

# **Immigration, Fiscal Policy, and Welfare in an Aging Population**

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## **Abstract**

I evaluate the welfare effects of exogenous changes in immigration policy by constructing a heterogeneous agent overlapping generations model with agents differing in age, origin, and skills. Calibrating the model to Germany, I match the main features of the social security and tax systems, and account for differences in inter-generational transmission of skills and fertility between immigrants and natives. I find that a prohibition on immigration reduces welfare for the natives, whereas a policy that allows an annual inflow equal to 0.4 percent of the population increases welfare for all agents on the new balanced growth path (by 0.1 to 2.8 percent depending on the type of the agent). Interactions between the social security system, taxes, and equilibrium prices are crucial: immigration reduces wages, but raises the rental rate of capital and the number of workers per retiree, allowing for higher pension benefits and a lower consumption tax rate.

JEL Classification: D6, D58, E62, F22, J11

Key Words: Immigration Policy, Social Security, Aging, Overlapping Generations Model, Welfare

# Immigration, Fiscal Policy, and Welfare in an Aging Population

## 1 Introduction

Does immigration reduce or increase the welfare of current and future generations when an economy experiences rapid aging of its population? Many studies in the literature provide mixed evidence by focusing on individual aspects of the problem, such as fiscal sustainability through immigration or the labor market effects of immigrants.

This paper quantifies the effects of changes in immigration policy on individual welfare by constructing a five-period life-cycle general equilibrium model. It contributes to the literature by explicitly accounting for the interactions between the labor market and the three pillars of fiscal policy: the tax system, the social security system, and the immigration policy. The model is calibrated to the German economy, which provides a natural experiment. Germany has historically been the major destination for immigrants in Europe and it provides rich micro-data on immigrants and natives that is necessary to identify main characteristics of the two groups, such as differences in skills and fertility. Moreover, Germany is one of the most prominent cases of aging: the ratio of the population aged 65 and older to those aged 15 to 65 (the dependency ratio) is estimated to increase from 28 percent to 50 percent in the next 45 years.<sup>1</sup>

Perhaps Germany is the most interesting economy to study due to recent changes in tax and social security policies in response to aging. First, marginal tax rates on labor income were reduced dramatically to increase the supply of labor (the top and bottom rates fell by 11 percent). Contemporaneously, immigration policy was reformed to favor inflow of high-skilled workers. Finally, the pension benefit formula was modified to include a “sustainability factor,” which reduces payments to retirees when the dependency ratio increases. The model presented here incorporates these elements.

We take each model period to be 20 years. Agents differ in origin (immigrant or native), age, and skill level (low or high). Life span is uncertain. The model replicates key features of tax and pension systems. In particular, there are marginal labor income

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<sup>1</sup> Börsch-Supan and Wilke (2003) report that before the pension system was reformed in 2001 and 2005, projections indicated that the payroll tax rate needed to finance German pensions would increase from today's 19.5 percent to more than 28 percent in 2040, if the benefit levels and labor force participation rates were maintained.

and payroll taxes, and pensions are indexed to lifetime earnings via a benefit calculation formula. A key element of the model is the number of people of each type in each period. To make the evolution of the distribution of population precise, I calculate skill- and origin-specific fertility from the data as well as a Markov skill transition matrix, which shows the probability that a child of a particular type of parent will have high or low skill. The model is calibrated by using individual level micro-data from the German Socio-Economic Panel.

In the baseline, I assume that the economy is on the balanced growth path with annual inflow of low- and high-skilled immigrants equal to 0.1 and 0.01 percent of the population, respectively. Combined, this corresponds to an initial annual net inflow of 83,000 people, which is in line with the data in 2005 provided by the German Statistical Institute.<sup>2</sup> I solve for the equilibrium transition path of the economy and report results from six different experiments in which either the skill composition or the total inflow of the immigrants change.

There are three important conclusions. First, higher immigrant inflows increase welfare by causing a decline in the dependency ratio, which allows the government to balance its budget with a lower consumption tax rate. For example, when annual inflows of young working-age immigrants of each skill level are 0.2 percent of the population, welfare of initial young natives is 2.8 percent higher compared to welfare under the baseline economy. The increase is around 2.0 percent for 40-59 year old natives, 0.9 percent for 60-79 year old natives, and 0.1 percent for the initial old generation. Second, increases in life-expectancy make the returns from immigration higher: under the same immigrant inflow, when the survival probability to age 80+ is doubled, welfare increases by a bigger factor (3.1 percent for the initial young). Third, keeping the total inflow constant, reversing the skill composition of immigrants improves welfare by the same order of magnitude.

In this paper, increased immigration not only raises the size of the labor force, but also lowers real wages. This negatively affects pension benefits, as benefits are strongly linked to past wages. However, there are two other impacts of immigration. The first is the rise in return to capital, which raises the return on savings. The second effect comes

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<sup>2</sup> Genesis (The Federal Statistical Office of Germany database system).

through “the sustainability factor” in the pension benefit calculation formula. Specifically, when aging leads to an increase in the retiree-contributor ratio, the sustainability factor reduces the value of each pension point earned, which in turn reduces the pension benefit. Hence, when the economy experiences a higher immigrant inflow, pension benefits rise, as the number of pensioners per worker declines. In equilibrium, these two positive effects dominate the negative effect of declining wages, and therefore the consumption of a retiree increases.

This study contributes to a large literature in public finance that focuses on the relationship between demographic transition, fiscal sustainability, and immigration. Storesletten (2000) and Bonin, Raffelhueschen, and Walliser (2000) ask whether immigrants can help sustain fiscal policy, without a specific emphasis on individual welfare. The former uses a calibrated dynamic general equilibrium model and concludes that a policy that admits 1.6 million 40-44-year-old high-skilled immigrants annually could resolve the fiscal problems associated with the aging of the baby boom generation in the U.S. The latter uses the generational accounting framework (Auerbach, Gokhale, and Kotlikoff, 1994) and finds that immigration can only partially decrease the fiscal burden of future generations induced by aging in Germany. Others study labor-market effects of immigrants with an emphasis on wages. Pischke and Velling (1997) finds no evidence of detrimental effects on native wages by using data on Germany, while Borjas (2003) concludes that a 10 percent increase in supply reduces wages by 3 to 4 percent in the U.S.

Krueger (1999), Fehr (2000), Imrohoroglu, Imrohoroglu, and Joines (1995), Imrohoroglu, Imrohoroglu, and Fuster (2003) focus on quantifying welfare effects of social security without explicitly modeling immigration policy. A three-region world economy (U.S., Japan, and the Euro-region) model with labor immobility appears in Fehr, Jokisch, and Kotlikoff (2005), which concludes that, independent of the skill level, an expansion of immigration will not alter the capital shortage, tax hikes, and the reduction in real wages expected along the demographic transition. My approach in this paper is unique, as the impact of immigration on welfare is studied by incorporating the details of the social security system and the tax policy in the model, which are necessary to identify the interaction among different components of fiscal policy.



## 2 The Model

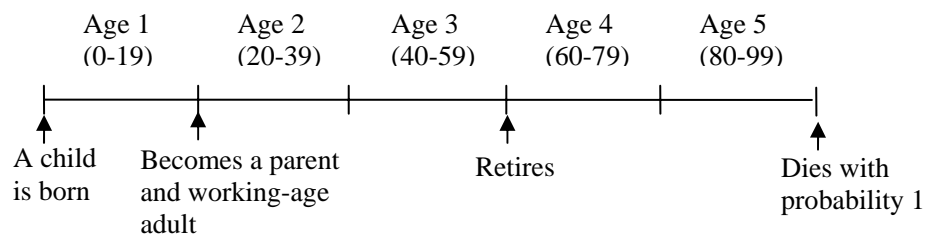
The economy consists of three sectors: heterogeneous individuals with elastic labor supply, a perfectly competitive representative firm with constant returns to scale production technology, and a government that balances its budget.

### 2.1 Individuals

Individuals live for a maximum of five periods and differ with respect to age, origin, and working ability. A model period is 20 years. In period one (youth), individuals do not work or save. They derive consumption from the transfer income paid by the government. In periods two and three (adulthood), individuals choose optimal consumption  $c$ , working hours  $n$ , and end-of-period wealth holding  $a$ , taking the taxes, factor prices, and the immigration policy as given. They pay payroll and labor income taxes with marginal rates  $\tau_p$  and  $\tau$ , respectively. Consumption and capital income are taxed at flat rates  $\tau_c$  and  $\tau_k$ . Individuals can have children only in the second period, between the ages of 20 and 39. The fertility rate is exogenous. In periods four and five (retirement), individuals do not work. Hence, consumption is derived from government transfers  $\chi$  and proceeds from assets. There is longevity uncertainty. The conditional probability of surviving from age  $i$  to  $i+1$  is  $\lambda_i$ . In case of accidental death, individual wealth becomes a part of the government's revenue.

The type of an agent alive in  $t$  is denoted by  $(i, o, s)$ , where  $i \in \{1, 2, 3, 4, 5\}$  is age;  $o \in \{m, n\}$  is origin,  $o = n$  for natives and  $o = m$  for immigrants; and  $s \in \{l, h\}$  is working ability (or productivity),  $s = l$  for low-ability,  $s = h$  for high-ability.

