*

Obviously $z(A) \geq 1$ for any reasonable filter. This excludes the likelihood of a filter predominantly picking junk. The last part of the assumption reflects the trade-off between tolerance and precision. A higher filter tolerance reduces the risk that valuable information will be rejected by increasing ψ_1. However, it increases the probability that junk will pass through even more, so that filter precision declines.

The following lemma describes the relationship between information exposure and perception if the consumer employs a filter of tolerance level A.

Lemma 2. *Let i be exposed to $\mathbf{S_i}$ of size S and quality q^S, as given by Lemma 1. Let $\mathbf{M_i}$ be selected by a filter of tolerance level A_i and denote by q_i the share of ideals in $\mathbf{M_i}$. Then we have: (a) $\frac{M_i}{S} = \psi_1(A_i)q^S + \psi_2(A_i)(1-q^S) \equiv \psi(A_i)$ and $q_i = \frac{\psi_1(A_i)}{\psi(A_i)}q^S$. (b) For $A < 1$, $d\psi/dA > 0$ and $dq_i/dA < 0$ if $z(A) > 1$.*

The lemma shows that loosening the filter increases perceived diversity M_i at the cost of the quality of perceived information q_i. Finally, capacity constraint (5) requires

$$\psi(A_i)S \leq \tau_0, \tag{6}$$

which places a limit on filter tolerance.

3 Consumer and producer behaviour

Individual agents take aggregate values as given. A consumer decides about two things: the filter setting and, for each perceived product, the quantity purchased. A firm decides about market entry and about the price to charge for its product.

3.1 Consumers

Given $\mathbf{M_i}$ and goods prices $p(\omega)$, the consumer chooses for $\omega \in \mathbf{M_i}$ quantities $x_i(\omega)$ so as to maximise (1) subject to the budget constraint

$$\int_{\mathbf{M}_i} p(\omega)x_i(\omega)d\omega \leq y. \tag{7}$$

This gives for i's demand functions

$$x_i(\omega) = \delta_i(\omega)^{\varepsilon} \frac{y}{P_i} p(\omega)^{-\varepsilon}, \tag{8}$$

with $\varepsilon \equiv \frac{1}{1-\rho} > 1$ and $P_i \equiv \int\limits_{\mathbf{M}_i} \delta_i(\omega)^{\varepsilon} p(\omega)^{1-\varepsilon} d\omega$. (The derivation of (8) is provided in the Appendix.)

The firms, facing iso-elastic demand, set $p(\omega) = \frac{1}{1-1/\varepsilon} = c/\rho \equiv p$ (see Section 3.2 for the market demand implied by (8)). Thus, (8) reduces to

$$x_i(\omega) = \frac{y}{pM_i} \frac{\delta_i(\omega)^{\varepsilon}}{D_i}, \tag{9}$$

where $D_i \equiv \frac{1}{M_i} \int\limits_{\mathbf{M}_i} \delta_i(\omega)^{\varepsilon} d\omega$ is the average quality of the items perceived and consumed by i. In view of (2), we have

$$D_i = q_i + \delta^{\varepsilon}(1 - q_i), \tag{10}$$

where q_i is the share of ideals in $\mathbf{M}_i$ given by Lemma 2.

Substituting (9) into (1), we obtain[8] for the utility of consumer i:

$$V^i = \frac{y}{p}(M_i D_i)^{\frac{1-\rho}{\rho}}. \tag{11}$$

According to Lemma 2, both the diversity M_i and the quality q_i of perceived items depend on the filter tolerance chosen by the consumer. A looser filter increases diversity at the cost of lower quality. The following proposition characterises the individually optimal filter tolerance.

Proposition 1. *Given information exposure S of quality q^S, every consumer chooses $A_i = A(S)$, where $A(S)$ is defined by the condition $\psi(A(S)) = min\{\tau_0/S, 1\}$.*

[8]Note that $1 + \rho\varepsilon = \varepsilon$.

The proposition shows that for the individual it is optimal to choose filter tolerance so as to perceive as many as possible of the advertisements to which he or she is exposed. For $A_i = A(S)$, $M_i = S$ if $S < \tau_0$ and $M_i = \tau_0$ otherwise. A tighter filter would increase the quality of information passing the filter – at the risk of losing information, however. According to Lemma 2, for $M_i = \tau_0$, $A(S)$ decreases with S. Growing information exposure allows the consumer to be more choosy. As a consequence, the quality of perceived information q_i rises. (Note that q_i is inversely related to A, according to Lemma 2.) In sum, consumers profit from a richer supply of information. However, the supply of information by firms depends on aggregate consumer behaviour and cannot be influenced by the individual consumer.

