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Socioeconomic Institute
Sozialökonomisches Institut

Working Paper No. 0302

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Multiple Losses, Ex-Ante Moral Hazard, and the Non-Optimality of the Standard Insurance Contract

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This Version: April 24, 2003

ABSTRACT: Under certain conditions the optimal insurance policy will offer full coverage above a deductible, as Arrow and others have shown long time ago. Interestingly, the same design of insurance policies applies in case of a single loss and ex-ante moral hazard. However, many insurance policies provide coverage against a variety of losses and the possibilities for the insured to affect the probabilities of each possible loss might be substantially different. The optimal design of a insurance contract providing coverage against different losses therefore should generally differ from the standard form under moral hazard.

The paper concentrates on the conditions under which the standard insurance contract holds under moral hazard and more than one loss. It gives some evidence that many insurance contracts should be split up. The main result is, that the relative changes of probabilities due to precautious activities are decisive. On the other hand, under moral hazard it is rarely ever optimal to combine two losses in one insurance contract prescribing only a single deductible for both losses if both losses can occur simultaneously.

Keywords: Insurance, Multiple Losses, Moral Hazard
JEL: D 80, D 82

1. Introduction

Since the early contribution by Arrow (1963), the optimal design of insurance contracts has been discussed widely in the literature. Arrow states that under certain conditions an optimal insurance policy will offer full coverage above a non-negative deductible. Authors like Raviv (1979) and others[1] have extended Arrow's result. At the end of his article Raviv (1979, 261) addresses the important question of how to deal with multiple losses and concludes that "the results regarding optimal insurance policies hold unchanged when the insured faces more than one risk, when the loss considered is the loss from all those risks".

Since moral hazard is recognized as being one of the most important limiting factors in insurance, it would be interesting to investigate if an optimal insurance policy under moral hazard would look substantially different from the standard contract described before, i.e. when one insurance policy covers many different kinds of losses. This question is relevant for practice in several aspects: First, it should give some hints on what kind of insurance policies we can expect to become feasible in future and what limits due to moral hazard might continue to exist. This applies especially to so called 'umbrella-policies' that are supposed to provide protection against a wide range of risks individuals are exposed to.[2] Second, we would be provided with a sort of yardstick to analyze whether insurance contracts we observe in practice are appropriate or whether they should be redesigned. Thinking for example of health insurance, one finds that it covers a wide range of risks reaching form curing a cold up to rescuing an insured from death by applying the latest medical technology in a single policy.

However, despite its importance, all the early articles on optimal insurance contracts are not explicitly concerned with moral hazard in insurance contracts, but simply assume that the insurer faces increasing costs per unit of coverage, which usually is the case if moral hazard is present. It was not until the 80s that moral

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[1] See e.g. Mossin (1968), Moffet (1977) or Schlesinger (1981). Gollier and Schlesinger (1996) show, that Arrows result can be proofed using only second-degree stochastic-dominance arguments.

[2]   In contrast to Gollier and Schlesinger (1995), who stress the advantages of a unique insurance contract to cover all sources of risk, this paper argues that, because of moral hazard, it might be preferable to cover risks by separate contracts rather than by one policy.

hazard problems in the design of insurance contracts were addressed directly. Winter (2000) provides an excellent survey. Specifically, he finds that regarding ex-ante moral hazard or (synonymously) self-protection, the optimal insurance policy for a single loss and one prevention activity is designed exactly as suggested by Arrow and Raviv: Full reimbursement above a deductible. Although Winter does not investigate multiple losses, it would be tempting to conclude that the design of an optimal insurance policy should not be influenced by self-protection and that Raviv's result for multiple losses should also hold when there is the possibility of self-protection.

However, moral hazard should make a difference as soon as the possibilities of influencing or the incentives to influence the probability of all these incidents are not the same. Just consider a household contents insurance policy, covering the risk of burglary as well as the risk of a lightning strike or a fire. In such a situation, our experience with insurance models would suggest different treatment of different types of risk in insurance contracts to be in the insurer's as well as in the insured's interest, since it would give scope for well-directed incentives for prevention in insurance arrangements. After all, it is the aim of a deductible to reduce moral hazard and to make at least partial insurance possible. On the other hand, indemnity payments, that depend on the type of loss suffered contradict the insured's preferences for a safe income in exchange for a single premium. The aim of this paper therefore is to investigate the conditions under which the optimal design of an insurance policy insuring for multiple losses under moral hazard continues to be full reimbursement above a deductible. It will be shown that generally the standard result for multiple losses no longer holds under moral hazard.

