

Modeling Pipeline Driving Behaviors

Hidden Markov Model Approach

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Driving behaviors at intersections are complex. At intersections, drivers face more traffic events than elsewhere and are thus exposed to more potential errors with safety consequences. Drivers make real-time responses in a stochastic manner. This study used hidden Markov models (HMMs) to model the driving behavior of through-going vehicles on major roads at intersections. Observed vehicle movement data were used to estimate the model. A single HMM was used to cluster movements when vehicles were close to the intersection. The reestimated clustered HMMs could more accurately predict vehicle movements compared with traditional car-following models.

Driving behavior is complex in terms of the amount of information being processed, the number of involved parties, and the chances of being affected by human errors. A driver has to perceive the status of his own and adjacent vehicles, road geometry and surface conditions, traffic control facilities and traffic signs, and even weather and lighting conditions. Further, driving is a process of correction. Drivers have to redress their own errors efficiently and recognize and respond to other drivers' errors. Their capability of addressing errors in driving varies across the population, changes over time, and depends on the temporal, physical, and psychological conditions. All these factors make driving behaviors stochastic rather than deterministic. However, this important aspect has not been well addressed to date in modeling driving behaviors.

Driving models are many. Each is used for a different purpose, and each has specific limitations. Control models are widely used in traffic simulation, which includes car-following models (1), lane-changing models (2), and emergency maneuver models (3). These models are designed to emulate highway vehicle movement and always oversimplify intersection maneuvers. Driver models developed in psychological research concentrate on drivers' perceptions (4) and operational tasks (5, 6). For the purposes of driver education and suggesting methods of collision avoidance, these models usually provide qualitative descriptions of driver behavior and are hardly strict in predicting driver maneuvers at intersections. Since Pipes (7) proposed a linear car-following model, there have been many improvements, including Chandler et al.'s (8) and Gazis et al.'s (9) improved Pipes model, Tyler's (10) optimal control model, Newell's (11) desired speed and shifted trajectory model, and Gipps's (12) psychodynamic car-following model. Other microscopic driving models include the cellular automaton model (13), the cell transmission model (14), and the intelligent driver model (15), among

others. Many of these models are proposed to facilitate traffic flow studies. Few of them have been thoroughly calibrated with real driving maneuver records. An even greater disadvantage of these models is that they are not designed to represent detailed, mistake-prone driving behaviors and the stochastic driver decision-making process. New developments, which take advantage of hidden Markov models (HMMs) to model mental states of drivers (6, 16), provide insights to our research.

The primary concentration in this research is on driving behaviors at intersections. Compared with normal road driving, vehicles close to intersections perform much more complicated maneuvers. In this study, discrete action states of drivers, which are partially observable, are used to represent driver maneuvers and choice-making procedures. For the first time, we have an abundant collection of measured vehicle movements that is sufficient for initial model estimation and calibration.

Measured vehicle movements comprise our raw observations for complex behavioral recognition. These behaviors are observed as patterns of events, which include continuous trajectories or discrete sequences of measurable properties such as position, direction, and speed from sensors or shape and color in images. However, these observations are just records of behaviors without any meaning, which is namely the first step of behavior modeling. A real understanding of behavior needs meanings or semantics of measured behaviors, which are usually predefined as a set of discrete events or states. For example, Neumann (17) describes car movements by a pyramidal hierarchy that includes complex semantics at the top and elementary ones on the base. Computer vision research aims to correlate video streams or images to symbolic activities (18, 19). The present research hints at driving behavior, which is human behavior intermediated by vehicle dynamics.

The authors' research develops a model of driver-vehicle combination at intersections and considers the observed vehicle movement as the consequences of drivers' actions. These actions eventually determine the occurrence of traffic conflicts and crashes. Drivers in conflicts behave in ways that are conditioned on the behavior of others. Ignoring this factor oversimplifies the understanding of driver behavior. This study identifies potential vehicle conflicts and includes them in the observable state set. Though it is naive to presume a constant influence between drivers, we will start with a simplified influence model in modeling vehicle interactions.

HIDDEN MARKOV DRIVING MODEL

A driver model relates the driver's behavior to his perception, physical and psychological conditions, driving experience, and preferences under traffic conditions. Driver behavior is affected by many internal and external factors. At intersections drivers perceive information

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from their own vehicles, other vehicles on the road, traffic facilities, and the environment and generate responses through a decision-making process that is hardly tractable. Vehicle dynamics vary from one vehicle to another. Drivers' understanding of their own vehicles and traffic environment varies also. These factors make the combination of driver and vehicle complex and hard to predict, even when the knowledge of vehicle and driver history is abundant. The uncertainty, deviation, and inconsistency may seem to dominate the output.

While difficult, predicting driver-vehicle behavior is possible. Vehicles have physical limitations in making maneuvers. Drivers also possess psychological and physiological limitations. Drivers' preferences under certain traffic circumstances are fairly consistent. Their skills and habits are observable or derivable from their previous behavior.

