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EQUILIBRIUM IN COMPETING NETWORKS WITH DIFFERENTIATED PRODUCTS

by

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1. Introduction

A number of goods are characterised by the fact that the utility a consumer derives from buying them increases with the number of consumers who are also buying them. Such goods are commonly referred to as "networks" to which consumers decide whether or not to get linked. If they get linked to one, they benefit from the so-called "network externality", which is the utility induced by the other consumers.

Network externalities can be generated in various ways. For example, buying a certain type of micro-computer is more attractive if the model is widely spread. Indeed, the decision to purchase a computer not only depends on its intrinsic characteristics, but also on the possibility to exchange software or informations with other users, or even because spare material is made more readily available. The decision to buy a camera can also be affected by the fact that it is easier to get lenses fitting the most common models. Clearly, this type of externality is more important the larger the market of the firm selling the good (or the network).

Similarly, consumers using a credit card or a telephone system will benefit from network externalities: a credit card tends to be more widely accepted as the number of owners increases; getting linked to a telephone system is certainly more attractive the more persons one can get in touch with. Another example is provided by the new French telecommunication network Minitel, which allows users to exchange informations, sell goods, etc.

When firms propose **compatible networks**, the presence of one consumer is equally valued by other consumers of the same good or of any compatible good. As a consequence, the externality associated with one network includes the effect of all the users of the other networks compatible with it. For example, the fact that Taïwanese clones are widely spread benefits the users of IBM PC's and vice-versa. Compatibility is certainly a key aspect in the study of network externalities. Indeed, each firm can have access to a larger externality and can make its product more attractive by increasing its compatibility with products sold by other firms. However, this also affects the degree of competition between firms because their products clearly become closer substitutes. The two extreme cases which are usually considered are full compatibility and total incompatibility, although in various situations of practical interest, goods are only partially compatible. For example, Digital is selling computers which can be linked to peripherals (printers, disc-drives, etc.) built by other companies, with the restriction that the most sophisticated features can only be used with Digital. More generally, intra-brand externality is usually larger than inter-brand externality.

The pricing problem of firms producing goods subject to network externalities has received increasing attention in recent years. Katz and Shapiro (1985) study various equilibrium situations with and without compatibility and consider the private and social incentive to build an adapter (a good which makes two networks compatible). In particular, they consider the benefit for an industry to adopt a standard. If a standard is adopted, all the goods produced in the industry are compatible with each other.For example, the compact disc industry has adopted the Philips - Sony standard. However, the adoption is not necessarily automatic and did not happen in the case of video discs.

The issue of standardisation has also been considered from the innovation process point of view (see David, 1985, Farrell and Saloner, 1985, 1986b, Katz and Shapiro, 1986a, 1986b and Dosi et al., 1988). If a network is already implemented, the scope for innovation is limited unless the new product is simultaneously better and fully compatible with existing material. This is so because consumers who are linked to the old network are reluctant to switch to a new one (even if it is better) which is not yet used by anybody. An alternative way of looking at this problem is in terms of learning or in terms of "increasing return to technology adoption". Suppose several incompatible standards are made available to consumers and suppose too that those standards are perfectly equivalent in terms of quality, so that consumers are indifferent between them as long as no externality is generated. In such a situation, it cannot be predicted which one will turn out to be adopted by consumers. The reason is that the first consumer has to buy one good of a specific standard (without any specific reason or because he thinks that it will be the standard adopted by all his fellow consumers for example). He is immediately followed by all other consumers who consider the externality already generated in that network and decide to buy it too. The trouble is that if the first consumer had chosen another network (or standard), that network would have been chosen by all other consumers as well, which is what is meant by the concept of "increasing return to technology adoption". In some sense, consumers as a whole lock into a particular standard. In a nice paper, David (1985) explains in a similar way why the QWERTY keyboard is still used when much better alternatives are available.

Clearly, the previous argument shows that any standard could have been adopted, so that any standardisation process can only be path dependant (i.e. depend on previous events in the sequence of adoption). Note that we have considered the case of networks which, as long as no externality is generated, are perceived as equivalent by consumers. Would they have been differentiated in some way, several standards could have survived because some consumers would have been ready to prefer one to the other, even with different levels of externality. Note also that we did not allow firms to have any kind of strategy. For example, a firm facing a shrinking demand, could try to regain consumers by lowering prices so as to compensate for a small externality. Another profitable strategy could be for one firm to stick to its own standard and even initially make losses so as to capture the whole market later and force all other firms to accept the same standard. It could even adopt an early-mover strategy which would make it the leader of the market with a low quality good. Later entrants, even with a better substitute, would not be in a position to attract any consumer because they would already all be locked in the first network. One could also imagine that consumers anticipate such situations and are able to make the right choice in terms of standard and time of adoption.

All previous comments suggest that a dynamic model would be most helpful in tackling the problem of network externalities. However, it would clearly be a difficult task and authors have preferred to explore simpler static models with their limitations.

The most common static model of competition in the presence of externalities relies on a Cournot framework (see Katz and Shapiro, 1985 and Bental and Spiegel, 1988). Each firm decides on the quantities to sell, taking as given the quantities sold by other firms. A Nash equilibrium is reached when no firm has an incentive to deviate from its strategy. It is also usually assumed that the goods are basically perfect substitutes leaving aside the network externality aspect: all consumers are indifferent between the goods if they are available at the same price. However, this does not imply that goods are still perfect substitutes at equilibrium since externality (and prices) may be different.

The difficulty with the Cournot framework is that one needs to assume that, at equilibrium, all consumers' expectations are fulfilled. This means that consumers correctly anticipate the size of each network and make their purchase decision according to that belief. Such an assumption is necessary since, as mentionned by Katz and Shapiro (1985) and as we mentionned before, there may be other equilibria. One problem is that the symmetric fullfilled expectations equilibrium assumed by Katz and Shapiro (1985) is not stable. Suppose the system is at the (candidate) symmetric equilibrium; suppose too that some consumers (who only observe prices even when firms play in quantities) deviate and do not make the "right" choice, shifting (say) from firm 1 to firm 2. By doing so, they increase the network externality of firm 2 and induce all consumers to do the same through a bandwagon effect. Such an equilibrium can only be sustained if firms comply with their original commitment on quantities sold. If they do not, there may be several demand equilibria. For example, if consumers expect that they will all buy good 2 (say), each of them will indeed buy good 2 (because it is the best choice for everybody , considering the expected externality) and all expectations will also be fulfilled. Intuitively, this shows that, in the presence of externalities, the simple observation of the price vector by consumers cannot does not give enough information to fully determine a unique demand system (the demand associated with each good). Consumers need some extra information on the size of the networks. Note that this also implies that a pure price game cannot be played. Actually, assuming a "fulfilled expectations" equilibrium corresponds to one way of selecting one equilibrium out of all available.

