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Theory and Experimental Evidence

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All-Pay Auctions with Negative Prize Externalities: Theory and Experimental Evidence*

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ABSTRACT: The paper characterizes the mixed-strategy equilibria in all-pay auctions with endogenous prizes that depend positively on own effort and negatively on the effort of competitors. Such auctions arise naturally in the context of investment games, lobbying games, and promotion tournaments. We also provide an experimental analysis of a special case which captures the strategic situation of a two-stage game with investment preceding homogenous Bertrand competition. We obtain overinvestment both relative to the mixed-strategy equilibrium and the social optimum.

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1 Introduction

In all-pay auctions, all players cast bids for a prize which only one player obtains. Contrary to ordinary auctions, even the losers have to pay their bids. All-pay auctions have received a lot of attention, as they reflect important aspects of the strategic interaction involved in many different economic activities. For instance, in innovation tournaments, firms' investments influence the probability of winning a patent, the value of which accrues to the winner. In lobbying contests, rival activists can exert effort to achieve the political outcome that is favorable for them. In promotion tournaments, the employees' efforts influence their chances of promotion. Independently of the precise application, the literature usually assumes that the strategic interaction relates exclusively to the chances of obtaining the prize rather than to the ex-post value of the prize, which is assumed to be exogenously fixed.

However, as Baye and Hoppe (2003) point out, there are many important examples where players' activities influence prizes. Specifically, they argue for investment tournaments that a high effort not only increases the chances of obtaining the prize, but also its value. Even though the formulation of their model is general enough to allow for the possibility, these authors do not mention an additional source of prize endogeneity. The efforts of one player may have adverse effects on the prize that another player obtains. For instance, consider a market that is sufficiently competitive that only a firm that is better than the others can earn positive profits. For a particularly stark example, consider a homogenous Bertrand market where firms can invest into cost reduction before product market competition. Then, the firm with the lowest marginal costs obtains the prize, that is, a positive product-market profit, but the size of the prize depends on the investments of the competitors. If the second-best firm has invested almost as much as the winner, the requirement of limit pricing will lead to very low profits for the winner.[1] Thus, investments involve negative externalities not only because they reduce their winning chances, but also because they reduce the prize that the winner will obtain.

For a similar example, suppose the bids in the all-pay auction are specific investments of job candidates (e.g., preparation for job interviews). Then,

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[1] In the homogeneous Bertrand case, the equilibrium profit margin of the most efficient firm corresponds to the difference between its costs and those of the second-best firm.

it is natural to assume that the second-best player's bid (effort) influences the outside options of the prospective employer. Therefore, the prize of the winner, that is, the difference between his wage in the new position and his outside option is likely to depend on the difference between his bid and the second-best bid. In particular, this prize is likely to become small as the second-best bid approaches the winning bid.

This paper therefore analyzes all-pay auctions where the prize (i) is a weakly decreasing function of the second-highest bid such that (ii) the value of the prize is zero when the bids are identical. Finally, we also assume that (iii) the prize is a positive function of the own efforts.

Our first set of contributions is theoretical. We characterize the equilibrium structure of all-pay auctions with negative prize externalities. First, contrary to standard fixed-prize auctions, all-pay auctions with bid-dependent prizes often have pure-strategy equilibria (PSEs) where exactly one player bids a positive amount. However, as there are as many of these (asymmetric) equilibria as there are players, their predictive value is limited. Second, like in the standard case, there are typically symmetric mixed-strategy equilibria (MSEs) where players put weight on all strategies up to a cut-off value. Third, the natural analogues of the asymmetric MSEs identified by Baye et al. (1996) for the fixed-prize case do not exist.[2] However, there are asymmetric MSEs where some players mix over all strategies up to a cut-off value and the others put all weight on zero.

The second contribution of the paper is an experimental analysis of a specific all-pay auction with negative prize externalities. We consider parameterized versions of the auction that is derived from the Bertrand investment game; with 2 and 4 players. In both games, players choose investments $Y_i \in \{0, 1, ..., 9\}$. The games have multiple PSEs where one player chooses a positive investment level of 5 and the other player(s) choose 0. In both games, the symmetric MSE has all players mixing between $0, 1, 2, 3, 4,$ and 5. The experimental analysis shows that the MSE predicts the percentage of zero investments quite well. However, low, but positive investments are chosen less than predicted; high investments are chosen more often than predicted, which results in negative profits. Interestingly, the frequency distribution has a lot of mass around 5, the non-zero bid in the asymmetric PSE.

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[2]In those equilibria, some players behave as in the symmetric MSE; but other players modify the strategy by not casting any positive, but small bids. Instead, they put all the weight that these strategies receive in the symmetric MSE onto zero.

As the standard MSE illustrated above is not a fully convincing predictor, we also try to explain the investment behavior through modified objective functions, capturing a "joy of winning" and a "fear of losing". To this end, we extend the profit function of the Bertrand investment game by two parameters: γ and β. The former takes into account the additional benefit from investing more than the others, the latter the additional loss from investing in vain. The MSE obtained in this fashion reflects the investment behavior better, particularly in the 2-player case, but the fit is still imperfect. Summing up, the best interpretation of the evidence is that some players play the symmetric MSE, whereas others speculate that the remaining bidders do not invest, and respond accordingly (like the active bidder in the asymmetric PSE).

Auctions have been discussed extensively in the experimental literature.[3] Experiments on all-pay auctions are comparatively rare, and exclusively concerned with the fixed-prize case. In spite of the differences in the equilibrium structure, our experimental observations are similar to those that are familiar from the fixed-prize case. Most closely related is Gneezy and Smorodinsky (2006) who consider symmetric all-pay auctions with 4, 8, and 12 players. Like us, they obtain overbidding that diminishes over time, but remains substantial even in later periods. Also, as in our case, the percentage of very low bids is close to the MSE prediction.[4] Also, one of the six treatments analyzed by Davis and Reilly (1998) corresponds to the fixed-prize all-pay auction.[5] In an experiment with 5 players, the authors observe overbidding that diminishes over time, but does not disappear. Davis and Reilly (1998) also consider the alternative probabilistic set-up that goes back to Tullock (1980).[6] This variant of the all-pay auction has a symmetric PSE. Davis and Reilly (1998) show that overbidding also occurs in the probabilistic case. Earlier experimental evidence on all-pay auctions with fixed prizes is mixed. Millner and Pratt (1989) also observed overbidding, whereas, in the simpler

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[3] See Kagel (1995) for a survey.

[4] They also observe positive effects of the number of bidders on revenue and negative effects on average bids, with the former arising only in first periods and the latter only in late periods.

[5] In contrast to Davis and Reilly (1998), the experiment of Gneezy and Smorodinsky (2006) takes the form of a repeated game, where subjects do not rotate among different treatments, playing the same all-pay auction.

[6] In this model, each player wins the prize with probability $b_i / \sum_j b_j$, where b_i is his bid.

setting of Shogren and Baik (1991) the Nash prediction is fairly accurate. Summing up, even though the equilibrium structure of the all-pay auction with negative prize externalities differs substantially from the fixed-prize case, the experimental observations, in particular, the overbidding phenomenon, are quite similar.

In this paper, we proceed as follows. Section 2 contains the general setting with the characterization of the MSE. Section 3 introduces the experimental design, including the analysis of the Bertrand investment game. Section 4 describes the experimental results, comparing the investment observations in the Bertrand game to the MSE. Section 5 discusses alternative MSEs. Section 6 concludes.

2   The Model

2.1   Assumptions

We analyze static games of the following type. Players $i = 1, ..., I$ simultaneously choose bids $b_i \in S = \{0, 1, .., N\} \subset \mathbb{N}^+$. The cost of submitting bid $b_i = n$ is k_n such that $k_0 = 0$ and k_n is increasing in n. This includes the standard case that $k_n = n$, but allows for greater generality.[7] Payoffs are given as follows. Let $g(n_i, n_j)$ be a function that is non-decreasing in n_i, non-increasing in n_j and satisfies $g(n_i, n_j) = 0$ whenever $n_i \leq n_j$. Let $b^{(2)}$ be the second-highest bid. Then the payoff of player i is given by

$$f\left(b_i, b^{(2)}\right) = \begin{cases} g(b_i, b^{(2)}) - b_i & \text{if} \quad b_i > b^{(2)} \\ -b_i & \text{if} \quad b_i \leq b^{(2)} \end{cases} . \tag{1}$$

Thus, as in a standard all-pay auction, only the highest bidder obtains a positive payoff. However, there is an important twist: The prize for a successful bidder is not fixed. It depends on the winning bid, and on the second-highest bid. The higher the winning bid, the higher the prize; the higher the second-highest bid, the lower the prize. In the limit, as the difference between the highest and the second-highest bid tends to zero, so does the prize. We further maintain the following assumption.