To address the problem, section 2 adopts a framework introduced by Schlesinger (1987) that allows for a wide range of interpretations. After reporting the fundamental result concerning the optimal design of an insurance contract for multiple losses without self-protection in section 2.1, section 2.2 introduces moral hazard; i.e. it is assumed that the insured's self-protection activities cannot be observed by the insurer, who has to rely on proper incentives for self-protection instead. Section 3 looks into more concrete interpretations of the model and their results. The analyses by Raviv (1979), Schlesinger (1987), and Winter (2000) will turn out to be special cases of the broader model used in this paper. Section 4 concludes.

2. A general model for analyzing the optimal design of insurance contract

2.1 Optimal insurance contract without moral hazard

In order to provide a fairly general framework for studying the optimal design of insurance contracts, Schlesinger (1987) introduces a discrete model in which there are one state of no loss and three states of loss with arbitrary size L_1, L_2, and L_3. This framework is flexible enough to allow analyses of a wide range of insurance contracts. For example one is free to define L_1, L_2, and L_3 as the occurrence of a loss A, a loss B, and the simultaneous occurrence of both losses, respectively. The probability of these four states is $p_1 \ldots p_4$ with $\sum_{i=1}^{4} p_i = 1$. The expected utility of an insured consequently reads as:

$$(1) \qquad EU = p_1 U(W - P) + \sum_{i=2}^{4} p_i U(W - P - L_i + I_i),$$

with W denoting the exogenous wealth of the individual, P the premium paid to the insurance, and $I_i \geq 0$ the indemnity received from the insurance in state i. Furthermore, the utility function U is assumed to be twice differentiable with $U' > 0$ and $U'' < 0$, indicating risk-aversion.

The insurance industry is assumed to be competitive; i.e. given the public information on the probability of losses, the insurance premium cannot exceed the actuarially fair premium times a proportional loading $(1 + \kappa)$, with $\kappa > 0$ representing the loading factor. This leads to the constraint:

$$(2) \qquad (1 + \kappa) \sum_{i=2}^{4} p_i I_i - P = 0.$$

The optimal insurance contract without moral hazard can be derived by maximizing (1) w.r.t. all I_i, s.t. (2). The first-order conditions read

$$(3) \qquad \frac{U'(W - P - L_i + I_i)}{(1 + \kappa)} = \mu \quad \text{for } i = 2 \ldots 4,$$

where μ is the Lagrange multiplier for the premium function (2). FOC (3) states that the individual's marginal utility should be the same in all states of loss if $I_i > 0$. However, due to the loading, full insurance cannot be optimal. This leads to the optimal indemnity function prescribing a deductible $d > 0$ and full reimbursement for losses that exceed the deductible:

$$(4) \qquad I_i^* = \begin{cases} 0 & \text{for} \quad L_i \leq d \\ L - d & \text{for} \quad L_i > d \end{cases}$$

This result has been proved by Arrow (1971), Raviv (1979), Schlesinger (1987), and Gollier and Schlesinger (1996). Note that losses L_i are arbitrary. As mentioned before, we are free to define L_1, L_2, and L_3 as the occurrence of a loss A, a loss B, and the simultaneous occurrence of both losses, respectively. For the optimal indemnity being zero or offering full reimbursement the sum of both losses is decisive.

2.2 Optimal insurance contract for multiple losses under moral hazard

To investigate the effects of moral hazard in a more general model, we now allow for self-protection activities. To be more concrete, let there be two self-protection activities called x_a and x_b. These activities appear as costs in the individual's utility function and affect the probabilities of the four states in the model, rather than the amount of loss in any particular state. A fairly general formulation to capture this effect is:

$$(5) \qquad \begin{aligned} EU &= p_1(x_a, x_b) \cdot U(W - P - x_a - x_b) \\ &+ \sum_{i=2}^{4} p_i(x_a, x_b) \cdot U(W - P - x_a - x_b - L_i + I_i) \end{aligned}$$'

$$\text{with} \quad \frac{\partial p_i}{\partial x_j} \leq 0, \quad \frac{\partial^2 p_i}{\partial x_j^2} \geq 0 \text{ for } i = 2 \ldots 4 \text{ and } j = a,b \text{ and}$$

$$\frac{\partial p_1}{\partial x_j} \geq 0, \quad \frac{\partial^2 p_i}{\partial x_j^2} \leq 0 \text{ for } j = a,b.$$