As long as goods are perfect substitutes, demand associated with each of them cannot be fully determined through the vector of prices. However, we show that when goods are no more perfect substitutes but are **horizontally differentiated**[1], demand functions will exist, even in a static framework. Again, assuming that goods are differentiated is reasonnable as we study the problem of partial compatibility. Note that we do not consider horizontal differentiation as decision variables for the firms, an assumption usually made in spatial competition (see Hotelling, 1929, and d' Aspremont et al., 1979). We rather consider that firms produce goods which are inherently different; a similar approach was used in location theory by de Palma et al. (1985) who show that when differentiation is large, firms agglomerate in the center of the market and nevertheless make positive profits. The fact that goods may only be partially compatible is all the more likely to happen when consumers are different in their tastes because it induces firms to provide differentiated (thus possibly not fully compatible) products.

The paper is organized as follows. In section 2, we reconsider Bertrand competition in the presence of network externalities. We give a simple argument to show why demand is not defined and we also show how externalities can play the role of a barrier to entry even when the potential entrant offers a product of better quality. In section 3, we propose a general model of duopolistic competition with horizontally differentiated products. We show that externalities

[1] Two goods are said to be horizontally differentiated if the criteria of differentiation is a purely subjective question of taste and not an objective measure of quality (see Phlips, 1983, and Gabszewicz and Thisse, 1986). For example, Apple and IBM sell horizontally differentiated computers. Indeed, would they be available at the same price (and for a given level of network externality), some consumers would prefer Apple and some IBM.

may entail non existence of a Bertrand equilibrium when the degree of differentiation is low. We derive a condition for the existence of a Nash equilibrium and compute it explicitly under a specific formulation. In section 4, we solve the general $n > 2$ firms case. Conclusions are presented in section 5.

2. *Bertrand competition and network externalities*

We assume that there are two firms, denoted by 1 and 2, selling an homogeneous good at prices p_1 and p_2 respectively. Each firm chooses its price to maximize its profit taking the price charged by the other firm as given.

Throughout the paper, we assume that demand is perfectly inelastic: every consumer buys one and only one unit of the good. We also assume that firms can produce the good at constant marginal cost, set equal to zero for reasons of convenience. In the absence of externalities, product homogeneity implies that each consumer buys from the cheapest firm. We normalize the size of the market to one. If the price charged by firm 2 is constant, quantity Q_1 sold by firm 1 is given by

$$
\begin{aligned}
Q_1 &= 1 && \text{if } p_1 < p_2, \\
Q_1 &= .5 && \text{if } p_1 = p_2, \\
Q_1 &= 0 && \text{if } p_1 > p_2.
\end{aligned}
$$

Obviously, $Q_2 = 1 - Q_1$.

The situation can be represented as in figure 1. The implication is that each firm has an incentive to undercut the price charged by its competitor so that, eventually, prices are driven to zero (or to the constant marginal cost). Naturally, this argument, and its implication, hold for homogeneous products only. As soon as products are horizontally differentiated, it is possible to prove, under fairly general assumptions, that a unique symmetric Nash equilibrium with strictly positive profits for both firms will exist (see Anderson and de Palma, 1988).

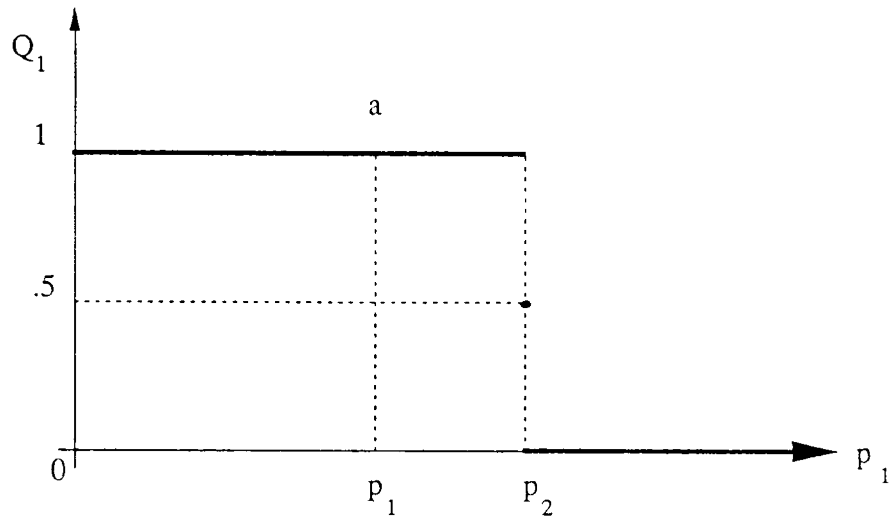

Figure 1

We now examine how the externality affects Bertrand's argument. We assume that the utility associated with each good strictly increases with its externality. This implies that, though goods are intrinsically perfect substitutes, they may no longer be so if quantities sold by each firm are different.

Without any loss of generality, we assume that the externality associated with good (or network) i is linear and given by

$$E_i = R_I.Q_i + R_E.(1 - Q_i), i = 1, 2. \quad (1)$$

It is important to notice that R_I (the intra-brand externality) and R_E (the extra-brand externality) are not firm dependant, so that the externality associated with each network only depends on the relative quantities sold. In that sense, it is a symmetric property.

We shall also assume that $R_I > R_E \geq 0$. This implies that the externality effect induced by a consumer is of a higher utility to the consumers of the same network than to those of the other one. In this paper, we restrict our analysis to the case of positive externalities (negative externalities would entail $R_I < 0$ and $R_E = 0$. They are studied in chapters 3 and 4). Networks are partially compatible if $R_I > R_E > 0$, and the following are limit cases:

$R_E = 0$ and $R_I > 0$: complete incompatibility,

$R_I = R_E > 0$: full compatibility,

$R_I = R_E = 0$: no network externality effect.

We now see how standard Bertrand competition is affected by the presence of externalities. Let us first consider the case with no externalities. Suppose we have $p_1 < p_2$ and $Q_1 = 1$, see point a in figure 1). We see that firm 2 cannot sell its good, but if it undercuts p_1 by a small amount, it can regain the whole market. We now consider the same situation in the presence of externalities. We assume that firm 1 sells $Q_1 = 1$; thus the externalities associated with each good are given by

$$E_1 = R_I.1 + R_E.0 = R_I,$$

and

$$E_2 = R_I.0 + R_E.1 = R_E.$$

In such a situation, it would not be sufficient for firm 2 to undercut p_1 by a small amount to regain all consumers. Indeed, the externality has induced an increased utility for consumers linked to network 1. As a consequence of the definitions of E_1 and E_2, firm 1 has now an advantage which depends on the difference in externalities $R_I - R_E$. Undercutting will prove usefull only if it can compensate for the externality.