3.2 Firms

According to Assumption 3, a firm advertising ω reaches consumers for which ω is ideal with probability φ_0. The probability that the advertisement passes the spam filter of a consumer i is $\psi_1(A)$. Moreover, according to Assumptions 1 and 2, the probability that ω is ideal for the consumer is $h\chi$. In this case $\delta_i = 1$ in i's demand function (8). With probability $\varphi_0\varphi_1(1-h)\chi + \varphi_0\varphi_2(1-\chi)$, the advertisement is sent to a consumer for whom it is spam, and the probability that it passes the spam filter is $\psi_2(A)$. In this case, $\delta_i = \delta$ in (8). In sum, the expected market demand for ω is given by

$$X(\omega) = p(\omega)^{-\varepsilon} y \varphi_0 \int\limits_{\mathbf{N}} \left[\frac{\psi_1(A) h\chi}{P_i} + \frac{\delta^{\varepsilon}\psi_2(A)}{P_i}(\varphi_1\chi(1-h) + \varphi_2(1-\chi)) \right] di, \qquad (12)$$

which is iso-elastic. Thus, for all ω, the monopoly price is given by $p = c/\rho$. Substituting this in (12) and calculating expected operating profit, we get the following result:

Proposition 2. *Suppose firm t expects that consumers spend their budget on M products and use a filter of strength A, then expected demand for product $\omega(t)$ is $X = \frac{yr}{pM}$ and expected operating profit is $\pi = \frac{(1-\rho)y}{M}r$, with $r \equiv \psi(A)R$, where $\psi(A)$ (with $d\psi/dA > 0$) and R are given by Lemma 2 and Lemma 1, respectively.*

A firm advertising good ω to consumer i knows that the advertisement passes i's filter with probability $\psi_1(A)$ if ω matches i's ideals and with probability $\psi_2(A)$ if it does not. Consumers whose filter rejects the advertisement ignore ω. The rejection rate increases if filter tolerance A is reduced. Therefore, the relevant firm range r and expected profits decline if tighter filters are used.

The expected operating profit π defines the willingness to pay for market entry. The possibilities of entry depend on whether consumer attention is scarce or not. The following assumption defines the required entry costs.

Assumption 6. *In an economy with a set of firms $\mathbf{T}$ and information exposure S: (i) If $S < \tau_0$, a firm $t' \notin \mathbf{T}$ can enter by spending f_0. (ii) If $S \geq \tau_0$, then $t' \notin \mathbf{T}$ can enter by spending more on advertising than $t \in \mathbf{T}$.*

Part (i) of the assumption deals with the case where there is no scarcity of consumer attention. In this case, some exogenous fixed cost – given by the feasible innovation and distribution technology – is necessary and sufficient to reach consumers. This reflects the traditional view of informative advertising. In contrast, if consumer attention is scarce, then a firm must edge other firms out of the consumer's mind by multiplying advertisements. This is captured by Part (ii). Under free entry, π

must equal the cost of entry. This leads to equalised entry cost f under scarcity of attention, too. Assumption 6 implies that $f > f_0$ only if $S \geq \tau_0$.

4 Equilibrium and welfare

An economy is in equilibrium if consumers and firms behave as described in the previous section, if the firms' expectations are correct, if the zero-profit condition holds and if the aggregate resource constraints are satisfied.

According to Proposition 1, consumer behaviour is in equilibrium if filter tolerance A and information exposure S satisfy the relationship

$$\psi(A) = \min\{\tau_0/S, 1\}. \tag{13}$$

Moreover, according to Lemma 2, diversity of perception M is related to information exposure by

$$M = \psi(A)S. \tag{14}$$

According to Proposition 2, under correct expectations each firm expects profit $\pi = \frac{(1-\rho)y}{M}\psi(A)R$. Thus, the zero-profit condition reads

$$\frac{(1-\rho)y}{M}\psi(A)R = f. \tag{15}$$

Finally, the aggregate income constraint requires that total consumption expenditure pXT equals total income y.[9] Using $X = \frac{y}{pM}\psi(A)R$ from Proposition 2, this condition reduces to

$$\psi(A)RT = M. \tag{16}$$

[9]The other aggregate constraint is that the diversity of distributed information RT must equal the diversity of information exposure S. However, this already follows from (14) and (16).