Assumption 1 $g(n_i, n_j)$ is concave in n_i for $n_i > n_j$, and k_n is convex in n.

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[7]For instance, we shall apply our framework below to the case that $b_i = n$ is a reduction of marginal costs by n, and k_n is the corresponding strictly convex investment cost.

Of course, the assumption is consistent with the special case that $k_n = n$.

2.2   Asymmetric PSE

In the following simple characterization of the asymmetric PSEs of the game, let $b^* = \arg\max_{b_i} (g(b_i, 0) - b_i)$.

Proposition 1 *(i) If there exists a PSE of the game such that at least one player i chooses $b_i > 0$, then $b_i = b^*$, all players $j \neq i$ choose $b_j = 0$, and there is one such equilibrium for each player.*
(ii) Such equilibria exist if and only if

$$\max_{b_j} \; (g(b_j, b^*) - b_j) \leq 0. \tag{2}$$

The proof is straightforward: (i) If there is more than one player with non-zero bids, at least one of them must earn negative payoffs. Also, the active player must best-respond to zero. (ii) is a simple statement of the best-response conditions for the candidate equilibria. Intuitively, condition (2) requires that those players who choose $b_j = 0$ do not find it profitable to leapfrog player i, that is, to choose $b_j > b_i^*$. Assumption 1 works in favor of this condition: Intuitively, with concave prizes and convex costs, earning positive payoffs by overbidding a player who has chosen the best response to 0 becomes increasingly difficult. However, in Section 3, we will provide an example where asymmetric PSEs even exist in the boundary case that the prize is linear in the own bid.

In spite of its simplicity, the result is interesting, because it stands in stark contrast with the case of fixed prizes. For deterministic all-pay auctions with continuous strategy spaces, PSEs typically do not exist (Baye et al., 1996). This result carries over to the case of discrete bids, as long as the prize is sufficiently large: With a fixed prize v, a PSE would still require that at most one player chooses $b_i > 0$. The best response condition of player i would require $b_i^* = 1$ because this is sufficient to overbid the other players. However, $b_i^* = 1$ would make leapfrogging to $b_j' = 2$ attractive for players with $b_j = 0$, unless $k_2 > v$. More generally, even with bid-dependent prizes, a PSE does not exist if $g(n_i, n_j)$ increases more rapidly in n_i than k_{n_i} near $n_i = n_j = n$.

Essentially, with bid-dependent prizes, it often makes sense to overbid the other players by a sufficiently large amount. This may make it unattractive for losing bidders to leapfrog the winner.

2.3 Symmetric MSE

Next, we provide a general characterization of symmetric MSEs. The result implies that such equilibria exist under very general conditions, and it provides an algorithm for calculating them. We use the following definitions.

Definition 1 *For any mixed strategy* $\mathbf{p} = (p_0, ..., p_n)$, $p^{-(n)} \equiv p^{-(n,I-1)}$ *is the probability that the highest bid of* $I - 1$ *players following this strategy is* n.

Next, we define a particularly important class of equilibrium candidates.

Definition 2 *Suppose* $M \in \{1, ..., N\}$. *An M-equilibrium is a symmetric MSE where all players put symmetric positive weights on strategies* $0, ..., M$, *and zero weights on all higher strategies.*

Proposition 2 provides a recursive formula for calculating symmetric MSEs for all-pay auctions with bid-dependent prizes. In particular, it provides conditions for the existence of such an equilibrium.

Proposition 2 *Suppose that Assumption 1 holds. (i) An M-equilibrium exists if and only if there exists a sequence* $(q_0, ..., q_{M-1})$ *such that:*

$$q_n = \frac{k_{n+1} - k_n - \sum_{m=0}^{n-1} q_m \left(g(n+1, m) - g(n, m) \right)}{g(n+1, n)}, \qquad (3)$$

where

$$q_n \geq 0 \text{ for } n \leq M - 1, \ \sum_{n=0}^{M-1} q_n < 1; \qquad (4)$$

and

$$\sum_{n=0}^{M-1} q_n g\left(M+1, n\right) + \left(1 - \sum_{n=0}^{M-1} q_n\right) g\left(M+1, M\right) - k_{M+1} \leq 0. \qquad (5)$$

For this equilibrium, $p^{-(n)} = q_n$ *for* $n \in \{0, ..., M - 1\}$.
(ii) If an M-equilibrium exists, it is the unique symmetric MSE.

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Proof. See Appendix. ∎

We illustrate the meaning of the result, and its proof for $M = 1$. Then, condition (5) becomes

$$q_0 g(2,0) + (1 - q_0) g(2,1) - k_2 < 0. \tag{6}$$

Condition (3) applied to $n = 0$ reads $q_0 = \frac{k_1}{g(1,0)}$; and, therefore, (4) merely requires that $g(1,0) - k_1 > 0$. By Proposition 2, the game has a symmetric MSE $(p_0, 1 - p_0, 0, ..., 0)$ where $p^{-(0)} = q_0 = \frac{k_1}{g(1,0)}$. This probability is such that players are indifferent between bidding 1 unit or not bidding. Also, (6) guarantees that bidding 2 units would lead to negative expected payoffs. Using concavity of g and convexity of the function k_n, choosing $b_i > 2$ is not profitable either. The standard characterization result for MSEs (Mas-Colell et al. 1995, Proposition 8.D.1) therefore yields the result.

2.4 Asymmetric MSE

Baye et al. (1996) have shown that, for fixed prizes v and continuous bidding, a symmetric equilibrium like the one just derived is not the only MSE of the all-pay auction. In addition, there are asymmetric equilibria where some players randomize over all strategies below a cut-off value, and all other players randomize in exactly the same fashion over all strategies starting from some lower bound above zero up to the cut-off value, but put all the remaining mass on 0. In the following, we show that natural analogues of such equilibria also exist in our discrete game when the prize is fixed. In our more general setting, however, all these equilibria disappear. Instead, there is another type of MSE where some of the players put all mass on zero.

2.4.1 Fixed prizes

In the degenerate case that the prize v is fixed, we show that there are also many asymmetric MSEs similar to those identified by Baye et al. (1996). To formulate this result, define $P_n = p_0 + ... + p_n$.

Proposition 3 *Suppose the prize is fixed, that is, $g(n_i, n_j) = v$ for some suitable constant $v > 0$ if and only $n_i > n_j$. Suppose there are $I \geq 3$ players. Define $n = M$ to be the maximal bid such that $v > k_n$. Suppose $J \in \{2, ..., I - 1\}$, $r \leq M$. Then there exist MSEs such that*
(i) J players choose $(p_0, ..., p_M, 0, ..., 0)$;

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(ii) $I - J$ *players choose* $\left(P_r, 0, ..., 0, p_{r+1}, ..., p_M, 0..., 0\right)$;

(iii) $P_{n-1} = \left(\frac{k_n}{v}\right)^{1/(I-1)}$ *for* $n \in \{r, ..., M-1\}$;

(iv) $P_{n-1} = \left(\frac{k_n}{v\left(\frac{k_r}{v}\right)^{(I-J)/(I-1)}}\right)^{1/(J-1)}$ *for* $n \in \{1, ..., r-1\}$;

(v) $P_M = 1$.

Proof. See Appendix. ■

To illustrate the proposition, suppose $I = 4$, $M = 3$. Then, the proposition says that there are four types of asymmetric equilibria, which differ with respect to the number of players who are not mixing over all strategies (1 or 2) and the minimal non-zero strategy played by those players (2 or 3).[8]

2.4.2   Endogenous prizes

The next result rules out the possibility that equilibria as derived in Proposition 3 exist when prizes are strictly decreasing in competitor bids. Thus, the equilibrium properties of all-pay auctions with negative prize externalities are dramatically different from those of all-pay auctions with fixed prizes.

