DISCUSSION PAPER NO. 172

TOWARDS THE GENERAL EQUILIBRIUM THEORY OF THE LABOR-MANAGED MARKET ECONOMY

by

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October 8,1975

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ABSTRACT

The purpose of this paper is to construct a model of a labor-managed market economy, in which preference relations of consumers are defined both on commodity bundles and on labor supply, and to observe economic mechanisms, inherent in the model, that determine which firms are formed and coexist in a market economy.

*/ The present work was initially inspired by J. Greenberg [5] and his unpublished manuscript on the application of the Lindahl solution to a labor-managed market economy. His discussion with the author stimulated this research. Of course, the author is solely responsible for any possible errors in this paper. The financial support for this research was provided, in part, by a National Science Foundation Grant No. GS 31346X through The Center for Mathematical Studies in Economics and Management Science, Northwestern University.

I. INTRODUCTION

An attempt was made in an earlier paper, T. Ichiishi [6], to explain the formation of firms in a labor-managed market economy, where the productivity of consumer coalitions (firms) was emphasized as one cause of this phenomenon. To make this viewpoint explicit, it was postulated in that paper that each economic agent can be characterized as an "economic animal"; i.e., he has a lexicographic preference ordering over nominal wealth levels and (marketed) commodity bundles.

Another model, more classical and standard in flavor, in which each economic agent has a preference relation defined not only on the commodity bundles but also on his supply of labor, was considered by J. Greenberg [5] and his unpublished manuscript. He observed a logical similarity between his model and the model of local public goods, and furthermore, introduced various game-theoretic concepts as possible solutions to his model. However, in his work the formation of firms was not satisfactorily explained.

The present paper combines the above two works: It constructs a model of a labor-managed market economy, in which preference relations of consumers are defined both on commodity bundles and on labor supply, and observes economic mechanisms, inherent in the model, that determine which firms are formed and coexist in a market equilibrium.

The model in this paper, however, cannot cover some important aspects of the economy, because of considerable technical difficulties which would arise: First, it assumes that the cardinality of the set of the economic agents is finite, so it cannot describe an idealized case in which no single agent can have an influence on the society. Second, and more important, it does not explicitly introduce uncertainty into the model. If one wants to apply a model of the labor-managed market economy to a capitalistic society, one cannot dispense with uncertainty, since stock is considered as a marketed commodity, and since stock ownership is explained by the portfolio analysis.

Section II is devoted to construction of a model and statement of the main results. Proof of the main theorem is given in Section III.

II. MODEL AND RESULTS

A model with marketed commodities and non-marketed labor is constructed. There are m types of commodities, where m is a positive integer, and a commodity is characterized by its physical properties, the date on which it will be available, and the location at which it will be available. Denote by M the set of integers $\{1,\dots,m\}$. The important feature about a commodity is that it is marketed, so that the dual concept, its price, is well-defined. Full-time labor is not marketed. A wage is not, therefore, a price but a reward that the laborer gets by participating a production unit. There are n economic agents, where n is a positive integer. Denote by N the set of integers $\{1,\dots,n\}$. Each economic agent $j \in N$ is able to supply ℓ^j types of labor. Denote by L^j the set of integers $\{m + \sum_{h<j} \ell^h + 1,\ m + \sum_{h<j} \ell^h + 2, \dots, m + \sum_{h<j} \ell^h + \ell^j\}$, and define $\ell \equiv m + \sum_{j \in N} \ell^j$. Note that $\{M, L^1, \dots, L^n\}$ is a partition of $\{1,\dots,\ell\}$. A positive integer i $(\leq \ell)$ signifies the type of commodity if $i \in M$, and the type of labor supplied by the j<u>th</u> agent if $i \in L^j$. For a vector x in the ℓ-dimensional Euclidian space $\mathbb{R}^\ell$, x_i denotes its i<u>th</u> component, x_M the subvector $\{x_i \mid i \in M\}$, and x_{L^j} and x_L are similarly defined, where $L \equiv L^1 \cup \dots \cup L^n$.