In addition to these equilibrium conditions, we must also keep in mind the feasibility constraints

$$M \leq \tau_0, f \geq f_0. \tag{17}$$

The system of equations (13) to (17) characterises equilibrium values for the variables A (or $\psi(A)$), S, M, f and T. It is worth noting that if these five variables are determined, then all other consumer variables of the model – $q_i, D_i, x_i(\omega)$ – are determined as well, namely by Lemma 2, Equation (10) and Equation (9), respectively. When considering equilibria we must distinguish between a situation in which firms are competing for scarce consumer attention and the conventional case of informative advertising (with no crowding). This is captured by the following definition, suggested by Falkinger [2008]:

Definition 1. *An equilibrium is information poor if $S \leq \tau_0$ and $f = f_0$. Otherwise the equilibrium is information rich.*

Lemma 3. *In an information-rich equilibrium, $S \geq \tau_0$.*

We will see that an economy has either an information-poor or an information-rich equilibrium. Therefore, it makes sense to call an economy information poor or information rich.

4.1 Information-poor economies

The following proposition characterises economies in which decentralised competition leads to an information-poor equilibrium. Since firm profits are zero in equilibrium, only consumer utility matters for welfare.

Proposition 3. *If* $(1-\rho)yR/f_0 \leq \tau_0$*, then: (i) The economy has a unique equilibrium. The equilibrium is information poor. (ii) In the equilibrium* $\psi(A) = 1$*,* $M = S = (1-\rho)yR/f_0$*. Consumer utility is given by* $V^P = U_0(RD_0)^{\frac{1-\rho}{\rho}}$*, where* $U_0 \equiv y^{1/\rho}(\frac{1-\rho}{f_0})^{\frac{1-\rho}{\rho}}/p$ *and* $D_0 \equiv \frac{h\chi}{\tilde{\varphi}(h,\chi)}(1-\delta^\varepsilon) + \delta^\varepsilon$*.*

Recalling $R = \varphi_0\tilde{\varphi}(h,\chi)$ from Lemma 1, we learn from the first part of the proposition that a small feasible range (φ_0) of information distribution or high costs f_0 are one reason why there is no problem of scarce attention in an economy. Another reason might be information targeting or homogenous consumer tastes – both reduce $\tilde{\varphi}$, which means a consumer is exposed to less information of little or no use. Welfare increases with the range of information distribution. This confirms the conventional view of informative advertising. In an information-poor economy, zero-profit condition (15) reduces to $M = (1-\rho)yR/f_0$. Perceived diversity is equal to the diversity of supply ($M = S$) and limited by the innovation and information possibilities of firms. If firms can inform a wider range of consumers, people can choose from a more diverse set of goods. However, whereas an increase in the technically feasible range (φ_0) is unambiguously positive, an increase in $\tilde{\varphi}$ has a negative side effect. According to Part (b) of Lemma 1, a rise in $\tilde{\varphi}$ means that the distributed information matches consumer tastes less precisely. Thus, the increased diversity comes with a lower average quality (D_0), which reduces utility. For the net effect, we calculate, by use of Lemma 1, the expression

$$RD_0 = \varphi_0 \left\{ h\chi + \delta^\varepsilon \left[\varphi_1(1-h)\chi + \varphi_2(1-\chi) \right] \right\}. \tag{18}$$

Obviously, a higher success rate of innovations (χ) is beneficial. So is homogeneity of tastes (h). Both make a good match between innovations and preferences more likely. They are – together with the technical range (φ_0) – the basic determinants of utility if non-ideals are pure junk. For $\delta = 0$, we have $RD_0 = \varphi_0 h\chi$. If $\delta > 0$, then also general junk suppression ($\varphi_2 < 1$) or information targeting $\varphi_1 < 1$ matter. As expression (18) shows, such measures are not desirable in an information-poor economy. It is true that they increase the quality of received information, but they do this at the cost of diversity. And as long as junk may convey some information, the latter effect dominates. The reason is that in an information-poor economy, junk has no opportunity costs in terms of crowding out more useful information. This changes, of course, if the economy is information rich. And, as we will see, less careful distribution of information – raising $\tilde{\varphi}$ and thus R – could be one of the reasons why an economy becomes information rich. Hence, the considerations about the role of careless distribution of information in an information-poor economy come with a caveat. Only if the economy is information poor for $\varphi_1 = \varphi_2 = 1$, that is, if $(1-\rho)y\varphi_0/f_0 \leq \tau_0$, do they apply without any reservations.

