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Deflation Caused by Bank Insolvency*

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Abstract

The Japanese economy has suffered from persistent deflation since the mid-1990s, when the banking system fell into serious undercapitalization. In Germany and in China, worries about impending deflation have emerged, along with fear of prospective or hidden bank insolvency.

In this paper I present a simple model in which bank insolvency causes deflation. During a period of bank insolvency, bank deposits in excess of bank assets continue to exist if the government (implicitly) guarantees them. I assume that bank deposits cannot exceed a certain multiple of the monetary base and that the government is prohibited to expand fiscal expenditures. A government that guarantees unbacked bank deposits without recapitalizing an insolvent banking system is forced to set the nominal interest rate at nearly zero and to let the price level fall.

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Keywords: Bank insolvency, deflation, forbearance policy, liquidity trap.

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1 Introduction

Japan has suffered from persistent deflation and has kept the short-term nominal rate of interest at zero since the mid-1990s, when the banking system fell into serious undercapitalization. The persistent deflation and low nominal interest rates in Japan have triggered a debate in macroeconomics about monetary policy in a deflationary environment. In this academic and policy debate, deflation and low nominal interest rates are assumed to be caused by exogenous shocks on productivity or preference.[1] There does not seem to be any research pointing to a possible linkage between Japan's deflation and its financial system disarray subsequent to the collapse of its asset-price bubble at the beginning of the 1990s. In point of fact, it was precisely when the fear of an impending banking crisis emerged in the mid-1990s that deflation in Japan set in.

In Germany, worry of impending deflation emerged in 2003, just when the vulnerability of its financial system surfaced. In China, where the scale of the nonperforming loan problem is alleged to be larger than Japan's, the prices of goods are falling whereas the prices of services are rising. The examples of Japan, Germany, and China seem to indicate the possibility of a linkage between banking system problems and deflation.[2] The unique characteristic of Japan's banking problem is the lengthy postponement of action to bail banks out. The Japanese government moved quickly to guarantee all bank deposits, but it did not come up with measures to restore the solvency of the banking system until several years had passed after bank insolvency had become apparent. My conjecture is that the postponed recapitalization of an insolvent banking system may have had a causal link with the protracted deflation.

The purpose of this note is to show a theoretical possibility that systemic bank insolvency can cause deflation if the government makes use of a forbearance policy: guaranteeing bank deposit without bailing out an insolvent banking system directly through

[1] Papers in this literature include, for example, Krugman (1998), Eggertsson and Woodford (2003), Auerbach and Obstfeld (2003), and Svensson (2003).

[2] In the literature of banking crises, it has recently been found that the inflation rate typically falls after the onset of bank insolvency in single-crisis countries (Boyd et al. [2001]).

an injection of taxpayers' money. During a period of bank insolvency, bank deposits in excess of bank assets continue to exist under this forbearance policy. Assuming that the banks need to hold liquid assets to produce transaction services associated with the deposits, the growth of unbacked deposits forces banks to increase their holdings of liquid assets and to decrease loans to firms, where the loans are proportional to the price level. This mechanism reduces the ratio of bank loans to cash, and induces deflation.

If deflation is caused by bank insolvency and the government's forbearance, the straightforward policy to stop deflation would be to bail out the insolvent banks through a one time lump-sum transfer from the consumers to them. Monetary policy (i.e., swapping cash for government bonds) alone will not be effective in halting deflation, since the increase of cash by means of this policy will decrease the outstanding amount of government bonds, resulting in a decrease in necessary tax revenue. If real activity does not change, the decrease in tax revenue implies a decrease in the price level. Therefore, monetary policy can be effective in stopping deflation only if fiscal expansion accompanies it. But in this case, the government debt will violate the transversality condition. In short, to cope with deflation without resolving bank insolvency is a very difficult task for policymakers to tackle.

The organization of this paper is as follows: In the next section, I present the basic structure of the model. In Section 3, I introduce bank insolvency caused by an unexpected macroeconomic shock, and I describe the development of deflation in an environment of deposit guarantees and postponed bank recapitalization. Section 4 provides some concluding remarks.