According to (5) the effect of each self-protection activity on both probabilities may be identical or different. It is also possible that one self-protection activity affects one probability only; as analysed below in section 3.2. However, since the insurer cannot observe the self-protection activities of the insured, the insured will invest in self-protection only if she has the incentive to do so. This leads to the incentive compatibility constraints

(6)
$$x_a = \arg \max_{x_a} p_1(x_a, x_b) \cdot U(W - P - x_a - x_b)$$
$$+ \sum_{i=2}^{4} p_i(x_a, x_b) \cdot U(W - P - x_a - x_b - L_i + I_i)$$

and

(7)
$$x_b = \arg \max_{x_b} p_1(x_a, x_b) \cdot U(W - P - x_a - x_b)$$
$$+ \sum_{i=2}^{4} p_i(x_a, x_b) \cdot U(W - P - x_a - x_b - L_i + I_i)$$

Besides the incentive compatibility constraint, the break-even constraint for the insurer has to be met. As before, the premium P must, at least, cover his expected costs including the loading:

(8) $$\left(1+\kappa\right)\sum_{i=2}^{4} p_i(x_a, x_b) I_i - P \le 0.$$

The final constraint is that the indemnity must not be negative:

(9) $$I_i \ge 0.$$

For an interior solution of the maximizing program, the incentive compatibility constraints can be replaced by the first order conditions. These read for (6) and (7) as

(10) $$\sum_{i=1}^{4} \frac{\partial p_i}{\partial x_a} U_i - EU' = 0$$

and

(11) $$\sum_{i=1}^{4} \frac{\partial p_i}{\partial x_b} U_i - EU' = 0,$$

with U_i being shorthand for the insured's utility in state i and $EU' = \sum_{i=1}^{4} p_i U_i'$ representing the insured's expected marginal utility. Equations (10) and (11) state, that in an optimum the change in expected utility due to an increase of the self-insurance activity has to equal the expected marginal utility of income. According to (5), in this model the latter is the marginal cost of self-insurance activities, since the prices of both self-insurance activities have been normalized to one.

The optimization problem can now be stated as a problem of Lagrange: Maximize (5) with respect to $I_2 \dots I_4$ s.t. (8), (9), (10), and (11). Disregard (9) for the moment and let $\lambda_a \geq 0$, $\lambda_b \geq 0$, and $\lambda_P \geq 0$ be the shadow prices on constraints (10), (11), and (8) respectively. The resulting first-order conditions are:

$$(12) \quad p_i U_i' - \lambda_a \left(\frac{\partial p_i}{\partial x_a} U_i' - p_i U_i'' \right)_i - \lambda_b \left(\frac{\partial p_i}{\partial x_b} U_i' - p_i U_i'' \right) - \lambda_P p_i (1+\kappa) = 0$$

$$\text{for} \quad i = 2 \dots 4,$$

which can be rewritten as

$$(13) \quad \lambda_P = \frac{\left(1 - \lambda_a \frac{\partial p_i / \partial x_a}{p_i} - \lambda_b \frac{\partial p_i / \partial x_b}{p_i} \right) \cdot U_i'}{1+\kappa} + \frac{(\lambda_a + \lambda_b) \cdot U_i''}{1+\kappa}$$

or

$$(14) \quad \lambda_P = \frac{\left\{ \left(1 - \lambda_a \frac{\partial p_i / \partial x_a}{p_i} - \lambda_b \frac{\partial p_i / \partial x_b}{p_i} \right) - (\lambda_a + \lambda_b) \cdot R_i \right\} \cdot U_i'}{1+\kappa}$$

$$\text{for} \quad i = 2 \dots 4,$$

with $R_i = -U_i'' / U_i'$ representing the Arrow-Pratt degree of absolute risk aversion. Since U_i' is assumed to be positive, it follows from (14) that

$$1 - \lambda_a \frac{\partial p_i / \partial x_a}{p_i} - \lambda_b \frac{\partial p_i / \partial x_b}{p_i} > R_i (\lambda_a + \lambda_b).$$

