Using Computer Algebra Packages to Complement the Spreadsheet Construction of Binomial Option Trees: The Example of Mathcad

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Abstract

In this paper we show how the mathematical programming package Mathcad can be used to complement the construction of a binomial option tree in Excel. A binomial option tree is first constructed in Excel using standard spreadsheet 'cut and paste' operations. The same binomial tree is then constructed in Mathcad. We conclude that spreadsheet construction of the tree provides students with a sound concept of the underlying mechanics of the option tree. Additionally, the Mathcad construction reinforces the mathematical notation found in many option pricing texts (e.g. summation signs and indices) and allows for the construction of a more flexible lattice that may be easily altered (e.g. the number of steps). In the process students are provided with an understanding of how to construct option trees in the increasingly important world of computer algebra packages.

Introduction

This paper shows how the computer algebra package 'Mathcad' can be used to complement the spreadsheet (Excel) construction of a binomial option tree. We first describe the basic option problem under consideration and construct a basic binomial tree with Excel. We then construct the same tree with Mathcad. The construction of binomial trees in Excel is performed through the common 'cut and paste' operations shown in many texts (e.g. Benninga, 1998). However, the Mathcad program is constructed using the mathematical symbols found in many financial derivative texts. As a result, Mathcad construction reinforces the mathematical notation underlying binomial option pricing and introduces the programming concepts of computer algebra packages. Also, the Mathcad program incorporates a high degree of flexibility, which allows students easily to examine how increases in the number of steps affect the option price and how the binomial price may converge to the Black and Scholes model price. The use of Mathcad is most appropriate for advanced undergraduates or graduates who have constructed option trees in a spreadsheet environment but who wish to extend their programming skills to the more powerful computer algebra packages increasingly used by financial institutions.

Building the model in Excel

1. Basic values

We consider a basic European call option. The example is based on Figlewski *et al*. (1990). The binomial option tree as constructed in Excel is provided in Exhibit 1A (a formula view is shown in Exhibit 1B). The first step is to define the basic values. These include the current stock price (p=\$100), the exercise price (x=\$100), the time until the expiration of the option (t=91 days), the annual risk-free rate (i=.06), the stock volatility (v=.15), and the number of steps (n=3).

2. Per step values

The second step is to calculate the 'per step' values that are used for building the option tree. The percentage of year represented by each step (st) is:
$$st=(t/365)/n = .083105.$$

The compounded rate of return is:
$$r=\ln(1+i)*st = .004842.$$

The volatility is normalised by the square root of the step:
$$vol=v*sqrt(st) = .043242.$$

Finally, the up and down movements are:
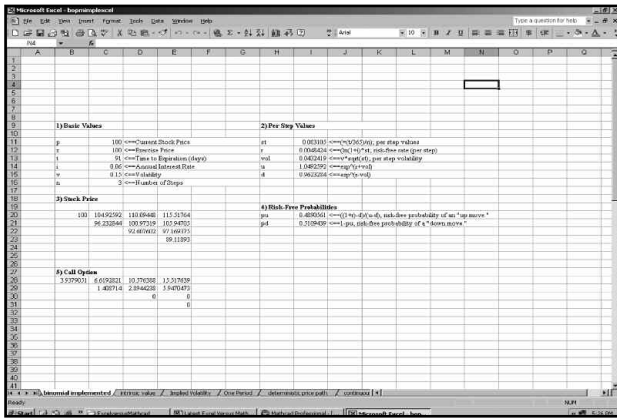$$u=\exp^{r+vol} =1.049259; \quad d= \exp^{r-vol} = .962328.$$
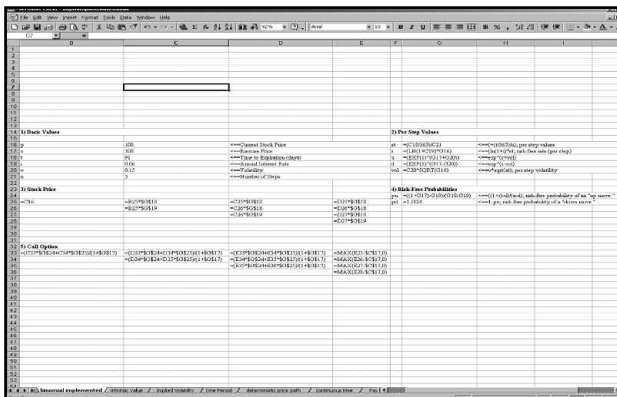
Exhibit 1A. Construction of option lattice on EXCEL