Driver behaviors at or near intersections are complex. In this study, only conflicts related to crossing and left-turn traffic will be studied. Right-turn, U-turn, and lane-changing behaviors are not considered. Traditional intersection safety studies for left-turn and crossing traffic concentrate on gap acceptance of drivers. An implicit assumption is used: drivers will be safe if they accept a gap that is long enough for their crossings. But the reality is more complicated. Many accidents happen after vehicles on a minor road have fully stopped and accepted a gap. One can argue that this is because the drivers accepted a bad gap. But even if a gap is big, if the crossing vehicle is slow, there is still a chance for a crash. Many factors, such as driver hesitation, or insufficient acceleration, failure to consider vehicle length, overestimating the vehicle's accelerating capability, long driver response time (especially with elderly drivers), underestimating vehicle load, or unrecognizable road surface conditions, may cause unexpected conflicts that may be avoided under average traffic conditions (average vehicle, average driver, and an ordinary road surface that help to generate the critical gap). Despite these uncertainties, there are relatively few road crashes compared with the number of opportunities for crashes. This is because drivers adjust their behavior after they recognize their initial failure in perception and action. Their adjustments may not be another one-shot decision. Instead, the adjustment is a sequence of behaviors until the conflict is over. All these behaviors are chosen from a finite set of driver behaviors. These adjustments can be modeled as Markov chains.

Recognizing driver-vehicle behaviors as stochastic processes helps us understand the odds in traffic safety. A Markov chain or process is a sequence of stochastic events or states. These events or states belong to a set with a finite number of elements. The probability of an event or state presenting at a moment depends only on the immediately previous event or state. A HMM represents a Markov process whose states are not directly observable. The state of the observed sequence is associated with the hidden states by a set of probability distributions. The use of HMMs in sign language recognition (20), shows their potential in modeling resemble and distinct behaviors in other areas.

This analysis assumes that, in a certain population, driver behavior is statistically consistent when facing certain level of conflicts. For instance, a portion of drivers will accept a specific gap at intersections, and a portion of drivers will accelerate their vehicles at a certain rate with a limited deviation after they accept a gap. On the basis of this assumption, HMMs are expected to capture the common driver behavior from several recorded sequences of vehicle movement. Figure 1 shows a stylized HMM used in this research. The circles in this figure are hidden states among which transitions may happen according to a probability matrix. In our application, the hidden states can be thought of as the attitudes of drivers. The

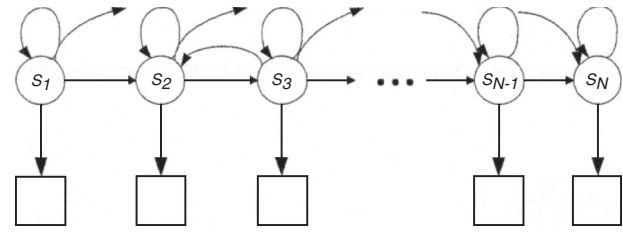


FIGURE 1 Structure of HMM.

squares in this figure are observable states that are generated by the model with certain probabilities. This model is complete if the transition matrix and the observed probabilities are obtained. The Baum-Welch estimation algorithm has been applied to estimate these probabilities from observations (21).

In this study of driver behavior, vehicle movements can only be observed on the road. It is hard to observe drivers' behaviors within vehicles. To obtain driver's attitudes on driving under certain circumstances is nearly impossible. To derive unobservable driver attitudes (A_n), a HMM is used, in which vehicle dynamics data (D) from the real world are observable states, as shown in Figure 2.

An HMM can be understood as representing a set of behaviors in the real world. A behavior is recognized by computing the probability that an HMM generates the observed event sequence. In the case of intersection traffic, behaviors could be sequences of acceleration (acc), deceleration (dec), cruising (crs), and various steering movements or combinations of them. Typically, the analysis will start with vehicle speed or acceleration as the observed states of HMM, and estimate driver attitudes that are the hidden states. In the model, the observed states statistically depend on the hidden states.

The driver behavior described by a HMM can be used directly in microscopic simulation that generates traffic conflicts and crashes. This provides more realistic traffic simulation at intersections, in which bad decisions of drivers are made possible.

VEHICLE DATA AND PREPROCESS

Data Source

The data were collected as part of the Intersection Decision Support (IDS) Project at the University of Minnesota, which aims to develop a collision-avoidance warning system for drivers at rural unsignalized intersections. The installation of the system began in May 2004 and was finished in January 2005. The data collection started in spring 2005. A set of sensors, computers, and communication systems is installed at the intersection of US-52 and County State Aid Highway (CSAH) 9 in Goodhue County, Minnesota. The IDS system uses

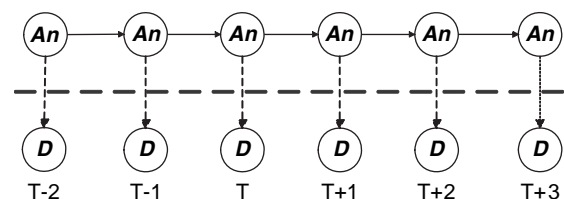


FIGURE 2 Temporal sequence of HMM.

sensing and communication devices to determine the safe gaps for vehicles on the minor roads and conveys the information to drivers. Fourteen radar sensors and two lidar sensors are placed along the roadside. Radar sensors collect vehicle speed and position; lidar sensors measure vehicle size. To achieve the goal of real-time collision warning, the system must be capable of working continuously and monitoring all vehicle movements across the intersection. A useful byproduct of this project is the collection of detailed information of vehicle movements. The data to be used in constructing the HMM driver model was obtained from these observations, which include the presence, type, location, velocity, acceleration and deceleration of vehicles, and the distance and time to the location where roads cross. Video cameras detect the lighting condition. Weather conditions are collected by an existing weather station. The updating rate was not consistent in the early stage of the experiment, but the aim was 10 Hz. This is the first study in which such detailed and so many vehicle trajectories, which are not only records of vehicle dynamics but also records of drivers' decision making, have become available. Those are exactly the data we need in our research on modeling driving behavior.