Conversely, p_2 being fixed, it would be possible for firm 1 to increase its price beyond p_2 without loosing any consumer. Indeed, the price of good 1 can be increased as long as the price differential does not totally compensate $R_I - R_E$.

We represent the situation in figure 2; we see that quantities sold are no more functions of the prices, but only correspondances : to the price vector (p_1, p_2) may correspond several quantities (Q_1, Q_2). Thus, the concept of demand function disappears.

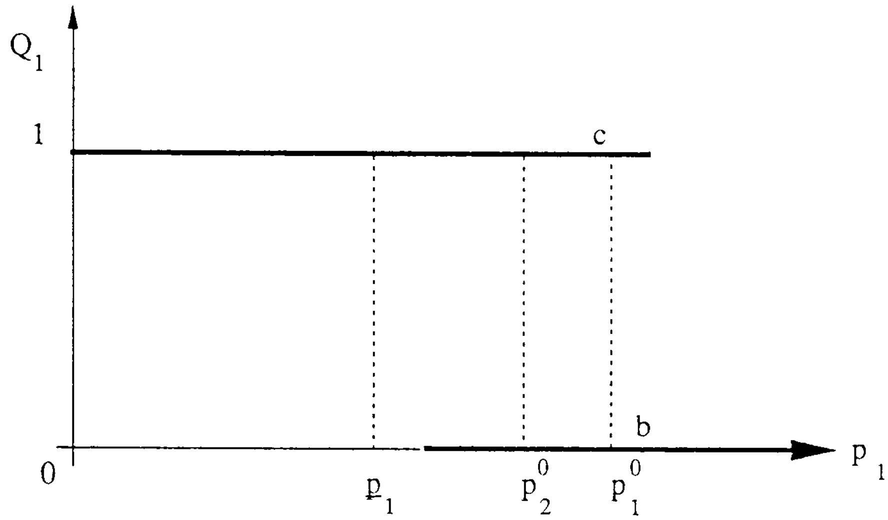

Figure 2

Indeed, suppose that the market is characterised by prices ($p_1°$, $p_2°$) and quantities ($Q_1°$, $Q_2°$) = (0, 1), which corresponds to point b in figure 2. Suppose now that firm 1 decreases its price to regain the whole market. It could do so by charging $p_1 = \underline{p}_1$. Suppose also that, afterwards, it decides to move back to $p_1°$. In that case, the new situation would be characterised by firm 1 selling $Q_1 = 1$ and firm 2 selling $Q_2 = 0$. The new quantities are now given by (Q_1, Q_2) = (1, 0), although the price vector is unchanged (point c in figure 2).

The fact that quantities are not uniquely determined by prices can be interpreted in terms of a phenomenon of hysteresis : i.e. a situation where changing one parameter and coming back to its original value leads to different levels of demand. In this case, decreasing a price and going back to the original situation, may lead to quantities which are different from those observed in the original situation.

Thus, firms have an incentive to decrease prices and come back to the original situation. If they have the possibility of doing so, it is clear that a static Nash equilibrium with firms competing in prices is no longer relevant. However, note that it can exist if the firm, after changing its price, is not allowed to come back to the original situation. In that case many configurations may be Nash equilibria (see Friedman, 1986) and the game would be characterised by a discontinuity in the payoff functions. Such situations have been considered by Dasgupta and Maskin (1986) where it is shown that equilibria may exist, but again, if firms are allowed to come back to the original situation and make more profit, the key problem is not the discontinuity of payoffs, but rather undefined demands. However, it seems difficult to limit ourself to such cases for two reasons; first, because a model where firms are allowed to come back to the original situation is clearly more realistic; and second because, as we show in the next section, it is possible to obtain well defined demands when products are differentiated, which leads to a well defined static game. Thus, product differentiation in the presence of network externalities does not only make the model more realistic, it is also essential for the study of price competition in a realistic static model.

The second important consequence is that network externalities can also play the role of barriers to entry. Suppose firm 1 is supplying the whole market and faces the threat of a potential competitor (firm 2). Since firms are producing at zero marginal cost, entry is deterred as long as, even by setting a price p_2 equal to zero, firm 2 is not able to get any consumer. This is happening when p_1 is low enough (but still positive) so as not to compensate for the difference in the externalities. In that case, no consumer is willing to buy product 2 because there would be more to lose in term of externality than to gain in terms of prices. Naturally, the larger the externality differential R_I - R_E, the higher the barrier. As a consequence, a monopolist would gain from preventing compatibility with potential competitors (so as to decrease R_E) and from increasing the "intra-brand" externality R_I.

Note that although consumers have no individual incentive to shift from firm 1 to firm 2, there always exists a coalition of consumers which could gain from doing so. The size of that coalition clearly depends on R_I - R_E. Through a bandwagon effect, all the consumers who were still buying good 2 will follow and buy good 1. This suggests that, in the absence of any switching cost and learning, consumers would all switch to good 2 if it is cheaper.

The previous argument that a monopolist would gain from preventing compatibility with a potential entrant does not necessarily hold when the products are vertically differentiated. Let us consider a firm which offers a product of high quality and is facing a competitive fringe selling a similar good but of a much lower quality. In such a case, the "strong" firm may find it profitable to encourage compatibility because it would gain more from increased externality than it would lose from increased competition (such cases have been studied by Esser and Leruth, 1989).

Conversely, it may also be the case that the barrier to entry is high enough to deter entry even from competitors who would be able to supply a good of better quality. This would typically be a situation where consumers are sticking to the "wrong" good because none of them is ready to make the move (as in David, 1985). In that sense, high externality effects can also reduce the speed of R&D (see also Farrell and Saloner, 1985).

3. Duopoly and horizontal differentiation

3.1. A general formulation

There are two firms (1 and 2) selling horizontally differentiated goods which are subject to network externalities. Using the same notation as in section 2, the externalities E_1 and E_2 associated with good 1 and good 2 are given by (1).

We assume that the demand function can be written

$$Q_1 = D\left[\frac{p_1 - p_2}{\mu}, \frac{E_1 - E_2}{\mu}\right], \qquad (2)$$

with $Q_1\,[.] \in C^2$, $Q_1 \in [0, 1]$ and $Q_2 = 1 - Q_1$.

As a first step, we impose two conditions on the demand function D [.] :

A1: demand is strictly decreasing in the first argument (downward sloping demand) and is strictly increasing in the second argument (positive network externality effect).

A2: $D[.] \rightarrow 0$ as $\frac{p_1 - p_2}{\mu} \rightarrow +\infty$.