Proposition 4 *Suppose* $g(n_i, n_j)$ *is strictly decreasing in* n_j *for* $n_i > n_j$. *Then, there can be no equilibrium such that there exists an* $r \geq 2$ *such that:*
(i) At least one player chooses $\mathbf{p}$ *with positive weights* p_0 *and* p_r;
(ii) At least one player chooses $\widetilde{\mathbf{p}}$ *with positive weights* $\widetilde{p}_0$ *and* $\widetilde{p}_r$ *such that* $\widetilde{p}_0 > p_0$, *but* $\widetilde{p}_1 = 0, ..., \widetilde{p}_{r-1} = 0$;
(iii) $\displaystyle\sum_{n=0}^{r-1} p_r = \sum_{n=0}^{r-1} \widetilde{p}_r$.

Proof. See Appendix. ■

This immediately rules out equilibria as in Proposition 3. The scope for asymmetric equilibria is further limited by the following result.

Proposition 5 *There can be no equilibrium such that there exists* $r \geq 1$, *where* $(p_0, ..., p_r) \neq (\widetilde{p}_0, ..., \widetilde{p}_r)$; $p_n > 0$ *and* $\widetilde{p}_n > 0$ *for all* $n \in \{0, ..., r\}$.

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[8]The condition that $n = M$ be maximal with the property that $v > k_n$ is easily seen to be necessary; for instance, when $M = 2$, there are no equilibria with some bidders randomizing over $0, 1$, and 2, and the remaining bidders randomizing over 0 and 2.

Proof. Suppose $s \leq r$ is minimal such that $p_s \neq \widetilde{p}_s$. Then,

$$\sum_{n=0}^{s} p^{-(s)} g(s, 0) \neq \sum_{n=0}^{s} \widetilde{p}^{-(s)} g(s, 0), \tag{7}$$

contradicting the requirement that both players are indifferent between playing s and 0. ∎

Note that this result also holds in the case of fixed prizes.

However, there is one class of asymmetric MSE that does exist. In these equilibria, some players mix over all strategies up to some value M. The remaining players all choose 0.

Proposition 6 *Suppose $I \geq 3$. Then, for every $J \in \{2, ..., I-1\}$, there exist equilibria such that:*
(i) J players randomize over strategies $0, 1, ..., M$, such that $p^{-(n,J-1)} = q_n$ for $n \in \{0, ..., M-1\}$, where q_n is defined as in (3) to (5);
(ii) The remaining players put all weight on 0.

Proof. See Appendix. ∎

In spite of the similarities in the strategies of the J active bidders with those played in the symmetric MSE, there is a crucial difference: As there are some players who put all weight on zero, q^n (for $n > 0$) is the highest remaining bid. The intuition for the result is that, if the J active bidders (who face $J - 1$ active bidders and $I - J$ bidders who always bid 0) obtain zero expected profits for all positive bids, the $I - J$ passive bidders (who face J active bidders and $I - J - 1$ bidders who always bid 0) must obtain negative expected profits.

3   The Experiment

3.1   The Bertrand Investment Game

In the following, we will show that a simple two-stage game can be reduced to an all-pay auction with negative prize externalities. In this Bertrand investment game (BIG), all firms $i = 1, ..., I$ are identical ex-ante with constant marginal costs $c > 0$. In the first stage, firms simultaneously choose investments $Y_i \in [0, c)$, resulting in marginal costs $c_i = c - Y_i$.[9] Investment costs

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[9]Even though we restrict the agents to finite choice sets in the experiment, the theoretical analysis is much more transparent if the choice set is a continuum.

are kY_i^2, where $k > 0$. In the second stage, firms compete in the product market as Bertrand competitors; with a demand function $D(p) = a - p$. Let $c_{-i}^m = \min_{j \neq i} c_j$, and denote the monopoly prices and payoffs (gross of investment costs) associated with marginal costs c_i as $p^M(c_i)$ and $\pi^M(c_i)$, respectively. It is well known that gross payoffs of the most efficient firm are

$$
\pi_i(c_1, ..., c_I, \alpha) = \begin{cases} (c_{-i}^m - c_i)D(c_{-i}^m), & \text{if } c_{-i}^m \leq p^M(c_i) \\ \pi^M(c_i), & \text{if } c_{-i}^m \geq p^M(c_i) \end{cases} . \tag{8}
$$

Intuitively, if efficiency differences are sufficiently small that the second-most efficient firm has costs below the monopoly price of the most efficient firm, this firm undercuts the competitors marginally, so that it obtains (approximately) a demand of $D(c_{-i}^m)$ and a markup corresponding to the cost differential; otherwise it sets the monopoly price. We assume that the efficiency differences are so small that no firm can earn the monopoly profit. Then, defining $Y^{(2)} = \max_{j \neq i} Y_j$, the net payoff of firm i is given by

$$
\Pi_i(Y_1, ..., Y_I) = \begin{cases} (Y_i - Y^{(2)})D(c - Y^{(2)}) - kY_i^2, & \text{if } Y_i > Y^{(2)} \\ -kY_i^2, & \text{if } Y_i \leq Y^{(2)} \end{cases} . \tag{9}
$$

Hence, with $g(n_i, n_j) = (n_i - n_j)D(c - n_j)$, the game corresponds exactly to our general set-up. Even though this is a two-stage game, by assuming that players play the Nash equilibrium in stage 2, we can reduce the game to the first stage. The one-stage game obtained in this fashion corresponds to an all-pay auction with negative prize externalities.

It is straightforward to calculate the equilibria for the BIG. First, as already suggested for the general case in Proposition 1, the game has multiple asymmetric PSEs. Define $\alpha \equiv a - c$.

Proposition 7 *For $k > \frac{1}{2}$, there are multiple asymmetric PSEs with one firm investing $Y_i^{IC} = \frac{\alpha}{2k}$ and firms $j \neq i$ investing $Y_j^{IC} = 0$.*

Proof. If firms $j \neq i$ invest $Y_j^{IC} = 0$, then the best response of firm i is $Y_i^{IC} = \frac{\alpha}{2k}$ for any $k > 0$. If firm i invests $Y_i^{IC} = \frac{\alpha}{2k}$, then the best response of the other firms is $Y_j^{IC} = 0$ for $k > \frac{1}{2}$. ∎

Intuitively, if more than one firm invests, then at least one firm obtains zero product market payoffs and therefore negative net payoffs; deviation is therefore profitable.

The BIG also has a symmetric MSE under very general conditions. We calculate this equilibrium in the Appendix.

3.2 Experimental Design and Procedures

The experimental design reflects the two-stage investment game which, as mentioned above, can be reduced to an all-pay auction with negative prize externalities, assigning the payoffs of the respective product market game to each investment vector. Apart from making the game more transparent to the experimental subjects, this design feature highlights the nature of the game as an all-pay auction, focusing attention on bidding (investment) rather than on behavior in the product market. This guarantees that deviations do not result from speculations about non-equilibrium behavior in the product market.[10]

Our two sessions concern two examples of the BIG. We ran a two-player (BIG2) and a four-player treatment (BIG4). The parameter values were $\alpha = 30$ and $k = 3$ for BIG2; $\alpha = 20$ and $k = 2$ for BIG4.[11] We restricted investment choice sets to $Y_i \in \{0, 1, ..., 9\}$ in both cases. Applying the results obtained above, the following holds.

Observation 1 *For BIG2 and BIG4, there are asymmetric PSEs, each with one player investing* 5 *and the other player(s) investing* 0.

Coordination on such equilibria is obviously problematic. The MSE is potentially more appealing as a predictor.