Combining (14) for two states i and j, eliminating λ_P, and solving for U_i' / U_j' yields the optimal relative marginal utility,

$$(15) \qquad \frac{U_i'}{U_j'} = \frac{1 - \dfrac{\lambda_a \dfrac{\partial p_j}{\partial x_a} + \lambda_b \dfrac{\partial p_j}{\partial x_b}}{p_j} - (\lambda_a + \lambda_b) \cdot R_j}{1 - \dfrac{\lambda_a \dfrac{\partial p_i}{\partial x_a} + \lambda_b \dfrac{\partial p_i}{\partial x_b}}{p_i} - (\lambda_a + \lambda_b) \cdot R_i} .$$

While the second term in the numerator and the denominator of (15) mirrors the marginal effectiveness of preventive effort (fixed by the insured) on the probabilities, the first capture the insured's risk preferences., i.e. her risk aversion. Different effectiveness of prevention in equilibrium has two effects that work into different directions. First, if at least one of the prevention activities has a higher effect on the probability of state i relative to state j, this increases the relative marginal utility U_i'/U_j' calling for relatively less insurance coverage in state i. Second, however, assuming decreasing absolute risk aversion, a decrease of net wealth in state i relative to net wealth in state j causes R_i to increase relative to R_j and relative marginal utility U_i'/U_j' to decrease again. Therefore, as is well known, an optimal insurance contract has to strike a balance between providing appropriate incentives for prevention and the individuals' demand for insurance protection.

A sufficient condition for the relative marginal utility U_i'/U_j' to be 1 is that the fractions in the nominator and the denominator are the same, i.e.,

$$(16) \qquad \frac{\lambda_a}{\lambda_b} = \frac{\dfrac{\partial p_j / \partial x_a}{p_j} - \dfrac{\partial p_i / \partial x_a}{p_i}}{\dfrac{\partial p_i / \partial x_b}{p_i} - \dfrac{\partial p_j / \partial x_b}{p_j}} .$$

Since $\lambda_a / \lambda_b \geq 0$, for $U_i' = U_j'$ a relative higher effect of prevention activity a on p_j has to be compensated by a relative higher effect of prevention activity b on p_i et vice versa. In this case, an optimal insurance contract would show one single deductible for both losses.

## *3. Some interpretations of the solution*

In this section two special cases are investigated that give some more insights into the limits of designing optimal insurance policies under moral hazard.

3.1 Self-protection affects probability of loss but not its amount

For the first special case, we explicitly model different amounts of a loss which cannot be affected by the insured. However, as before, the insured's prevention activities affect the probability of a occurrence of the loss. Let the probability of occurrence be $pr(x_a, x_b)$ and let state 1 in section's 2.2 model represent the state of no loss which consequently has the probability $(1 - pr(x_a, x_b))$. The model's three other states then represent states of loss with different amounts of losses that occur with probabilities $pr(x_a, x_b) \cdot p_{h,2}$, $pr(x_a, x_b) \cdot p_{h,3}$, and $pr(x_a, x_b) \cdot p_{h,4}$ with $\sum_{i=2}^{4} p_{h,i} = 1$ (see figure 1).

