

Simulating a Simple Real Business Cycle Model using Excel

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Abstract

Simulating the real business cycle models is a popular topic in first-year graduate courses on macroeconomics. Usually, Maple and Matlab are used for this purpose, mainly because they can be used both for solving and for simulating the models. Strulik (2004) demonstrates that Excel can be used both for solving and for simulating a standard RBC model. In this paper, we propose a more elementary approach that might be suitable for undergraduate courses. We illustrate (i) how to solve a simple RBC model by hand and (ii) how to use Excel to simulate the solution.

Introduction

Simulating the real business cycle models (Kydland and Prescott, 1982; Long and Plosser, 1983) is a popular topic in first-year graduate courses on macroeconomics. Usually, Maple and Matlab are used for this purpose, mainly because they can be used both for solving and for simulating the models. Strulik (2004) demonstrates that Excel can be used both for solving and for simulating a standard RBC model.

In this paper, we propose a more elementary approach that might be suitable for undergraduate courses. We illustrate (i) how to solve a simple RBC model by hand and (ii) how to use Excel to simulate the solution.

A simple real business cycle model

Let $\alpha, \beta, \gamma, \delta, \rho \in (0,1)$ and $z_0, k_0, \sigma \in \Re^+$ be given. Let $\{\varepsilon_t\}_{t=1}^{\infty}$ and $\{z_t\}_{t=1}^{\infty}$ be stochastic processes such that for each t , ε_t is an *i.i.d.* drawn (at the beginning of period t) from the normal distribution $N(0, \sigma^2)$, and z_t is a random variable defined by

$$\log z_t = \rho \log z_{t-1} + \varepsilon_t.$$

Then, consider the problem of choosing a ‘plan’ $\{c_t, l_t, k_t\}_{t=0}^{\infty}$ that maximises the expected value (evaluated at the end of period 0) of

$$\sum_{t=0}^{\infty} \beta^t [\log c_t + \gamma \log(1 - l_t)]$$

subject to

$$c_t + k_{t+1} \leq z_t k_t^\alpha l_t^{1-\alpha} + (1 - \delta) k_t, \text{ for } t = 0, 1, 2,$$

The first order conditions to this problem are given by

$$(1) \quad \frac{1}{c_t} = \beta E_t \left[\frac{\alpha z_{t+1} k_{t+1}^{\alpha-1} l_{t+1}^{1-\alpha} + 1 - \delta}{c_{t+1}} \right],$$

$$(2) \quad c_t + k_{t+1} = z_t k_t^\alpha l_t^{1-\alpha} + (1 - \delta) k_t,$$

$$(3) \quad \frac{\gamma}{1 - l_t} = \frac{(1 - \alpha) z_t k_t^\alpha l_t^{-\alpha}}{c_t},$$

$$(4) \quad \log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}.$$

where $E_t[X]$ represents the expected value of the random variable X evaluated at the end of period t .

We linearise (1)–(4) around the ‘deterministic steady-state’ (c, l, k, z) defined by

$$\frac{1}{c} = \beta \cdot \frac{1 - \delta + \alpha z k^{\alpha-1} l^{1-\alpha}}{c},$$

$$c + k = (1 - \delta) k + z k^\alpha l^{1-\alpha},$$

$$\frac{\gamma}{1 - l} = \frac{(1 - \alpha) z k^\alpha l^{-\alpha}}{c},$$

$$\log z = \rho \log z.$$

From these equations, we get

$$\frac{k}{l} = \left(\frac{\alpha}{\beta(1 - \delta)} \right)^{\frac{1}{1-\alpha}} = \frac{(1 - \alpha) \left(\frac{k}{l} \right)^\alpha}{\left(\left(\frac{k}{l} \right)^\alpha - \delta \left(\frac{k}{l} \right) \right) \gamma + (1 - \alpha) \left(\frac{k}{l} \right)^\alpha},$$