The geometry of the intersection is shown in Figure 3. The southbound traffic from Port 1 and the northbound traffic from Port 5 are the sources of major road traffic on US-52. The eastbound traffic from Port 7 and the westbound traffic from Port 3 are the sources of minor road traffic on CSAH 9. There are significant differences between major road vehicles and minor road vehicles because minor road vehicles have a lower priority right-of-way and have to stop before stop signs and yield to major road vehicles when entering the major road. Furthermore, some minor road vehicles that intend to

make a left turn or go through will have to cross the four-lane (2+2) divided highway. The drivers have to choose whether to finish the maneuver in one step or two steps. This will affect the observed vehicle dynamics significantly and make the modeling effort more challenging. In this part of the research, we concentrate on modeling major road vehicles.

The first step of data processing was to classify the vehicle movements. Figure 4 presents the traffic counts of major movements in 5 h. The lower number above each bar is the entrance and exit port of the traffic. For example, "52" represents traffic entering from Port 5 and exiting Port 2. The upper numbers are traffic counts for all major movements. As one can see, the number of through-going vehicles is much greater than others, which provides a good sample set for model estimation. Because the number of left-turn and right-turn vehicles on the major roads is small and their behaviors are much more complicated than those of through-going vehicles, they were not included in this phase of the study.

With vehicle movement data in a 5-h period, the trajectories of 1963 major road vehicles were extracted and were used to estimate the HMMs for main road vehicles.

Preprocessing Vehicle Data

The first step was to estimate a single HMM for all samples. The basic procedure of data processing and modeling includes

- Preprocessing vehicle data, which includes reading the raw data according to the given format, recognizing vehicle origins and

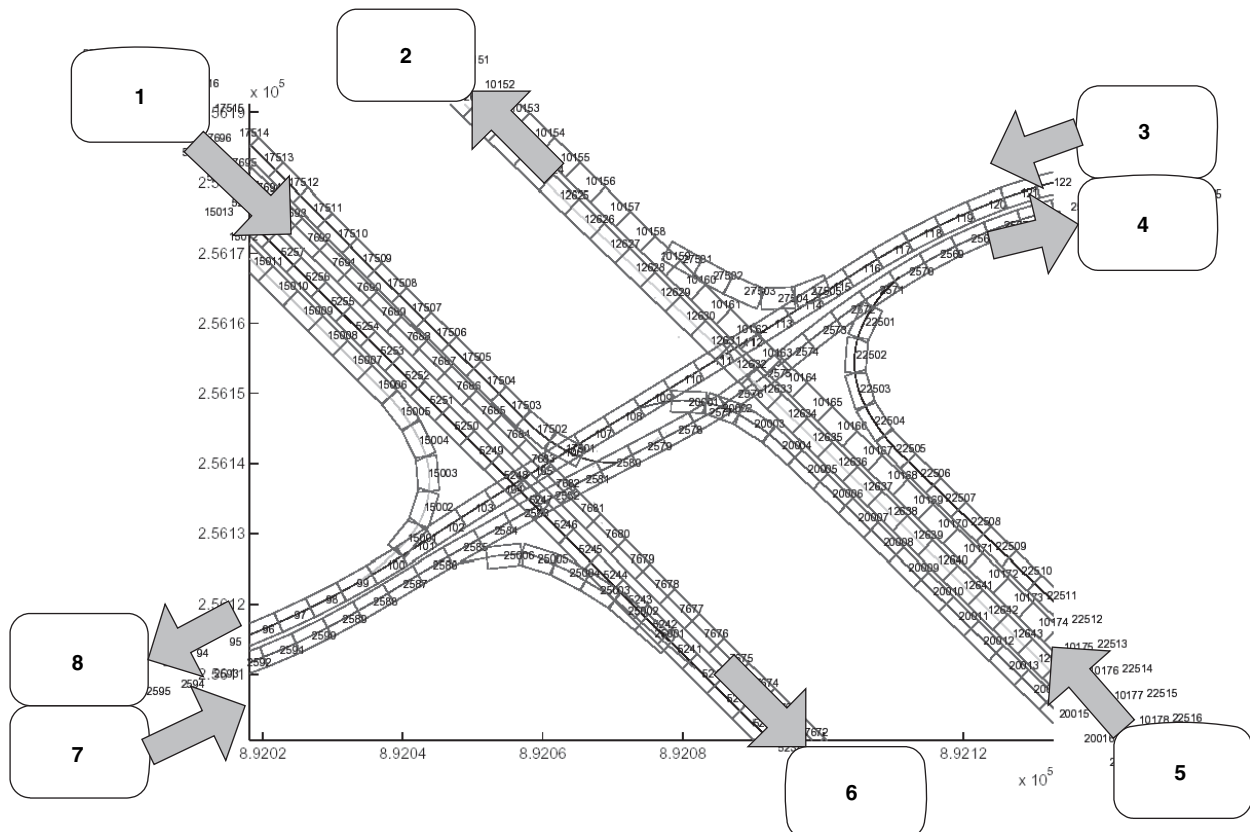


FIGURE 3 Geometry of intersection and ports.