2 The Basic Model

The structure of the model is similar to that in Kobayashi (2003), with a few essential differences: I introduce *cash* in a version of the previous model and simplify the production technology such that the output is equal to the endowment at each date, and there is no capital accumulation.

The economy continues for an infinite period from date 0: $t = 0, 1, 2, \cdots, +\infty$. This economy consists of one government and continua of consumers, firms, and banks. Each continuum (of consumers, firms, and banks) is of measure 1. I assume that consumers, firms, and banks act as price takers as a result of competition in each sector. I also make the following assumption about the medium of exchange in this economy:

Assumption 1 *Money consists of cash, bank deposits, or government bonds. All transactions between a consumer and a firm must be mediated by money. A consumer and a firm cannot directly lend to or borrow from each other.*

Cash is intrinsically useless paper provided by the government.

Government The government issues bonds (B_t) and cash (M_t), collects revenue from a consumption tax ($\tau_t P_t c_t$), and makes a lump-sum transfer to consumers (G_t), satisfying the following budget constraint:

$$\begin{aligned} &B_0 + M_0 = B + M + G_0, \\ &\tau_t P_t c_t = R_{t-1} B_{t-1} + M_{t-1} + G_t - B_t - M_t, \ \text{ for } t \geq 1, \end{aligned} \tag{1}$$

where B and M are the initial amount of government bonds and cash owned by consumers, P_t is the price of the consumer goods, c_t is the consumption at date t, and R_t is the nominal return between date t and date $t+1$.

Consumers The consumers are infinitely long-lived and maximize the following utility: $\sum_{t=1}^{\infty} \beta^t u(c_t)$, where c_t is consumption at date t, β is the discount factor, and $u(c)$ is a concave and increasing function of c. I assume for simplicity that $u(c) = \ln c$. Each consumer is endowed with one unit of labor at each date t. At each date t, consumers sell their labor to firms, buy consumer goods from firms, and consume them at the same date t. The consumer's maximization problem is as follows:

$$\max_{c_t, n_t, B_t, D_t} \sum_{t=1}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{cases} B_0 + D_0 \leq B + M + W_0 n_0 + G_0, \\ (1+\tau_t) P_t c_t + B_t + D_t \leq R_{t-1}(B_{t-1} + D_{t-1}) + W_t n_t + \Pi_t^F + G_t, \quad \text{for } t \geq 1, \\ 0 \leq n_t \leq 1, \ \text{ for } \forall t, \end{cases} \tag{2}$$

where B and M are the initial amounts of government bonds and cash owned by consumers, D_t is bank deposits at date t, R_t is the nominal return on bank deposits and government bonds, n_t is the labor supply, W_t is the wage, and Π_t^F is the dividend from firms.

Note that consumers deposit all cash M_t provided by the government in banks, since the deposit rate R_t is no less than one. The arbitrage between bonds and deposits guarantees that the nominal returns on government bonds and on bank deposits are equal: Arbitrage occurs because they are perfectly equivalent assets for consumers (see Assumption 1). Note also that wage income $W_t n_t$ and dividend Π_t^F are given in the form of bank deposits (see below).

Firms and banks Firms continue operating for one period. Firms are established at date t, they produce output at date $t+1$, sell the output, pay out the dividends (if any) to consumers, and are liquidated at date $t+1$. I assume a simple production technology: one unit of labor at date t is transformed into one unit of consumer goods at date $t+1$. Since Assumption 1 holds, a firm needs to have money (cash or bank deposits) to buy labor from consumers. Since it does not have money, it must borrow from banks. Given the output price P_{t+1} at date $t+1$ and the wage W_t at date t, a firm solves the following profit maximization:

$$\max_{L_t, n_t^d, y_t} \Pi_{t+1}^F \equiv P_{t+1} y_{t+1} - R_t^L L_t \tag{3}$$

subject to

$$\begin{cases} y_{t+1} \leq n_t^d, \\ W_t n_t^d \leq L_t, \end{cases}$$

where L_t is the amount of money that the firm borrows from a bank, R_t^L is the nominal return on the bank loan, n_t^d is the amount of labor employed, and y_{t+1} is the amount of consumer goods produced. Note that the bank provides loan L_t for the firm by giving the bank deposits L_t.