4.2 Information-rich economies

According to Lemma 3, $S \geq \tau_0$ if an equilibrium is information rich. Thus, the equilibrium condition (13) takes the form

$$\psi(A) = \tau_0/S, \tag{19}$$

and (14) gives $M = \tau_0$. Moreover, with $M = \tau_0$, condition (15) reads

$$\frac{(1-\rho)y}{\tau_0}\psi(A)R = f. \tag{20}$$

The next proposition characterises economies in which competition leads to information-rich equilibria.

Proposition 4. *If* $(1-\rho)yR/f_0 > \tau_0$*, then: (i) All equilibria of the economy are information rich. (ii) Any tuple* A, S, f *satisfying* (19)*,* (20) *and* $f \geq f_0$ *is an equilibrium. (iii) Consumer utility depends on the filter tightness adopted in equilibrium. It is given by* $V^R = \frac{y}{p}(\tau_0 D)^{\frac{1-\rho}{\rho}}$*, where* $D = \frac{\psi_1(A)}{\psi(A)} \frac{h\chi}{\tilde{\varphi}(h,\chi)}(1-\delta^\varepsilon) + \delta^\varepsilon$.

The proposition shows that an information-rich economy differs with respect to a series of important elements from the information-poor one. First, in contrast to an information-poor economy, IT progress (an increase in φ_0 or a decrease in f_0) plays no direct role for welfare in an information-rich equilibrium, as shown by Part (iii) of the proposition. The reason can be seen by looking at Equilibrium Condition (20). For given filter tolerance A, any increase in the range of information distribution raises advertising expenditure because firms then have a stronger incentive to compete for consumer attention. However, Condition (20) also implies that if R is high, then the constraint $f \geq f_0$ holds for lower values of $\psi(A)$. That means that an increase in R allows equilibria with tighter filters. I will come back to this point when discussing policy in Section 5. Second, for given filter tolerance, more careful distribution of information (reducing $\tilde{\varphi}$) is unambiguously beneficial for consumers in an information- rich economy. Under random distribution of information (Example 1 in Section 2), and in particular under global distribution of advertisements to all consumers, we have $\tilde{\varphi}(h,\chi) = 1$ and thus $D = \frac{\psi_1(A)}{\psi(A)} h\chi(1-\delta^\varepsilon) + \delta^\varepsilon \equiv D_1$. Comparing this with the case of general junk suppression (Example 2) – for instance, if providers eliminate advertisements considered generally as junk, we have

$\tilde{\varphi}(h,\chi) = \chi < 1$ and $D = \frac{\psi_1(A)}{\psi(A)} h(1-\delta^{\varepsilon}) + \delta^{\varepsilon} > D_1$.

Third, the most important implication of Proposition 4 is that an information-rich economy has multiple equilibria. Higher filter tolerance is harmful in an information-rich economy because it reduces the quality of information exposure.[10] The problem is that the individual consumer has no influence on the filter tolerance realised in equilibrium. According to Proposition 1, the consumer adjusts his/her filter tolerance so as to bring the information exposure in line with the information-processing capacity. Deviating from this filter choice to a tighter one would mean unused capacity and a loss of potential information. By choosing A_i so that $\psi(A_i) = \tau_0/S$, the consumer exploits the received information optimally. But there is no way to influence the information supplied. For this to happen, the expectations of firms would have to be changed, and these obviously do not depend on the behaviour of a single consumer. The following illustration summarises the basic mechanisms involved. Suppose that firms expect lax filtering. Then, according to Proposition 2, the expected range of consumers they can effectively reach increases so that market entry becomes more attractive. This leads to a rise in advertising expenditure so as to bring the cost of entry in line with the willingness to pay for entry according to (20). At the same time, firm size increases, so as to cover the higher entry cost, and the number of firms declines.[11] Thus, every consumer is exposed to less diverse information ($S = RT$, according to Lemma 1). The best choice remaining for the

[10]Note that $\frac{\psi_1(A)}{\psi(A)} = \frac{1}{q^S + (1-q^S)\psi_2(A)/\psi_1(A)}$ and that $z(A) = \psi_1(A)/\psi_2(A)$ decreases with A (Assumption 5). Thus, $\psi_1(A)/\psi(A)$ and D decline with A. (Recall that $q^S < 1$ under Assumption 4.)