Exhibit 1B. Construction of option lattice on EXCEL (formula view)

3. Stock price tree

The next step is to build the binomial tree for the stock price. The initial stock price (x) is $100. The up price at t=1 (104.9259) is calculated by multiplying the price at t=0 (i.e $100) by the 'up movement' (i.e. u=1.049259). The 'down price' at t=1 (96.23284) is calculated by multiplying the price at t=0 (i.e $100) by the 'down movement' (i.e. d=.962328). The other stock prices are calculated in a similar manner.

4. Risk-free probabilities

Next, the risk-free probabilities (part 4) must be defined prior to constructing the lattice for the call option. The 'up' risk-free probability is: pu=[(1+r)-d]/(u-d)=.489056.

The 'down' risk-free probability is:
pd=1-pu=.510944.

5. Call option tree

The binomial lattice for the call option is shown in part 5. The terminal prices for the call at t=3 are calculated by the formula: Max(s-x,0). The terminal price for the option after 'zero' up moves and three down moves is 0. This amount is

Max(s-x,0)=Max((89.11893-100),0)= 0.

The remainder of the call option prices are calculated by using 'backward induction' and applying the formula:

Callprice(t=n) = ((pu * callprice (n+1)) + (pd * callprice (n+1)).
$$\frac{}{1+r}$$

For example, the call price after two up moves and one down move is 2.894424.

This is calculated by:

Callprice(t=n) = (pu * callprice (n+1) + pd * callprice (n+1)).
$$\frac{}{1+r}$$

$$= \frac{(.489056*5.947047+.510944*0)}{1.004842}$$

= 2.894424.

The option tree provides a current option value of 3.937905. After the construction of the model, students may vary basic values to examine the impact of parameter changes on the option price.

Building the model in Mathcad

As shown above, using standard, copy, cut and paste operations, Excel can be used to teach students the basics of binomial option pricing models. Exhibit 2 shows how the same model can be built in Mathcad. As stated above, Mathcad construction is based on mathematical notation found in many finance texts.

1. Basic values

The basic values are entered directly into the worksheet as shown in Exhibit 2.
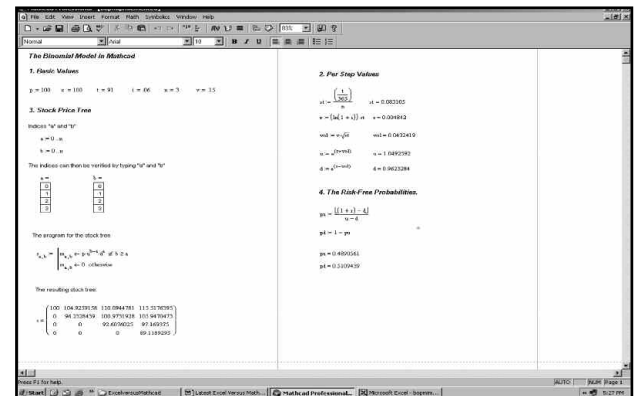

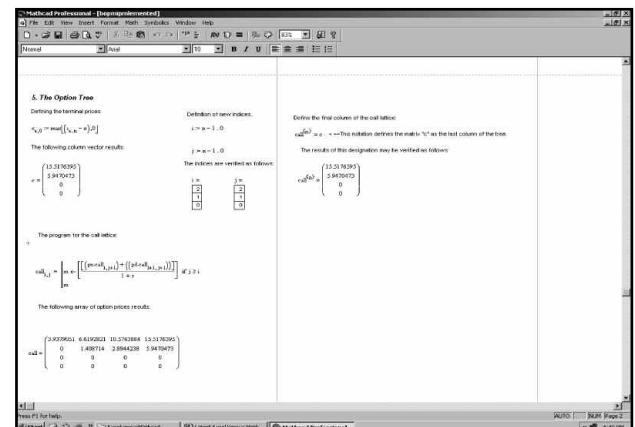Exhibit 2. Construction of the model in Mathcad

Exhibit 2. Continued

2. Per step values

Part 2 of Exhibit 2 shows the per step values. In Mathcad the formulas are written directly onto the worksheet. The results can then be checked by typing the variable.