Observation 2 (i) *For BIG2, there is a symmetric MSE given by*

$$\left(p_0^{BIG2}, ..., p_9^{BIG2} \right) = (0.1, 0.193, 0.187, 0.182, 0.176, 0.160, 0, 0, 0, 0). \quad (10)$$

(ii) *For BIG4, there is a symmetric MSE given by*

$$\left(p_0^{BIG4}, ..., p_9^{BIG4} \right) = (0.464, 0.198, 0.116, 0.086, 0.069, 0.067, 0, 0, 0, 0). \quad (11)$$

Hence, in both cases, players randomize over all strategies up to and including 5, the non-zero bid arising in the asymmetric PSE. (i) follows directly

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[10] For instance, Dufwenberg and Gneezy (2000) have shown that experimental subjects tend to choose prices above marginal costs in symmetric Bertrand games. If subjects anticipate this, the investment incentives will differ from a situation with marginal-costs pricing.

[11] Because BIG2 and BIG4 also differ with respect to α and k, the treatments cannot be compared to identify number effects (see Sacco and Schmutzler, 2008 for a discussion of number effects in investment games).

from Corollary 1 in the Appendix, because $p_i = q_i$ with two players. As to (ii), Corollary 1 yields

$$(q_0, ..., q_9) = (0.1, 0.190, 0.182, 0.174, 0.167, 0.187, 0, 0, 0, 0), \qquad (12)$$

from which we obtain

$$p_0^{BIG4} = (q_0)^{1/3} = (0.1)^{1/3} = 0.464. \qquad (13)$$

The probability $p_1^{BIG4} = 0.198$ follows from

$$q_1 = 3 \left(p_0^{BIG4} \right)^2 p_1^{BIG4} + 3 \left(p_1^{BIG4} \right)^2 p_0^{BIG4} + \left(p_1^{BIG4} \right)^3 = 0.190. \qquad (14)$$

Recursively, the other probabilities are obtained.

We note the following immediate implication of Observation 2.

Observation 3 (i) *For BIG2, the expected investment is* 2.62. (ii) *For BIG4, the expected investment is* 1.30.

The experiments were conducted in February and June 2006 at the University of Zurich. The participants were undergraduate students from various disciplines. Each treatment was run for 20 periods. There were 34 subjects in BIG2 and 36 in BIG4. This led to a total of 1400 investment observations. No subject participated in both sessions. The participants were randomly matched into groups of size 2 or 4 after each period (Stranger design).[12] At the end of each period, subjects were informed about the investment level of the other group member(s) and their own net payoff for that period. All participants received an initial endowment of CHF 35 ($\approx$EUR 22) under BIG2 and CHF 45 ($\approx$EUR 28) under BIG4. Average earnings including the endowment were CHF 32 ($\approx$EUR 20) for BIG2 and CHF 38 ($\approx$EUR 24) for BIG4. Sessions lasted about 90 minutes each. The experiment was programmed and conducted with the software z-Tree (Fischbacher, 2007).

4 Experimental Results

4.1 The 2-player Case

Our first observations concern the relation between the symmetric MSE and realized mean investments.

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[12]The subjects thus take their decisions based on one-shot considerations.

Result 1 *Under BIG2, mean investments are higher than in the symmetric MSE.*
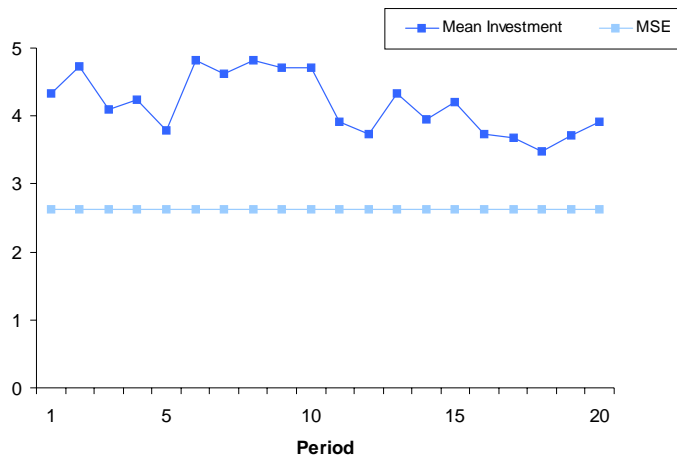
Figure 1: Mean investment for BIG2.

Figure 1 reveals that the mean investment level exceeds the equilibrium investment level of 2.62 throughout the 20 periods. A regression over a constant and a Wilcoxon rank sum test show high significance ($p < 0.01$) when considering the difference between predicted and observed investments over all periods. This still holds when taking into account either the last ten or the last five periods. That is, there is no convergence to the Nash equilibrium, even though the investments in the first ten periods are significantly higher than those in the last ten periods (Wilcoxon rank sum test, $p = 0.016$).[13]

A further interesting aspect concerns the investment distribution. The properties of this distribution over all periods are summarized in Result 2.

Result 2 *Under BIG2, (i) the frequency distribution exhibits a global maximum at 5. (ii) There is a local maximum at 0. (iii) A substantial fraction of the subjects chooses strategies that are not part of the symmetric MSE, that is, invests more than 5.*

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[13]Gneezy and Smorodinsky (2006) report qualitatively similar results for the symmetric all-pay auction. As described in the introduction, however, there are important differences between the structure of the Bertrand investment game and the all-pay auction.

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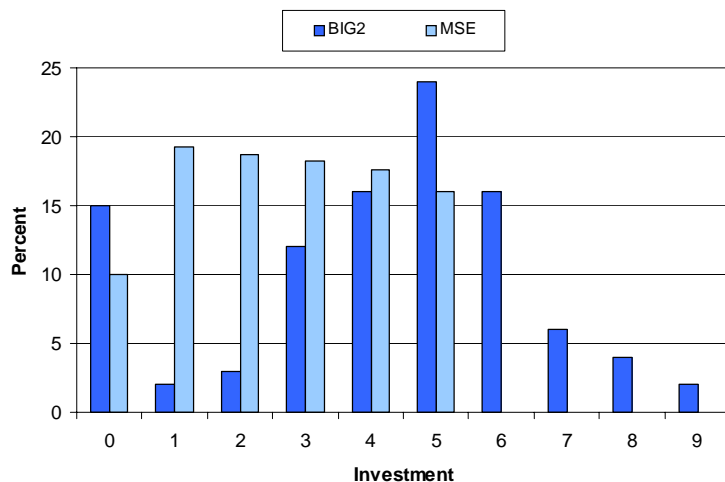
Figure 2: Investment distribution for BIG2.

Figure 2 shows that (i) the investment level of 5 is played in 24% of the cases. (ii) The investment level of 0 is chosen in 15% of the cases. (iii) In 28% of the cases a strategy that is not part of the symmetric MSE is played. We see that the observed investment levels are higher than predicted. Except for the investment level of 0, low investments are chosen less than predicted; high investments more often than predicted.

Qualitatively, the properties summarized in Result 2 also hold in most individual periods, not just in the aggregate.[14]

Interestingly, the heterogeneity of investments represents differences in individual investment propensities as much as heterogeneity in investments across time. Table 1 shows that the distribution of the mean investments per subject displays similar heterogeneity as the overall distribution of investments.[15] However, the two frequency distributions have qualitatively different features. Specifically, the former has a single global maximum in $[4, 5)$, whereas the latter has a local maximum in 0 apart from the global

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[14](i) In 19 periods the investment distribution exhibits a global maximum at 4 or 5. (ii) In 15 periods there is a local maximum at 0. (iii) The fraction of subjects investing more than 5 lies between 15% and 35% per period.

[15]Decomposing the variance into the variance of the average investments of players and the variances of individual players' investments shows that 39% come from the former source.

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maximum in 5.

Interval	$[0, 1)$	$[1, 2)$	$[2, 3)$	$[3, 4)$	$[4, 5)$	$[5, 6)$	$[6, 7)$	$[7, 9]$
Frequency	1	1	6	6	11	6	2	1

Table 1: Subject distribution for BIG2.