Figure 1: The Life-cycle of an Individual



An agent's efficiency units,  $e_{i,o,s}$ , is exogenously given. Wage per efficiency unit is  $w$ . Therefore,  $w_t e_{i,o,s} n_{i,o,s,t}$  is the total labor income of an agent who supplies  $n$  units of labor at  $t$ . A tax function  $T$  computes total taxes paid on labor income (the sum of payroll and income taxes). A pension function  $P$  computes the benefit for an individual as a function of his labor earnings and average earnings  $\bar{y}$  in the economy during adulthood. Tables 1 and 2 show the main variables and functions used in the model.

### 2.1.1 The Individual's Problem

An individual born at  $t$  chooses  $\{n_{i,o,s,t}, c_{i,o,s,t}, a_{i+1,o,s,t}\}_{i,o,s,t}$  to maximize expected utility

$$E \left( \frac{[c_{1,o,s,t}^\alpha]^{1-\gamma}}{1-\gamma} + \sum_{j=1}^4 \left( \prod_{k=2}^{j+1} \lambda_k \right) \frac{\beta^j [c_{j+1,o,s,t+j}^\alpha (1-n_{j+1,o,s,t+j})^{1-\alpha}]^{1-\gamma}}{1-\gamma} \right)$$

subject to

$$c_{1,o,s,t} (1 + \tau_{c,t}) = \chi_{1,o,t}$$

$$c_{2,o,s,t+1} (1 + \tau_{c,t+1}) + a_{3,o,s,t+1} \leq w_{t+1} e_{2,o,s} n_{2,o,s,t+1} + \chi_{2,o,t+1} - T(w_{t+1} e_{2,o,s} n_{2,o,s,t+1})$$

$$c_{3,o,s,t+2} (1 + \tau_{c,t+2}) + a_{4,o,s,t+2} \leq w_{t+2} e_{3,o,s} n_{3,o,s,t+2} + \chi_{3,o,t+2} - T(w_{t+2} e_{3,o,s} n_{3,o,s,t+2}) \\ + (1 + (1 - \tau_k) r_{t+2}) a_{3,o,s,t+2}$$

$$c_{4,o,s,t+3} (1 + \tau_{c,t+3}) + a_{5,o,s,t+3} \leq (1 + (1 - \tau_k) r_{t+3}) a_{4,o,s,t+3} + \chi_{4,o,t+3} \\ + P(w_{t+1} e_{2,o,s} n_{2,o,s,t+1}, w_{t+2} e_{3,o,s} n_{3,o,s,t+2}, \bar{y}_{t+1}, \bar{y}_{t+2})$$

$$c_{5,o,s,t+4} (1 + \tau_{c,t+4}) \leq (1 + (1 - \tau_k) r_{t+4}) a_{5,o,s,t+3} + \chi_{4,o,t+3} \\ + P(w_{t+1} e_{2,o,s} n_{2,o,s,t+1}, w_{t+2} e_{3,o,s} n_{3,o,s,t+2}, \bar{y}_{t+1}, \bar{y}_{t+2})$$

$$n_{4,o,s,t+2} = n_{5,o,s,t+3} = 0.$$

### 2.1.2 Skill Transmission and the Measure of Newborns

I assume that children of immigrants are natives and that individuals can have children only in the second period of their lives. Transmission of skills from parents to children follows a Markov process. Let  $\varphi_{o,s}$  denote the number of children per person of origin- $o$  and skill- $s$ . Let  $\mu_{2,o,s,t}$  denote the number of child-bearing individuals at time  $t$ . Let  $\pi_{o,s}$

be the probability that a parent of origin- $o$  and skill- $s$  will have a high-skilled child. Then, the number of newborns of each skill level is:

$$\begin{aligned}\mu_{1,n,h,t} &= \sum_{s \in \{l,h\}} \left( \varphi_{n,s} \mu_{2,n,s,t} \pi_{n,s} + \varphi_{m,s} \mu_{2,m,s,t} \pi_{m,s} \right) \\ \mu_{1,n,l,t} &= \sum_{s \in \{l,h\}} \left( \varphi_{n,s} \mu_{2,n,s,t} (1 - \pi_{n,s}) + \varphi_{m,s} \mu_{2,m,s,t} (1 - \pi_{m,s}) \right).\end{aligned}$$

Skills are assumed to be exogenous and fixed during an agent's lifetime.

## 2.2 The Firm's Problem

The representative firm hires labor ( $N_t$ ) and capital ( $K_t$ ) to produce output. The production function is Cobb-Douglas:

$$Y_t = K_t^\theta (z_t N_t)^{1-\theta}, \quad (1)$$

where  $z_t$  is an exogenous labor augmenting productivity process with deterministic growth rate  $\Gamma$ . The aggregate labor input  $N_t$  is the sum of efficiency units supplied by each agent in the economy:

$$N_t = \sum_{i \in \{2,3\}} \sum_{o \in \{m,n\}} \sum_{s \in \{l,h\}} e_{i,o,s} n_{i,o,s,t} \mu_{i,o,s,t}. \quad (2)$$

The aggregate capital input  $K_t$  is the sum of total private wealth:

$$K_t = \sum_{i \in \{3,4,5\}} \sum_{o \in \{m,n\}} \sum_{s \in \{l,h\}} a_{i,o,s,t} \mu_{i,o,s,t}. \quad (3)$$

Let  $\delta$  be the depreciation rate of the capital stock. Given the rental rate of capital  $r_t$  and the wage rate  $w_t$ , the firm maximizes profits:

$$\max_{\{K_t, N_t\}} \left\{ K_t^\theta (z_t N_t)^{1-\theta} - (r_t + \delta) K_t - w_t N_t \right\}. \quad (4)$$

## 2.3 The Government

Fiscal policy consists of flat tax rate  $\tau_k$  on capital income and  $\tau_c$  on consumption; marginal labor income tax rate  $\tau$  and payroll tax rate  $\tau_p$ ; per-capita age-specific public expenditures  $g_i$ ; per capita pension benefit  $P$  that is conditional on a worker's



productivity, and public transfers (excluding pensions)  $\chi$  that are age- and origin-specific.<sup>3</sup>

Age-specific per-capita government expenditures grow at the growth rate of the per-capita Gross National Product,  $\Gamma$ .<sup>4</sup> Hence, aggregate public expenditures are:

$$G_t = (1 + \Gamma)^t \sum_{i,o,s} g_i \mu_{i,o,s,t} . \quad (5)$$

Let  $T(w_t e_{i,o,s} n_{i,o,s,t})$  denote the total income and payroll taxes paid out of labor income by a type  $(i, o, s)$  individual. Denote the aggregate tax revenues of the government by  $Rev_t$ , aggregate non-pension related transfers by  $TR_t$ , and aggregate pension benefits as  $Pen_t$ . Then,

$$Rev_t = \sum_{i,o,s} T(w_t e_{i,o,s} n_{i,o,s,t}) \mu_{i,o,s,t} + \sum_{i,o,s} \tau_c c_{i,o,s,t} \mu_{i,o,s,t} + r_t \tau_k K_t \quad (6)$$

$$TR_t = (1 + \Gamma)^t \sum_{i,o} \chi_{i,o} \mu_{i,o,s,t} \quad (7)$$

$$Pen_t = \sum_{i,o,s} P_{i,o,s,t} \mu_{i,o,s,t} . \quad (8)$$

Government balances budget in each period. The consumption tax adjusts to maintain balance. Hence,

$$G_t + TR_t + Pen_t = Rev_t . \quad (9)$$

## 2.4 Immigration Policy

Immigration policy  $\psi = \{\psi_{2,l}, \psi_{2,h}\}$  determines the number of immigrants of age 20-39 (model age 2) of each ability level (low and high) as a fixed fraction of the size of the population in the previous period. I do not consider an inflow of immigrants aged 40 to 59. Akin (2006) shows that such a policy will not be welfare improving because those agents, on average, contribute for fewer years to the social security system compared to the longer years of retirement benefits they receive. Therefore, I analyze a class of immigration policies that allow the entry of younger workers.

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<sup>3</sup> The cost of bringing one more agent to the economy is  $g_i$ . This parameter depends only on the age of an agent. It is crucial to consider different costs of immigrants and natives on the government's budget; however such disaggregated data on government expenditures is not available.

<sup>4</sup> See Auerbach, Kotlikoff, Hagemann, and Nicoletti (1989).