3. The Stock Tree

Part 3 of Exhibit 2 shows the stock price tree. For this purpose, two indices 'a' and 'b' are created. The index 'a' refers to the number of rows in the array and the index 'b' is the number of columns (steps) in the array. Each of the indices starts at '0' and ends at 'n' (the number of periods). The indices can be verified simply by typing in the letters 'a' and 'b.'

The stock price tree is then constructed on the basis of the two-line program labelled 's'. The stock price tree, shown in a matrix labelled 's,' is located directly under the program.

The tree is based on the indices ('a' and 'b') and on the first line of the program 's'. For example, the stock price for the cell a=2 b = 3 is 97.1693. This price is p (100) multiplied by u (1.0492) raised to 'b-a' (i.e. 3-2) times d (.9623) raised to 'a' (i.e. 2).

4. Risk-free probabilities

Part 4 of Exhibit 2 defines the risk-free probabilities as in the Excel worksheet.

5. The option tree

Part 5 of Exhibit 2 shows the construction of the option lattice. The next step is to define a vector of terminal prices labelled 'c.' The equation (labelled 'c') is based on the same equation for the terminal prices in EXCEL. The vector 'c' results. The first step is to define two new indices, 'i' and 'j.' However, since the construction of the option lattice is performed through backward induction, the indices (are 'backwards' and) now start at 'n' and end at '0'. The indices may be verified simply by typing in the variables 'i' and 'j'.

Finally, the option tree is constructed. Two steps are needed here. First, a line of code shown as 'call$^{<n>}$:= c' defines the vector 'c' as the last column of the tree. Next the two-line program labelled 'call' is defined. The program directly incorporates the logic of the backward induction in the pricing of call options. For example, the option value for the cell i=1 j=2 is 2.8944. This cell is based on a fraction defined by the first line of code in the program 'call'. The numerator is the sum of the 'up' probability (pu=.489) times the cell corresponding with 'i' (i.e. 1) and 'j+1' (i.e. 3) (value = 5.947) and the 'down' probability (pd = .5109) times the cell corresponding with 'i+1' (i.e. 2) and 'j+1' (i.e. 3) (value = 0). The denominator is the 'one' plus the interest rate 'r' (.004842). The formula for this cell is: $call_{1,2}$ = (.4891*5.947 + .5109 * 0)/1.004842 = 2.8944. The other option prices are similarly defined.

	0	1	2	3	4	5	6	7	8	9	10	11
0	3.79	5.213	6.983	9.094	11.499	14.116	16.858	19.674	22.564	25.532	28.579	
1	0	2.412	3.499	4.942	6.772	8.976	11.479	14.157	16.907	19.731	22.63	
2	0	0	1.358	2.101	3.168	4.639	6.558	8.896	11.512	14.199	16.957	
3	0	0	0	0.637	1.064	1.741	2.779	4.294	6.366	8.922	11.546	
4	0	0	0	0	0.222	0.406	0.733	1.306	2.284	3.889	6.385	
5	0	0	0	0	0	0.043	0.087	0.176	0.366	0.722	1.463	
6	0	0	0	0	0	0	0	0	0	0	0	
7	0	0	0	0	0	0	0	0	0	0	0	
8	0	0	0	0	0	0	0	0	0	0	0	
9	0	0	0	0	0	0	0	0	0	0	0	
10	0	0	0	0	0	0	0	0	0	0	0	

(call =)

Exhibit 3. Lattice with ten steps

As with Excel, the initial parameters may be changed to show the impact of changes in these parameters on the option price. However, with the Mathcad program, the lattice tree can be extended simply by modifying the variable 'n'. (Exhibit 3 shows the lattice with n=10).Thus, students can see the impact of parameter changes on larger trees and explore how the number of steps affects convergence toward the Black and Scholes price.

We have found that Excel provides a good environment for teaching the basics of option pricing. However, we have used Mathcad to extend students' understanding of how derivative pricing is performed in computer algebra packages (e.g. Mathematica, Matlab). Also, students have indicated an appreciation of how Mathcad reinforces the mathematical notation found in textbooks for financial derivatives.

References

Benninga, S. (1998) *Financial Modeling*. Cambridge MA, MIT Press.

Figlewski, S. (1990) 'Theoretical Valuation Models', 77–134, in Figlewski, S., Silver, W.L. and Subrahmanyam, M.G., *Financial Options: From Theory to Practice*. New York: McGraw-Hill.

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