Each economic agent plays two roles in the economy: that of consumer and that of member of a production unit. To each agent $j \in N$, as a consumer, are assigned his consumption set X^j, his utility function u^j on X^j, and his initial endowment ω^j. The consumption set X^j, a subset of $\mathbb{R}^\ell$, consists of all consumptions possible for the agent j, and ω^j is a point in $\mathbb{R}^m$. The usual sign convention (see, e.g., G. Debreu [3]) is made here, so that for $x \in X^j$, $x_i > 0$ (< 0, resp.) means demand (supply, resp.) by the agent j. For each two vectors x,y in a Euclidean space, $x \geq y$ means $x_i \geq y_i$ for every i; $x > y$ means $[x \geq y$ and $x \neq y]$; $x >> y$ means $x_i > y_i$ for every i. Assumptions on the consumption sector $\{(X^j,u^j,\omega^j)\}_{j\in N}$:

(X.1) For every $j \in N$, $x \in X^j$ implies $[x_{L^h} = 0$ for every $h \neq j$, and $x_{L^j} \leq 0]$.

(X.2) For every $j \in N$, the interior of $\{x_{L^j} \in \mathbb{R}^{\ell^j} \mid x \in X^j\}$ in $\mathbb{R}^{\ell^j}$ is non-empty.

(X.3) For every $j \in N$, X^j is convex and closed in $\mathbb{R}^\ell$.

(X.4) For every $j \in N$, $[x,x' \in X^j]$ implies $(x_M,x'_L) \in X^j$.

(X.5) For every $j \in N$, there exists $c^j \in \mathbb{R}^\ell$ such that $X^j + \{c^j\} \subset \mathbb{R}^\ell_+$, where $\mathbb{R}^\ell_+$ denotes the non-negative orthant of $\mathbb{R}^\ell$.

(X.6) For every $j \in N$, the function $u^j:X^j \to \mathbb{R}$ is continuous.

(X.7) For every $j \in N$ and for every neighborhood U in X^j of any $x \in X^j$, there exists $x' \in U$ such that $[x'_L = x_L$, and $u^j(x') > u^j(x)]$.

(X.8) For every $j \in N$, for every $x, x' \in X^j$, and for every t such that $0 \leq t \leq 1$, $u^j(tx + (1-t)x') \geq \text{Min}\,[u^j(x),u^j(x')]$.

(X.9) For every $j \in N$ and for every $p \in \Delta^{m-1}, p \cdot \omega^j > \mathrm{Min}\,\{p \cdot x_M | x \in X^j\}$, where $\Delta^{m-1} \equiv \{p \in \mathbb{R}^m_+ \mid \sum_{i=1}^{m} p_i = 1\}$, and $p \cdot \omega$ is the Euclidian inner product of p and ω.

The assumptions (X.1) and (X.2) are natural in view of the above discussion on labor; (X.4) is standard, in fact, it is frequently assumed that a consumption set is $\mathbb{R}^\ell_+$ for a pure exchange economy; (X.7) is a strong version of local non-satiation. (See, e.g., G. Debreu [3], for discussion on the other assumptions.)

Let $\mathcal{N}$ be an algebra of subsets of N. Each element S of $\mathcal{N}$, called a coalition, plays a role as a production unit, where an agent works for this production unit if and only if he is a member of S. The production set $Y(S)$, a subset of $\mathbb{R}^\ell$, is assigned to each $S \in \mathcal{N}$. It reflects the technological knowledge and skill of the members of S; it also reflects non-marketed and non-labor production factors owned by the members of S. The usual sign convention is taken to distinguish input and output (see, e.g., G. Debreu [3]), so that under a price system $p \in \mathbb{R}^m$, $p \cdot y_M$ is the value-added made by the activity $y \in Y(S)$. A coalition structure $\mathcal{T}$ is a partition of N such that every member of $\mathcal{T}$ belongs to $\mathcal{N}$. Denote by $\underset{\sim}{\mathcal{T}}$ any fixed, non-empty collection of coalition structures. A member $\mathcal{T}$ of $\underset{\sim}{\mathcal{T}}$ describes which production units coexist in the economy. Denote by Y the society's total production possibility set $\bigcup_{\mathcal{T} \in \underset{\sim}{\mathcal{T}}} (\sum_{T \in \mathcal{T}} Y(T))$.