To show how player heterogeneity translates into net payoff differences, we first consider the relation between mean investments and mean losses. Figure 3 reveals that there is a clear positive relation.[16] This is closely related to the evolution of the mean net payoff over time. Figure 4 shows that the mean net payoff is negative in all periods, even towards the end of the game. Over all periods and subjects, the ratio between total bids (investment costs) and the prize (gross payoffs) is 1.56. The ratio in the first ten periods is higher than in the last ten periods ($1.85 > 1.35$). While still substantial, these values are considerably lower than those reported by Gneezy and Smorodinsky (2006) for the case of fixed prizes. There, depending on the treatment, total bids where still 2-3 times higher than the prize in the last period.
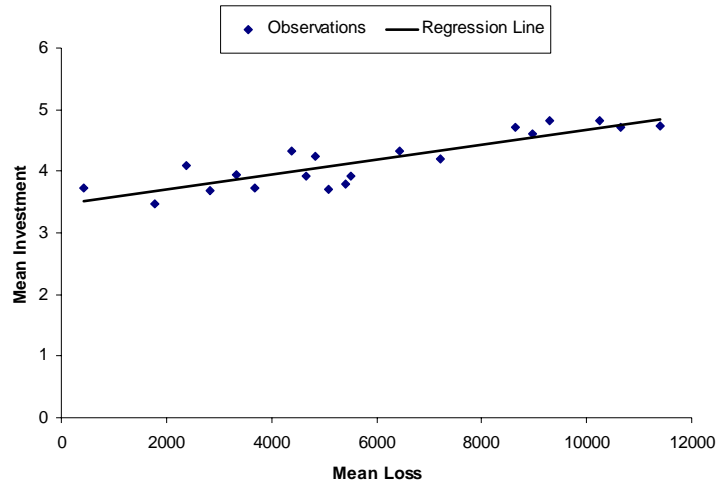
Figure 3: The relation between mean investments and losses for BIG2.

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[16] The regression analysis shows an R^2 of 0.77.
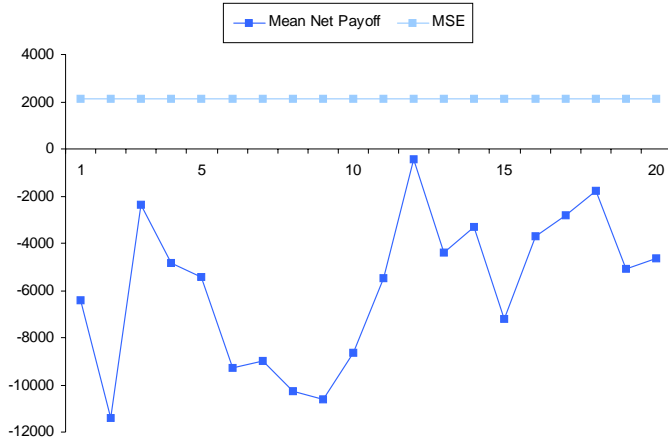
Figure 4: Mean net payoff for BIG2.

4.2 The 4-player Case

The analysis of the 4-player case leads to similar results as in the 2-player case, confirming the overinvestment behavior. We start with the comparison of predicted and observed mean investments.

Result 3 *Under BIG4, mean investments are higher than in the symmetric MSE.*

Figure 5 reveals that the mean investment level lies above the equilibrium investment level of 1.30 throughout the 20 periods. Note, however, that there is a downward tendency. The investments in the first ten periods are significantly higher than those in the last ten periods (Wilcoxon rank sum test, $p = 0.018$). Even in the final periods, investments stay above the equilibrium investment, though there is no significant difference for the last 5 periods (Wilcoxon rank sum test, $p = 0.116$).

Next, we deal with player heterogeneity. The properties of the investment distribution over all periods are summarized in Result 4.

Result 4 *Under BIG4, (i) the frequency distribution exhibits a global maximum at 0. (ii) There is a local maximum at 6. (iii) A substantial fraction of the subjects chooses strategies that are not part of the symmetric MSE, that is, invests more than 5.*

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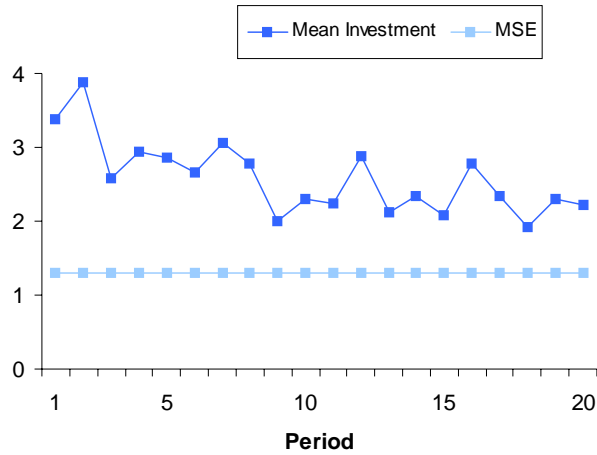
Figure 5: Mean investment for BIG4.

Figure 6 shows that (i) the investment level of 0 is played in 42% of the cases. (ii) The investment level of 6 is chosen in 11% of the cases. (iii) In 22% of the cases a strategy that is not part of the symmetric MSE is played. Again, there is overinvestment. Nevertheless, one aspect of the MSE is well reflected in behavior, namely the fact that the investment level of 0 is chosen in almost half of the cases.

The general patterns shown in Result 4 also hold in most individual periods.[17]

Table 2 shows that, as in BIG2, the distribution of investments reflects player heterogeneity to a large extent. However, contrary to BIG2, the majority of players now chooses very low average investments, with the mode in $[0, 1)$. 10 of the 36 players invest at most one unit on average. Similarly, 10 of the 36 players invest between 4 and 6 units on average.[18]

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[17]Specifically, (i) in all periods except the first one, the largest fraction of subjects chooses zero. (ii) In 14 periods, there is a local maximum at 5 or 6. (iii) The fraction of subjects investing more than 5 lies between 12% and 32% per period.

[18]The variance's decomposition shows that 48.5% comes from the mean investments.
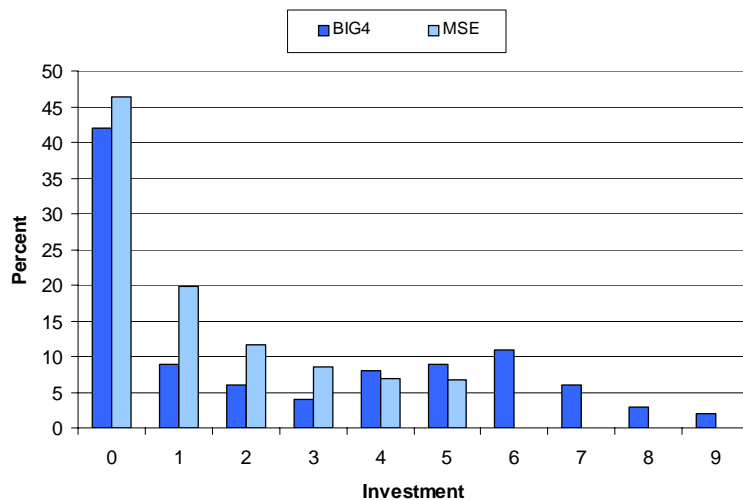
Figure 6: Investment distribution for BIG4.

Table 2: Subject distribution for BIG4.

Interval	$[0,1)$	$[1,2)$	$[2,3)$	$[3,4)$	$[4,5)$	$[5,6)$	$[6,7)$	$[7,9]$
Frequency	10	6	6	3	4	6	1	0

Finally, we consider the effect of overinvestment on net payoffs. Figure 7 plots mean investments against mean losses. A clear positive relation emerges.[19] The mean net payoffs over the 20 periods are shown in Figure 8. The mean net payoff is negative in all periods, implying that the overinvestment is not profitable. Over all periods and subjects, the ratio between bids and prizes is 1.92. Again, the ratio in the first ten periods is higher than in the last ten periods (2.12 > 1.75).