[11]According to (16), for $M = \tau_0$, $T = \frac{\tau_0}{\psi(A)R}$. $d\psi/dA > 0$ implies $dT/dA < 0$.

individual consumer is also to loosen filtering in order to get the maximum out of the reduced information supply. This illustrates a possible vicious circle. Obviously, the circle also works the other way round. In any case, it shows that in an information-rich economy, a decentralised equilibrium will not be efficient except by coincidence.

5 Policy

The equilibrium analysis has shown that in an information-rich economy there are multiple equilibria. Moreover, the various equilibria lead to different welfare levels. This points to an important role for policy intervention. According to Proposition 4, $V^R = \frac{y}{p}(\tau_0 D)^{\frac{1-\rho}{\rho}}$, where D is a decreasing function of equilibrium filter tolerance A. As shown by Equilibrium Condition (20), high filter tolerance goes hand in hand with high advertisement expenditure by firms. On the other hand, a high cost of advertising requires large firms. Thus, the diversity of goods advertised to consumers is comparatively smaller than under conditions of lower advertising costs. In other words, loose filtering goes hand in hand with wasteful advertising, which reduces the diversity of supply. This has a direct and an indirect effect on the average quality of goods both perceived and consumed. The direct effect comes from the fact that the set of ideals advertised to a consumer shrinks. The size of this set is given by $I_i = q^S S$, where $q^S = h\chi/\tilde{\varphi}(h, \chi)$ depends on the heterogeneity of tastes (among other things). If S declines, I_i is also reduced. However, there is a further, indirect effect on the average quality of perceived goods. Under scarcity of attention, a consumer does not see the full set of advertisements, rather only the subset passing his/her filter. Now, if the diversity of supply declines due to wasteful advertising expenditure, the consumer will relax his/her filter tightness. This means that more of everything, and in particular also more junk, will pass through.

The fact that lax filtering by the mass of consumers triggers a vicious circle of wasteful advertising by firms and adoption of a lax filter by each individual consumer suggests that regulating filter tolerance might be a remedy. The following proposition describes the optimal regulation.

Proposition 5. *Suppose that the economy is information rich. Let A_{min} be defined by the equation $\psi(A_{min}) = \frac{\tau_0 f_0}{(1-\rho)yR}$. An equilibrium with maximal consumer welfare results if $A_i \leq A_{min}$ is imposed on consumers. In the resulting equilibrium, $f = f_0$.*

According to Proposition 1, a consumer facing signal exposure $S \geq \tau_0$ has an interest in raising filter tolerance A_i up to the level where $M_i = \psi(A_i)S = \tau_0$. Now, under $A_{\min}$ the effective range ($r = \psi(A)R$) of a firm is $\frac{\tau_0 f_0}{(1-\rho)y}$, so that, according to (16), $T = (1-\rho)y/f_0$ firms find it profitable to enter the market. This gives for information exposure $S = RT = R(1-\rho)y/f_0$. Thus, optimal individual filter choice is indeed $A_i = A_{\min}$.

According to Proposition 4, average perceived quality D – and thus utility – is a decreasing function of equilibrium filter tolerance. This is why it is desirable to implement an equilibrium with tight filtering. The limit comes from the constraint $f \geq f_0$. (Firms must be able to cover the innovation and information costs.) If $A_i = A_{\min}$, then $f = f_0$. Wasteful advertising is avoided and consumer welfare is maximal.

Moreover, under $A_{\min}$, equilibrium quality q_i is given by

$$q^* \equiv \frac{\psi_1(A_{\min})}{\psi(A_{\min})} q^S, \quad q^S = \frac{h\chi}{\tilde{\varphi}(h,\chi)}. \tag{21}$$

According to Lemma 2, $\psi_1(A_{\min})/\psi(A_{\min})$ decreases with $A_{\min}$, where $A_{\min}$ is in turn a decreasing function of $R = \varphi_0 \tilde{\varphi}(h,\chi)$. Thus, under optimal filter regulation, an extension of the technically feasible range of information distribution is positive for product quality and utility. Information targeting (lowering φ_1) or central filtering of general junk (lowering φ_2) decreases $\tilde{\varphi}$. This has a direct positive effect on q^* but an indirect negative effect by increasing $A_{\min}$. For given parameter values h and χ, the total effect can be easily seen by substituting $\psi(A_{\min}) = \frac{\tau_0 f_0}{(1-\rho)yR}$ into (21) and using $R = \varphi_0 \tilde{\varphi}(h,\chi)$. We get $q^* = \psi_1(A_{\min}) \frac{(1-\rho)y\varphi_0 h\chi}{\tau_0 f_0})$. Thus, reducing $\tilde{\varphi}$ is good for quality. It allows looser individual filtering, which increases $\psi_1(A_{\min})$, that is, the probability of not missing a useful piece of information.