Assumptions on the production sector $(\{Y(S)\}_{S \in \mathcal{N}}, \underset{\sim}{\mathcal{T}})$:

(Y.1) For every $S \in \mathcal{N}$, $y \in Y(S)$ implies $[y_{L^j} = 0$ for any $j \notin S$, and $y_{L^j} \leq 0$ for every $j \in S]$.

(Y.2) For every $S \in \mathcal{N}$, $0 \in Y(S)$; and $Y(\phi) = \{0\}$.

(Y.3) For every $S \in \mathcal{N}$, $[y' \leq y \in Y(S)$, $y'_{L^j} = 0$ for every $j \notin S]$ implies $y' \in Y(S)$.

(Y.4) For every $S \in \mathcal{N}$, $Y(S)$ is closed in $\mathbb{R}^{\ell}$.

(Y.5) For every $S \in \mathcal{N}$, $[y \in Y(S)$, and $\epsilon > 0]$ implies that there exists $\delta > 0$ such that if $[y' \in \mathbb{R}^{\ell}$, $y'_{L^j} << 0$ for every $j \in S$, $y'_{L^j} = 0$ for every $j \notin S$, and $\sum_{i \in L} |y'_i - y_i| < \delta]$, then $(y''_M, y'_L) \in Y(S)$ for some $y''_M \in \mathbb{R}^m$ for which $\sum_{i \in M} |y''_i - y_i| < \epsilon$.

(Y.6) If $\{\lambda_b, S_b\}_{b \in B}$ is a finite sequence in $\mathbb{R}_+ \times \mathcal{N}$ such that $\sum_{b \in B: S_b \ni j} \lambda_b = 1$ for every $j \in N$, then $\sum_{b \in B} \lambda_b Y(S_b) \subset Y$.

The assumption (Y.1) is natural in view of the above discussion on labor; (Y.5) is true if the (implicit) production function is continuous; the balancedness assumption (Y.6) has recently been discussed extensively in the literature (see, e.g., V. Böhm [1] and T. Ichiishi [6]). The assumptions (Y.2) - (Y.4) are standard (see, e.g., G. Debreu [3]).

An economy $\mathscr{E}$ is a datum $(N, \eta, \{(X^j, u^j, \omega^j)\}_{j \in N}, \{Y(S)\}_{S \in \eta}, \underset{\sim}{\mathscr{T}})$. A <u>competitive equilibrium</u> of an economy $\mathscr{E}$ is a quintuple, $(\{x^{j*}\}_{j \in N}, \mathscr{T}^*, \{y^*(T)\}_{T \in \mathscr{T}^*}, \{\pi^{j*}\}_{j \in N}, p^*)$ of members of $(\prod_{j \in N} X^j, \underset{\sim}{\mathscr{T}}, \prod_{T \in \mathscr{T}^*} Y(T), \mathbb{R}^n_+, \Delta^{m-1})$, respectively, such that:

(i) $p^* \cdot x^{j*}_M \leq p^* \cdot \omega^j + \pi^{j*}$, for every $j \in N$;

(ii) $\sum_{j \in T} x^{j*}_L \leq y^*_L(T)$, for every $T \in \mathscr{T}^*$;

(iii) $\sum_{j \in T} \pi^{j*} \leq p^* \cdot y^*_M(T)$, for every $T \in \mathscr{T}^*$;