$$c = \frac{(1-\alpha)\left(\frac{k}{l}\right)^\alpha (1-l)}{\gamma},$$

$$k = \frac{l\left(\frac{k}{l}\right)^\alpha - c}{\delta},$$

$$z = 1.$$

By linearising (1)-(4) around (c, l, k, z) , we get

$$(5) \quad -\frac{1}{c} \cdot \hat{c}_t = -\frac{\beta(1-\delta + \alpha z k^{\alpha-1} l^{1-\alpha})}{c} \cdot E_t[\hat{c}_{t+1}] + \frac{\beta(1-\alpha)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot E_t[\hat{z}_{t+1}]$$

$$+ \frac{\beta(\alpha-1)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \hat{k}_{t+1} + \frac{\beta\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot E_t[\hat{z}_{t+1}],$$

$$(6) \quad c\hat{c}_t + k\hat{k}_{t+1} = (1-\alpha)z k^\alpha l^{1-\alpha} \cdot \hat{l}_t + \frac{k}{\beta} \cdot \hat{k}_t + z k^\alpha l^{1-\alpha} \cdot \hat{z}_t,$$

$$(7) \quad -\frac{\gamma l}{(1-l)^2} \cdot \hat{l}_t = -\frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{c}_t + \frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{k}_t$$

$$- \frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{l}_t + \frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{z}_t,$$

$$(8) \quad \hat{z}_{t+1} = \rho \hat{z}_t + \varepsilon_{t+1},$$

where $\hat{c}_t \equiv \frac{c_t - c}{c}$, $\hat{l}_t \equiv \frac{l_t - l}{l}$, $\hat{k}_t \equiv \frac{k_t - k}{k}$, and $\hat{z}_t \equiv \frac{z_t - z}{z}$.

From (7), we get

$$\hat{l}_t = \frac{-\frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{c}_t + \frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{k}_t + \frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c} \cdot \hat{z}_t}{\frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c} - \frac{\gamma l}{(1-l)^2}}.$$

Thus, we can use it to eliminate \hat{l}_t and \hat{l}_{t-1} from (5) and (6).

Then, (5), (6) and (8) can be written in a matrix form as

$$(9) \quad \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \hat{c}_t \\ \hat{k}_{t+1} \\ \hat{z}_t \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & b_{11} & b_{13} \\ 0 & 0 & 0 \\ 1 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{t+1} \\ E_t[\hat{c}_{t+1}] - \hat{c}_{t+1} \\ E_t[\hat{z}_{t+1}] - \hat{z}_{t+1} \end{bmatrix},$$

where

$$a_{11} \equiv -\frac{1}{c},$$

$$b_{11} \equiv -\frac{\beta(1-\delta + \alpha z k^{\alpha-1} l^{1-\alpha})}{c} + \frac{\beta(1-\alpha)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c},$$

$$b_{12} \equiv \frac{\beta(\alpha-1)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} + \frac{\beta\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c},$$

$$b_{13} \equiv \frac{\beta\alpha z k^{\alpha-1} l^{1-\alpha}}{c} + \frac{\beta(1-\alpha)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c},$$

$$a_{21} \equiv c + \frac{(1-\alpha)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \frac{(1-\alpha)z k^\alpha l^{-\alpha}}{c},$$

$$a_{22} \equiv -\frac{k}{\beta} - \frac{(1-\alpha)\alpha z k^{\alpha-1} l^{1-\alpha}}{c} \cdot \frac{\alpha(1-\alpha)z k^\alpha l^{-\alpha}}{c},$$

$$b_{22} \equiv -k,$$

$$a_{33} \equiv \rho,$$

$$b_{33} \equiv -1.$$

In what follows, we use Farmer's (1999) method to solve the linearised system (9). From (9), we get

$$(10) \quad \begin{bmatrix} \hat{c}_t \\ \hat{k}_{t+1} \\ \hat{z}_t \end{bmatrix} = M_0 \begin{bmatrix} \hat{c}_{t+1} \\ \hat{k}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} + M_1 \begin{bmatrix} \varepsilon_{t+1} \\ E_t[\hat{c}_{t+1}] - \hat{c}_{t+1} \\ E_t[\hat{z}_{t+1}] - \hat{z}_{t+1} \end{bmatrix},$$

where

$$M_0 \equiv \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix},$$

$$M_1 \equiv \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}^{-1} \begin{bmatrix} 0 & b_{11} & b_{13} \\ 0 & 0 & 0 \\ 1 & 0 & b_{33} \end{bmatrix}.$$