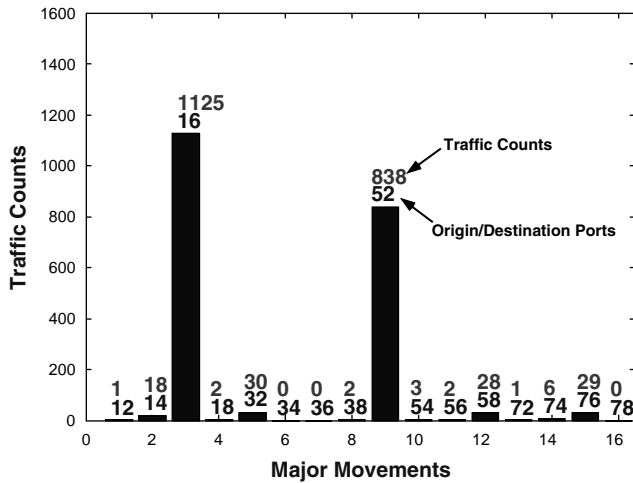


FIGURE 4 Traffic counts of road vehicle movements in 5 h.

destinations, and modifying data structure for further operation and transforming vehicle data to specified structure;

- Interpolating vehicle movement data to the 10-Hz updating rate;
- Extracting vehicle data in need, for example, all through-going vehicles on major road; and
- Extracting more vehicle information, including lane use, lead vehicle, and vehicles in conflict with each vehicle.

Generating Vehicle State Sequences

Then the observed vehicle trajectories are transformed to sequences of vehicle states, which are used in model estimation. To simplify the analysis, a small state set was used to represent vehicle behavior in extracting observable states. A basic state set of vehicle dynamics is defined as {ACC, CRS, DEC}.

Within the data set of vehicle movement, the most accurate information was vehicle speed, which was directly measured by radar sensors. Other information, such as position and acceleration rate, was less accurate because it was obtained by integrating data from many radar sensors and is thus affected by multiple noise sources. To obtain the acceleration rate, which was used to generate vehicle state sequences, we used the speed difference in each time step, that is, a positive change denotes state {acc}, zero change denotes state {crs}, and a negative change denotes state {dec}.

The state set of traffic condition experienced by each vehicle is defined as {in conflict, not in conflict}. A conflict exists when there is a potential that two vehicles may collide with each other. This definition is broader than traditional ones, in which a conflict is a situation that forces drivers to maneuver to avoid collisions. A broader definition was used because the difference between maneuvers caused by potential crash and normal driving maneuvers is relatively fuzzy. There was not enough information to distinguish between drivers' normal driving habits when they are close to intersections, cautious driving maneuvers, and alerted maneuvers in response to conflicts. Moreover, a broader definition helps us understand the general effect of traffic conflicts on driving behaviors, not exclusively severe conflicts and crashes. The procedure of extracting vehicles in conflicts includes

- Defining major movements of vehicles that potentially conflict with the object vehicle,

- Extracting all vehicles with movements in conflict with the object vehicle, and
- Comparing the operating time of each pair of vehicles in conflicts. If their time duration is overlapped, they are a pair of vehicles in conflict and the object vehicle is denoted as {conf}; otherwise, the object vehicle is denoted as {nonconf}.

After the maneuver state and the traffic state for each vehicle on the major road are defined, the observable vehicle states can be generated. The observable vehicle state set includes six states that are generated by {acc, crs, dec} × {in conflict, not in conflict}, which results in {(dec, nonconf), (dec, conf), (crs, nonconf), (crs, conf), (acc, nonconf), (acc, conf)}.

Defining hidden states is not easy without information about the object, such as driving attitudes. However, one can learn from testing the same sample set with different numbers of hidden states. If, in the results, one state behaves similar to another one, it is a redundant state and can be eliminated. After some experiments, three hidden states were found sufficient to represent distinct driving attitudes for the sample set.

ESTIMATION

Estimating Single HMMs

A single HMM can be estimated from a 5-h subset of the sample. For each sample, there is a probability that this sequence of movement is generated by the HMM. We calculate the log likelihood for each sequence, as shown in Figure 5. In this figure, each point represents the log likelihood of a state sequence of a vehicle on the estimated HMM. The higher the value, the less likely that the sequence is generated by the estimated HMM. In this figure, if a vehicle is involved in conflicts, the number of vehicles that are in conflict with the object vehicle is shown next to the point. As one can see, most vehicles on the major road were not involved in any conflict, and their likelihood of being generated by the estimated model is higher. In contrast, some major road vehicles in conflict with one or two other vehicles are less likely to be generated by the estimated HMM. In other words, the estimated model does not fit vehicles in conflicts as well as those

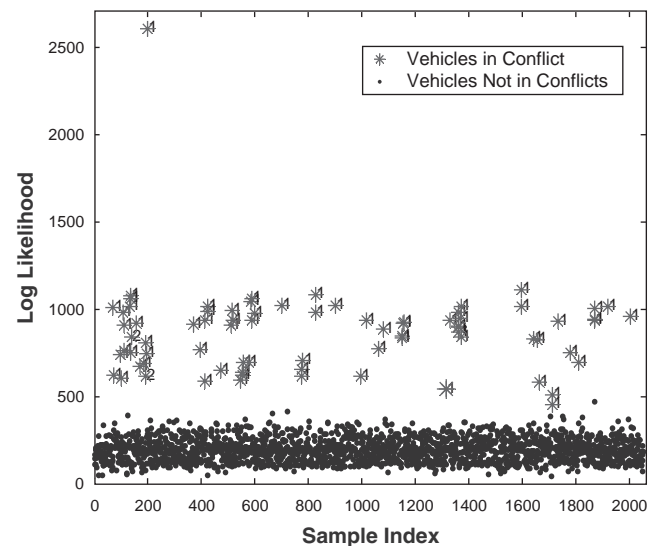


FIGURE 5 Log likelihood of samples in a single HMM (5 h).

not in conflicts. An outlier in the figure represents a vehicle involved in conflict with four other vehicles. This is not a reasonable situation. An individual check of this data point suggests that data error could be the major reason.