For given φ_1 and φ_2, range $\tilde{\varphi}$ and thus $A_{\min}$ depend on homogeneity h and the success rate of innovations χ. It is straightforward to check: (i) $\partial q^S/\partial h > 0$ and (ii) $\partial \tilde{\varphi}/\partial \chi \geq 0$ (with equality for $\varphi_2 = 0$). Moreover, $\partial \tilde{\varphi}/\partial h \geq 0$ (with equality for $\varphi_1 = \varphi_2 = 1$). Thus, both the direct and the indirect effect on q^* of homogeneity and innovation success rate are non-negative and the total effect is positive. However, while wasteful advertising can be eliminated by optimal filter regulation, and while the targeting of information distribution may be improved by firms, the innovation process is probably less easy to control. Definitely out of the range of any policy control is the heterogeneity of preferences. This heterogeneity is the ultimate reason why each of us will be exposed to junk information even under the best policies and under conditions of optimal individual behaviour.

Finally, it is worth noting that for optimal filter regulation, the regulator does not need to know individual preferences, that is, who has which ideals. $A_{\min}$ depends only on the parameters responsible for information richness, in particular on the ratio of fixed costs over market size. ($(1-\rho)$ can be inferred from the price-cost margin.) The regulation proposed in Proposition 5 recommends to consumers a minimal rejection rate, not a specific content to be rejected. The personalisation with respect to the content of the items that should be accepted or rejected is left to the individual. This personalisation defines the assignment of junk probabilities and thus the functions $\psi_1(A)$ and $\psi_2(A)$. As long as these functions have the properties described, in particular the fact that perfect filtering is excluded because the exact content of information is revealed only after perception, the role of filter tolerance remains.

6 Conclusion

Progress in information technologies provides producers with the opportunity to distribute information about their product to a wide range of consumers. Not all information is equally useful, however. In particular, if there is heterogeneity, what is valuable for one individual may be considered useless by another. The larger the range of consumers who can be addressed by a producer, the more consumers will be exposed to the same set of advertisements. Under heterogeneous preferences, this implies that consumers receive, along with useful advertisements, additional information which is useless to them. As long as consumers have free capacity to perceive each supplied piece of information, exposure to more advertisements is welcome, even if the mix of useful and useless items is poor. In contrast, if attention is scarce, junk information has opportunity costs, and measures to improve the mix of perceived information are definitely beneficial for consumers. Better targeting of information distribution is clearly one instrument that can be used to improve the match between advertisements and preferences. However, the asymmetric nature of the relationship between producers and users of information puts crucial limits on the possibilities for targeting information. While the producer knows the content of the information sent, the user has the knowledge about how to value this content according to his/her preferences. The definite value can only be assessed by processing the information.

By employing a personalised filter, the user can save information-processing capacity. However, perfect filtering is unfeasible. On the one hand, under a very tolerant filter much junk will pass through in addition to the useful information. On the other hand, a tighter filter may block useful information along with the junk.

This paper analysed the equilibrium effects of individual filters in an economy with limited information-processing capacity. The basic characteristic of a filter is its tolerance. The consumer can control the quality of perceived items by employing a filter which is more or less in line with his/her preference profile and by setting a threshold for the tolerated junk probability. However, an individual consumer has no control over the quality of information supplied in the first place. Since producers address a mass of consumers, this quality is a function of aggregate behaviour. It was shown that the aggregate diversity of distributed information, which is the basis on which a filter can sample, depends on the expected filter tolerance used in the population. The individual consumer has no influence on this. (S)he simply adjusts individual filter tolerance so as to bring information exposure in line with her/his information-processing capacity. This gives rise to multiple equilibria in the information-rich economy. If everybody applies a low rejection rate, firms will engage in fierce competition for attention. This will lead to wasteful advertising and drive product variants out of the market. In contrast, if everybody chooses a high rejection rate, the crowding between single firms' advertisements will be reduced. This will increase the diversity of aggregate supply.

The aggregate effects imply that in general the market equilibrium of an information-rich economy is inefficient. It was shown that a social optimum can be implemented by regulating filter tolerance: Each consumer has to use a filter that rejects at least the prescribed rate of received advertisements. This rate depends on the feasible range of information distribution and the heterogeneity of consumers, but involves no knowledge of individual preferences. The personalisation of the profile on which the rejection or acceptance of items is based is left to the consumer, provided that the required filter tightness is achieved.