Note that

$$\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 \\ -\frac{a_{21}}{a_{11}a_{22}} & \frac{1}{a_{22}} & -\frac{a_{23}}{a_{22}a_{33}} \\ 0 & 0 & \frac{1}{a_{33}} \end{bmatrix}$$

Thus,

$$M_0 = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}^{-1} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{a_{11}} & 0 & 0 \\ -\frac{a_{21}}{a_{11}a_{22}} & \frac{1}{a_{22}} & -\frac{a_{23}}{a_{22}a_{33}} \\ 0 & 0 & \frac{1}{a_{33}} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{b_{11}}{a_{11}} & \frac{b_{12}}{a_{11}} & \frac{b_{13}}{a_{11}} \\ -\frac{a_{21}b_{11}}{a_{11}a_{22}} & -\frac{a_{21}b_{12}}{a_{11}a_{22}} + \frac{b_{22}}{a_{22}} & -\frac{a_{21}b_{13}}{a_{11}a_{22}} - \frac{a_{23}b_{33}}{a_{22}a_{33}} \\ 0 & 0 & \frac{b_{33}}{a_{33}} \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{bmatrix}.$$

Note that

$$M_0 - \lambda I = \begin{bmatrix} m_{11} - \lambda & m_{12} & m_{13} \\ m_{21} & m_{22} - \lambda & m_{23} \\ 0 & 0 & m_{33} - \lambda \end{bmatrix}$$

and

$$|M_0 - \lambda I| = (m_{11} - \lambda)(m_{22} - \lambda)(m_{33} - \lambda) - m_{12}m_{21}(m_{33} - \lambda)$$

$$= (m_{33} - \lambda)(m_{11}m_{22} - m_{12}m_{21} - (m_{11} + m_{22})\lambda + \lambda^2).$$

Thus, the eigen values of M_0 and a corresponding matrix Q of eigen vectors are given by

$$\lambda_1 \equiv \frac{m_{11} + m_{22} - \sqrt{(m_{11} + m_{22})^2 - 4(m_{11}m_{22} - m_{12}m_{21})}}{2},$$

$$\lambda_2 \equiv \frac{m_{11} + m_{22} + \sqrt{(m_{11} + m_{22})^2 - 4(m_{11}m_{22} - m_{12}m_{21})}}{2},$$

$$\lambda_3 \equiv m_{33},$$

and

$$Q = \begin{bmatrix} m_{12} & m_{12} & \frac{m_{12}m_{23} - m_{13}(m_{22} - \lambda_3)}{(m_{11} - \lambda_3)(m_{21} - \lambda_3) - m_{12}m_{21}} \\ \lambda_1 - m_{11} & \lambda_2 - m_{11} & \frac{m_{21}m_{13} - m_{23}(m_{11} - \lambda_3)}{(m_{11} - \lambda_3)(m_{21} - \lambda_3) - m_{12}m_{21}} \\ 0 & 0 & 1 \end{bmatrix} \equiv \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ 0 & 0 & 1 \end{bmatrix}.$$