(iv) $\sum_{j \in N} x^{j*}_M \leq \sum_{j \in N} \omega^j + \sum_{T \in \mathscr{T}^*} y^*_M(T)$, and

$$p^* \cdot \sum_{j \in N} x^{j*}_M = p^* \cdot \sum_{j \in N} \omega^j + p^* \cdot \sum_{T \in \mathscr{T}^*} y^*_M(T); \text{ and}$$

(v) For any $S \in \eta$, there exist no $\{x^j\}_{j \in S} \in \prod_{j \in S} X^j$, no $y \in Y(S)$, no $\{\pi^j\}_{j \in S} \in \mathbb{R}^{\# S}_+$, such that

$u^j(x^j) > u^j(x^{j*})$, for every $j \in S$,

$p^* \cdot x^j_M \leq p^* \cdot \omega^j + \pi^j$, for every $j \in S$,

$\sum_{j \in S} x^j_L \leq y_L$, and

$\sum_{j \in S} \pi^j \leq p^* \cdot y_M$.

The condition (i) is the wealth constraint for a consumer; (ii) means that the activity $y^*(T)$ is feasible with the collaboration of the members in T; (iii) means that the value-added distribution $\{\pi^{j*}\}$ is feasible; (iv) means that all the markets are cleared; and (v) means that the coalition structure $\mathscr{T}^*$ and the value-added distribution $\{\pi^{j*}\}$

are stable in the sense that no new coalition S is formed to bring about by its own effort a higher utility level to each member of S. The following theorem, whose essence is found in the unpublished manuscript of J. Greenberg, gives a rationale for the behaviors of economic agents and of production units:

<u>Theorem I.</u> <u>Suppose that</u> $(\{x^{j*}\}, \mathcal{T}^*, \{y^*(T)\}, \{\pi^{j*}\}, p^*)$ <u>is a competitive equilibrium of an economy</u> $\mathcal{E}$ <u>in which</u> (X.3), (X.4), (X.6), (X.7), (X.9), <u>and</u> (Y.2) <u>are satisfied, and</u> $\{j\} \in \mathcal{N}$ <u>for every</u> $j \in N$. <u>Then, for every</u> $j \in N$, x^{j*} <u>is a maximal element of the budget set</u> $\{x \in X^j \mid p^* \cdot x_M \leq p^* \cdot \omega^j + \pi^{j*}\}$ <u>with respect to</u> u^j; <u>and for every</u> $T \in \mathcal{T}^*$, $p^* \cdot y^*_M(T)$ <u>is the maximum of</u> $\{p^* \cdot y_M \mid y \in Y(T),\ y_L = y^*_L(T)\}$.

<u>Proof of Theorem I.</u> Suppose there exists $j \in N$ such that, for some x^j in his budget set, $u^j(x^j) > u^j(x^{j*})$. Let $j \in T \in \mathcal{T}^*$. Considering the coalition $\{j\}$ for the condition (v), one can easily show that (Y.2) implies $\pi^{j*} > 0$ for every such j. Then, using (X.3), (X.4), (X.6), (X.7) and (X.9), one can construct a value-added distribution within the coalition T that contradicts (v). The second statement of the theorem is obvious in view of (X.7).

Q.E.D.

The main result of the present paper is now stated:

<u>Theorem II.</u> <u>Let</u> $\mathcal{E}$ <u>be an economy which satisfies</u> (X.1) - (X.9) <u>and</u> (Y.1) - (Y.6). <u>Assume also</u>:

(Z.1) $\underset{\sim}{A}\, Y \cap \mathbb{R}^{\ell}_{+} = \{0\}$.

(Z.2) <u>For every</u> $j \in N$, $[\underset{\sim}{A}\, Y + (\omega^j, 0)] \cap X^j \neq \emptyset$.

<u>where</u> $\underset{\sim}{A}\, C$ <u>is the asymptotic cone of</u> C (<u>see, e.g., G. Debreu</u> [3]). <u>Then there exists a competitive equilibrium</u> of $\mathcal{E}$.