Both for BIG2 and BIG4, we have seen that, apart from zero investments, the symmetric MSE discussed above is not a perfect predictor for the observed investment behavior. BIG4 also has asymmetric MSEs (see Proposition 6) where some players put all weight on zero. This does not improve the fit, as this would lead to more weight on 0, rather than on 5. For instance, if

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[19]The regression analysis shows an R^2 of 0.87.
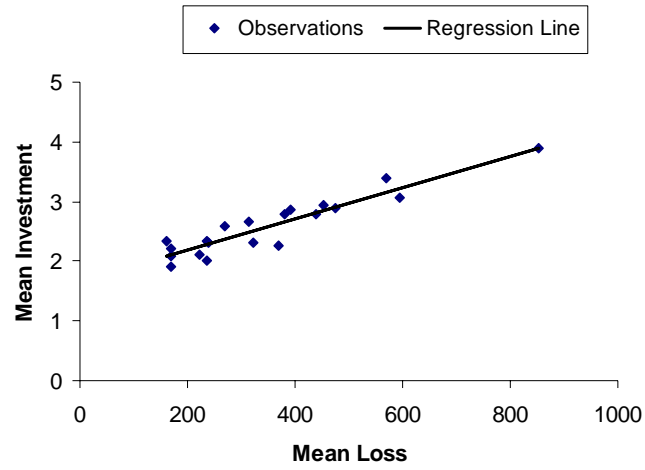
Figure 7: The relation between mean investments and losses for BIG4.
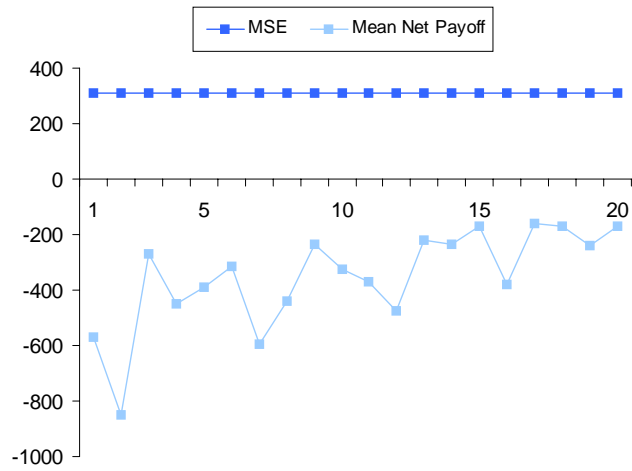
Figure 8: Mean net payoff for BIG4.

$J = 2$, the expected frequencies are given by

$$\left(\hat{p}_0^{BIG4}, ..., \hat{p}_9^{BIG4} \right) = (0.55, 0.095, 0.091, 0.087, 0.083, 0.094, 0, 0, 0, 0) . \quad (15)$$

The next section discusses MSEs based on alternative objective functions which shall help us to understand the investment behavior better.

5   Alternative Objective Functions

For the Bertrand game, we now consider the following modified objective function. The net payoff of firm i given in (9) is replaced by

$$\tilde{\Pi}_i(Y_1, ..., Y_I) = \begin{cases} (Y_i - Y^{(2)})D(c - Y^{(2)}) - kY_i^2 + \gamma, & \text{if } Y_i > Y^{(2)} \\ -kY_i^2 - \beta, & \text{if } Y_i \leq Y^{(2)} \wedge Y_i \neq 0 \end{cases} , \quad (16)$$

where $\gamma > 0$ and $\beta > 0$.

(16) captures the idea that one may derive utility from winning the auction (captured by the parameter γ), and disutility from bidding a positive amount in vain (captured by the parameter β). As we intend to explain overinvestment, we consider parameterizations where $\gamma > \beta$. Subjects might overinvest because they focus on winning on the investment race, neglecting investment costs. In the following, we illustrate the symmetric MSEs of this modified game for two parameterizations. We start with $\gamma = 100$, $\beta = 20$. For BIG2, the investments are shown in Figure 9.

We see that the predicted zero investments essentially coincide with the observed ones. Further, in contrast to the symmetric MSE of the previous section, the frequency distribution corresponding to the symmetric MSE of the modified game also has two maxima. In spite of this great advantage, the fit is far from perfect: The symmetric MSE has too much mass on very high investments, and it fails to predict the observed global maximum at 5.

The investments for $\gamma = 50$ and $\beta = 20$ are shown in Figure 10. The lower value of the γ-parameter implies that the equilibrium does not overpredict high values as much as in the case reflected in Figure 9. The symmetric MSE is shifted to the left. The global maximum at 0 is more pronounced (23% instead of 15%), whereas the local maximum is at 6. However, this improvement comes at a cost: The percentage of subjects choosing zero is now predicted less accurately. This trade-off also shows up in other parameterizations. It thus appears that, in spite of the additional degrees of freedom,
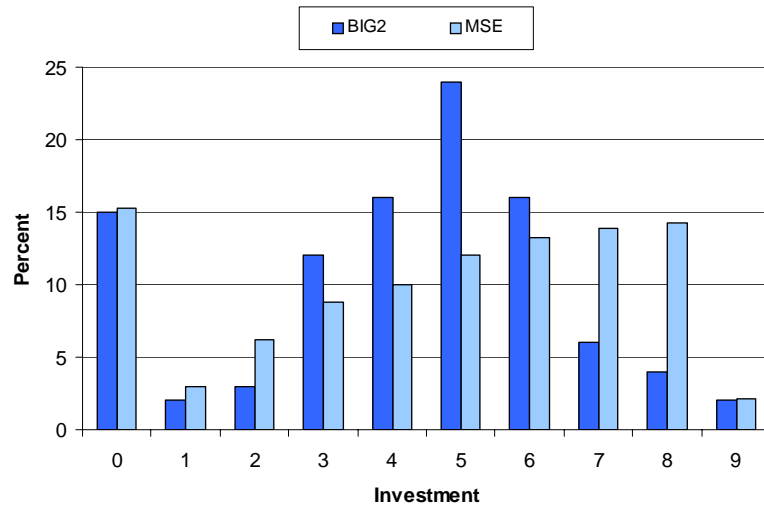
Figure 9: Investment distribution for BIG2 $(\gamma = 100, \beta = 20)$.
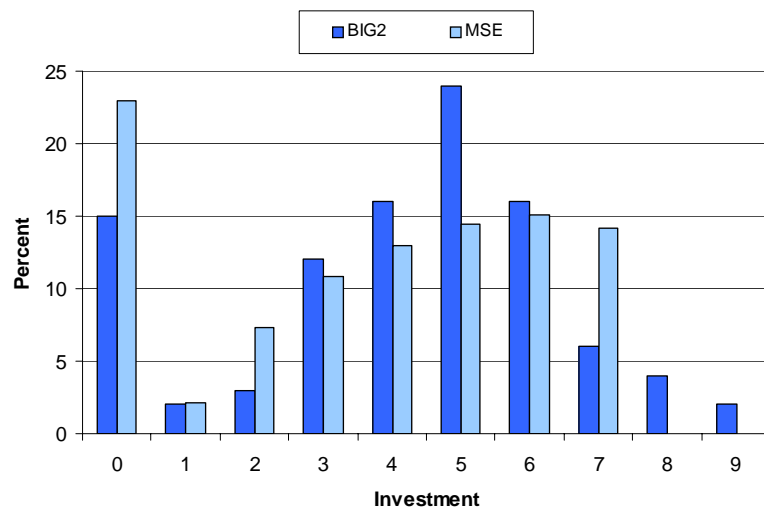

Figure 10: Investment distribution for BIG2 $(\gamma = 50, \beta = 20)$.

the modified equilibrium does not capture behavior in a fully satisfactory manner.

In the modified approach just described, subjects obtain some utility from investing more than the others even if the net payoff is negative.[20] As an alternative, we assume that the additional benefit γ arises only if the net payoff is positive. For the additional loss, the same as above holds. Figure 11 shows for $\gamma = 100$ and $\beta = 20$ the frequency distribution in the BIG2. In contrast to Figure 9, except for zero investments, the symmetric MSE is more concentrated on the left; the global maximum is at 3. A decrease to $\gamma = 50$ would shift the global maximum from 3 to 0.[21]
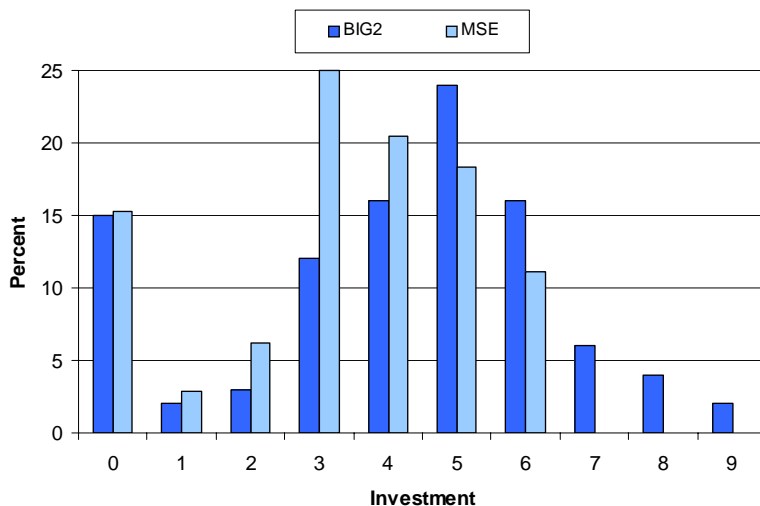
Figure 11: Investment distribution for BIG2 $\left(\gamma = 100, \beta = 20\right).$

Next, consider the 4-player case. In contrast to the 2-player case, the difference between the symmetric MSE with modified payoffs as in (16) and the standard symmetric MSE is very small. Figure 12 shows the investments for

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[20]Obviously, even then if the monetary losses from investing more than the others are sufficiently high relative to γ, the net payoff according to (16) may be negative.