I assume that banks continue operating indefinitely. Banks accept deposits from consumers and provide loans to firms. Here I assume that the amount of deposits that a bank can lend is limited by its production technology for transaction services:

Assumption 2 *A bank holds cash, government bonds, and loans to firms as its assets, and deposits as its liabilities. It cannot have any liabilities other than deposits. A bank that has cash reserve M_t can create deposits D_t, where*

$$D_t \leq \frac{1}{\rho} M_t, \;\; (0 < \rho < 1). \tag{4}$$

This production technology is a simplified version of the technology for producing demand deposits in Chari, Christiano, and Eichenbaum (1995).[3] This assumption can be interpreted as a requirement that a bank must produce transaction services for all its liabilities. A crucial point is that the government bonds cannot be used as banks' reserves: It is assumed that a bank produces transaction services only from cash, not from government bonds.

A bank's profit maximization is

$$\max_{L_t, B_t^B D_t} \sum_{t=1}^{\infty} \left(\prod_{s=0}^{t} R_s^{-1} \right) \left\{ R_t^L L_t + R_t B_t^B + M_t - R_t D_t \right\} \tag{5}$$

subject to

$$\begin{cases} D_t = L_t + B_t^B + M_t, \\ D_t \leq \frac{1}{\rho} M_t, \\ B_t^B \geq 0, \end{cases}$$

where L_t is the amount of bank loans, B_t^B is the amount of government bonds held by the bank, and D_t is the amount of deposits. The third constraint ($B_t^B \geq 0$) necessitates some

[3] The microfoundation of this constraint can be built on the setting introduced by Smith (2002): At a time between date t and date $t+1$, a portion of the depositors (of measure ρ) are forced to move to a place where they lose communication with their original banks. The ρ depositors must withdraw their entire deposits D_t, take them in the form of cash to the new location, and deposit them in other banks there. Thus the banks must provide ρ depositors with cash ρD_t at a time between t and $t+1$. Therefore, the banks must hold cash $M_t \geq \rho D_t$ at date t to fulfill the demand deposit contract.

Although I assume that the upper bound of deposit creation is a technological constraint, it can be interpreted alternatively that the required reserve imposed by the government determines the upper bound, and that ρ is the rate of required reserves.

explanation. Since in reality a bank can sell government bonds short, it seems plausible to assume that a bank can set B_t^B at a negative value. And in this model the banks have strong incentive to set $B_t^B < 0$, because $R_t^L > R_t$ in the equilibrium as long as $R_t > 1$ (see equation [9]): The banks can earn infinite profits by selling government bonds short and increasing loans to firms. However, Assumption 2 sets a limit on this arbitrage. Assumption 2 says that the *gross* liabilities of a bank must be smaller than or equal to $\rho^{-1}M_t$. Thus if a bank sets B_t^B at a negative value, it must set aside cash reserves $-\rho B_t^B$, since the bank bears liabilities $-B_t^B$ when it sells $-B_t^B$ units of bonds short: In this model, selling government bonds short is equivalent for a bank to increasing bank deposits in the same amounts. Therefore, we can set the constraint $B_t^B \geq 0$ without loss of generality.

Constraint on fiscal policy I consider the case where the following constraint on fiscal policy is imposed:

$$\tau_t = \tau > 0, \text{ and } G_t = 0, \text{ for } \forall t. \tag{6}$$

In Section 3 (page 12) I discuss the case where the government can choose $\{G_t\}_{t=0}^{\infty}$ freely.

Competitive equilibrium The first-order conditions (FOCs) for consumers, firms, and banks imply

$$\frac{P_{t+1}c_{t+1}}{\beta P_t c_t} = R_t, \tag{7}$$

$$P_{t+1} = R_t^L W_t, \tag{8}$$

$$R_t = (1-\rho)R_t^L + \rho. \tag{9}$$

The market-clearing conditions are $n_t^d = 1$, $c_t = 1$, and $W_t = L_t$. Note that $B_t^B = 0$ in the equilibrium since $R_t^L \geq R_t$.[4] There exists a competitive equilibrium with constant inflation:

[4]If $R_t = 1$, the nominal returns on bank loans and on the government bonds are equal. In this case banks are indifferent between loans and bonds, but I assume for simplicity that banks choose to provide loans in this case.