A final caveat is in order. The analysis was based on the assumption that all advertising is informative in the sense that knowledge about the identity of a product is transmitted. This knowledge may be junk insofar as the product is useless to a consumer or even to all consumers. Moreover, it was assumed that all firms have zero mass. Obviously, persuasive advertising or strategic use of information would make a big difference.

7 Appendix

Proof of Lemma 1. (a) $R(\omega) = \varphi_0 \text{prob}\{\omega \in \mathbf{I_i}\} + \varphi_0\varphi_1 \text{prob}\{\omega \in \mathbf{I} - \mathbf{I_i}\} + \varphi_0\varphi_2 \text{prob}\{\omega \notin \mathbf{I}\}$. According to Assumptions 1 and 2, $\text{prob}\{\omega \in \mathbf{I_i}\} = h\chi, \text{prob}\{\omega \in \mathbf{I} - \mathbf{I_i}\} = (1-h)\chi$ and $\text{prob}\{\omega \notin \mathbf{I}\} = 1 - \chi$. In an analogous way, we get S_i.

(b) The measure of ideals in $\mathbf{S_i}$ is given by $\int\limits_{\mathbf{\Omega_T \cap I_i}} \varphi(\omega, i) d\omega = \varphi_0 h\chi T$. Dividing this by $S_i = \varphi_0 \tilde{\varphi}(h, \chi)T$, we have q^S. QED.

Proof of Lemma 2. (a) $M_i = \int\limits_{\mathbf{S_i \cap I_i}} \psi_1 d\omega + \int\limits_{\mathbf{S_i - I_i}} \psi_2 d\omega = \psi_1 q^S S + \psi_2(1 - q^S)S$. (b) $d\psi/dA > 0$ follows immediately from $d\psi_j/dA > 0$. $dq_i/dA < 0$ is equivalent to $dz/dA < 0$. QED.

Derivation of (8). The Lagrangian for $\max\limits_{x_i(\omega)} U^i$ subject to (7) is

$\mathcal{L} = \left[\int\limits_{\mathbf{M_i}} \delta_i(\omega) x_i(\omega)^\rho d\omega\right]^{1/\rho} + \lambda \left[y - \int\limits_{\mathbf{M_i}} p(\omega) x_i(\omega) d\omega\right]$. Solving the first-order condition for $x_i(\omega)$ gives us $x_i(\omega) = \delta_i(\omega)^\varepsilon p(\omega)^{-\varepsilon} \lambda^{-\varepsilon} U^i, \varepsilon \equiv \frac{1}{1-\rho}$. Using this in the budget constraint $\int\limits_{\mathbf{M_i}} p(\omega) x_i(\omega) d\omega = y$, we obtain $\lambda^{-\varepsilon} U^i = y/P_i$. Hence, $x_i(\omega) = \delta_i(\omega)^\varepsilon p(\omega)^{-\varepsilon} y/P_i$. QED.

Proof of Proposition 1. Substituting (10) into (11) and using Part (a) of Lemma 2, we get $V^i = \frac{y}{p}\left[S\left[\psi_1(A_i) q^D (1 - \delta^\varepsilon) + \psi(A_i)\delta^\varepsilon\right]\right]^{\frac{1-\rho}{\rho}}$, which increases in A_i because of Assumptions 4 and 5. Thus, i chooses the maximal A consistent with (6). QED.

Proof of Proposition 2. With $A_i = A$ in Lemma 2 and (10), we have $q_i = \frac{\psi_1(A)}{\psi(A)} q^S \equiv q(A)$ and $D_i = q(A) + \delta^\varepsilon(1 - q(A)) \equiv D(A)$. Applying the definition of ψ, we get $1 - q(A) = \frac{\psi_2(A)(1-q^S)}{\psi(A)}$, where $q^S = \frac{h\chi}{\tilde{\varphi}(h,\chi)}$ and $1 - q^S = \frac{\varphi_1(1-h)\chi + \varphi_2(1-\chi)}{\tilde{\varphi}(h,\chi)}$, according to Lemma 1 and the definition of $\tilde{\varphi}(h, \chi)$. Moreover, for $M_i = M$ and $p(\omega) = p$, we have $P_i = p^{1-\varepsilon} MD(A)$. Using these facts in (12), we obtain

$X(\omega) = \frac{y\varphi_0}{pMD(A)}[q(A)\psi(A)\tilde{\varphi}(h,\chi) + \delta^\varepsilon[1-q(A)]\tilde{\varphi}(h,\chi)\psi(A)] = \frac{y\varphi_0\psi(A)\tilde{\varphi}(h,\chi)}{pM}$, which reduces to $\frac{y\psi(A)R}{pM}$ since $R = \varphi_0\tilde{\varphi}(h,\chi)$, according to Lemma 1. Finally, $p = c/\rho$ gives us $(p-c)X = (1-\rho)yr/M$. QED.