The parameter $\mu \geq 0$ is designed to capture heterogeneity of tastes among consumers. For high values of μ, price difference becomes relatively less important in the decision process. The limit case $\mu \rightarrow +\infty$ corresponds to independant demands: each firm can behave as a monopolist. In the absence of externalities, the other limit case ($\mu = 0$) would correspond to Bertrand competition.

We compute the externalities according to equation (1) and, using the fact that $Q_1 + Q_2 = 1$, we obtain

$$E_1 - E_2 = (R_I - R_E).(2Q_1 - 1).$$

Thus, demand equation (2) can be written as

$$Q_1 = D\,[\frac{p_1 - p_2}{\mu} ; \frac{(R_I - R_E).(2Q_1 - 1)}{\mu}] \tag{3}$$

and we see that the externality effect introduces demand itself as an argument in the demand equation. However, note that the situation is basically very different from what is happening with fulfilled expectations. Here, if equation (3) can be solved for a unique Q_1, expectations will always and automatically be fulfilled. In other words, if Q_1 is unique, it can be seen as the (unique) outcome of a stationnary process where consumers join the networks one after the other. In the long run, no other demand equilibrium can be achieved.

We make a further assumption on D; we assume that:

A3: for any level of prices, D is convex (respectively concave) with respect to Q_1 as long as $D \leq 0.5$ (respectively $D \geq 0.5$).

In other words, we assume that D is logistic in the number of consumers Q_1: when $p_1 > p_2$ (resp. $p_1 < p_2$), D[.] increases slower (resp. faster) in the externality. Such a function is represented in figure 3. Intuitively, this assumption implies that if one good is more expensive than the other, an increased externality affects less the demand for that good than it does for the cheaper one.

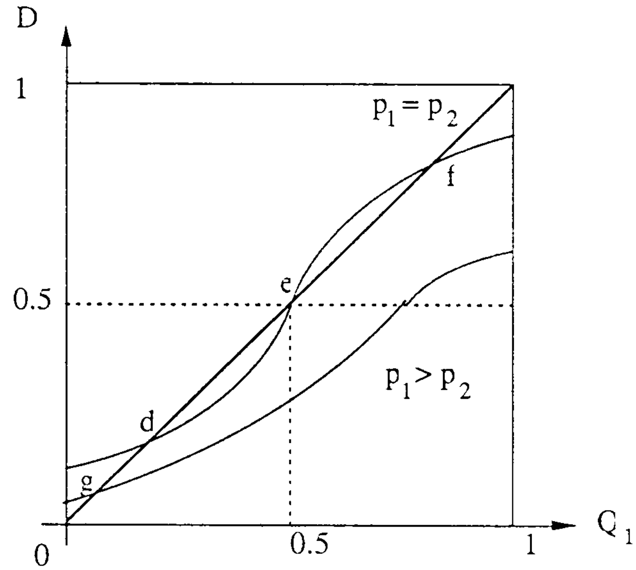

Figure 3

The question is now to find conditions under which (3) has only one solution Q_1*, with

$$Q_1^* = G\left[\frac{p_1 - p_2}{\mu}; \frac{R_I - R_E}{\mu}\right] \tag{4}$$

Mathematically, we are looking for conditions under which (3) has only one fixed point, which can then be written as in (4).

We first derive these conditions in the case $p_1 = p_2$.

Lemma 1: Demand equation (3) can be inverted when $p_1 = p_2$ if and only if D satisfies the following condition:

$$D_2[.] \leq \frac{\mu}{2(R_I - R_E)} \text{ at } Q_1 = 0.5, \tag{5}$$

where $D_2[.]$ is the first derivative of D with respect to its second argument.

Proof: The slope of D[.] with respect to Q_1 is given by $\{\mu.\ D_2[.] / 2(R_I - R_E)\}$. At $Q_1 = 0.5$, this slope is maximum by A3. If it is smaller or equal to 1 at that point, condition (5) is satisfied and the function D[.] will intersect the main diagonal 00' at (0.5, 0.5) only (see figure 3). When condition (5) is not satisfied, the function D has three fixed point, $Q_1^- < 0.5$, $Q_1^\circ = 0.5$ and $Q_1^+ > 0.5$; this is a direct consequence of A1 and A3.

We now consider the case $p_1 \neq p_2$ and prove

Lemma 2 : Demand equation (3) can be inverted when $p_1 \neq p_2$ if condition (5) is satisfied.

Proof. Let us first consider the case $p_1 > p_2$. Let q_1 be defined by : $D[(p_1 - p_2) / \mu; (R_I - R_E).(2q_1 - 1) / \mu] = 0.5$. We know that $q_1 > 0.5$ (assumption A1). For any value of $Q_1 < q_1$, D[.] is convex, its slope is less than 1 (condition 5) and therefore, it will intersect the main diagonal 00' once only. For values $Q_1 \geq q_1$, D[.] is strictly below the diagonal as it is already for $p_1 = p_2$. (see figure 4)

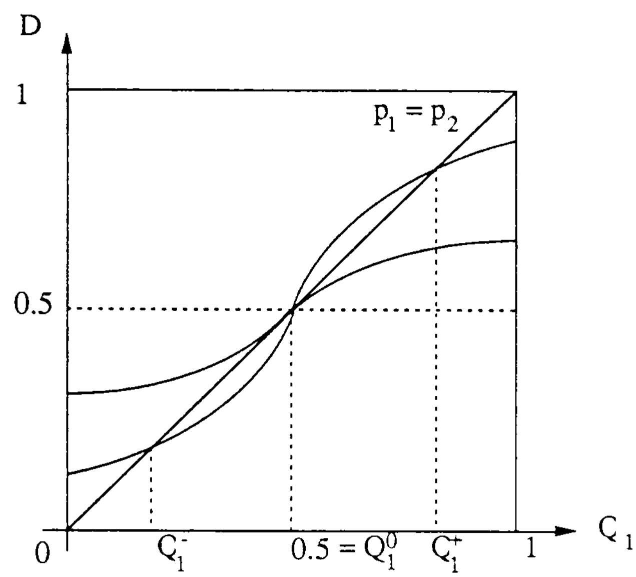

Figure 4

As a consequence of lemmas 1 and 2 and of the implicit function theorem, we state

Proposition 1 : Given assumptions A1-A3, demand equation (3) can be inverted if and only if condition (5) is satisfied. Moreover, the solution Q*[.] is C^2 in prices and can be written as

$$Q^*[.] = Q^*[\frac{p_1 - p_2}{\mu}, \frac{R_I - R_E}{\mu}] .$$

Condition (5) shows that the degree of heterogeneity (measured by μ) has to be sufficiently large compared to the externality effect in order to obtain demand functions which are properly defined. If $R_I = R_E$, there is full compatibility and demand is well defined. However, if firms can generate a lot of intra-brand externality which does not benefit to the consumers of the other good, the probability that almost all consumers will patronize one firm increases. It is only when the goods are sufficiently differentiated that such an effect disappears.