## 2.5 Law of Motion for Population

Let  $\mu_t$  denote the total population in the economy at time  $t$ . Given the immigration policy  $\psi$ , children per person  $\{\varphi_{o,s}\}_{o \in \{m,n\}}$ , skill transition probabilities  $\{\pi_{o,s}\}_{o \in \{m,n\}, s \in \{l,h\}}$ , and survival probabilities  $\{\lambda_i\}_{i \in \{1,2,3,4\}}$ , population evolves according to:

$$\begin{aligned}
\mu_{1,n,h,t} &= \sum_{s \in \{l,h\}} (\varphi_{n,s} \mu_{2,n,s,t} \pi_{n,s} + \varphi_{m,s} \mu_{2,m,s,t} \pi_{m,s}) \\
\mu_{1,n,l,t} &= \sum_{s \in \{l,h\}} (\varphi_{n,s} \mu_{2,n,s,t} (1 - \pi_{n,s}) + \varphi_{m,s} \mu_{2,m,s,t} (1 - \pi_{m,s})) \\
\mu_{i+1,n,s,t} &= \lambda_i \mu_{i,n,s,t-1}, \text{ for } i \in \{1,2,3,4\} \text{ and } s \in \{l,h\} \\
\mu_{2,m,s,t} &= \psi_{2,s} \mu_{t-1}, \text{ for } s \in \{l,h\} \\
\mu_{i+1,m,s,t} &= \lambda_i \mu_{i,m,s,t-1}, \text{ for } i \in \{2,3,4\} \text{ and } s \in \{l,h\} \\
\mu_{t-1} &= \sum_{(i,o,s)} \mu_{i,o,s,t-1}.
\end{aligned} \tag{10}$$

The skill transition probabilities are important in determining the number of agents who are high- or low-skilled at any time in the economy. Several studies in the literature established that many countries experience a low intergenerational mobility in schooling and income. For example, by using German micro-data, Dustmann (2005) reports that parental background is strongly related to the school choice and school achievement of the child. He also finds little convergence for individuals from different parental backgrounds. Acemoglu and Pischke (2001) find that a 10 percent increase in family income generates a 1.4 percent increase in the probability of attending a four-year college in the U.S. This rigidity is incorporated to the model.

## 3 Competitive Equilibrium

**Definition (Competitive Equilibrium)** Given the initial distribution of assets  $a_0$ , population  $\mu_0$ , government transfers  $\{\chi_{i,o}\}_{i,o}$  and expenditures  $g_i$ ; tax rates  $\tau_c, \tau_k, \tau_p$ , and  $\tau$ , fertility rates and skill transition probabilities  $\{\varphi_{o,s}, \pi_{o,s}\}_{o,s}$ , survival probabilities

$\{\lambda_i\}_i$ , and immigration policy  $\psi$ , an equilibrium is a sequence

$\left\{w_t, r_t, N_t, K_t, Rev_t, Pen_t, G_t, TR_t, \tau_{c,t}, \left\{n_{i,o,s,t}, c_{i,o,s,t}, a_{i,o,s,t}, \mu_{i,o,s,t}\right\}_{i,o,s}\right\}_{t=0}^{\infty}$  such that

(i)  $\left\{n_{i,o,s,t}, c_{i,o,s,t}, a_{i+1,o,s,t}\right\}_{i,o,s,t}$  solves the individual's problem,

(ii)  $\{K_t, N_t\}$  solves the firm's problem,

(iii) The goods market clears:

$$K_t^\theta (z_t N_t)^{1-\theta} = K_{t+1} - (1-\delta)K_t + G_t + \sum_{i,o,s} \mu_{i,o,s,t} c_{i,o,s,t},$$

(iv) The labor market clears (Equation (2) holds.),

(v) Aggregate capital equals aggregate private wealth (Equation (3) holds.),

(vi) The consumption tax rate balances the government's budget (Equation (9) holds.),

(vii) Population evolves according to (10).

## 4 Calibration

I calibrate preference and production function parameters using the necessary conditions of the detrended version of the model. Efficiency units, children per person, skill transition probabilities, age-origin distribution of non-pension transfers, and total working hours are calculated using the German Socio-Economic Panel Data (GSOEP) that is published by the German Institute for Economic Research (DIW Berlin). I used the European Commission's Economic and Financial Affairs report on the German pension system (2005) to replicate the pension system. The income and social security tax schedule information comes from OECD's 2006 report on taxing wages.

### 4.1 The data

The GSOEP is a representative longitudinal micro-data on persons and households in Germany that starts in 1984. It contains information on education levels, sources of income (labor income, transfers from the government and other sources of income), number of children, nationality, and work hours for the different groups that constitute the German population. Individuals over age 16 are interviewed every year. For the purposes of this paper, I use three sub-samples of the data: Sample A "Residents in the Federal Republic of Germany," Sample B "Foreigners in the Federal Republic of Germany," and Sample C "German residents in the German Democratic Republic."

Sample A (the West-German sample) includes data on persons in private households with a household head who does *not* belong to the main foreigner groups of “guestworkers” (Turkish, Greek, Yugoslavian, Spanish, or Italian). Sample B has information on persons in private households with a household head from main foreigner guestworker groups. This sample consists of five autonomous samples for the five numerically largest foreign nationality groups living in West-Germany as immigrants in 1984. Sample C, the East-German sample, covers persons in private households where the household head was a German Democratic Republic citizen.

An immigrant in the model is an individual who belongs to the main foreigner guestworker groups.

The distribution of aggregate public transfers among different transfer categories, as well as public expenditures other than transfers, is published by the German Statistical Institute (Statistisches Bundesamt). In this paper, I use the 2005 data.

**4.2 Preference parameters.** The coefficient of relative risk aversion,  $\gamma$ , is assumed to be 2.0 in the main calibration, which is commonly used in the literature for overlapping generations models.<sup>5</sup> The time preference parameter  $\beta$  is chosen so that the steady state equilibrium of the model replicates the capital-output ratio in the German economy. The consumption share parameter  $\alpha$  is chosen so that the steady state average annual working hours of individuals between the ages 20 and 59 are consistent with the GSOEP. On average, annual working hours are 1612. I suppose that the maximum working hours of an individual is 100 hours per week and 5200 hours per year.

**4.3 Efficiency units.** The efficiency units for working-age individuals are estimated from the annual labor income profiles of immigrants and natives of each skill level and age. For the purposes of this paper, individuals with at least 15 years of schooling (the equivalent of three or more years of college education) are assumed to be high-skilled. Conditional on participation in the labor market (more than 450 hours a year), I calculate the average hourly wage for both immigrants and natives of working age for each skill level. Note that individuals in the model work only in the second and third periods of their lives, ages 20 to 59. Table 4 lists the estimated average hourly wages in Euros in 2005.

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<sup>5</sup> For example, see Nishiyama and Smetters (2003, 2007).

**4.4 Skill Transition.** In order to calculate skill transition probabilities, I identify parents and children in the first sample of the micro panel data, 1984. I then divide parents into four groups: low-skilled natives, high-skilled natives, low-skilled immigrants, and high-skilled immigrants. In order to find the skill level of their children, I follow the children in the data each year until the last sample in 2005. Children and parents with incomplete information on years of schooling over time are dropped from the sample. Given a category of parents, I find the skill transition probabilities by calculating the number of children of each skill level as a fraction of the total number of children who belong to those parents. Findings are presented in Table 5.

#### **4.5 Fertility Rates and the Initial Distribution of the Population**

**Fertility rates.** Children per person are estimated from the GSOEP for both immigrants and German natives of each skill level. Individuals can have children only in the second period of life in the model. Hence, the estimates in Table 6 correspond to children per person of ages 20-39 in the data.

The challenge in calculating fertility across origins and skill levels is how to allocate children with one high-skilled and one low-skilled parent, or children who have one German and one immigrant parent. In this study, all children who have at least one immigrant parent are treated as children of immigrants. Once children have been allocated to German native or immigrant categories, the allocation of children to skill levels is done in accordance with the skill intensity in the household.

The following steps give the details of the procedure to calculate the number of children per fertile-age adult of each skill- $s$  and origin- $o$ :

1. Link children and adults by matching their household numbers. Allocate children who are matched with both native and immigrant adults to the immigrant adults. This matching process forms “pseudo-households” consisting of fertile-age adults and the children who are allocated to them on the basis of origin.
2. For each pseudo-household compute the proportion of adults who are of skill- $s$  and multiply this by the number of children in the pseudo-household.
3. Sum the results in Step 2 across pseudo-households in the data. This sum is the total number of children allocated to skill- $s$  and origin- $o$ .
4. Divide the total number of children from Step 3 by the number of fertile-age adults of skill- $s$  and origin- $o$  in the micro-data.

The numbers in Table 6 are calculated as an average of figures between 1984 and 2005.

**The Initial Distribution of the Population.** The German Statistical Office Population Statistics summarizes the distribution of population across ages and nationalities. Data by education groups is not available. Therefore, I assume that the distribution of aggregate population across skills mimics the corresponding distribution from the micro-data, which is a representative sample of the whole population. Table 7 shows the calculated distribution of population for 2005.

**Survival Probabilities.** I use data published by the Human Mortality Database at the University of California Berkeley. The last data available for Germany is for the year 2005. Therefore, I assume that the survival probabilities are fixed at their 2005 levels. Calculated probabilities are shown in Table 8.