Lemma 1 *Assume that* $\beta > 1 - \left(\frac{1}{\rho} - 1\right)\tau$. *For any value of* π *greater than or equal to* β*, there exists a competitive equilibrium with a constant rate of inflation:* $\frac{P_{t+1}}{P_t} = \pi$.

(Proof) In an equilibrium with a constant inflation rate π, the dynamics of the system are described by $R_t = \frac{\pi}{\beta}$, $P_t = \pi^t P_0$, $R_t^L = \frac{\frac{\pi}{\beta}-\rho}{1-\rho}$, $W_t = \frac{(1-\rho)\pi^{t+1}P_0}{\frac{\pi}{\beta}-\rho}$, $M_t = \frac{\rho}{1-\rho}W_t$, and $\tau\pi^{t+1}P_0 = \frac{\pi}{\beta}B_t + M_t - B_{t+1} - M_{t+1}$. Solving this system, it can be shown that

$$B_t = \frac{\pi^t}{\beta^t}\left[B_0 - \left\{\tau - \frac{\rho(1-\pi)}{\frac{\pi}{\beta}-\rho}\right\}\beta P_0\left(\frac{1-\beta^t}{1-\beta}\right)\right]. \tag{10}$$

Since I prohibit fiscal expenditure, the value of B_t must be nonnegative in an equilibrium. Note that $\pi \geq \beta$, since the nominal return $R_t = \frac{\pi}{\beta}$ cannot be less than one in the presence of cash. Note also that $\left\{\tau - \frac{\rho(1-\pi)}{\frac{\pi}{\beta}-\rho}\right\} > 0$ for $\pi(\geq \beta)$, since $\beta > 1 - \left(\frac{1}{\rho} - 1\right)\tau$. Therefore, setting P_0 at a value less than or equal to $\frac{B_0}{\left\{\tau - \frac{\rho(1-\pi)}{\frac{\pi}{\beta}-\rho}\right\}\frac{\beta}{1-\beta}}$, we have $B_t > 0$ for all $t < +\infty$. The transversality condition for the government debt uniquely determines P_0:

$$P_0 = \frac{B_0}{\left\{\tau - \frac{\rho(1-\pi)}{\frac{\pi}{\beta}-\rho}\right\}\frac{\beta}{1-\beta}}.$$

It has been shown that for any $\pi \geq \beta$ there exists a competitive equilibirium with constant inflation π. (End of Proof)

This lemma says that inflation at any given rate can occur as an equilibrium outcome if the government sets $\{B_t, M_t\}_{t=0}^{\infty}$ appropriately. In particular, there exists a steady state equilibrium with a constant price level (i.e., $\pi = 1$) in which the government chooses $B_t = B$ and $M_t = M$ such that

$$B = \frac{\tau(1-\rho\beta)}{\rho(1-\beta)}M. \tag{11}$$

In this equilibrium, $P_t = P^* \equiv \frac{1-\rho\beta}{\rho\beta}M$, $R_t = \frac{1}{\beta}$, $R_t^L = \frac{1}{\beta}\frac{1-\rho\beta}{1-\rho}$, and $W_t = L_t = (\frac{1}{\rho} - 1)M$.

3 Bank Insolvency and Deflation

I assume that at date 0, all agents in the economy expected that the economy would stay in the steady state equilibrium described above, and that an unexpected macroeconomic shock hit the economy at date 1 that rendered banks insolvent.

Bank insolvency At date 1, after firms sold goods (produced between date 0 and date 1) to the consumers and before they repaid their bank loans, a macroeconomic shock λ ($0 < \lambda < 1$) hit the economy unexpectedly. I assume that this shock enabled λ firms to transfer their revenue (P^*) to their owners (consumers) and to go bankrupt and default on their bank borrowings. Thus the consumers obtained unexpected profits λP^* in the form of bank deposits at date 1, while the banks incurred irrecoverable loans $N_1 = \lambda P^*$. This shock λ can be interpreted as a model of the emergence and collapse of the asset-price bubble.[5]