Intuitively, proposition 1 can be explained as follows. When condition (5) is not satisfied, we know that for $p_1 = p_2$, D[.] has three intersections with the main diagonal and thus three fixed points (see figure 4). We also know that for very large values of p_1, there can only be one such intersection (by A2). Thus, the "demand" D[.] associated to p_1 for p_2 fixed (and when condition (5) is violated in $p_1 = p_2$) displays hysteresis as shown in figure 5. The limit case for $\mu = 0$ is also presented in the same figure. As μ gets larger, hysteresis decreases and vanishes when μ is large enough to satisfy condition (5) in $p_1 = p_2$. Note that the presence of hysteresis implies that when condition (5) is violated, only a dynamic model can tackle the externality effects unless, as done in Katz and Shapiro (1985), one imposes restrictions on the equilibrium configuration. In particular, the concept of fulfilled expectations they use, is actually equivalent to consider, out of the three possible configurations presented in figure 4 and in figure 5, the one in the middle (e). It is only when condition (5) is not violated that their concept leads to the configuration we consider.

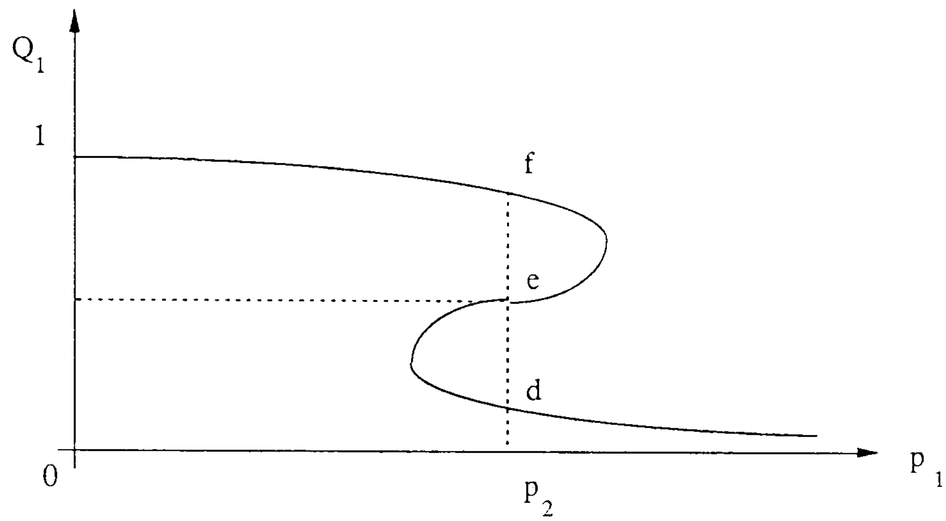

Figure 5

We now turn to show that a Nash equilibrium exists.

General conditions ensuring existence of a Nash equilibrium for a duopoly selling horizontally differentiated products have been examined by Anderson and de Palma (1988) along the lines suggested by Perloff and Salop (1985). We list those conditions below:

C1. $Q_i = G[p_i - p_j, \mu]$; $i, j = 1, 2, i \neq j$, $G[.] \in C^2$;

C2. $\frac{\partial Q_i}{\partial p_i} < 0$;

C3. $\frac{\partial^2 Q_i}{\partial p_i^2} > (<) \, 0$ as $p_i > (<) \, p_j$;

C4. $Q_i . \frac{\partial^2 Q_i}{\partial p_i^2} - 2.\left(\frac{\partial Q_i}{\partial p_i}\right)^2 < 0$;

C5. lim Q_i for $\mu \rightarrow +\infty$ equals 0.5 for p_i and p_j finite

C6. $\frac{\partial Q_i}{\partial \mu} > (<) \, 0$ as $p_i > (<) \, p_j$.

Some of these conditions are obviously satisfied by the demand function (4). They are direct consequences of the assumptions imposed on (3). C1 is verified because of property 1.

C2 is verified because if D[.] is decreasing with respect to the price (the first argument), so will Q_i*. C5 is satisfied because as μ gets large, the first argument of D[.] tends to zero and D[.] has the same behavior as if $p_1 = p_2$. We have seen that in that case, the only inersection with the diagonal is in 0.5. The same argument can be used to check that C6 is satisfied.

However, C3 and C4 cannot be derived from the assumptions imposed on D[.]. C3 and C4 are imposed in order to insure the concavity of the demand functions and are thus expressed in terms of the behavior of the derivatives. Though we know that if D[.] is C^2, Q_i is also C^2, the behavior of the derivatives is not necessarily the same. One way to cope with this problem would be to impose a set of conditions on D[.] and its derivatives so that Q_i* verifies C3 and C4. This is however not very natural. Indeed, either (3) cannot be solved, demand is not defined and any condition would be irrelevant; or it can be solved and it is Q_i* which matters. Thus, we impose

A4: Q_i* satisfies conditions C3 and C4.

We can now prove:

Proposition 2 : If D[.] verifies A1-A3 and (5), and if Q_i* satisfies A4, there exists a unique symmetric Nash equilibrium in prices given by:

$$p_i = -\frac{Q_i^*}{\frac{\partial Q_i^*}{\partial p_i}}$$

Proof. The proof of this proposition is a direct consequence of the results derived in Anderson and de Palma (1988).

Note that if condition (5) is violated, no Nash equilibrium will exist. Indeed, let us consider a pair of prices (p_1, p_2) as a candidate equilibrium, with $p_1 < p_2$. The demand functions may be defined if p_1 is much smaller than p_2, but will not be any more if one firm decides to charge the same price as its competitor. As a consequence, that pair of asymmetric prices cannot be a Nash equilibrium. For the same reason, there cannot be any symmetric price equilibrium if (5) is violated. Under the conditions stated in property 2, (2) can be inverted (see property 1) and Q_i* satisfies conditions C1- C6.

Note however that Anderson and de Palma (1988) have proved existence and uniqueness of a Nash equilibrium in a situation without externalities. Here, using (2), we have extended this result and introduced additional conditions (A1-A3) to specify the way externalities can be incorporated in the demand function to guarantee existence and uniqueness of a (static) Nash equilibrium in prices.

We now provide an example of a demand function satisfying C1-C6. A natural candidate is the CES, which is widely used for analysing issues in product differentiation (see Spence, 1976 or Dixit and Stiglitz, 1977). However, there is no obvious way of integrating externalities in that model. Therefore, we use a Logit formulation. The theoretical foundations of this model as a model of product differentiation are discussed in Anderson et al. (1989).