**4.6 Depreciation rate of fixed capital.** Let  $\eta$  be the population growth rate on the balanced growth path. The depreciation rate of fixed capital,  $\delta$ , is chosen such that:

$$\delta = \frac{\text{Total Gross Investment}}{\text{Fixed Capital}} - \Gamma - \eta.$$

In 2001, gross fixed capital formation (investment), GDP and total net capital stock for Germany were 420.8, 1979.6, 7165.5 billion Euros in current prices, respectively.<sup>6</sup> Given these values, I match a steady state capital-output ratio of 3.2 and investment-output ratio of 0.21. Hence, the ratio of gross investment to fixed capital is 6.5 percent. I assume that the GDP per capita growth rate,  $\Gamma$ , is 1.89 percent, which is the average annual growth rate of GDP per capita in Germany for the last two decades.<sup>7</sup> Note that the population growth rate on the balanced growth path depends on the immigration policy. For the baseline economy on a balanced growth path with 0.2 percent immigration of each age-skill category annually,  $\eta$  is approximately 1 percent per year.

**4.7 Share of capital in the production function:**  $\theta$  is chosen such that  $\delta + r = \theta \frac{Y}{K}$ .

Given  $Y/K = 0.31$ ,  $r = 4.16$  percent,  $\delta = 4.66$  percent, we get  $\theta = 0.284$ .

Table 8 shows the values of the calibrated parameters other than those of tax schedule, fixed transfers, and social security.

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<sup>6</sup> See Kamps (2005).

<sup>7</sup> SourceOECD, Online Database, National Accounts Data (2006).

#### 4.8 Taxes, Pensions, and Fixed-transfers

**Taxes.** All taxes in the model are collected at the individual level. The tax system includes progressive taxes on labor income as well as a capital income tax and a consumption tax. The capital income tax rate is assumed to be a flat-rate of 15 percent, which is the standard rate of corporate tax in Germany. The consumption tax adjusts in each period in order to balance the government budget. Therefore, it is a variable that is determined in equilibrium.

The labor income tax rate replicates the income tax schedule in Germany in 2005. The basic tax allowance is €7,664. The marginal tax rate increases linearly from 15 percent to 24 percent until €12,739. Between €12,740 and €52,151, it increases linearly to 42 percent.<sup>8</sup> For incomes higher than €52,152, the tax rate is constant at 42 percent (See Figure 2.). More specifically, let  $X$  be the taxable income rounded to the next full euro amount. Define  $Y = (X - 7,664)/10,000$ ;  $Z = (X - 12,739)/10,000$ . The income tax liability (in Euros) is calculated according to the following formula:

$$\text{Tax liability} = \begin{cases} 0 & \text{if } X \leq 7,664 \\ (883.7Y + 1,500)Y & \text{if } 7,665 \leq X \leq 12,739 \\ (228.7Z + 2,397)Z + 989 & \text{if } 12,740 \leq X \leq 52,151 \\ 0.42X - 7,914 & \text{if } X \geq 52,152 \end{cases}$$

Compulsory social security contributions also replicate the marginal rates in 2005. They consist of a tax for pension and unemployment benefits (26 percent) up to a gross income ceiling €62,400, and tax for sickness and long-term care (14.7 percent) up to a gross income ceiling of €42,300 (See Figure 3.). Both the employer and the employee pay equal shares of the payroll tax. Since the incidence of the payroll tax does not affect the equilibrium results, I assume that it is fully paid by the employee. Figure 4 shows the sum of income and social security tax rates on income. Since all these rates are yearly and one period in the model is 20 years, the tax schedule in the calibration is adjusted to be compatible with the model.

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<sup>8</sup> See OECD (2006).

**Pension Benefits.** The general pay-as-you-go, earnings-related statutory pension scheme covers around 85 percent of the employed population.<sup>9</sup> For each year of contributions, a worker in the statutory pension scheme receives pension points that reflect his relative earnings position. The average wage in a particular year is equal to one pension point. The individual pension benefit at  $t$  (denoted as  $P_t$ ) is calculated as the product of the sum of pension points earned through working-lifetime ( $pp$ ) and the value of one pension point,  $ppv_t$ , measured in Euros per month. That is,

$$P_t = pp * ppv_t.$$

Pension point in working-year  $j$  is determined by the earning of an individual in that year ( $w_j e_{j,o,s} n_{i,o,s,j}$ ) relative to the average earnings of all the workers in the economy net of taxes ( $\bar{y}_j$ ). Therefore, for an individual who works for  $I$  years, the sum of pension points earned is:

$$pp = \sum_{j=1}^I \frac{w_j e_{j,o,s} n_{i,o,s,j}}{\bar{y}_j}.$$

As an example, a person retiring with a contribution period of 40 years based on an average income earns 40 pension points over his working years. These pension points are multiplied by the current pension point value (€6.13 for pensioners from Western Germany), which gives a gross pension of  $40 * €6.13 = €1,045.2$  per month.

The pension point value is adjusted annually. The adjustment factor depends on growth rate of gross earnings and "the sustainability factor," which reduces benefits if the number of contributors to the system decreases relative to the number of pensioners.<sup>10</sup>

In the model, each individual works for 40 periods (between the ages of 20 to 59) and retires at age 60.<sup>11</sup> I approximate the Sustainability Factor by the change in the ratio of pensioners to contributors from one period to another. Therefore, in the calibration, value of one pension point at time  $t$  is calculated as:

<sup>9</sup> See Börsch-Supan and Wilke (2003).

<sup>10</sup> The adjustment formula also includes the "Riester Factor," which considers transition to a multi-pillar pension system in which contributions to tax subsidized voluntary private pension scheme (second pillar) reduce benefits in the public scheme (first pillar). However, Bonin (2001) and Börsch-Supan (2002) report that since the labor-market assumptions underlying the Riester Factor are unrealistic, its effects on the sustainability of the pension system will be minimal. Hence, I abstract away from it.

<sup>11</sup> In 2005, the average effective age of retirement in Germany was 61.7 for men and 60.7 for women. (See *Statistics on Average Age and Official Age of Retirement in OECD Countries*, OECD, 2006.)



$$vpp_t = vpp_{t-1} \times \frac{\bar{y}_{t-1}}{\bar{y}_{t-2}} \times \left( \left( 1 - \frac{\left( \frac{\text{pensioners}}{\text{contributors}} \right)_{t-1}}{\left( \frac{\text{pensioners}}{\text{contributors}} \right)_{t-2}} \right) 0.25 + 1 \right).$$

The constant, 0.25, that multiplies the Sustainability Factor is called the "allocation factor" in the German Pension System.

**Non-pension Transfers per Person.** Age-origin profiles of total transfers per capita are estimated from the GSOEP. The data includes questions about monthly old-age and disability benefits, child benefits, maternity benefits, general welfare, health and long-term care benefits, housing allowances, subsistence allowances, and unemployment benefits. Since the pension, health and long-term care benefits are specifically modeled, transfers per person exclude those items. Table 9 summarizes the results.

## 5 Results

### 5.1 Experiments

I assume that the economy is initially on a balanced growth path (in 2005) with an annual inflow of 20-39 year old low- and high-skilled immigrants equal to 0.1 percent and 0.01 percent of the German population, respectively. This corresponds to an annual net inflow of around 83,000 immigrants, which is in line with the data of 2005 and 2005 published by the German Statistical Institute. I investigate the resulting effects on allocations, prices, and welfare of an exogenous one-time change in immigration policy. I report welfare results from six different experiments in which either the skill distribution of immigrants or the total size of immigrant inflows change. All simulations are executed by detrending the model. The algorithm used to solve for the transition path is given in the Appendix.

### 5.2 Equilibrium allocations and prices

To set the background for the welfare results, I first summarize the effects on allocations and prices of a new immigration policy under which annual inflow of immigrants of each

type is 0.2 percent of the total population.<sup>12</sup> Figures 5-10 show the movement of the economy on the transition path. The year 2005 is the initial balanced growth path, 2025 is the first period after the policy change, and 2165 is the year in which the economy reaches the new balanced growth path.

**Consumption:** This policy increases the consumption of individuals of every age, origin, and skill group. The main factor that affects equilibrium consumption is the decline in the consumption tax rate. Although immigrants increase government expenditures and pensions in the aggregate, government's tax revenue goes up by a bigger factor. Thus, the consumption tax rate that balances the budget goes down, allowing for an increase in consumption. For retirees, the increase in pension benefits generated through a lower dependency ratio is another factor that contributes to higher consumption.

**Labor supply:** Labor supply is quite robust to the change in immigration policy. We observe a slight increase (0.1 percent) in hours worked for individuals aged 20-39 on the new balanced growth path. 40-59 year old workers experience a decline in the working hours by 0.2 percent. Note that 40-59 year old agents face two opposite effects. First, the rise in the rental rate increases the asset income, which reduces incentive to work. Second, the decline in wages provides incentive to work more. Here, the first effect dominates the second.<sup>13</sup>

**Prices:** With increased immigration, the rate of return on capital increases from 5.4 percent in 2005 to 5.7 percent in 2165. The wage rate is lower on the transition path. An increased inflow of immigrants reduces the capital-labor ratio, which causes a decrease in the wage rate and an increase in the interest rate.