Then,

$$Q^{-1} = \frac{1}{q_{11}q_{22} - q_{12}q_{21}} \begin{bmatrix} q_{22} & -q_{12} & -q_{22}q_{13} + q_{12}q_{23} \\ -q_{21} & q_{11} & q_{21}q_{13} - q_{11}q_{23} \\ 0 & 0 & q_{11}q_{22} - q_{12}q_{21} \end{bmatrix},$$

and

$$M_0 = Q \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} Q^{-1}.$$

By pre-multiplying both sides of (10) by Q^{-1} , and by putting

$$\begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \end{bmatrix} \equiv Q^{-1} \begin{bmatrix} \hat{c}_t \\ \hat{k}_t \\ \hat{z}_t \end{bmatrix},$$

we obtain

$$\begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} x_{t+1}^1 \\ x_{t+1}^2 \\ x_{t+1}^3 \end{bmatrix} + Q^{-1} M_1 \begin{bmatrix} \varepsilon_{t+1} \\ E_t[\hat{c}_{t+1}] - \hat{c}_{t+1} \\ E_t[\hat{z}_{t+1}] - \hat{z}_{t+1} \end{bmatrix}.$$

Taking the expected value of both sides evaluated at the end of period t ,

$$\begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} E_t[x_{t+1}^1] \\ E_t[x_{t+1}^2] \\ E_t[x_{t+1}^3] \end{bmatrix}.$$

It follows that for each $T > 0$, $x_t^j = \lambda_1^T E_t[x_{t+T}^j]$. Also, it can be shown that $|\lambda_j| < 1$. Thus,

$$x_t^j = \lim_{T \rightarrow \infty} \lambda_1^T E_t[x_{t+T}^j] = 0,$$

so that

$$q_{22}\hat{c}_t - q_{12}\hat{k}_t + (q_{12}q_{23} - q_{22}q_{13})\hat{z}_t = 0.$$

Thus, the solution to the linearised system (10) is given by

$$(11) \quad \hat{c}_t = \frac{q_{12}}{q_{22}} \cdot \hat{k}_t + \left(q_{13} - \frac{q_{12}q_{23}}{q_{22}} \right) \cdot \hat{z}_t,$$

$$(12) \quad \hat{k}_{t+1} = \frac{a_{21}\hat{c}_t + a_{22}\hat{k}_t + a_{23}\hat{z}_t}{b_{22}},$$

$$(13) \quad \hat{z}_{t+1} = \rho\hat{z}_t + \varepsilon_{t+1}.$$

Recall that from (7), we have

$$(14) \quad \hat{l}_t = d_1\hat{c}_t + d_2\hat{k}_t + d_3\hat{z}_t,$$

where $d_1 = \frac{1}{c} \cdot \frac{\alpha(1-\alpha)zk^\alpha l^{-\alpha}}{\alpha(1-\alpha)zk^\alpha l^{-\alpha} - \gamma l} \cdot \frac{(1-\alpha)zk^\alpha l^{-\alpha}}{c}$,

$$d_2 = \frac{1}{c} \cdot \frac{\alpha(1-\alpha)zk^\alpha l^{-\alpha}}{\alpha(1-\alpha)zk^\alpha l^{-\alpha} - \gamma l} \cdot \frac{\alpha(1-\alpha)zk^\alpha l^{-\alpha}}{c},$$

$$d_3 = \frac{1}{c} \cdot \frac{(1-\alpha)zk^\alpha l^{-\alpha}}{\alpha(1-\alpha)zk^\alpha l^{-\alpha} - \gamma l} \cdot \frac{(1-\alpha)zk^\alpha l^{-\alpha}}{c}.$$

Given the values of z_0 , k_0 , and $\{x_t^j\}_{t=1}^\infty$, we can use (11)–(14) to simulate the evolution of c_t , l_t , k_t , and z_t .

Simulating the model using Excel

Step 1. (Figure 1) Choose the values of parameters, α , β , γ , δ , ρ , and σ . Here, we use the same values as in Table 2 of King and Rebelo (1999: 955).