[21]As a further alternative, we briefly mention the case where the additional benefit γ is given if the net payoff is positive and the additional loss β if the net payoff is negative. The results are qualitatively similar to those illustrated in Figure 11. However, a decrease in γ would not shift the global maximum, which is at 3 for both considered parameterizations.

$\gamma = 5$ and $\beta = 1$. In the symmetric MSE, players mix between all investment choices up to 6 instead of 5. Apart from that, there are no consistent differences: Low investments are chosen less than predicted, high investments more often. Further, a parameter change does not have a large impact. For $\gamma = 2.5$ and $\beta = 1$, the symmetric MSE-distribution is very similar to that of Figure 12.[22]
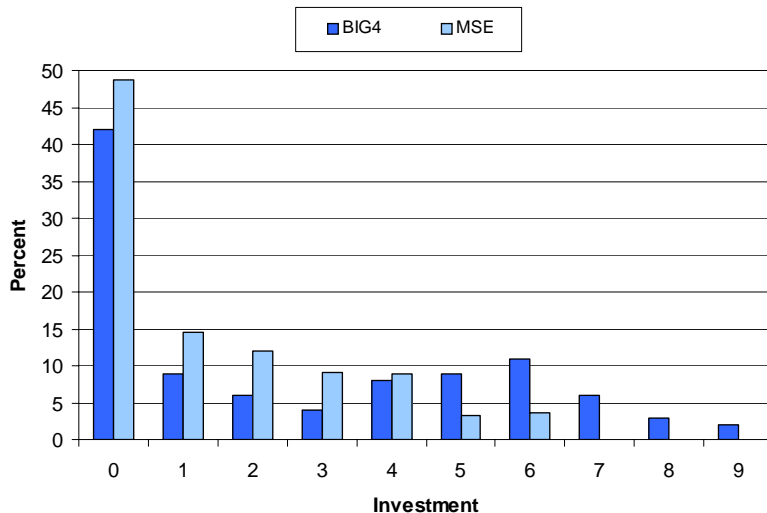

Figure 12: Investment distribution for BIG4 $(\gamma = 5, \beta = 1)$.

Summing up, the observed deviations from the symmetric MSE cannot be explained perfectly by a "joy of winning" or "a fear of losing", as proposed here. One reason may be that the approach does not allow for the asymmetries between players suggested by the experimental observations. It might therefore be useful to consider an alternative approach where the β and γ-parameters are allowed to vary across players, and calculate the Bayesian equilibrium for alternative parameterizations.[23] An alternative, more casual

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[22]Like for the two-player setting, we also considered the case where the additional benefit γ arises only if the net payoff is positive and another case, where γ is given if the net payoff is positive and β if the net payoff is negative. The MSE does not show remarkable differences with respect to Figure 12; we therefore omit additional considerations.

[23]Standard fixed-prize all-pay auctions have been analyzed as Bayesian games by Amann and Leininger (1996).

explanation that is also based on player heterogeneity could start from the observation that, at least in BIG2, there is a large concentration of players at 5, which is the best response to 0. This suggests that some players speculate that the opponent abstains from investing, and responds optimally to their own belief.

6   Conclusion

We have analyzed all-pay auctions, where the prize is a positive function of the own bid and a negative function of the other players' bids; it is zero when players' bids are identical. That is, the effort of one player has negative externalities on the prize that another player obtains. This negative effect of the own effort on the prize that another bidder gets differentiates our setting from standard all-pay auctions.

We showed that, contrary to the fixed-prize case, the game often has asymmetric PSEs. Like the fixed-prize auction, it has a symmetric MSE. The asymmetric MSEs that loom large in the fixed-prize case analyzed by Baye et al. (1996) do not exist, however.[24] We then provided an experimental analysis that is motivated by a particular example that corresponds to a reduced version of a Bertrand investment game. It turned out that, the symmetric MSE of this game predicts the percentage of zero bids very well. However, like in the fixed-prize case analyzed in earlier experiments (e.g., Gneezy and Smorodinsky, 2006), there is overinvestment, but it is less pronounced.

As the symmetric MSE resulting from the Bertrand investment game does not predict the investment behavior well, we considered alternative payoff functions. We extended the analysis to account for "joy of winning" and "fear of losing". For the 2-player setting, the symmetric MSEs obtained in this fashion reflect the investment behavior better, but not perfectly. For the 4-player setting, the symmetric MSEs based on the modified net payoff functions do not lead to substantial improvements.

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[24]However, there are alternative asymmetric MSE where some players mix over strategies up to a cut-off value and others always play zero.

Appendix

Proof of Proposition 2

(i) First, consider sufficiency. By (4),

$$\left(q_0, ..., q_{M-1}, 1 - \sum_{n=0}^{M-1} q_n, 0, ..., 0 \right) \tag{17}$$

defines a probability distribution. Together with the requirement that $q_n = p^{(n)}$, (3) for $n = 0$ guarantees that players are indifferent between strategies 0 and 1. A simple induction argument yields indifference between all strategies $0, 1, ..., M$: Suppose indifference obtains for some $n = m$, that is,

$$\sum_{n=0}^{m-1} q_n g(m, n) - k_m = 0. \tag{18}$$

Then,

$$\sum_{n=0}^{m} q_n g(m+1, n) - k_{m+1} = \sum_{n=0}^{m-1} q_n g(m, n) - k_m +$$

$$\sum_{n=0}^{m-1} q_n \left(g(m+1, n) - g(m, n) \right) + q_m g(m+1, n) - (k_{m+1} - k_m) = 0, \tag{19}$$

where the last equation follows from (3) and (18). The left hand side of (5) is the expected payoff that a player would obtain by choosing $M+1$ units, when the other players play the proposed equilibrium $(p_0, ..., p_{M-1}, 1 - p_{M-1}, 0, ..., 0)$ such that $p^{-(n)} = q_n$. Concavity of $g(n_i, n_j)$ and convexity of the function k_n imply that choosing arbitrary $n > M$ would lead to negative expected payoffs. By the standard characterization result for the MSE (Mas-Colell et al. 1995, Proposition 8D1), an MSE obtains. Necessity is immediate in view of this characterization result.

(ii) We show that (a) there exists no MSE without weight on zero, and (b) no equilibrium with $p_0 > 0$, $p_r > 0$ for some $r > 0$, and $p_s = 0$ for some $s \in \{1, ..., r - 1\}$; (c) At most one M-equilibrium can exist.

(a) Let $n > 0$ be minimal in p such that $q_n > 0$. Then the net payoff from choosing n when all other players choose p is $-k_n < 0$.