**Pensioner-contributor ratio:** The ratio on the new balanced growth path (0.57) is smaller relative to its initial value (0.41). As more working-age immigrants are allowed, the imbalance created by rapid aging improves.

**Individual pensions:** In this experiment, pension benefits increase. Two opposing factors contribute to this behavior through the indexation formula. First, declining wage earnings on the transition path push pension benefits down. Second, the reduction in the

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<sup>12</sup> I only report the welfare results for all experiments. The graphs for the other five experiments are available upon request.

<sup>13</sup> Individuals do not have asset income in the second period of the lifecycle. Hence, for them, the only relevant effect is the second one.

pensioner-contributor ratio due to increased immigration creates a rise in benefits through the sustainability factor. In this experiment, the second effect dominates.

In summary, there are three interesting results. First, the exact modeling of the social security system enables us to assess the opposing interactions between changes in wages and changes in the pensioner-contributor ratio. The inclusion of the sustainability factor raises pension benefits via increasing the pension point value, compensating for a decline in benefits caused by declining wage earnings.

### **5.3 Welfare**

In order to measure the change in welfare, I calculate percentage change in real income that is needed to achieve lifetime utility under the new immigration policy. Since individual preferences are of constant elasticity of substitution type, the change in real income is a perfect index of change in welfare. Table 10 summarizes the results for the new balanced growth path.

There are two main conclusions. First, when there is a prohibition on immigration (Experiment 1 in Table 10), native welfare goes down by 3 percent. Second, an increase in welfare may be achieved regardless of the skill type of immigrants. Keeping the total inflow constant, we see that policies admitting more low-skilled immigrants achieve a similar welfare improvement as policies admitting more high-skilled immigrants. The results indicate that the main channel through which immigration influences equilibrium allocations is the pensioner-contributor ratio. Bigger inflows, independent of the skill level, improve the dependency ratio by allowing more workers to enter the labor market. Low-skilled immigrants help slightly more, as their fertility rate is the highest among all types of agents in the economy.

On the new balanced growth path, relative to the initial path, we observe that consumption is higher. Labor supply is slightly higher for agents aged 20-39 and is lower for those aged 40-59. The increase in consumption is mostly due to a decline in the consumption tax rate. As an example, under Experiment 2, consumption tax rate goes down from its initial level of 20 percent to 9 percent.

Similar welfare conclusions hold on the transition path. Table 11 shows the results for cohorts alive at the first period of the policy change (2025). Figures 5 to 10

illustrate the behavior of some of the key equilibrium variables on the transition from the initial balanced growth path to the new one under Experiment 2. Welfare of all types of agents goes up under increased immigration scenarios.

#### 5.4 The Role of the Sustainability Factor

To isolate the role of the sustainability factor in the results, I consider what would occur if the allocation factor were set to zero. Hence, the indexation formula becomes:

$$vpp_t = vpp_{t-1} \times \frac{\bar{y}_{t-1}}{\bar{y}_{t-2}},$$

where the pension point value at  $t$  depends only on its value at  $t-1$  and the growth rate of earnings.

I evaluate the results from two experiments. First, I consider a shift to a policy that prohibits immigration. Second, I change the immigration policy such that annual inflow of each skill type of immigrants is 0.2 percent of the population. For each experiment, I first calculate two different equilibrium balanced growth paths - one with the sustainability factor in effect, and one without it. Then, I compare the outcomes.

Under a shift to a policy that prohibits immigration, consumption and welfare of each agent decline, regardless of whether or not the sustainability factor is included in benefit calculations. The reason is the rise in the pensioner-contributor ratio due to low native fertility. However, when the benefit formula includes the sustainability factor, the rise in the dependency ratio causes a decline in individual pensions. Hence, in the aggregate, government expenditures on pensions are lower compared to a no-sustainability-factor scenario, which makes it possible for the budget to balance with a lower consumption tax rate. As a result, when the two balanced growth paths are compared, the one that incorporates the sustainability factor delivers lower individual pension benefits and consumption tax rate, but higher consumption and welfare.

In the case of positive immigrant inflows, the conclusion is reversed: individual consumption and welfare are higher in the absence of the sustainability factor. With more immigrants in the economy, the pensioner-contributor ratio declines. Hence, when the sustainability factor is included, the value of each pension point goes up, increasing the benefit for each individual. However, this also raises aggregate government spending on

pensions, necessitating a higher consumption tax rate to balance the budget. Therefore, the new balanced growth path consumption as well as welfare are lower compared to their levels when the sustainability factor is excluded from calculations. In this experiment, exclusion of the sustainability factor improves welfare 0.5 percent more relative to the outcome with the sustainability factor.

In summary, the inclusion of a sustainability factor is beneficial for an economy that tries to minimize immigration; but detrimental to one that would like to pursue a more liberal immigration policy.

### **5.5 An increase in life-expectancy**

In the developed world, life-expectancy has increased substantially over time. For example, over the last century, U.S. life-expectancy at birth rose from 48 to 75 years among men, and 51 to 80 years among women.<sup>14</sup> In Germany, between 1962 and 2002 the average life expectancy has increased from 67.1 years to 75.6 years among men, and 72.7 to 81.3 among women.<sup>15</sup> Therefore, it is important to understand the implications of immigration in an economy where individuals live longer.

To achieve this, I double the conditional probability of surviving from age 60+ to 80+, that is, I set  $\lambda_4 = 0.658$  in the new experiment. It is no surprise that increased life expectancy results in higher positive influence of immigration. As more people reach older ages, number of elderly relative to those of the working-age rises. Therefore, more immigration helps the economy by increasing the size of the contributors. On the transition path, welfare of each type increases by 0.1 to 0.3 percent relative to the level observed when  $\lambda_4 = 0.329$ .

### **5.6 Sensitivity Analysis**

In the model, the only parameter that is not calibrated is the coefficient of relative risk aversion  $\gamma$ , whose value is assumed to be 2. The values that are used in the literature range from 0.5 to 4 for life-cycle models. To evaluate the robustness of the results to the degree of risk aversion, I solve for equilibrium where risk aversion is equal to 0.5, 1, or 3

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<sup>14</sup> Center for Disease Control and Prevention (2005).

<sup>15</sup>See Klenk, Rapp, Buchele, Keil, and Weiland (2007).

and report the percentage change in welfare on the new balanced growth path relative to 2005. Under the new policy, the annual inflow of each type of immigrants is 0.2 percent of the population (Experiment 2). The results are presented in Table 13.

Although lower degrees of risk aversion lead to a smaller increase in welfare on the new balanced growth path, the behavior of the equilibrium allocations do not change. For all values of  $\gamma$ , consumption and welfare increase for all types, labor supply is steady, the wage rate declines, and the interest rate rises.

## **6 Concluding Remarks**

In this paper I analyzed how a change in immigration policy would affect government finances and individual choices by incorporating key elements of the German social security and income tax systems in an overlapping generations model. One of the main findings is that allowing an annual immigrant inflow equal to 0.4 percent of the population will increase consumption and welfare. This is due to a rise in the return to savings and a decrease in the consumption tax rate. There are two opposite effects of immigration on an individual's pension benefits. First, a decline in the wage rate reduces benefits, as the latter is strongly linked to the former through the indexation formula. Second, immigration leads to an improvement in the retiree-contributor ratio, which raises benefits through the sustainability factor. The net effect on pensions is positive.

Another interesting result is the improvement in welfare regardless of the skill composition of immigrants. Many heated debates on immigration in the developed world are based on the argument that low-skilled immigrants lower the well-being of natives because they bring wages down. This paper proves otherwise. Although native wages decline after an immigrant inflow, the improvement in the dependency ratio not only leads to a rise in pension benefits; but also reduces taxes on consumption.

Finally, the paper delivers a clear policy recommendation. Nations with aging populations would benefit from opening borders to young, working-age immigrants, as long as those individuals contribute to the system as tax-payers.