3.2. An example: the logit formulation

We now present a specific demand function which can be constructed as follows. We assume that the utility U_i derived by a consumer who buys good i is equal to

$$U_i = -p_i + R_I.Q_i + R_E.(1 - Q_i) + \mu.\varepsilon, \quad i = 1, 2,$$

where ε is a random variable of zero mean and unit variance, designed to capture the heterogeneity in consumers' tastes. The other parameters have the same interpretation as before. Note that when $\mu \to 0$, we obtain the model studied in section 2.

Assuming that the consumers are utility maximisers and that the ε are identically, independently Gumbel distributed, the resulting demand is given by (see Manski and McFadden, 1981)

$$Q_1 = \frac{1}{1 + e^{\left[\frac{p_1 - p_2}{\mu} + \frac{(R_I - R_E).(1 - 2Q_1)}{\mu}\right]}}, \tag{6}$$

and $Q_2 = 1 - Q_1$. This expression has the same functionnal form as the Logit formula although, of course, the corresponding demand G[.] is not of the Logit type. It is a matter of simple algebra to check that (6) satisfies A1-A3. Condition (5) can, in this case, be written as

$$\mu > \frac{R_I - R_E}{2} . \tag{7}$$

Under condition (7), (6) can be solved for Q_1 and it is easy to verify that conditions C3 and C4 are also satisfied. Using proposition 2, we know that there exists a unique symmetric Nash equilibrium in prices; it is given by

$$p_1 = p_2 = 2\mu - (R_I - R_E). \tag{8}$$

At equilibrium, the profits of each firm are

$$\pi_1 = \pi_2 = \mu - \frac{R_I - R_E}{2} .$$

Prices are always positive (from (7) and (8)). When $R_I = R_E = 0$, the equilibrium prices are equal to 2μ, a result derived by de Palma and Anderson (1988). It is easy to check the following properties

P1. Equilibrium prices and profits increase with μ, the degree of heterogeneity.

P2. Equilibrium prices and profits decrease with R_I and increase with R_E.

Given that $R_I \geq R_E$, the global effect of incompatibility (measured here by $R_I - R_E$) tends to decrease prices and profits. However, the specific effects of the intra-brand and extra-brand externalities are different.

When R_I increases, firms have an incentive to attract as many consumers as possible to make them benefit from a larger externality. Indeed, a higher externality can be interpreted as a better quality and thus, the marginal benefit to decrease the price increases since the consumer who joins a network induces other consumers to do the same because of the increased externality. The mechanism is similar to what would be a snow ball effect in a dynamic framework or as is sometimes used an "increasing return to adoption". As a result, competition is more intense and prices (and profits) are driven down.

The opposite is true when R_E is increasing. In that case, if a consumer is attracted from firm 1 (say) to firm 2, the gain in externality for firm 2 is lower. The reason is that any extra consumer also induces some externality in the other network and this slows down the incentive for firm 2 to cut down its price. By doing so, the firm would attract less consumers because the snow ball effect is weakened. As a result, increasing the value of R_E reduces competition. Of course, as expected, the exernality plays no role when $R_I = R_E$.

Our results suggest that, though firms have an incentive to differentiate their products (or increase μ), they will try to increase the compatibility between them. Indeed, firms certainly try to be perceived differently by consumers. This is the case for Apple and IBM, one specializing in user friendly computers and the other in more scientifically oriented material. But both firms try to increase their compatibility so that they both gain in externality: it is now possible to read on a McIntosh a text written for an IBM Pc.

4. An Oligopoly Model.

The general formulation presented in (2) cannot straightforwardly be extended to the case of N (> 2) firms. We need to use a specific model and, as in the previous section, we choose the logit specification.

We assume that there are N goods. The model is still symmetric: the intra brand externality R_I is equal across firms and the inter brand externality R_E is the same between any pair of firms. Let Q_i be the number of consumers patronizing firm i and p_i be the price of good i, i = 1, ..., N. The utility derived by a consumer purchasing good i is

$$U_i = -p_i + R_I.Q_i + R_E.\Sigma_{j\neq i} Q_j + \mu.\varepsilon; \quad i = 1, ..., N.$$

Assuming, as before, that the random variables ε are identically, independently Gumbel distributed, we obtain the following demand model

$$Q_i = \frac{1}{1 + \Sigma_{j\neq i}\, e^{\left[\frac{p_i - p_j}{\mu} + \frac{(R_I - R_E).(Q_j - Q_i)}{\mu}\right]}}, \quad i = 1, ..., N. \qquad (9)$$

We restrict our analysis to the symmetric solution and see under which condition it is a Nash equilibrium. It is thus not necessary to solve the whole system (9), which would be a formidable task. We just have to ensure the existence of a well defined demand function for one firm (for example firm 1), when all other firms set a price p*. In this case, (9) can be written as

$$Q_1 = \frac{1}{1 + (N - 1).e^{\left[\frac{p_1 - p^*}{\mu} + \frac{(R_I - R_E).(1 - N.Q_1)}{\mu.(N - 1)} \right]}} \tag{10}$$

We now look at the conditions under which (10) can be solved in Q_1. We use (10) to compute the value of the derivative of Q_1 with respect to p_1. This leads to

$$\frac{dQ_1}{dp_1}.\{1 - \frac{(R_I - R_E).N}{(N - 1)}.\frac{Q_1.(1 - Q_1)}{\mu}\} = - \frac{Q_1.(1 - Q_1)}{\mu}. \tag{11}$$

As the RHS of equation (11) is negative, demand will be defined if and only if the coefficient of dQ_1/dp_1 is positive. We see that if that condition is satisfied for $Q_1 = 0.5$, it will always be true. Thus, the necessary and sufficient condition is given by

$$\mu > \frac{(R_I - R_E).N}{4.(N - 1)}. \tag{12}$$

Note that for $Q_1 = 0$ or 1, (11) simply states that dQ_1/dp_1 is equal to zero. As expected, for $N = 2$, this condition is equivalent to (7). However, one should keep in mind that in the general case, condition (12) is far more limited. Indeed, it only ensures the existence of a properly defined demand function once all the remaining firm have set the same price. It does not hold for any set of prices. Moreover, we could prove that a unique Nash equilibrium exists in the case $N = 2$ by using proposition 2; but there exists no such result in the general case. As a consequence, now that the demand function is well defined, we have to prove that the symmetric price structure is indeed a Nash equilibrium. This can be done by checking that the profit of firm 1 is quasi-concave in p_1 (see Friedmann, 1986). In the appendix, we prove that (12) **does not** guarantee quasi-concavity. Indeed, we show that it is guaranteed for

$$\mu > (R_I - R_E).\frac{2.(N - 1)}{N^2}. \tag{12'}$$

We also show in the appendix that for $2 < N < 6$, (12') is a stronger condition than (12) and is thus the one to be satisfied.