Forbearance policy At date 1 after cash is paid out to consumers, banks' only assets are the irrecoverable loans N_1, and their liabilities are the uncleared deposits N_1 from consumers. If the government does not guarantee the uncleared deposits N_1, bank runs occur and the irrecoverable loans N_1 are immediately written off, while consumers bear the cost as a lump sum. But, as in Kobayashi (2003), the government does not allow the occurrence of defaults on bank deposits, since it wants to maintain the public's confidence in banks or in deposit money. The government's guarantee of deposits N_1 is an implicit liability of the government. If the government decides that it is unwilling to let depositors bear the losses resulting from bank insolvency, it has no other choice than to make up for the losses itself, ultimately through the use of taxpayers' money. In what follows, I use the term *bank recapitalization* to refer to this form of banking system recapitalization through the injection of taxpayers' money. To inject taxpayers' money into the banking system is an unpopular policy and politically difficult to implement. The government tries to put off recognizing bank insolvency and making up for losses. Thus the government can be said to have undertaken a forbearance policy: guaranteeing the uncleared deposits N_1 and postponing bank recapitalization. I consider a perfect foresight equilibrium where the government guarantees bank deposits under the following assumption:

[5]Suppose that consumers own useless assets, e.g., land, the fundamental price of which is zero. An exogenous boom makes land prices rise, and firms invest their revenue P^* in land expecting a further rise of prices. Then, the subsequent price collapse makes firms go bankrupt, leaving the consumers unexpected profits.

Assumption 3 *The government declares that and people expect that it will recapitalize the banking sector through a one time lump-sum transfer from consumers to banks at some date T (where T can be a very large integer), i.e., the government levies a lump-sum tax on consumers at date T, and gives the tax revenue to the banks at the same date.*

Note that bank recapitalization through a lump-sum transfer does not distort the decision making of the consumers. Therefore, in the model of this paper the postponement of bank recapitalization does not have any effects on welfare, while in the model of Deckle and Kletzer (2003), Barseghyan (2002), and Kobayashi (2003), where recapitalization through a lump-sum transfer is prohibited, the postponement of recapitalization causes welfare loss under a distortionary tax system.

Deflation under forbearance policy Banks have no other choice than to let the uncleared deposits N_1 evolve at the nominal rate of return R_t, since otherwise depositors will make a run on banks to withdraw cash and to invest it in government bonds, resulting in banks' defaults on deposits.

Therefore, as long as the government postpones bank recapitalization, the uncleared deposits at date t (N_t) must satisfy

$$N_t = (\prod_{s=1}^{t-1} R_s) N_1 = (\prod_{s=1}^{t-1} R_s) \lambda P^*. \tag{12}$$

The consumer's problem from date 1 onward is

$$\max_{c_t, n_t, B_t, D_t} \sum_{t=1}^{\infty} \beta^t u(c_t)$$

subject to

$$\begin{cases} (1+\tau)P_1 c_1 + D_1 + B_1 \leq R_0(B_0 + D_0) + W_1 n_1 + N_1, \\ (1+\tau)P_t c_t + D_t + B_t \leq R_{t-1}(B_{t-1} + D_{t-1}) + W_t n_t + \Pi_t^F, \quad \text{for } t \geq 2 \text{ and } t \neq T, \\ (1+\tau)P_T c_T + D_T + B_T \leq R_{T-1}(B_{T-1} + D_{T-1}) + W_T n_T + \Pi_T^F - N_T, \quad \text{for date } T, \\ 0 \leq n_t \leq 1, \ \text{for } \forall t. \end{cases} \tag{13}$$

The banks take it for granted that they have the unbacked deposit liabilities N_t from consumers and the same amount of assets, i.e., the government's guarantee of the deposits N_t. Therefore, the banks' profit maximization problem is

$$\max_{L_t, B_t^B, D_t} \sum_{t=1}^{\infty} \left(\prod_{s=0}^{t} R_s^{-1} \right) \left\{ R_t^L L_t + R_t B_t^B + M_t + R_t N_t - R_t D_t \right\} \tag{14}$$

subject to

$$\begin{cases} D_t = L_t + B_t^B + N_t + M_t \text{ for } 1 \leq t < T, \\ D_t = L_t + B_t^B + M_t \text{ for } t \geq T, \\ D_t \leq \frac{1}{\rho} M_t, \\ B_t^B \geq 0. \end{cases}$$