Clustering Major Road Vehicles

The single HMM does not fit all sample sequences well; this hints that samples representing vehicles in conflicts may have a behavior set different from those not in conflicts. The solution is to use different HMMs to model behavior characterized by different clusters. A heuristic clustering method is used, which separates vehicles by their log likelihood ratio from the single HMM. Too many clusters may cause the model to be too complicated and eliminate the variety among real clusters. Two clusters represented the samples well. Each sample was assigned to a cluster by comparing its log likelihood with thresholds, for example, a sample with a log likelihood value between two thresholds was assigned to a specified cluster.

Then the two clusters were reestimated individually, which generates two HMMs. The log likelihoods of the single HMM and the clustered model are shown in Table 1. The increase in log likelihood indicates the improvement of the model in representing the samples.

To illustrate the clustered HMMs further, the differences in a set of estimated cluster HMMs will be discussed below. People will expect to see diversity in behaviors of vehicles not in conflict and vehicles in conflict. From the estimated cluster HMMs, one sees that drivers behave differently when facing different traffic conditions. And within a group of people facing similar traffic conditions, their behavior patterns are relatively consistent. Table 2 shows an example of estimated clustered HMMs that distinguishes Clusters A and B. The left portion of the table is the transition matrix of the HMMs. For instance, the matrix of {A1, A2, A3} × {A1, A2, A3} is the transition matrix of Cluster A, in which A1 denotes the first hidden state. The right portion is the confusion matrix for Clusters A and B, which represents the probability of observing a certain maneuver from a given hidden state. As one can see, samples in Cluster A were not involved in any conflicts. They have higher tendency to change from States 1 and 2 to State 3 (0.63 and 0.53). State 3 represents a higher probability of decelerating (0.73). But for vehicles in Cluster B, they are more likely to stay in their current attitude (state) (0.82, 0.94, 0.95). The three attitudes (states) are distinct in that their probabilities of deceleration, cruising, and acceleration are significantly different.

APPLICATIONS: PREDICTING MOVEMENTS OF MAJOR ROAD VEHICLES

To test the effectiveness of the clustered HMMs, the estimated clustered HMMs were used to generate vehicle movement sequences and compare the results against real data. The assumption is that if

TABLE 1 Log Likelihood Comparison of Samples: Single HMM and Cluster HMMs

Cluster	A	B	Total
Number of vehicles	1,859	103	1,962
Total log likelihood with clustered HMMs	-346,642	-9,430	-356,072
Total log likelihood with single HMM	-366,599	-83,796	-450,395

TABLE 2 HMM Comparison for Clusters A and B

	A1	A2	A3	DEC	CRS	ACC
Nonconf						
A1	0.26	0.11	0.63	0.54	0.36	0.10
A2	0.18	0.29	0.53	0.37	0.23	0.4
A3	0.15	0.15	0.70	0.73	0.23	0.04
Conf						
	B1	B2	B3			
B1	0.82	0.08	0.1	0.45	0.05	0.5
B2	0.04	0.94	0.02	0	1	0
B3	0.05	0	0.95	0.99	0.006	0.004

NOTE: Bold numbers denote probabilities that are more significant than others, which represent higher tendencies of certain transitions.

it is known which clusters the testing samples belong to, the clustered HMMs should generate prediction sequences that include the numbers of maneuvers that are statistically close to the sequences from the real data. For instance, the number of acceleration maneuvers for vehicles not in conflict should be close to observed data. The procedure includes

- Reading vehicle movement sequences from a duration different from the one used to estimate the model;
- Determining the cluster a vehicle sequence belongs to—done by calculating the log likelihood of the sample with each clustered HMM and finding the HMM with maximum likelihood; and
- Generating vehicle maneuver sequence from the clustered HMM of this vehicle.

Sixty minutes of vehicle movement data were randomly picked from the database, from which 569 through-going major road vehicles are extracted. These 569 state sequences of vehicle movement include 105,928 observable states. Vehicles take an average of 18.6 s running time to cross the sensing area at the intersection. The comparison is shown in Table 3. As one can see, for some maneuvers like {dec, nonconf} and {acc, nonconf}, the differences are small, which means that the model can statistically provide a good prediction of maneuvers. For all maneuvers with conflicts, the errors are relatively large. Also the errors are proportional to the original numbers. It means that the samples that are correctly clustered are nicely predicted.

VALIDATION

Testing Clustering

An application of clustered HMMs is to assign vehicles to clusters with limited information. The testing samples introduced in the previous section were used to test the model. For each sequence, the first portion was extracted and used to recognize the cluster. This was done by calculating the log likelihood of the sample in each cluster HMM and finding the cluster with the maximum likelihood. Then the second portion of the sequence was used to determine which cluster this vehicle really belongs to. Because this is a stochastic approach, the complete information (100% length) of sequence cannot guarantee 100% accuracy of clustering. Sequence lengths of 90%, 50%, and 10% were tested. The higher this number is, the more likely the

TABLE 3 Comparison of Predicted Sequences and Real Sequences

	DEC		CRS		ACC		Total
	Nonconf.	Conf.	Nonconf.	Conf.	Nonconf.	Conf.	
Number of states generated by clustered HMMs	59,592	7,288	23,686	3,441	10,212	1,709	105,928
Number of observed states	60,433	4,276	27,515	2,001	10,965	738	105,928
Relative errors	-1.4%	70%	-14%	72%	-6.9%	131.6%	0

clustering result is right. The results of the test are shown in Figure 6. The 90% length in the legend means that 90% portion of the whole sequence is used to recognize the real cluster the sequence belongs to. The results show that when the length of the partial sequence used in cluster estimation is longer than 30%, the error rate becomes very small. We observe the first 30% length of a vehicle movement along a road and determine the type of driver with more than 90% confidence. This trend is consistent.