Proof of Lemma 3. Non-IP is equivalent to $S > \tau_0$ or $f > f_0$. According to Assumption 6, $f > f_0$ implies $S \geq \tau_0$. This and $f \geq f_0$ imply that if an equilibrium is information rich, then $S \geq \tau_0$ and $f \geq f_0$, with one inequality holding strictly. QED.

Proof of Proposition 3. (i) Suppose that an information-rich equilibrium exists despite $(1-\rho)yR/\tau_0 \leq f_0$. Then $S \geq \tau_0$ (Lemma 3), which implies $\psi(A_i) = \tau_0/S$ (Proposition 1) and $M_i = \tau_0$ (Lemma 2). Combining this with (14) and Proposition 2, we get $\pi = \frac{(1-\rho)y}{S}R \leq \frac{(1-\rho)y}{\tau_0}R \leq f_0$, which contradicts the equilibrium condition $\pi = f$ and the fact that either $S > \tau_0$ or $f > f_0$ in an information-rich economy. This proves that $S \leq \tau_0$ and $f = f_0$ if $(1-\rho)yR/\tau_0 \leq f_0$. Using this in (13) - (17), we get the unique solution derived in (ii).

(ii) In view of (13), $S \leq \tau_0$ leads to $\psi(A) = 1$ so that (14) reduces to $M = S$ and (15) gives us $S = (1-\rho)yR/f_0$. Moreover, $\psi(A) = 1$ implies, according to Lemma 2, $\psi_1(A) - (1-q^S)(\psi_1(A) - \psi_2(A)) = 1$ and thus $\psi_1(A) = \psi_2(A) = 1$, since $\psi_2(A) \leq \psi_1(A) \leq 1$. ($\psi_2(A) \leq \psi_1(A)$ follows from Assumption 5 and $\psi_1(A) = z(A)\psi_2(A)$.) Using this in Lemma 2 and in (10), we have $D_i = q^S + \delta^\varepsilon(1-q^S)$. Substituting $q^S = \chi h/\tilde{\varphi}(h,\chi)$ from Lemma 1, we get D_0. Moreover, $R = \varphi_0\tilde{\varphi}(h,\chi)$. Using this, $M = S$ and $S = (1-\rho)yR/f_0$ in (11), we obtain V^P. QED.

Proof of Proposition 4. (i) Suppose that an information-poor equilibrium exists, that is, $S \leq \tau_0$ and $f = f_0$. Then, in view of (13), $\psi(A) = 1$ and, according to (14) and (15), $M = S = (1-\rho)yR/f_0$. Thus, $S \leq \tau_0$ is a contradiction of $(1-\rho)yR/f_0 > \tau_0$.

(ii) If $S \geq \tau_0$, then (13) and (14) imply that $M = \tau_0$. Moreover, $RT = S$, according to Lemma 1. Thus, (13) - (17) reduces to $\psi(A)S = \tau_0$, $(1-\rho)y\psi(A)R = f\tau_0$ and $f \geq f_0$.

(iii) Use Lemma 1, Lemma 2, (10) and $A_i = A$ to get D. Substitute this and $M_i = \tau_0$ into (11) to prove V^R. QED.

Proof of Proposition 5. In view of Proposition 4, the optimal equilibrium is given by $\max\limits_A V^R$ subject to $f \geq f_0$, where $f = \frac{(1-\rho)y}{\tau_0}\psi(A)R$, according to (20). According to Assumption 5, D and thus V^R are declining in A. Hence, V^R is maximal at $\frac{(1-\rho)y}{\tau_0}\psi(A)R = f_0$. This defines $A_{\min}$. Moreover, following the line of reasoning of Proposition 1, $\max V^i = \frac{y}{p}(\tau_0 D_i)^{\frac{1-\rho}{\rho}}$ subject to $A_i \leq A_{\min}$ leads to $A_i = A_{\min}$. Thus, $A_i \leq A_{\min}$ implements the optimal equilibrium.

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