III. PROOF OF THEOREM II

For every $p \in \Delta^{m-1}$ and for every $S \in \eta$, define:

$$V_p(S) \equiv \left\{ \{u^j\}_{j\in S} \;\middle|\; \begin{array}{l} \forall j \in S ; \exists x^j \in X^j ; \exists \pi^j \in \mathbb{R}_+ ; \\ \exists y \in Y(S); \\ u^j \le u^j(x^j), \text{ and } p \cdot x^j_M \le p \cdot \omega^j + \pi^j, \\ \sum_{j\in S} x^j_L \le y_L \\ \sum_{j\in S} \pi^j \le p \cdot y_M \end{array} \right\}$$

$$H_p \equiv \bigcup_{\mathcal{T} \in \underset{\sim}{\mathcal{T}}} \left\{ \{u^j\}_{j\in N} \;\middle|\; \begin{array}{l} \forall j \in N ; \exists x^j \in X^j ; \exists \pi^j \in \mathbb{R}_+ ; \\ \forall T \in \mathcal{T}; \exists y(T) \in Y(T); \\ u^j \le u^j(x^j), \text{ and } p \cdot x^j_M \le p \cdot \omega^j + \pi^j, \\ \sum_{j\in T} x^j_L \le y_L(T) \\ \sum_{j\in T} \pi^j \le p \cdot y_M(T) \end{array} \right\}.$$

Define also H'_p as the set of all $\{u^j\}_{j\in N}$ that satisfies all the conditions of H_p except that the last condition for each $T \in \mathcal{T}$ is replaced by: $\sum_{j\in N} \pi^j \le p \cdot \sum_{T\in\mathcal{T}} y_M(T)$. Then, $(\{V_p(S)\}_{S\in\eta}, H_p)$ and $(\{V_p(S)\}_{S\in\eta}, H'_p)$ are cooperative games without side-payment. Denote by $C(p)$ the set of all $(\mathcal{T},\{x^j\}_{j\in N},\{\pi^j\}_{j\in N},\{y(T)\}_{T\in\mathcal{T}})\in\mathcal{T}\times\prod_{j\in N}X^j\times\mathbb{R}^n_+\times\prod_{T\in\mathcal{T}}Y(T)$ such that $\{u^j(x^j)\}_{j\in N}$ is in H_p with $\mathcal{T}$, π^j's and $y(T)$'s associated with it, but that for any $S\in\mathcal{N}$, $\{u^j(x^j)\}_{j\in S}$ is not an interior point of $V_p(S)$ in $\mathbb{R}^{\# S}$. Every member of $C(p)$ gives rise to a utility allocation in the core of the game $(\{V_p(S)\}_{S\in\mathcal{N}},H_p)$. The set $C'(p)$ is also defined similarly, where H'_p replaces H_p.

(1) <u>Assume</u> (X.7). <u>Then</u>, $C(p) = C'(p)$, <u>for every</u> $p\in\Delta^{m-1}$.

<u>Proof of (1).</u> Take any $(\mathcal{T},\{x^j\},\{\pi^j\},\{y(T)\})$ in $C'(p)$. If there exists $T'\in\mathcal{T}$ such that $\sum_{j\in T'}\pi^j > p\cdot y_M(T')$, then from the condition, $\sum_{j\in N}\pi^j \le p\cdot\sum_{T\in\mathcal{T}} y_M(T)$, one concludes: $\exists T\in\mathcal{T};\ \sum_{j\in T}\pi^j < p\cdot y_M(T)$. Each member in T could get more wage, and by (X.7) $\{u^j(x^j)\}_{j\in T}$ would be in the interior of $V_p(T)$ in $\mathbb{R}^{\# T}$; a contradiction. The other inclusion is obvious.

Q.E.D.

(2) <u>Assume</u> (X.1), (X.3), (X.8), (Y.1) <u>and</u> (Y.6). <u>Then, the game</u> $(\{V_p(S)\},H'_p)$ <u>is balanced for every</u> $p\in\Delta^{m-1}$.