Comparing Vehicle Trajectories

Another evaluation of the clustered HMMs is to compare the predicted speed and position sequences against the real vehicle movement

sequences and the sequences predicted by traditional car-following models. The procedure includes

- Extracting real vehicle movement sequences,
- Assigning vehicles to the existing clusters with the first 30% length of movement sequences,
- Generating the prediction sequences by the clustered HMMs,
- Generating the prediction sequences by the car-following model, and
- Comparing the prediction errors.

Peng and Lee (22) show that the Gipps model fits the highest number of maneuvers, compared with other longitudinal driving models. This result is supported by the comparison study of Brockfeld et al. (23).

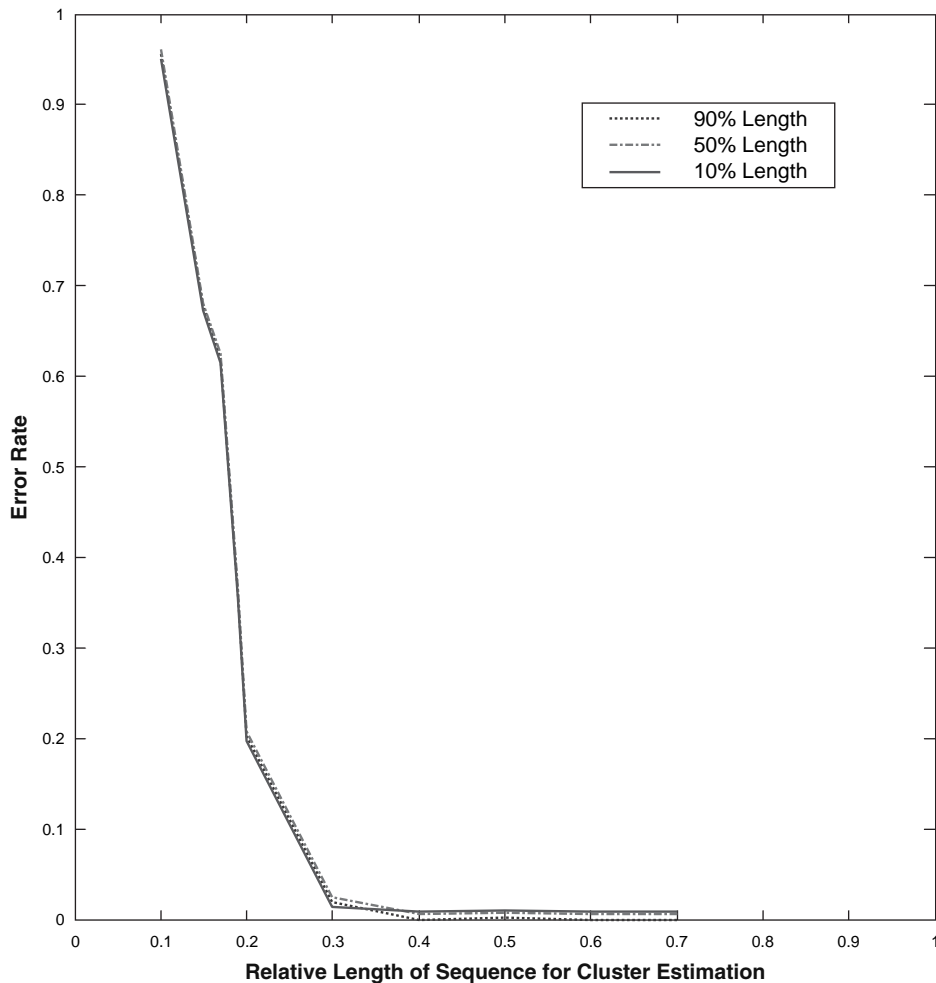


FIGURE 6 Error rates of clustering estimation.

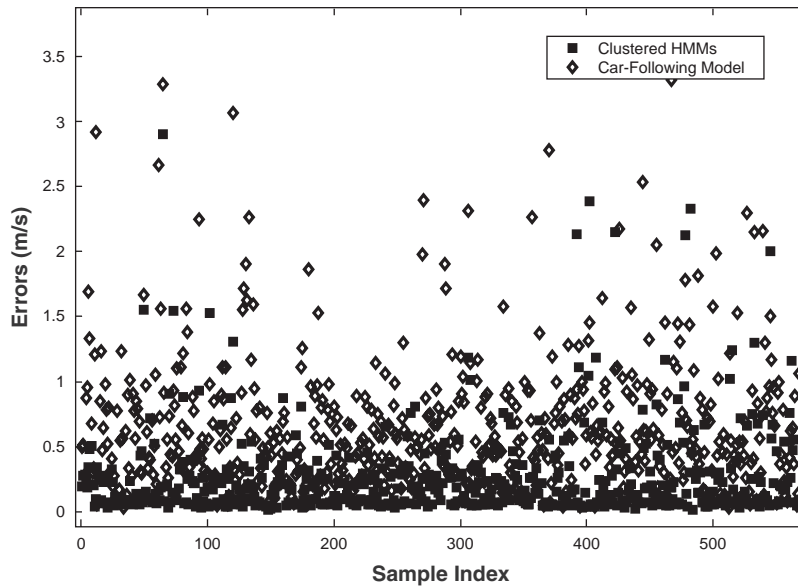


FIGURE 7 Comparing average-speed errors in clustered HMMs and car-following model.