Under condition (12) and (12') the resulting prices and profits characterising the Nash equilibrium are given by

$$p^* = \frac{\mu.N}{N - 1} - \frac{R_I - R_E}{N - 1}, \qquad (13)$$

$$\pi^* = \frac{\mu}{N - 1} - \frac{R_I - R_E}{N.(N - 1)}, \qquad (14)$$

The conclusions in terms of the parameters μ, R_I and R_E are obviously the same as in the case $N = 2$. We shall thus focus here on the properties of equilibrium prices as N varies. As is easily seen from (13), we have

P3. Equilibrium prices **increase** with N if $\mu < R_I - R_E$ and **decrease** with N if $\mu > R_I - R_E$.

The reason for which prices can increase with N for low values of μ is the following. As the number of firms increases, competition tends to drive prices down. But, on the other hand, the fact that each firm only enjoys a small market tends to decrease the effect of the externality, thus also its negative effect on the prices. It is only when the degree of differentiation (μ) is low that the global effect tends to increase prices. As μ gets larger, the externality plays a less important role in the competitive process and we get the usual result. In figure 6, we represent the regions (in terms of μ and N) where prices increase and decrease.

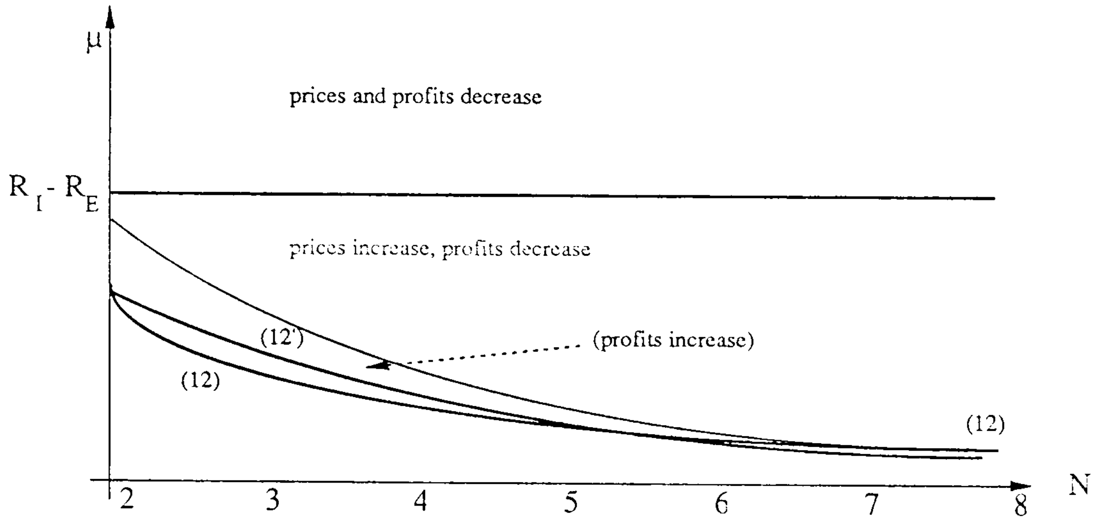

Figure 6

Note that figure 6 has to be understood in terms of discrete values of N. On the same figure, it can also be noticed that there is a small region where profits **increase** with the number of firms. This happens because prices increase more than quantities decrease. This result depends on the linear form of the utility function . As N gets large, the effect slows downs and then goes the other way. Thus, as firms enter the market, profits may raise first but then tend to decrease again until the expected profit is equal to the cost of entry. We have the following property

P4. If the cost of entry is K, the equilibrium number of firms N* is

$$N = \text{integer}\left[\mu + K + \frac{\sqrt{(\mu + K)^2 - 4K.(R_I - R_E)}}{2K} \right].$$

The proof of this proposition is a direct consequence of (14). In order to ensure that the above expressionb is well defined, we assume that profits are positive for N = 2, which is equivalent to

$$\mu > K + \frac{R_I - R_E}{2} .$$

Note that the presence of externalities tends to decrease the number of firms on the market. The reason is that the presence of externalities induces a phenomenon of agglomeration:

consumers tend to favour large networks. As a consequence, it is more difficult for an entrant to attract consumers; prices go down and, for a given cost of entry, so does the number of firms. When $R_I - R_E$ is equal to zero, the number of firms is maximum and equal to $N = 1+ \mu/K$, a result obtained by Anderson and de Palma (1987).

5. *Conclusions*

We have studied the problem of partial compatibility in the presence of network externalities using a model of price competition. As a first step, we consider the homogeneous goods case. We have shown that the externalities induce a phenomenon of hysteresis which prevents the existence of a static Nash equilibrium.

As a second step, we propose a general duopoly model with differentiated networks characterised by a partial compatibility with each other. We derive a necessary and sufficient condition for a static Nash equilibrium to exist (product heterogeneity is large enough compared to the degree of incompatibility). We provide an example based on the Logit formula and show that the presence of externality tends to decrease the level of the prices. We also show that firms benefit from an increased compatibility.

Finally, we extend this last specific model to the case of an oligopoly. We show that at the equilibrium, the prices may increase as the number of firms increases. However, at equilibrium, the number of firms tends to be lower in the presence of externalities.

We conjecture that the same approach could be used in a Cournot game (it would then of course be necessary to introduce an outside good in the model). We think that, even if consumers do not have expectations about the size of the networks, it would lead to a stable symmetric equilibrium.

We have purposedly restricted our analysis to symmetric equilibria. There are two reasons for this. First, we do not think that there are any other equilibria in the case of an horizontally differentiated oligopoly. Secondly, we think that asymmetric networks would be best studied in a vertical differentiation framework (such examples can be found in Esser and Leruth, 1989). Indeed, if networks are not symmetric, it implies that the larger ones induce a better utility (prices not being taken into account) than the smaller ones. Such a configuration is more likely to emerge when some consumers are ready to spend more in order to get linked to a large network while some others think the other way because they care more for the price.

We have analysed the case of positive network externalities. The results we have derived remain valid in the case of negative network externalities (congestion). In such a case, R_E is equal to zero and R_I is negative. Among others, it is worthwile mentionning that demand is always well defined and that a static Nash equilibrium always exists. Moreover, it is easy to see that prices increase as the impact of the externality (also called congestion in this case and represented by $R_I < 0$) gets larger.

Appendix

We now derive the condition under which no firm has an incentive to deviate from the symmetric equilibrium, i.e. there exists no p_1 (say) such that

$$\pi_1 (p_1, p^*, ..., p^*) \geq \pi_1 (p^*, p^*, ..., p^*),$$

where p* is given by (13).