<u>Proof of (2).</u> Let $\{\lambda_b,S_b\}_{b\in B}$ be a balanced collection, and $\{u^j\}_{j\in N}$ be such that for each $b\in B$, $\{u^j\}_{j\in S_b}\in V_p(S_b)$. Let $x^{jb}\in X^j$, $\pi^{jb}\in\mathbb{R}_+$,

and $y^b \in Y(S_b)$ be associated with $\{u^j\}_{j \in S_b}$. Define:

$x^j \equiv \sum_{b \in B: S_b \ni j} \lambda_b x^{jb}$, $\pi^j \equiv \sum_{b \in B: S_b \ni j} \lambda_b \pi^{jb}$. By (Y.6) there is $\mathcal{T} \in \underset{\sim}{\mathcal{T}}$ such that $\sum_{b \in B} \lambda_b y^b = \sum_{T \in \mathcal{T}} y(T)$ with $y(T) \in Y(T)$ for each $T \in \mathcal{T}$. With $(\mathcal{T}, \{x^j\}, \{\pi^j\}, \{y(T)\})$, it is straightforward to check that $\{u^j\}_{j \in N} \in H'_p$.

Q.E.D.

(3) Assume (X.1), (X.3), (X.8), (Y.1), and (Y.6). Assume also that H'_p is bounded from above. Then, $C'(p)$ is non-empty for every $p \in \Delta^{m-1}$.

Proof of (3). Apply V. Böhm's version [1, Theorem 1] of H. Scarf's Theorem [7] to (2).

Q.E.D.

Remark that for every member of $C(p)$, (i)(ii)(iii) and (v) of the equilibrium conditions are satisfied. The proof of (1) has shown that the second condition of (iv) is also satisfied by this member. To prove Theorem II, therefore, it suffices to show that there exists $p^* \in \Delta^{m-1}$ and a member of $C(p^*)$ such that the first condition of (iv) is satisfied: This is done by the usual technique of fixed-point argument. For this purpose, one needs:

(4) Assume (X.1) - (X.4), (X.6), (X.9), (Y.3) and (Y.5). Fix any $S \in \eta$ and any $p \in \Delta^{m-1}$. If $\{u^j(x^j)\}_{j \in S}$ is in the interior of $V_p(S)$ in $\mathbb{R}^{\# S}$,

then there exist $\tilde{x}^j \in X^j$ and $\tilde{\pi}^j \in \mathbb{R}_+$ for every $j \in S$, and $\tilde{y} \in Y(S)$ such that $u^j(\tilde{x}^j) > u^j(x^j)$, $p \cdot \tilde{x}^j_M < p \cdot \omega^j + \tilde{\pi}^j$ for every j, $\sum_{j\in S} \tilde{x}^j_L \leq \tilde{y}_L$, $\sum_{j\in S} \tilde{\pi}^j \leq p \cdot \tilde{y}_M$, and $\tilde{x}^j_{L^j}$ is in the interior of $\{x_{L^j} \in \mathbb{R}^{\ell^j} \mid x \in X^j\}$ in $\mathbb{R}^{\ell^j}$ for every $j \in S$.

Proof of (4). There exist $\bar{x}^j \in X^j$, $\bar{\pi}^j \in \mathbb{R}_+$, $\bar{y} \in Y(S)$ such that

$$u^j(\bar{x}^j) > u^j(x^j),\ p \cdot \bar{x}^j_M \leq p \cdot \omega^j + \bar{\pi}^j,\ \sum_{j\in S} \bar{x}^j_L \leq \bar{y}_L, \text{ and } \sum_{j\in S} \bar{\pi}^j \leq p \cdot \bar{y}_M.$$