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## Appendix An Equilibrium Transition Path

I solve the detrended version of the model. I assume that the economy is in the corresponding initial steady state in period 0, and the new immigration policy  $\tilde{\psi} = \{\tilde{\psi}_{2,l}, \tilde{\psi}_{2,h}, \tilde{\psi}_{3,l}, \tilde{\psi}_{3,h}\}$  is announced. I use the fixed point iteration algorithm (Judd, Kubler, and Schmedders 2003; Rios-Rull, 1997) to compute a transition path to the new steady-state equilibrium (thereafter, the final steady-state equilibrium) is as follows:

1. Assume that the economy reaches the new steady state within a large number of periods, 20 in this case.<sup>16</sup> Set the initial guess on the interest rate sequence  $\{r_t^0\}_{t=1}^{20}$ , consumption tax rate  $\{\tau_{c,t}^0\}_{t=1}^{20}$ , and average earnings  $\{\bar{y}_t^o\}_{t=1}^{20}$
2. For periods  $t = 1, 2, \dots, 19$ , compute forward the measure of individuals according to the law of motion for population. Given the initial guesses, compute the new equilibrium interest rate  $\{r_t^1\}_{t=1}^{20}$ , tax rate  $\{\tau_{c,t}^1\}_{t=1}^{20}$ , and  $\{\bar{y}_t^1\}_{t=1}^{20}$  sequences implied by the decision rules that would prevail if  $\{r_t^0\}_{t=1}^{20}$ ,  $\{\tau_{c,t}^0\}_{t=1}^{20}$ , and  $\{\bar{y}_t^o\}_{t=1}^{20}$  were the true equilibrium sequences.
3. New guesses of the interest rates, the consumption tax, and the average earnings are generated as an average of the previous guesses and the sequences implied by the individual and firm decision rules and the government's budget constraint.
4. Stop when  $\|r_t^1 - r_t^0\| / (1 + \|r_t^0\|)$ ,  $\|\tau_{c,t}^1 - \tau_{c,t}^0\| / (1 + \|\tau_{c,t}^0\|)$ ,  $\|\bar{y}_t^1 - \bar{y}_t^o\| / (1 + \|\bar{y}_t^o\|)$  is less than 0.006.
5. Update guesses of interest rate, consumption tax, and average earnings by
$$\{r_t^0\}_{t=1}^{20} = \lambda \{r_t^1\}_{t=1}^{20} + (1 - \lambda) \{r_t^0\}_{t=1}^{20}, \quad \{\tau_{c,t}^0\}_{t=1}^{20} = \lambda \{\tau_{c,t}^1\}_{t=1}^{20} + (1 - \lambda) \{\tau_{c,t}^0\}_{t=1}^{20},$$

$$\{\bar{y}_t^o\}_{t=1}^{20} = \lambda \{\bar{y}_t^1\}_{t=1}^{20} + (1 - \lambda) \{\bar{y}_t^o\}_{t=1}^{20}, \text{ where } \lambda \in (0, 1).$$

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<sup>16</sup> After 7 periods, which corresponds to 140 years, the annualized rate of return on capital is 0.001 percent away from its steady state value.

Figure 2. Marginal Income Tax Rates, 2005

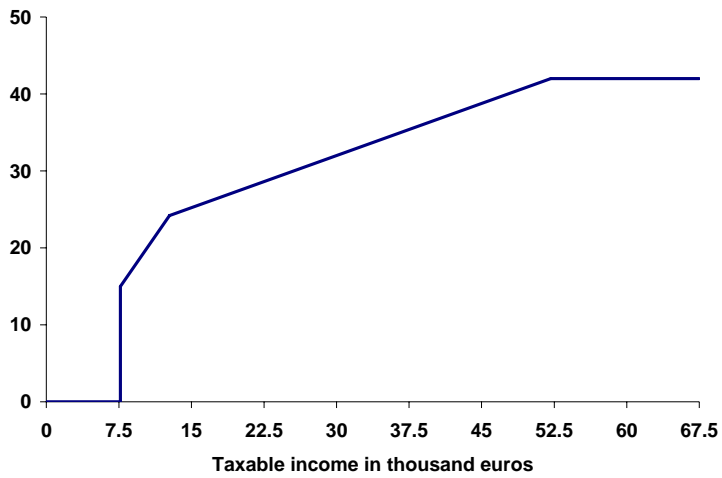


Figure 3. Marginal Payroll Tax Rates, 2005

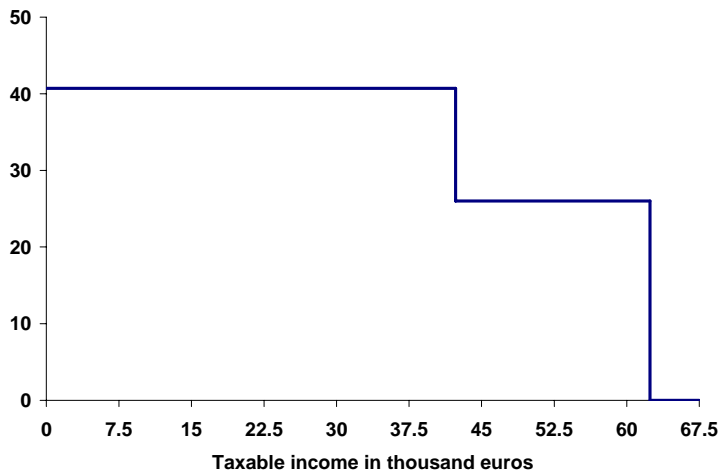


Figure 4. Total Marginal Tax on Labor Income, 2005

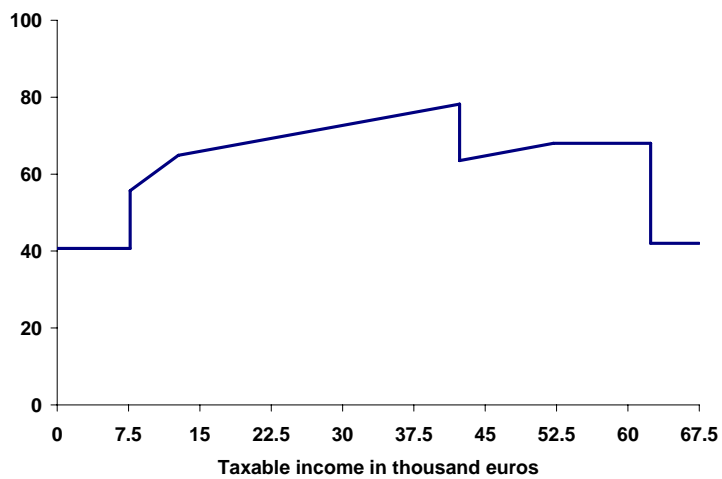


Figure 5. Consumption per Capita (consumption of 20-39 year old high-skilled native in 2005=100)

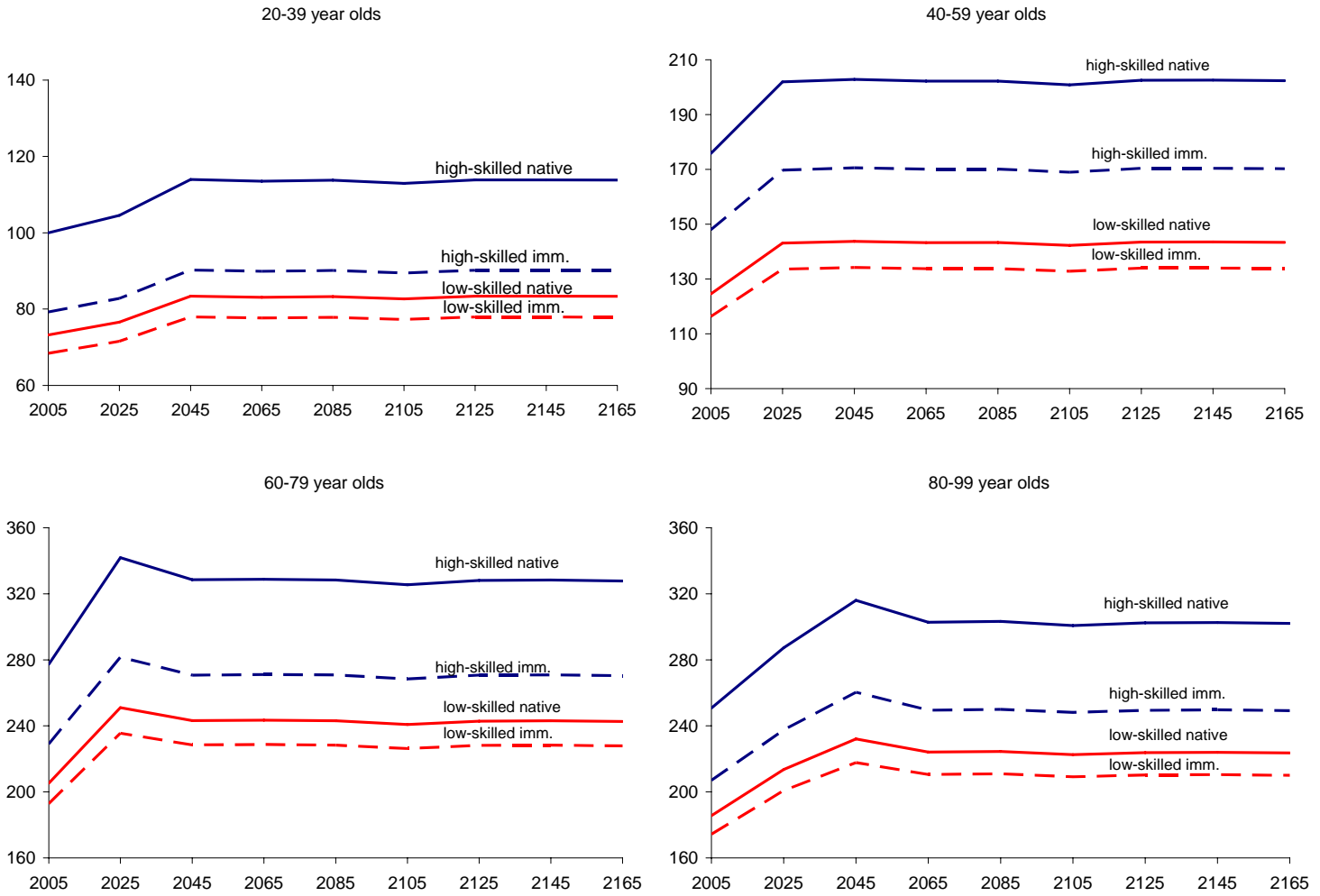


Figure 6. Labor Supply (Percentage of Time Worked)