The Gipps model was chosen as the representative of the traditional car-following model. First the prediction error of vehicle speeds was evaluated because the output of the Gipps model is vehicle speed. The differences are significant in Figure 7, where the average prediction errors of vehicle speed are shown. The average speed error of clustered HMMs is 0.47 m/s or 1.52%, while the average speed error of the Gipps model is 1.38 m/s or 4.5%. As one can see, the prediction error of clustered HMMs is much lower than that of the Gipps model. The errors of the predicted sequences are evaluated as

$$e_n = \sqrt{\sum_i \left[\log \left(\frac{\hat{x}_{n,i}}{x_{n,i}} \right) \right]^2} \quad (1)$$

where

- e_n = prediction error of sequence n ,
- $x_{n,i}$ = i th element of sequence n , and
- $\hat{x}_{n,i}$ = prediction of i th element of sequence n .

If the prediction sequence is close to the real sequence, e_n should be close to zero. In Figure 8, the relative error in predicting vehicle positions is compared with the clustered HMMs and Gipps model. It shows that the cluster HMMs provide good prediction of vehicle positions compared with Gipps model. The average relative error is 1.4% for the clustered HMMs, and 4.7% for the Gipps model. The results for the Gipps model, though inferior in this comparison, are relatively better than results of Brockfeld et al. (23), which find an average of 16% to 17%. These comparisons show the potential of the cluster HMMs in predicting sequences of vehicle movements.

CONCLUSION

This research took advantage of Markov dynamic theory to derive driving behavior models. Observed vehicle movement data were used to estimate HMM. A clustered HMM approach is proposed to model the variety of driving behaviors; this provides a new understanding

of driving behavior in conflicts. The estimated HMMs are capable of recognizing the driving style (cluster) of each vehicle before it enters the intersection. Clustering vehicles on the basis of the HMMs enables discovering different driving attitudes from observations. The estimated cluster HMMs are capable of predicting driving behaviors based on previous observed data, which shows potential for application in microscopic simulation.

This approach provides a new understanding of complex driving behaviors. We believe this research will lead us to an understanding of traffic conflicts in general, which eventually will help us in understanding severe conflict and crashes. The quantification of driver interactions improves the understanding and modeling of driver behavior and provides information for safety countermeasure design. The new driver model enables simulation that includes driver interactions at intersections and is more realistic than traditional counterparts. In subsequent research, the connections between driver behaviors, conflicts, and crashes will be analyzed on the basis of the proposed HMMs. Some improvements of the proposed model may include

- Modeling: including other movements on major and minor approaches;
- Calibration: improving fitness of cluster HMMs and including more traffic conditions and more environmental factors;
- Simulation: deploying estimated models in microscopic traffic simulation for evaluating safety and efficiency; and
- Interaction: enhancing the model by refining interactive decision-making processes among vehicles in conflicts.

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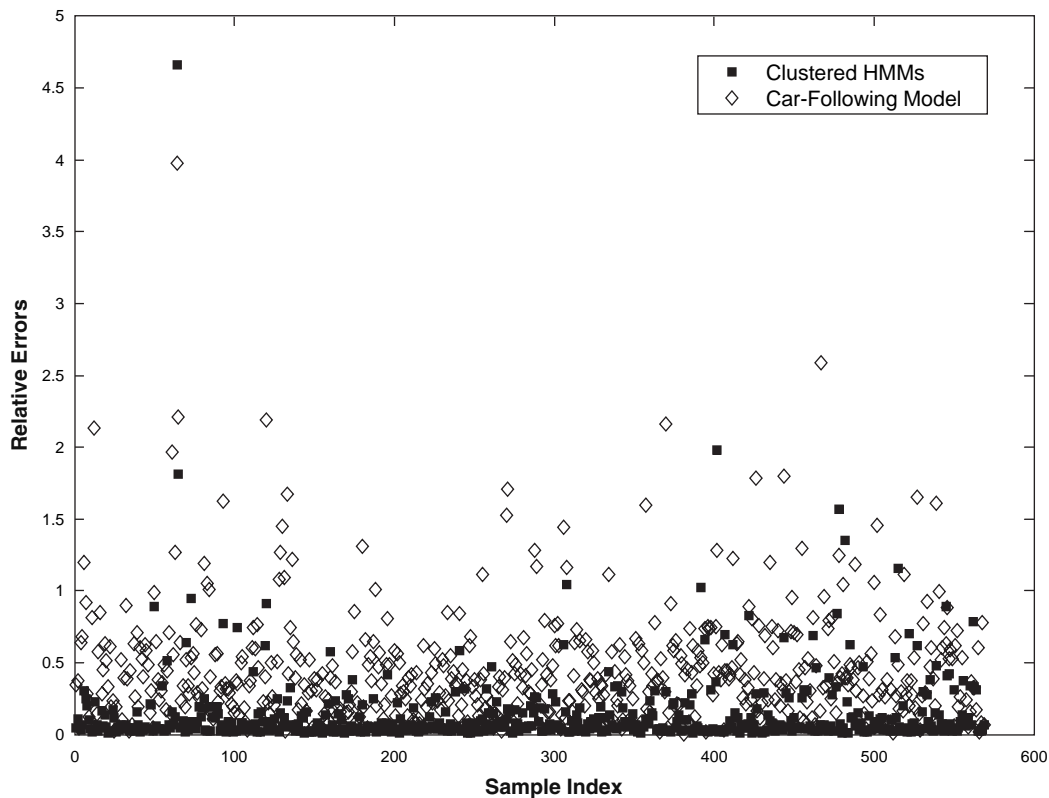


FIGURE 8 Comparing prediction errors for vehicle position.