By (X.3), (X.4), (X.6) and (X.9), there exist $\delta > 0$, $\bar{\bar{x}}^j \in X^j$ with $\bar{x}^j_L = \bar{\bar{x}}^j_L$ such that $p \cdot \bar{\bar{x}}^j_M \leq p \cdot \omega^j + \bar{\pi}^j - \delta$, and that $u^j(\bar{\bar{x}}^j) > u^j(x^j)$. By (X.2) and (X.4), there exists $\tilde{x}^j_{L^j}$ in the interior of $\{x_{L^j} \in \mathbb{R}^{\ell^j} \mid x \in X^j\}$ in $\mathbb{R}^{\ell^j}$, arbitrarily close to $\bar{\bar{x}}^j_{L^j}$. Put $\tilde{x}^j \equiv (\bar{\bar{x}}^j_M, \tilde{x}^j_L)$, where $\tilde{x}^j_{L^h} = 0$ for $h \neq j$. By (X.6), one may assume that $u^j(\tilde{x}^j) > u^j(x^j)$. In view of (X.1), $\tilde{x}^j_{L^j} << 0$. So, by (Y.5), there exists $\tilde{y} \in Y(S)$ such that $\sum_{j\in S} \tilde{x}^j_L \leq \tilde{y}_L$ and $p \cdot \bar{y}_M - p \cdot \tilde{y}_M < \delta \cdot \# S$.

Q.E.D.

Define the excess demand correspondence $\zeta : \Delta^{m-1} \to \mathbb{R}^m$, by $\zeta(p) \equiv \{ \sum_{j\in N} (x^j_M - \omega^j) - \sum_{T\in\mathcal{T}} y_M(T) \mid (\mathcal{T}, \{x^j\}, \{\pi^j\}, \{y(T)\}) \in C(p).\}$. To complete the proof of Theorem II, one makes use of G. Debreu's theorem [2]. In view of (Z.1) and (Z.2), one only needs to establish the following (5) - (7). (Define ζ' similarly to ζ, where C' replaces C. First, consider the case in which (X.7) is dropped but X^j's are bounded, and show existence of points that satisfy all the equilibrium conditions except that (iii) is replaced by $\sum_{j\in N} \pi^{j*} \leq p^* \cdot \sum_{T\in\mathcal{T}^*} y^*(T)$, by using the game $(\{V_p(S)\},H_p')$, the correspondence ζ', (3) and (5) - (7); then apply the usual limit argument, keeping in mind that the attainable set is compact; finally apply (1).) Proof of (5) is straightforward. To prove (6), one only has to check that (Y.6) implies convexity of Y. (7) is a consequence of (7') because $\#\underset{\sim}{\mathcal{T}} < \infty$.

(5) _For every_ $p \in \Delta^{m-1}$, $p \cdot \zeta'(p) \leq 0$.

(6) _Assume_ (X.3), (X.8) _and_ (Y.6). _Then,_ $\zeta'(p)$ _is convex for every_ $p \in \Delta^{m-1}$.

(7) _Assume_ (X.1) - (X.4), (X.6), (X.9), _and_ (Y.2) - (Y.5). _Assume also that_ X^j _is compact for each_ $j \in N$. _Then the graph of_ ζ' _is closed in_ $\Delta^{m-1} \times \mathbb{R}^m$.

(7') _Assume_ (X.3), (X.4), (X.6), (Y.2) - (Y.4), _and the conclusion of_ (4). _Fix any_ $\mathcal{T} \in \underset{\sim}{\mathcal{T}}$. _Suppose_ $\{p^\nu, \{x^{j\nu}\}_j, \{\pi^{j\nu}\}_j, \{y^\nu(T)\}_T\}_\nu$ _is a sequence in_ $\Delta^{m-1} \times \prod_{j\in N} X^j \times \mathbb{R}^n_+ \times \prod_{T\in\mathcal{T}} Y(T)$ _such that it converges to_ $(p^o, \{x^{jo}\}, \{\pi^{jo}\}, \{y^o(T)\})$ _and that_ $(\mathcal{T}, \{x^{j\nu}\}, \{\pi^{j\nu}\}, \{y^\nu(T)\}) \in C'(p^\nu)$ _for every_ ν. _Then,_ $(\mathcal{T}, \{x^{jo}\}, \{\pi^{jo}\}, \{y^o(T)\}) \in C'(p^o)$.