The dynamics of the system in which the unbacked deposits N_t are guaranteed are described by the following system. (Note that $B_t^B = 0$ for all t.)

$$R_t = \frac{1}{\beta} \frac{P_{t+1}}{P_t} \text{ for all } t \geq 1, \tag{15}$$

$$R_t^L = \frac{R_t - \rho}{1 - \rho} \text{ for all } t \geq 1, \tag{16}$$

$$W_t = \frac{P_{t+1}}{R_t^L} \text{ for all } t \geq 1, \tag{17}$$

$$\tau P_{t+1} = R_t B_t + M_t - B_{t+1} - M_{t+1} \text{ for all } t \geq 1, \tag{18}$$

$$W_t + N_t + M_t = \frac{1}{\rho} M_t \text{ for } 1 \leq t < T, \tag{19}$$

$$W_t + M_t = \frac{1}{\rho} M_t \text{ for } t \geq T. \tag{20}$$

I focus the analysis on the equilibrium where the inflation rate is constant for $t \leq T - 2$ and the price level is stable from date T onward: $\frac{P_{t+1}}{P_t} = \pi$ for $1 \leq t \leq T - 2$ and $P_t = \overline{P}$ for $t \geq T$.

Lemma 2 *Assume that* $\beta > 1 - \left(\frac{1}{\rho} - 1 \right) \tau$ *and that the shock* λ *satisfies*

$$0 < \lambda \leq \frac{\rho(1 - \rho)}{1 - \beta\rho}. \tag{21}$$

Assume also that the government sets the date of bank recapitalization T *at a large number. There exist* $\eta(T)$ *and* $\pi(T)$ *such that the government must set* $\eta(T) \leq \pi \leq \pi(T)$ *in order to have a constant inflation* $\frac{P_{t+1}}{P_t} = \pi$ *for* $t \leq T - 2$ *and a constant price* $P_t = \overline{P}$

for $t \geq T$, where $\beta \leq \eta(T) < \pi(T)$ and $\lim_{T\to\infty} \eta(T) = \lim_{T\to\infty} \pi(T) = \beta$. Therefore, if the government sets T at a sufficiently large number, deflation occurs for $t \leq T-2$ because $\pi(T) < 1$ for a large T, and the nominal interest rate ($R_t - 1 = \frac{\pi}{\beta} - 1$) becomes nearly zero.

(Proof) Since the economy was in the steady state before the shock λ hit, we have $P_0 = P_1 = P^* = \frac{1-\rho\beta}{\rho\beta} M_0 = \frac{1-\beta}{\tau\beta} B_0$. The condition $W_1 + N_1 + M_1 = \frac{1}{\rho} M_1$ implies $M_1 = M_0 \frac{1-\beta\rho}{1-\frac{\beta\rho}{\pi}} + \frac{N_1}{\frac{1}{\rho}-1}$. Therefore, $B_1 = B_0 - M_0 \frac{\frac{\rho}{\pi}-\rho}{\frac{1}{\beta}-\frac{\rho}{\pi}} - \frac{N_1}{\frac{1}{\rho}-1}$. The dynamics of the system for $1 \leq t \leq T-1$ are described by $R_t = \frac{\pi}{\beta}$, $P_t = \pi^{t-1} P^*$, $R_t^L = \frac{\frac{\pi}{\beta}-\rho}{1-\rho}$, $W_t = \frac{(1-\rho)\pi^t P^*}{\frac{\pi}{\beta}-\rho}$, $N_t = \frac{\pi^{t-1}}{\beta^{t-1}} N_1$, $W_t + N_t = \left(\frac{1}{\rho} - 1\right) M_t$, and $\tau\pi^t P^* = \frac{\pi}{\beta} B_t + M_t - B_{t+1} - M_{t+1}$. The last difference equation is for $1 \leq t \leq T-2$. The solution is, for $1 \leq t \leq T-2$,