REFERENCES

- Rothery, R. W. Car Following Model. Chapter 4 of *Traffic Flow Theory*. U.S. Department of Transportation. www.tfhrc.gov/its/tft/tft.htm. Accessed Feb. 2004.
- Ahmed, K., E. Moshe, H. Koutsopoulos, and R. Mishalani. Models of Freeway Lane Changing and Gap Acceptance Behavior. In *Proceedings of 13th International Symposium on Transportation and Traffic Theory*, Lyon, France, 1996.
- Allen, R., and T. Rosenthal. A Computer Simulation Analysis of Safety Critical Maneuvers for Assessing Ground Vehicle Dynamic Stability. *Vehicle Dynamics and Simulation*, SAE SP-950, 1993.
- Vanstrum, R., and G. Caples. Perception Model for Describing and Dealing with Driver Involvement in Highway Accidents. *Highway Research Record*, Vol. 365, 1972, pp. 17–24.
- McKnight, J., and B. Adams. *Driver Education and Task Analysis. Volume 1: Task Descriptions*. Technical Report. National Highway Safety Bureau, Department of Transportation, Nov. 1970.
- Reece, D. *Selective Perception for Robot Driving*. PhD thesis. Carnegie Mellon University, Pittsburgh, Pa., 1992.
- Pipes, L. A. An Operational Analysis of Traffic Dynamics. *Journal of Applied Physics*, Vol. 24, 1953, pp. 271–281.
- Chandler, F. E., R. Herman, and E. W. Montroll. Traffic Dynamics: Studies in Car Following. *Operational Research*, Vol. 6, 1958, pp. 165–184.
- Gazis, D. C., R. Herman, and R. W. Rothery. Nonlinear Follow-the-Leader Models of Traffic Flow. *Operational Research*, Vol. 9, 1961, pp. 545–566.
- Tyler, J. S. The Characteristics of Model Following Systems as Synthesized by Optimal Control. *IEEE Transactions on Automatic Control*, Vol. AC-9, 1964, pp. 485–498.
- Newell, G. F. Nonlinear Effects in the Dynamics of Car Following. *Operational Research*, Vol. 9, 1961, pp. 209–229.
- Gipps, P. G. A Behavioral Car-Following Model for Computer Simulation. *Transportation Research Part B*, Vol. 15, 1981, pp. 105–111.
- Nagel, K., and M. Schreckenberg. A Cellular Automation Model for Freeway Traffic. *J. Physique I*, Vol. 2, 1992, p. 2221.
- Daganzo C. F. The Cell Transmission Model: A Simple Dynamical Representation of Highway Traffic. *Transportation Research Part B*, Vol. 28, 1994, pp. 269–287.
- Treiber M., A. Hennecke, and D. Helbing. Congested Traffic States in Empirical Observation and Numerical Simulations. *Physical Review E*, Vol. 62, 2000, 1805–1824.
- van der Molen, H., and A. Botticher. Risk Models for Traffic Participants: A Concerted Effort for Theoretical Operationalizations. In *Road Users and Traffic Safety* (J. Rothengatter, and R. de Bruin, eds.), Van Gorcum, Assen, Netherlands, 1987.
- Neumann, B. *Semantic Structures: Advances in Natural Language Processing* (D. L. Waltz, ed.). Lawrence Erlbaum Associates, Inc., Hillsdale, N.J., 1989, pp. 167–206.
- Davis, J. W. and A. F. Bobick. The Representation and Recognition of Action Using Temporal Templates. *Proc. of IEEE Conference on Computer Vision and Pattern Recognition*, June 1997, Puerto Rico, pp. 928–934.
- Davis, L., R. Chelappa, A. Rosenfeld, D. Harwood, I. Haritaoglu, and R. Cutler. Visual Surveillance and Monitoring. *Proc. of DARPA Image Understanding Workshop*, 1998, Monterey, Calif., pp. 73–76.
- Starner, T., and A. Pentland. Visual Recognition of American Sign Language Using Hidden Markov Models. *Proc. International Workshop on Automatic Face and Gesture Recognition*, Zurich, Switzerland, 1995.
- Rabiner, L. R. A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition. *Proceedings of the IEEE*, Vol. 77, No. 2, 1989.
- Peng, H., and K. Lee. Identification and Verification of a Longitudinal Human Driving Model for Collision Warning and Avoidance Systems. *International Journal of Vehicle Autonomous Systems*, Vol. 2, Nos. 1 and 2, 2004.
- Brockfeld, E., R. D. Kühne, A. Skabardonis, and P. Wagner. Toward a Benchmarking of Microscopic Traffic Flow Models. In *Transportation Research Record: Journal of the Transportation Research Board*, No. 1852, Transportation Research Board of the National Academies, Washington, D.C., 2003, pp. 124–129.