<u>Proof of (7')</u>. It is trivial that $\{u^j(x^{jo})\} \in H'_{p^o}$. Suppose $\{u^j(x^{jo})\}_{j\in S}$ were in the interior of $V_{p^o}(S)$ in $\mathbb{R}^{\# S}$ for some $S \in \mathcal{N}$. Then there exist $\tilde{x}^j \in X^j$, $\tilde{\pi}^j \in \mathbb{R}_+$, $\tilde{y} \in Y(S)$ satisfying the properties in (4). Then, by (X.4) and (X.6), for every $j \in S$ there exists $x^{j*} \in X^j$ such that $x^{j*}_M = \tilde{x}^j_M$, $x^{j*}_{L^j} << \tilde{x}^j_{L^j}$ and $u^j(x^{j*}) > u^j(x^{jo})$. Put: $\bar{x}^{j\nu} \equiv \frac{1}{\nu} x^{j\nu} + (1 - \frac{1}{\nu})x^{j*}$, $\bar{y}^\nu \equiv \tilde{y}$. Then it is easy to show that $\sum_{j\in S} \bar{x}^{j\nu}_L \leq \bar{y}^\nu_L$ and $u^j(\bar{x}^{j\nu}) > u^j(x^{j\nu})$, eventually. If $p^o \cdot \tilde{y}_M > 0$, then define

$$\bar{\pi}^{j\nu} \equiv \begin{cases} \tilde{\pi}^j - \frac{1}{\# S^+} |p^\nu \cdot \tilde{y}_M - p^o \cdot \tilde{y}_M|, & \text{if } j \in S^+, \\ 0, \text{ if } j \notin S^+, \end{cases}$$

where $S^+ \equiv \{j \in S | \tilde{\pi}^j > 0\}$. Then $\bar{\pi}^{j\nu} \geq 0$ eventually, and $\sum_{j\in S} \bar{\pi}^{j\nu} \leq p^\nu \cdot \tilde{y}_M$. By $x^{j*}_M = \tilde{x}^j_M$, it is easy to show $p^\nu \cdot \bar{x}^{j\nu}_M \leq p^\nu \cdot \omega^j + \bar{\pi}^{j\nu}$, eventually: a contradiction to $(\mathcal{T}, \{x^{j\nu}\}, \{\pi^{j\nu}\}, \{y^\nu(T)\}) \in C'(p^\nu)$ for every ν. If $p^o \cdot \tilde{y}_M = 0$, we may assume without loss of generality that $\tilde{y} = 0$. Put $\bar{\pi}^{j\nu} \equiv 0$. Then, one can get a contradiction again.

Q.E.D.

REFERENCES

[1] Böhm, V.,The Core of an Economy with Production, Review of Economic Studies, 41 (1974), 429-436.

[2] Debreu, G., Market Equilibrium, Proceedings of the National Academy of Sciences of the U.S.A., 42 (1956), 876 - 878.

[3] Debreu, G.,"Theory of Value,"John Wiley and Sons, New York, N. Y., 1959.

[4] Debreu, G., New Concepts and Techniques for Equilibrium Analysis, International Economic Review, 3 (1962), 257-273.

[5] Greenberg, J., Pure and Local Public Goods: A Game-Theoretic Approach, Research Report No. 65, Department of Economics, Hebrew University of Jerusalem, 1975.

[6] Ichiishi, T., Coalition Structure in a Labor-Managed Market Economy, CORE Discussion Paper, No. 7503, Université Catholique de Louvain, 1975.

[7] Scarf, H., The Core of an n-Person Game, Econometrica, 35 (1967), 50-69.