$$\frac{B_{t+1}}{B_0} = \frac{\pi^t}{\beta^t} \left[a(\pi) - tb(\pi) - c + \beta^t \left\{ 1 - \frac{\rho(1-\pi)}{\tau(\frac{\pi}{\beta} - \rho)} \right\} \right], \tag{22}$$

where $a(\pi) = \frac{\rho(1-\pi)(1-\rho)}{\tau(\frac{\pi}{\beta}-\rho)(1-\beta\rho)}$, $b(\pi) = \frac{\lambda(1-\beta)\rho}{\tau\pi(1-\rho)} \left(\frac{\pi}{\beta} - 1\right)$, and $c = \frac{(1-\beta)\rho}{\tau\beta(1-\rho)} \lambda$. The function $a(\pi)$ is decreasing in π and $b(\pi)$ is increasing in π. In the equilibrium B_t must be nonnegative for all $t \geq 1$. Since $\beta^t \left\{1 - \frac{\rho(1-\pi)}{\tau(\frac{\pi}{\beta}-\rho)}\right\} \leq \beta \left(1 + \frac{\rho\beta}{\tau}\right)$, (22) implies

$$\frac{B_{t+1}}{B_0} \leq \frac{\pi^t}{\beta^t} \left[f(\pi, t) + \beta \left(1 + \frac{\rho\beta}{\tau} \right) \right], \tag{23}$$

where $f(\pi, t) = a(\pi) - tb(\pi) - c$. Define $\pi(T)$ as the solution to $f(\pi, T-2) + \beta \left(1 + \frac{\rho\beta}{\tau}\right) = 0$. It can be shown that $f(\beta, T-2) = a(\beta) - c > 0$ from (21), and that $\lim_{\pi\to\infty} f(\pi, T-2) = -\frac{(1-\rho)\rho\beta}{(1-\rho\beta)\tau} - (T-2)\frac{(1-\beta)\lambda\rho}{(1-\rho)\tau\beta}$, which is less than $-\beta \left(1 + \frac{\rho\beta}{\tau}\right)$ for a large T. Since $f(\pi, T-2)$ is a decreasing function of π, $\pi(T)$ uniquely exists. It is easily shown that $\beta < \pi(T)$ and $\lim_{T\to\infty} \pi(T) = \beta$. The necessary condition for $B_{T-1} > 0$ is that $\pi \leq \pi(T)$. Therefore, the government must set π such that

$$\beta \leq \pi \leq \pi(T). \tag{24}$$

The constant price level, $\overline{P}$, after the bank bailout is determined uniquely as follows. Equation (18) implies

$$\tau\overline{P} = R_{T-1} B_{T-1} + M_{T-1} - \overline{B} - \overline{M}, \tag{25}$$

where $\overline{B}$ and $\overline{M}$ are the steady state levels of bonds and cash. Lemma 1 implies that in a steady state equilibrium $\overline{B} = \frac{\tau\beta}{1-\beta} \overline{P}$ and $\overline{M} = \frac{\rho\beta}{1-\rho\beta} \overline{P}$. Since $R_{T-1} = \frac{\overline{P}}{\beta P_{T-1}}$, equation (25) implies

$$\overline{P} = \left[\frac{\tau}{1-\beta} + \frac{\rho\beta}{1-\rho\beta} - \frac{B_{T-1}}{\beta P_{T-1}} \right]^{-1} M_{T-1}. \tag{26}$$

The necessary condition for $\overline{P} > 0$ is $f(\pi, T-2) \leq \beta^{T-1}d$, where $d = \frac{P^*}{B_0}\left(\frac{\tau}{1-\beta} + \frac{\rho\beta}{1-\rho\beta}\right)$. This condition can be rewritten as

$$\pi \geq \xi(T), \tag{27}$$

where $\xi(T)$ is defined by $f(\xi(T), T-2) = \beta^{T-1}d$. By definition, $\xi(T) < \pi(T)$. It is also easily shown that $\lim_{T\to\infty} \xi(T) = \beta$. Combining conditions (24) and (27), the necessary condition for existence of the equilibrium is $\eta(T) \leq \pi \leq \pi(T)$, where $\eta(T) = \max\{\beta, \xi(T)\}$.