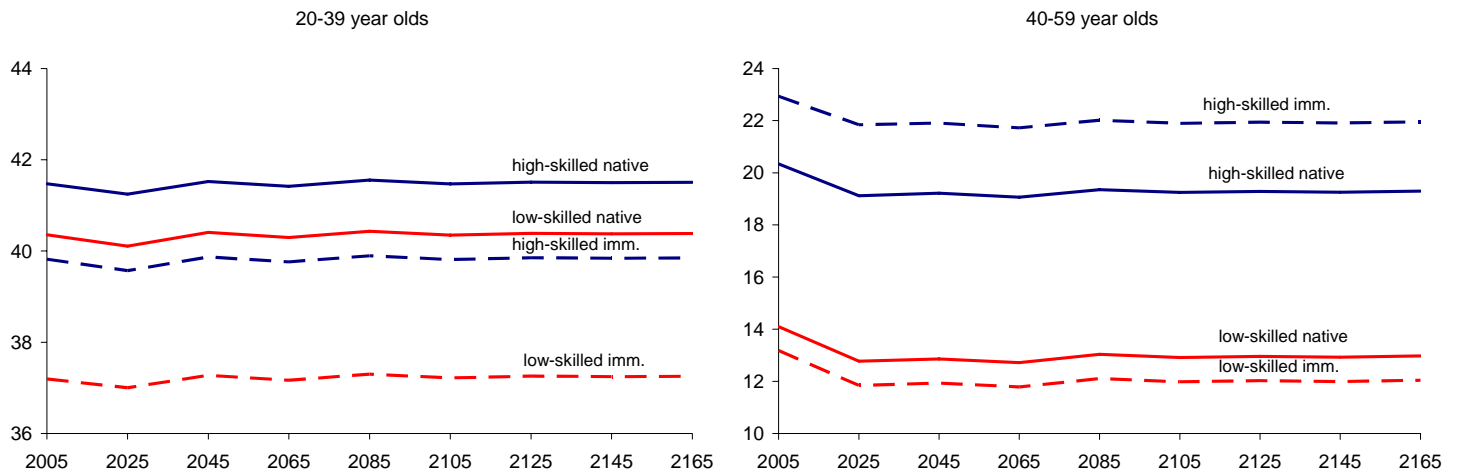


Figure 7. Individual Pension Benefits

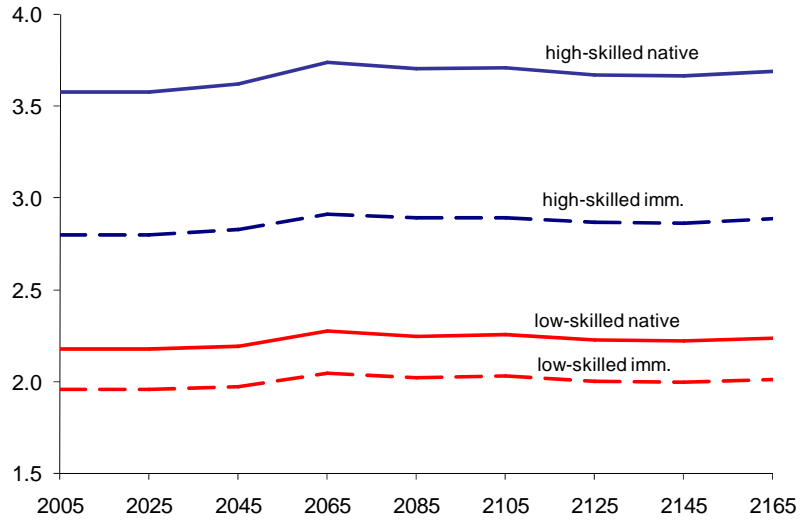


Figure 8. The Pensioner-Contributor Ratio

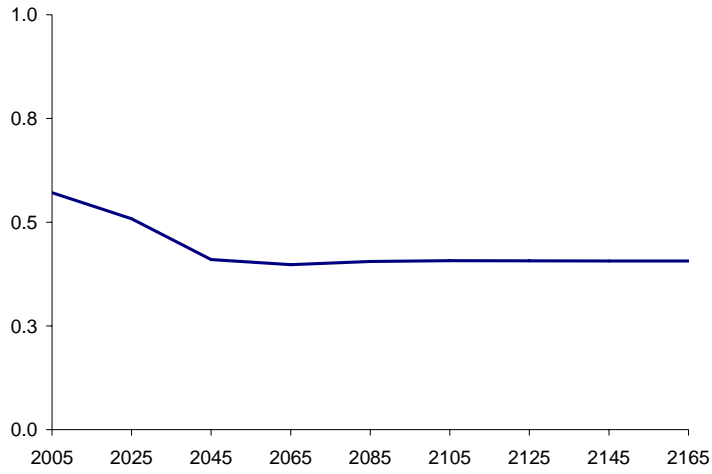


Figure 9. Annual Rate of Return on Capital (in percentage)

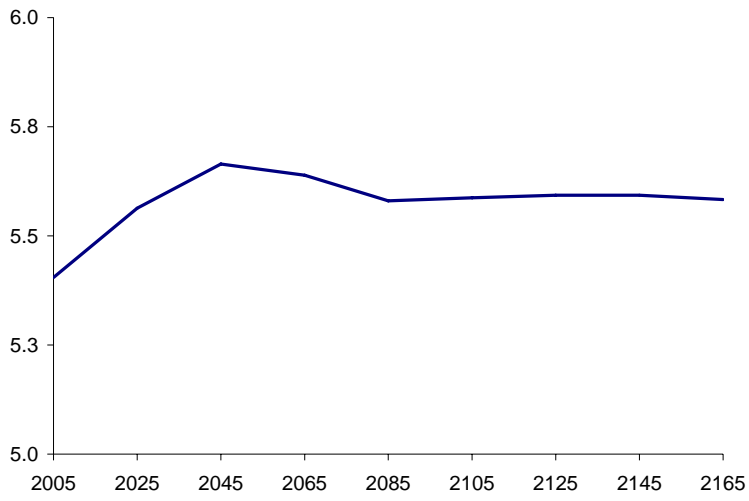


Figure 10. The Consumption Tax Rate (in percentage)

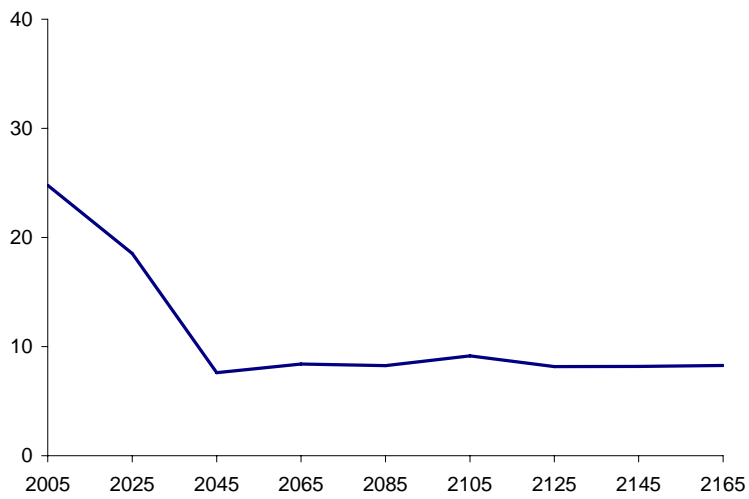


Table 1. Main Variables and Functions in the Model

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<b>Individual type <math>(i, o, s)</math>:</b>		
$i$	$\in \{1, 2, 3, 4\}$	Age
$o$	$\in \{m, n\}$	Origin (migrant, native)
$s$	$\in \{l, h\}$	Working ability (low, high)
<b>Individual decisions:</b>		
$c_{i,o,s}$		Consumption
$n_{i,o,s}$		Working hours
$a_{i+1,o,s}$		End-of-period wealth
<b>Main parameters and other variables:</b>		
$\beta$	$\in \mathfrak{R}_+$	Time preference
$\alpha$	$\in \mathfrak{R}_+$	Share of consumption in the utility function
$\gamma$	$\in \mathfrak{R}$	Coefficient of relative risk aversion
$\Gamma$	$\in \mathfrak{R}$	Productivity growth rate of the economy
$\eta$	$\in \mathfrak{R}$	Population growth rate
$\theta$	$\in \mathfrak{R}_+$	Share of capital in the production function
$\delta$	$\in \mathfrak{R}_+$	Depreciation rate
$\varphi_{o,s}$	$\in \mathfrak{R}$	Number of children per person of origin- $o$ , skill- $s$
$w_t$	$\in \mathfrak{R}_+$	Wage rate
$r_t$	$\in \mathfrak{R}_+$	Interest rate
$e_{i,o,s}$	$\in \mathfrak{R}_+$	Efficiency
$\pi_{o,s}$	$\in \mathfrak{R}_+$	Probability that individual $(o, s)$ has a high-skilled child
$\lambda_i$	$\in \mathfrak{R}_+$	Conditional probability of surviving from age $i$ to $i+1$
<b>Government policy:</b>		
$g_i$	$\in \mathfrak{R}_+$	Age specific government expenditures per capita
$\chi_{i,o,t}$	$\in \mathfrak{R}_+$	Transfer per person of age- $i$ , origin- $o$
$\tau$	$\in \mathfrak{R}_+$	Marginal income tax rate
$\tau_p$	$\in \mathfrak{R}_+$	Marginal payroll tax rate
$\tau_c$	$\in \mathfrak{R}_+$	Consumption tax rate (flat)
$\tau_k$	$\in \mathfrak{R}_+$	Capital income tax rate (flat)
<b>Immigration policy <math>\psi</math>:</b>		
$\{\psi_{2,s}\}_{s \in \{l,h\}}$		Number of immigrants aged 20-39 of working ability $s \in \{l, h\}$ as a fraction of population at $t-1$
<b>Joint distribution of individuals <math>\mu_i</math>:</b>		
$\{\mu_{i,o,s,t}\}_{i,o,s,t}$		Measure of individuals of type $(i, o, s)$