(b) Let s be minimal such that $p_s = 0$. Hence,

$$\sum_{n=0}^{s-1} q_n g(s-1, n) - k_{s-1} \geq 0 \geq \sum_{n=0}^{s} q_n g(s, n) - k_s = \sum_{n=0}^{s-1} q_n g(s, n) - k_s. \quad (20)$$

Therefore,

$$\sum_{n=0}^{s-1} q_n (g(s, n) - g(s-1, n)) - (k_s - k_{s-1}) \leq 0. \quad (21)$$

By concavity of g and convexity of k_n, we have

$$\sum_{n=0}^{s-1} q_n (g(s+1, n) - g(s, n)) - (k_{s+1} - k_s) \leq 0. \quad (22)$$

Using (20) and (22),

$$\sum_{n=0}^{s-1} q_n g(s+1, n) - k_{s+1} \leq 0. \quad (23)$$

Next, suppose

$$p_s = 0, ..., p_{s+l-1} = 0, \text{where } l = 2, ..., N - s. \quad (24)$$

Then,

$$\sum_{n=0}^{s-1} q_n g(s+l, n) - k_{s+l} =$$

$$\sum_{n=0}^{s-1} q_n g(s+l-1, n) - k_{s+l-1} +$$

$$\sum_{n=0}^{s-1} q_n (g(s+l, n) - g(s+l-1, n)) - (k_{s+l} - k_{s+l-1}) \leq 0. \quad (25)$$

(25) is non-positive by (24) and (22). Thus, $p_{s+l} = 0$.

(c) Suppose an M-equilibrium exists. Hence,

$$\sum_{n=0}^{M-1} q_n g(M, n) - k_M = 0. \quad (26)$$

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An L-equilibrium $(L < M)$ would require

$$\sum_{n=0}^{L-1} q_n g(M,n) + (1 - \sum_{n=0}^{L-1} q_n) g(M,L) - k_M \leq 0. \tag{27}$$

But,

$$(1 - \sum_{n=0}^{L-1} q_n) g(M,L) > \sum_{n=L}^{M-1} q_n g(M,L) > \sum_{n=L}^{M-1} q_n g(M,n). \tag{28}$$

Proof of Proposition 3

The J players who mix between all strategies between 0 and M must be indifferent between all these strategies. To see this, note that (iii) and (iv) imply

$$(P_{n-1})^{I-1} v - k_n = 0 \text{ for } n \in \{r, ..., M\}; \tag{29}$$

$$(P_r)^{I-J} (P_{n-1})^{J-1} v - k_n = 0 \text{ for } n \in \{1, ..., r-1\}. \tag{30}$$

As the left-hand sides of (29) and (30) are the expected payoffs of the corresponding strategies, the required indifference conditions hold. For $n > M$, expected payoffs are negative because $v > k_M$.

The $I - J$ remaining players must be indifferent between strategies 0 and $r, ..., M$. As strategies $n \in \{r, ..., M\}$ yield expected payoffs $(P_{n-1})^{I-1} v - k_n = 0$, the indifference condition holds. For strategies $n \in \{1, ..., r-1\}$, these players face a lower chance of having submitted the highest bid than those players that randomize over all strategies. Hence, using (30), their expected payoff is negative.

Proof of Proposition 4

Let $p^{-(n)}(\widetilde{p}^{-(n)})$ denote the probability that the highest bid of the opponents of a player who chooses $\mathbf{p}$ $(\widetilde{\mathbf{p}})$ is n. We shall show that, violating the requirement that both types of players obtain expected payoffs equal to k_r when they choose $b_i = r$, the following condition holds:

$$\widetilde{p}^{-(0)} g(r,0) + ... + \widetilde{p}^{-(r-1)} g(r,r-1) < p^{-(0)} g(r,0) + ... + p^{-(r-1)} g(r,r-1). \tag{31}$$

To see this, first note that (iii) implies

$$p^{-(0)} = \left(\sum_{n=0}^{r-1} p^{-(n)} - \sum_{n=1}^{r-1} p^{-(n)} \right); \tag{32}$$

27

$$\widetilde{p}^{-(0)} = \left(\sum_{n=0}^{r-1} p^{-(n)} - \sum_{n=1}^{r-1} \widetilde{p}^{-(n)} \right). \qquad (33)$$

Thus,

$$\sum_{n=0}^{r-1} \left(p^{-(n)} - \widetilde{p}^{-(n)} \right) g(r,n) =$$

$$\left(p^{-(0)} - \widetilde{p}^{-(0)} \right) g(r,0) + \sum_{n=1}^{r-1} \left(p^{-(n)} - \widetilde{p}^{-(n)} \right) g(r,n) =$$

$$\left(\sum_{n=1}^{r-1} \widetilde{p}^{-(n)} - \sum_{n=1}^{r-1} p^{-(n)} \right) g(r,0) + \sum_{n=1}^{r-1} \left(p^{-(n)} - \widetilde{p}^{-(n)} \right) g(r,n) =$$

$$\sum_{n=1}^{r-1} \left(p^{-(n)} - \widetilde{p}^{-(n)} \right) \left(g(r,n) - g(r,0) \right) > 0, \qquad (34)$$

where the last expression holds because $g(n_i, n_j)$ is strictly decreasing in n_j and (i) and (ii) imply $p^{-(n)} < \widetilde{p}^{-(n)}$ for all $n \in \{1, ..., r-1\}$.

Proof of Proposition 6

(i) Following the argument in the proof of Proposition (2), the conditions in (i) for the J active bidders show that these players obtain zero expected profits on strategies $0, 1, ..., M$.

(ii) Let $\hat{p}^{-(n)}$ denote the probability that the highest of the remaining bids for $I - J$ passive bidders is n. Because, compared with an active bidder, each passive bidder faces one more active bidder and one less passive bidder, $\hat{p}^{-(n)}$ stochastically dominates $p^{-(n)}$, that is, there exists an $r \in \{1, ..., M\}$ such that:

$$\hat{p}^{-(n)} < p^{-(n)} \text{ for } n \in \{0, ..., r-1\}; \qquad (35)$$

$$\hat{p}^{-(n)} > p^{-(n)} \text{ for } n \in \{r, ..., M\}; \qquad (36)$$

By (35),

$$\sum_{n=0}^{s-1} \hat{p}^{-(n)} g(s,n) < \sum_{n=0}^{s-1} p^{-(n)} g(s,n) \ \forall s \in \{0, ..., r\}. \qquad (37)$$

By (35) and (36),

$$\sum_{n=0}^{s-1} \hat{p}^{-(n)} g(s,n) - \sum_{n=0}^{r-1} p^{-(n)} g(r,n) =$$

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$$\sum_{n=0}^{r-1} \left(\hat{p}^{-(n)} - p^{-(n)} \right) g(s,n) - \sum_{n=r}^{s-1} \left(p^{-(n)} - \hat{p}^{-(n)} \right) g(s,n) <$$

$$g(s, r-1) \sum_{n=0}^{r-1} \left(\hat{p}^{-(n)} - p^{-(n)} \right) - g(s,r) \sum_{n=r}^{s-1} \left(p^{-(n)} - \hat{p}^{-(n)} \right) < 0. \qquad (38)$$

(38) holds because

$$g(s, r-1) > g(s,r) \qquad (39)$$

and

$$\sum_{n=0}^{r-1} \left(\hat{p}^{-(n)} - p^{-(n)} \right) > \sum_{n=r}^{s-1} \left(p^{-(n)} - \hat{p}^{-(n)} \right). \qquad (40)$$

The Bertrand Investment Game

Proposition 2 immediately allows us to characterize the symmetric MSE as follows.

Corollary 1 *A symmetric MSE of the BIG exists if and only if, for some $M \in \{1, ..., N\}$, there exists a sequence $(q_0, ..., q_{M-1})$ satisfying*

$$q_n = \frac{k_{n+1} - k_n - \sum_{n=0}^{M-1} q_m \left(\alpha + m \right)}{\left(\alpha + n \right)}, \qquad (41)$$

where

$$q_n \geq 0 \text{ for } n \leq M - 1, \ \sum_{n=0}^{M-1} q_n < 1; \qquad (42)$$

and

$$\sum_{n=0}^{M-1} q_n \left(M - n \right) \left(\alpha + n \right) + \left(1 - \sum_{n=0}^{M-1} q_n \right) \left(\alpha + M - 1 \right) - k_M < 0. \qquad (43)$$

The equilibrium is given as $(p_0, ..., p_{M-1}, 1 - p_{M-1}, 0, ..., 0)$ such that for $n = 0, ..., M - 1$ q_n is the probability that, given $(p_0, ..., p_n)$, the highest of $I - 1$ bids is n.

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