Figure 1: Prevention affects probability but not size of loss
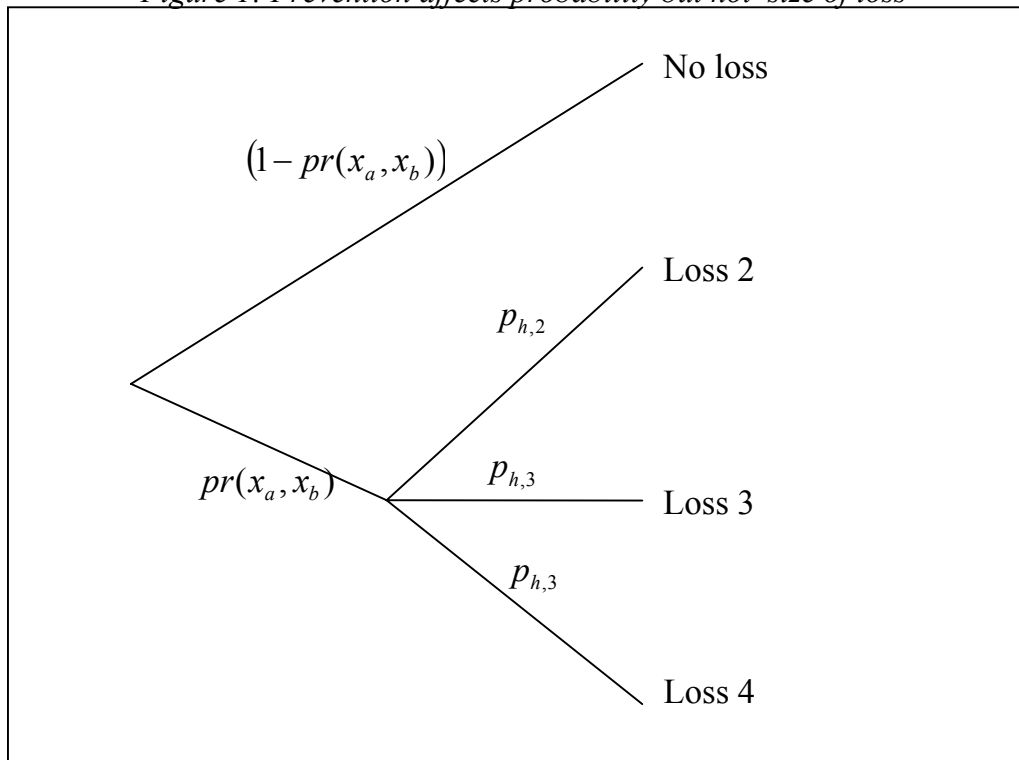
Note that self-protection has no effect on any $p_{h,i}$. The four states of our model therefore have the following probabilities:

$$
(17) \quad
\begin{aligned}
p_1 &= \left(1 - pr(x_a, x_b)\right) \\
p_2 &= pr(x_a, x_b) \cdot p_{h,2} \\
p_3 &= pr(x_a, x_b) \cdot p_{h,3} \\
p_4 &= pr(x_a, x_b) \cdot p_{h,4}
\end{aligned}
\ .
$$

Differentiating p_i, $i = 2..4$, w.r.t. x_j, $j = a,b$, yields

$$
(18) \quad \frac{\partial p_i}{\partial x_j} = \frac{\partial pr(x_a, x_b)}{\partial x_j} p_{h,i} \quad \text{for } i = 2..4 \text{ and } j = a,b.
$$

Plugging (18) into (14) reveals the critical terms in the first bracket of (14) to be the same for all $i = 2..4$. Therefore, ignoring restriction (9), the insured should always suffer the same out-of-pocket loss. If restriction (9) is taken into account, the optimal insurance contract again has the form

$$
(19) \quad I = \begin{cases} L - D, & \text{if } L \geq D \\ 0, & \text{otherwise} \end{cases},
$$

that is full reimbursement above a nonnegative deductible D. This is exactly the solution Winter (2000) presents for one prevention activity only. It turns out to be a special case of our more general analysis.

3.2 Two different losses and one cumulative loss

For the second special case suppose that there are two different kinds of losses which can occur alone or can happen both in a certain period of time. For example, think of two different kinds of illness. Let pr_1 and pr_2 be the probability of sickness 1 and sickness 2 respectively. The four states of the model then are defined as follows:

$$
(20) \quad
\begin{aligned}
p_1 &= \left(1 - pr_1(x_a, x_b)\right) \cdot \left(1 - pr_2(x_a, x_b)\right) \\
p_2 &= pr_1(x_a, x_b) \cdot \left(1 - pr_2(x_a, x_b)\right) \\
p_3 &= \left(1 - pr_1(x_a, x_b)\right) \cdot pr_2(x_a, x_b) \\
p_4 &= pr_1(x_a, x_b) \cdot pr_2(x_a, x_b)
\end{aligned}
\ .
$$

Differentiating these probabilities w.r.t. x_a and to x_b yields

(21)
$$
\begin{aligned}
\partial p_1/\partial x_i &= -(\partial pr_1/\partial x_i)(1-pr_2)-(\partial pr_2/\partial x_i)(1-pr_1)\\
\partial p_2/\partial x_i &= \;\;-(\partial pr_2/\partial x_i)pr_1+(\partial pr_1/\partial x_i)(1-pr_2)\\
\partial p_3/\partial x_i &= \;\;-(\partial pr_1/\partial x_i)pr_2+(\partial pr_2/\partial x_i)(1-pr_1)\\
\partial p_4/\partial x_i &= \;\;\;\;(\partial pr_1/\partial x_i)pr_2+(\partial pr_2/\partial x_i)pr
\end{aligned}
$$

$$\text{for}\quad i=a,b.$$

To see if the out-of-pocket loss in case of sickness 1 in an optimal insurance contract should be the same as in case of sickness 2, $\partial p_j/\partial x_i$ and p_i for $j=2\ldots4$ and $i=a,b$ from (20) and (21) have to be plugged into (14). The resulting equations read as:

(22)
$$
\lambda_P = \left\{ \left(1 - \lambda_a \frac{\dfrac{(-(\partial pr_2/\partial x_a)pr_1+(1-pr_2)(\partial pr_1/\partial x_a))}{pr_1\cdot(1-pr_2)}}{1+\kappa} - \lambda_b \frac{\dfrac{(-(\partial pr_2/\partial x_b)pr_1+(\partial pr_1/\partial x_b)(1-pr_2))}{pr_1\cdot(1-pr_2)}}{1+\kappa} \right) - \frac{(\lambda_a+\lambda_b)}{1+\kappa}R_2 \right\}\cdot U_2',
$$

(23)
$$
\lambda_P = \left\{ \left(1 - \lambda_a \frac{\dfrac{(-(\partial pr1/\partial x_a)pr_2+(\partial pr_2/\partial x_a)(1-pr_1))}{(1-pr_1)\cdot pr_2}}{1+\kappa} - \lambda_b \frac{\dfrac{(-(\partial pr_1/\partial x_b)pr_2+(\partial pr_2/\partial x_b)(1-pr_1))}{(1-pr_1)\cdot pr_2}}{1+\kappa} \right) - \frac{(\lambda_a+\lambda_b)}{1+\kappa}R_3 \right\}\cdot U_3'.\text{n}
$$

d

(24)
$$
\lambda_P = \left\{ \left(1 - \lambda_a \frac{\dfrac{((\partial pr_1/\partial x_a)pr_2+(\partial pr_2/\partial x_a)pr_1)}{pr_1\cdot pr_2}}{1+\kappa} - \lambda_b \frac{\dfrac{((\partial pr_1/\partial x_b)pr_2+(\partial pr_2/\partial x_b)pr_1)}{pr_1\cdot pr_2}}{1+\kappa} \right) - \frac{(\lambda_a+\lambda_b)}{1+\kappa}R_4 \right\}\cdot U_4'
$$

A sufficient condition for the out-of-pocket loss to be the same in all three states is again that the values in the parentheses are the same in (22), (23), and (24). However, combining the terms in parentheses in (22) with those in (24) and terms in (23) with those in (24) yields:

$$(25) \quad \frac{\lambda_a}{\lambda_b} = -\frac{\partial pr_2 / \partial x_b}{\partial pr_2 / \partial x_a} \quad \text{and}$$

$$(26) \quad \frac{\lambda_a}{\lambda_b} = -\frac{\partial pr_1 / \partial x_b}{\partial pr_1 / \partial x_a} .$$

Neither of the two conditions, (25) and (26), can be met if both activities, a and b, are prevention activities reducing the probability of a sickness. It is therefore unreasonable to justify the same out-pocket-loss in all states which calls for some elements of co-insurance to be incorporated in an optimal policy. Furthermore, according to the considerations in section 2.2., deductibles and co-insurance rates will generally differ for both losses.

4. Conclusion

The literature on the optimal design of insurance policies concentrates on settings where there is no moral hazard at all. Unambiguous optimal contracts in case of ex-ante moral hazard are derived for single losses only. Since this situation is unsatisfactory in the light of existing insurance contracts this paper investigates multiple losses and two self insurance activities.

Although it was not possible to derive an optimal insurance contract under moral hazard in general, the model presented here provides some useful insights. The most important one is that a policy offering full insurance above a deductible is appropriate under certain conditions only, but not in general. It has been shown i.e. that it is never optimal to combine two losses in one insurance contract with the same deductible for all losses if both losses can occur simultaneously. This gives rise to the idea that many insurance contracts that provide for a single deductible for different losses might be ill-designed.

Furthermore, the model gives some evidence that 'umbrella-policies', providing a stop-loss insurance against a complete range of losses, will probably remain unfeasible despite of their attractive properties for the insured. Since no general optimal policy for covering several losses under moral hazard could be derived, it might remain more sensible to cover these several losses by separate policies, each designed to give appropriate incentives for prevention.

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