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Table 2. Other Aggregate Variables in the Model

$K_t$	$\in \mathfrak{R}_+$	Capital stock
$N_t$	$\in \mathfrak{R}_+$	Total effective units of labor
$Y_t$	$\in \mathfrak{R}_+$	Gross national product
$G_t$	$\in \mathfrak{R}_+$	Aggregate government expenditures
$Pen_t$	$\in \mathfrak{R}_+$	Aggregate pension benefits
$Rev_t$	$\in \mathfrak{R}_+$	Aggregate government tax revenues
$TR_t$	$\in \mathfrak{R}_+$	Aggregate government fixed transfers

Table 3. Parameters and Target Variables

Parameter	Notation	Target variable	Value
Share parameter for consumption	$\alpha$	Average annual working hours in the steady state equals 1612h	0.26
Time preference	$\beta$	Capital-output ratio in the steady state equals 3.2	0.98
Coefficient of relative risk aversion	$\gamma$		2.00
Depreciation rate of capital stock	$\delta$	Capital-output ratio equals 3.2, investment-output ratio equals 0.21 in the steady state	0.046
Capital share of output	$\theta$	Capital-output ratio in the steady state equals 3.2	0.29
Long-term real growth rate	$\Gamma$	Average annual growth rate of GDP per capita in the last two decades	1.89

All rates are annual.

Table 4. Estimated Average Hourly Wages (before-tax), 2005

	Low-skilled	High-skilled
German natives:		
Ages 20-39	25.9	36.0
Ages 40-59	29.9	44.0
Immigrants:		
Ages 20-39	24.0	26.6
Ages 40-59	26.7	38.2

In Euros.



Table 5. Skill Transition Probabilities

	Probability of having a low-skilled child	Probability of having a high-skilled child
German natives:		
Low-skilled	0.83	0.17
High-skilled	0.58	0.42
Immigrants:		
Low-skilled	0.92	0.08
High-skilled	0.66	0.34

Table 6. Number of Children per Fertile-Age Person

	German natives	Immigrants
Low-skilled	0.84	1.14
High-skilled	0.80	0.84

Table 7. Initial Distribution of Population, 2005

	0-19	20-39	40-59	60-79
Immigrants:				
Low-skilled	3.59	1.82	1.11	0.36
High-skilled	0.23	0.11	0.07	0.04
Natives:				
Low-skilled	13.08	17.48	17.48	14.33
High-skilled	2.99	4.01	4.12	1.75

In millions.

Table 8. Parameter values (excluding taxes, transfers, and social security)

Parameter	Notation	Value
Share parameter for consumption	$\alpha$	0.26
Time preference	$\beta$	0.98
Coefficient of relative risk aversion	$\gamma$	2.00
Depreciation rate of capital stock	$\delta$	0.046
Capital share of output	$\theta$	0.29
Long-term real growth rate	$\Gamma$	1.89
Gov't expenditures per person aged 0-19 (€)	$g_1$	8,848
Gov't expenditures per person aged 20-39 (€)	$g_2$	5,503
Gov't expenditures per person aged 40-59 (€)	$g_3$	5,008
Gov't expenditures per person aged 60-79 (€)	$g_4$	5,008
Children per high-skilled native	$\varphi_{h,n}$	0.80
Children per low-skilled native	$\varphi_{l,n}$	0.84
Children per high-skilled immigrant	$\varphi_{h,m}$	0.84
Children per low-skilled immigrant	$\varphi_{l,m}$	1.14
Efficiency of high-skilled native aged 20-39	$e_{2,n,h}$	36.0
Efficiency of low-skilled native aged 20-39	$e_{2,n,l}$	25.9
Efficiency of high-skilled native aged 40-59	$e_{3,n,h}$	44.0
Efficiency of low-skilled native aged 40-59	$e_{3,n,l}$	28.9
Efficiency of high-skilled immigrant aged 20-39	$e_{2,m,h}$	26.6
Efficiency of low-skilled immigrant aged 20-39	$e_{2,m,l}$	24.0
Efficiency of high-skilled immigrant aged 40-59	$e_{3,m,h}$	38.3
Efficiency of low-skilled immigrant aged 40-59	$e_{3,m,l}$	26.7
Probability of having a high-skilled child for a high-skilled native	$\pi_{n,h}$	0.42
Probability of having a high-skilled child for a low-skilled native	$\pi_{n,l}$	0.17
Probability of having a high-skilled child for a high-skilled immig.	$\pi_{m,h}$	0.34
Probability of having a high-skilled child for a high-skilled immig.	$\pi_{m,l}$	0.08
Probability of surviving from age 0-19 to age 20-39	$\lambda_1$	0.990
Probability of surviving from age 20-39 to age 40-59	$\lambda_2$	0.957
Probability of surviving from age 40-59 to age 60-79	$\lambda_3$	0.786
Probability of surviving from age 60-79 to age 80-99	$\lambda_4$	0.329

All rates are annual.

Table 9. Total Non-Pension Transfers per Person, 2005

Age	Germans	Immigrants
0-19	2,481	1,997
20-39	3,372	3,079
40-59	4,392	5,219
60-79	16,469	11,816

Annual, in Euros.

Table 10. Welfare Change on the New Balanced Growth Path

Experiment	Annual Immigration (%)		Percentage Change in Welfare			
	High	Low	Native		Immigrant	
			High	Low	High	Low
1	0.0	0.0	-3.3	-3.3	--	--
2	0.2	0.2	3.7	3.7	3.7	3.7
3	0.3	0.1	3.6	3.7	3.6	3.7
4	0.1	0.3	3.9	3.9	3.9	4.9
5	0.4	0.1	4.5	4.6	4.5	4.6
6	0.1	0.4	5.1	5.0	5.0	4.9

Table 11. Welfare Change during the Transition to the New Balanced Growth Path

Experiment	Annual Immigration (%)		Percentage Change in Native Welfare (by age in 2025)							
	High	Low	20-39		40-59		60-79		80-99	
			High	Low	High	Low	High	Low	High	Low
2	0.2	0.2	2.8	2.8	2.1	1.7	0.9	0.9	0.1	0.1
3	0.3	0.1	2.8	2.8	2.0	1.8	0.8	0.9	0.2	0.2
4	0.1	0.3	2.9	2.9	2.1	1.7	0.9	0.9	0.1	0.1
5	0.4	0.1	3.5	3.7	2.5	2.2	1.1	1.1	0.2	0.2
6	0.1	0.4	3.8	3.7	2.6	2.2	1.1	1.1	0.2	0.2

Table 12. The Welfare Effects of Immigration when the Probability of Surviving to the Age of 80+ is Doubled

Experiment	Annual Immigration (%)		Percentage Change in Native Welfare (by age in 2025)							
			20-39		40-59		60-79		80-99	
	High	Low	High	Low	High	Low	High	Low	High	Low
2	0.2	0.2	3.1	3.1	2.4	2.0	1.1	1.1	0.2	0.2
3	0.3	0.1	3.0	3.1	2.3	2.0	1.1	1.1	0.3	0.3
4	0.1	0.3	3.2	3.2	2.4	2.0	1.1	1.1	0.2	0.2
5	0.4	0.1	3.8	3.9	2.9	2.5	1.5	1.5	0.3	0.3
6	0.1	0.4	4.0	4.0	3.0	2.5	1.4	1.4	0.3	0.3

Table 13: Sensitivity Analysis: Percentage Change in Welfare on the New Balanced Growth Path for Various Risk Aversion Coefficients under Experiment 2

Coefficient of Risk Aversion	Welfare (%)			
	Native		Immigrant	
	High	Low	High	Low
0.5	1.7	1.7	1.7	1.7
1.0	0.4	0.3	0.4	0.4
2.0	3.7	3.7	3.7	3.7
3.0	5.5	5.5	5.5	5.5