	A	B	C	D	...
1	A simple real business cycle model				
2	Parameter values				
3	$\alpha =$	0.333			
4	$\beta =$	0.984			
5	$\gamma =$	3.48			
6	$\delta =$	0.025			
7	$\rho =$	0.979			
8	$\sigma =$	0.0072			
⋮					

Figure 1

Step 2. (Figure 2) Calculate the deterministic steady-state (c , l , k , z). Since

$$\frac{k}{l} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \right)^{\frac{1}{1-\alpha}},$$

enter

$$= (\$B\$3 / (1 / \$B\$4 - 1 + \$B\$6)) ^ {1 / (1 - \$B\$3)}$$

in cell D3. Similarly, enter the formulas for l , c , k , and z in cells D4–D7.

	A	B	C	D	...
1	A simple real business cycle model				
2	Parameter values		Deterministic steady-state		
3	$\alpha =$	0.333	$k/ell =$	22.892336	
4	$\beta =$	0.984	$ell =$	0.1936226	
5	$\gamma =$	3.48	$c =$	0.4383911	
6	$\delta =$	0.025	$k =$	4.4324736	
7	$\rho =$	0.979	$z =$	1	
8	$\sigma =$	0.0072			
⋮					

Figure 2

Step 3. (Figure 3) Enter the formulas for the components of matrices A , B , and M_0 , the eigen values of m_0 , and the components of matrix Q . For instance, since

$$\hat{z}_1 = \rho\hat{z}_0 + z\varepsilon_1,$$

$$\hat{k}_1 = \frac{a_{21}\hat{c}_0 + a_{22}\hat{k}_0 + a_{23}\hat{z}_0}{b_{22}},$$

enter

$$= -\$B\$17 * \$B\$15 / (\$B\$13 * \$B\$18) + \$B\$20 / \$B\$18$$

in cell D17.

Step 4. (Figure 4) Enter 0 in the cells from A30 to J30.

Step 5. Enter =A30+1 in cell A31.

Step 6. Enter =NORMINV(RAND(),0,\$B\$8^2) in cell B31.

Step 7. (Figure 5) Enter the formulas for \hat{z}_1 , \hat{k}_1 , \hat{c}_1 , and \hat{l}_1 in cells C31–F31. For instance, since

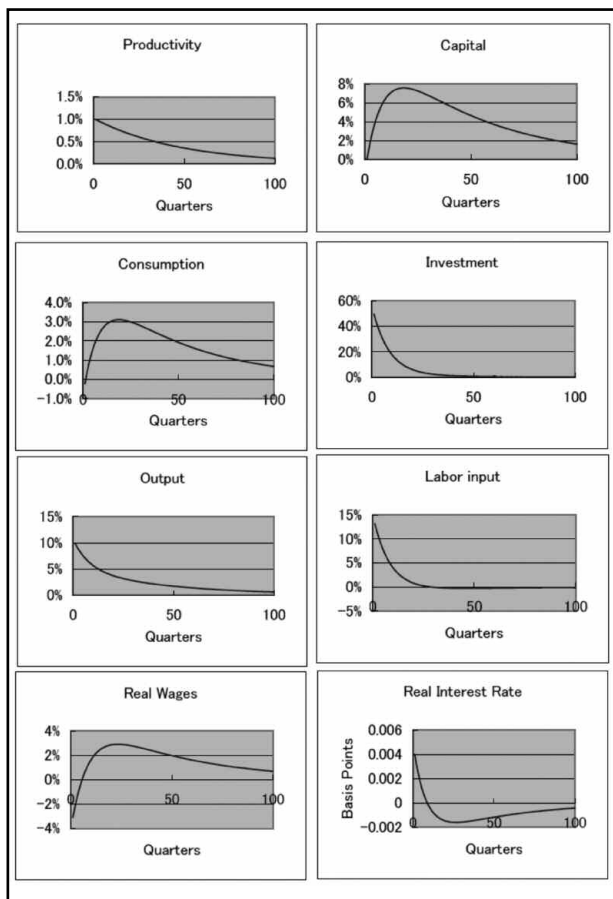


Figure 7

Notes

¹ Sample Excel files are available at <http://member.social.tsukuba.ac.jp/hokari/>

References

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