In the equilibrium, it must be the case that $\frac{\overline{P}}{P_{T-1}} \geq \beta$; otherwise the nominal interest rate becomes negative. Equation (19) implies that $M_{T-1} = \frac{\rho}{1-\rho}(W_{T-1} + N_{T-1}) > \frac{\rho}{1-\rho}R_{T-1}N_{T-2} \geq \frac{\rho}{1-\rho}N_{T-2} = \frac{\rho}{1-\rho}\frac{\pi^{T-3}}{\beta^{T-3}}N_1$. Thus, assuming that $\eta(T) \leq \pi$, equation (26) implies that

$$\frac{\overline{P}}{P_{T-1}} \geq \frac{\frac{\rho}{1-\rho}\lambda}{\left(\frac{\tau}{1-\beta} + \frac{\rho\beta}{1-\rho\beta}\right)\pi}\frac{1}{\beta^{T-3}}. \tag{28}$$

That T is sufficiently large guarantees that $R_{T-1} \geq 1$ and that there exists an equilibrium. (End of proof)

Discussion In this model nonperforming loans N_t are equivalent to government debt, since the government guarantees bank deposits. Therefore, one might expect that an increase in nonperforming loans would cause an increase in the inflation rate, as an increase in government debt usually does (Sargent and Wallace [1981]). The inflationary effect of nonperforming loans appears in the jump from P_{T-1} to $P_T = \overline{P}$. Equation (28) implies that the inflation rate at date $T-1$ (just before the bank bailout) becomes quite large if T is large. Nonperforming loans cause deflation "temporarily" for $t < T-1$, since banks must hold the assets and corresponding liabilities, N_t, under a technological constraint on production of transaction services (Assumption 2). But when banks are recapitalized, nonperforming loans work as government debt and do indeed cause one-time inflation.

The straightforward policy to prevent deflation in this model is to recapitalize the banking system immediately through a lump-sum transfer from consumers to banks. If the unbacked deposits N_t are eliminated from banks' balance sheets, the government recovers its ability to control inflation, as shown in Lemma 1. But if the recapitalization of an insolvent banking system by injecting taxpayers' money is politically difficult or infeasible, as it was in Japan in the 1990s, the government has no other choice than to set nominal interest rates at nearly zero and to let deflation occur.

The above arguments show that monetary policy (swaps between government bonds and cash) alone is not effective in halting deflation when fiscal policy has the constraint: $G_t = 0$. The price level becomes constant if the government can set $\{G_t\}_{t=0}^{\infty}$ to make $B_t \geq (\prod_{s=0}^{t} R_s)B'$, where B' is an appropriate positive constant. In this case, the government can postpone bank recapitalization indefinitely, and can keep the price level constant for all t. But the government debt B_t violates the transversality condition, since $\lim_{t\to\infty} \left(\prod_{s=0}^{t} \frac{1}{R_s}\right) B_t \geq B' > 0$. Thus escaping from deflation without recapitalizing insolvent banking system is infeasible for the government in the equilibrium.

4 Concluding Remarks

In this paper I demonstrated that deflation can occur if the government undertakes forbearance policy vis-à-vis bank insolvency by guaranteeing deposits and postponing bank recapitalization.

A forbearance policy turns nonperforming loans into de facto government debt. Therefore, nonperforming loans have the same inflationary effect on the economy as the government debt, although this effect is sealed until the time of bank recapitalization. Until then banks are forced to hold deposit liabilities that correspond to the nonperforming loans under the technological constraint that total deposit liabilities cannot exceed a certain multiple of cash reserves. Since cash reserves become scarcer as nonperforming loans grow, price level falls until the bank bailout. But at the time of the bailout, when the scarcity of cash reserves diminishes abruptly, a big inflationary spurt occurs.

Since I focused on the changes in price level in this paper, I simplified the production technology in a way that resulted in no welfare loss. But it should be easy to generalize this model so as to incorporate a distortionary tax system and capital accumulation. If that were done, surely the forbearance on bank insolvency would generate a welfare loss, just like in Dekle and Kletzer (2003), Barseghyan (2002), and Kobayashi (2003), in addition to deflation and a fall in nominal interest rates.

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