Using (10), we have

$$Q_1 = \frac{1}{1 + \Omega_1} \; ; \quad \Omega_1 = (N - 1).\exp[\frac{p_1 - p^*}{\mu} + \frac{(R_I - R_E).(1 - N.Q_1)}{\mu.(N - 1)}] \qquad (A.1)$$

(A.1.) can be rewritten as

$$p_1 = \mu.\log(\frac{\Omega_1}{N - 1}) + \frac{\mu.N}{N - 1} + \frac{R_I - R_E}{N - 1} . (\frac{N}{1 + \Omega_1} - 2).$$

Setting $\Delta = \frac{\mu}{R_I - R_E}$, we have

$$\pi_1 = \frac{R_I - R_E}{1 + \Omega_1} . \{\Delta . [\log(\frac{\Omega_1}{N - 1}) + \frac{N}{N - 1}] - \frac{1}{N - 1} . [2 - \frac{N}{1 + \Omega_1}]\}. \qquad (A.2)$$

Since π_1 is continuous and differentiable, $p_1 = p^*$ is a symmetric solution if there is no $\Omega_1 \neq N - 1$ solution of $d\pi_1 / d\Omega_1 = 0$ and leading to $\pi_1(\Omega_1) > \pi_1(N - 1)$. Indeed, Ω_1 varies continuously between 0 (for $p_1 = -\infty$) and $+\infty$ (for $p_1 = +\infty$). Note that we can indifferently consider π_1 as a function of p_1 or Ω_1 because p^* is fixed for all other firms.

From (A.2), we see that $\partial\pi_1 / \partial\Omega_1$ has the same sign as $F(\Omega_1) - G(\Omega_1)$, with

$$F(\Omega_1) = - \frac{1}{\Delta.(N - 1)} . \left(\frac{2N}{1 + \Omega_1} - 2 \right),$$

$$G(\Omega_1) = -1 - \frac{1}{\Omega_1} + \frac{N}{N - 1} + \log \left(\frac{\Omega_1}{N - 1} \right).$$

It can be seen that the first order condition is satisfied at $\Omega_1 = N - 1$ (the symmetric configuration) and that π_1 is decreasing for $\Omega_1 \to +\infty$ and for $\Omega_1 \to 0$. We now consider (the following analysis is based on figure A1) the first derivatives of $F(\Omega_1)$ and $G(\Omega_1)$.

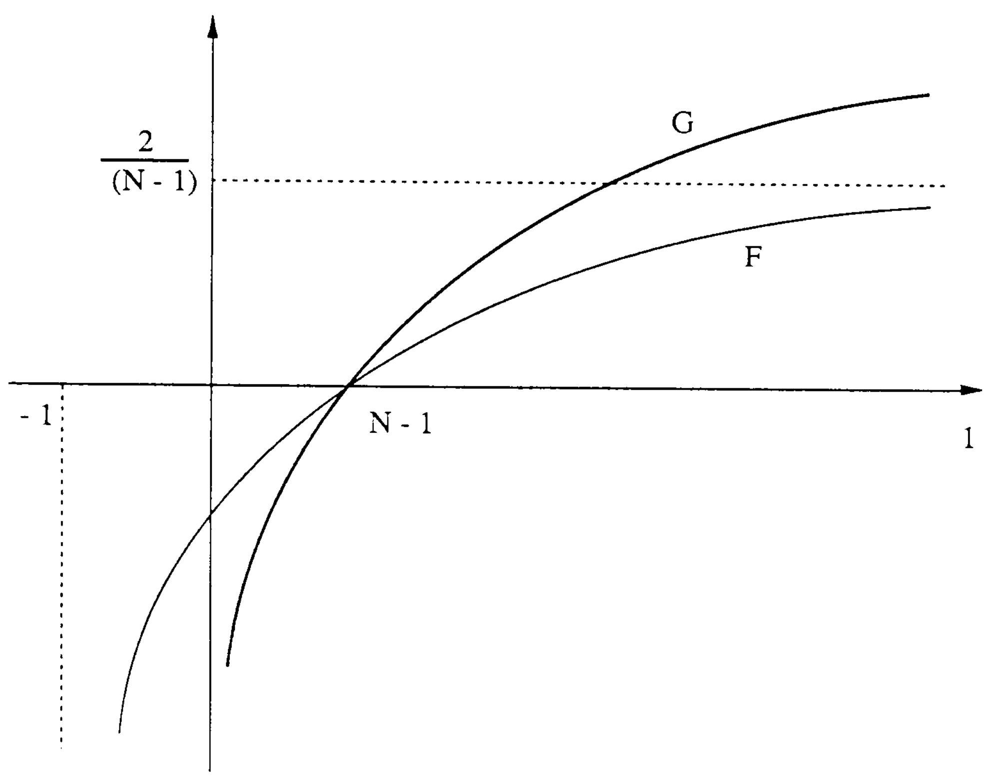

Figure A1

We have

$$\frac{d\,F\,(\Omega_1)}{d\,\Omega_1} = \frac{2N}{\Delta.(N - 1).(1 + \Omega_1)^2},$$

$$\frac{d\,G\,(\Omega_1)}{d\,\Omega_1} = \frac{\Omega_1 + 1}{\Omega_1{}^2}.$$

Note that if in $\Omega_1 = N - 1$, one has

$$\frac{d\,F\,(\Omega_1)}{d\,\Omega_1} = \frac{2}{\Delta.N.(N - 1)} > \frac{d\,G\,(\Omega_1)}{d\,\Omega_1} = \frac{N}{(N - 1)^2}, \qquad \text{(A.3)}$$

F and G intersect at least one more time for $\Omega_1 > N - 1$ and thus π_1 must have other extrema. The reason is that F (Ω_1) has an horizontal asymptote for $\Omega_1 \rightarrow +\infty$, while G ($\Omega_1$) does not. As a consequence, any other extrema may correspond to a maximum preventing $\Omega_1 = N - 1$ to be a Nash equilibrium. In any case, it is enough to check when (A.3) is not satisfied, i.e.

$$\Delta > \frac{2.(N - 1)}{N^2} \text{ or } \mu > (R_I - R_E).\frac{2.(N - 1)}{N^2}. \qquad \text{(A.4)}$$

Note that for N = 2, (A.4) is equivalent to (7), so that we can also limit the analysis to $N \geq 3$.

We show that if Δ satisfies (A.4), F (Ω_1) and G (Ω_1) do not intersect. It is enough to prove that

$$\frac{d\,F\,(\Omega_1)}{d\,\Omega_1} < \frac{d\,G\,(\Omega_1)}{d\,\Omega_1}, \text{ for any } \Omega_1,$$

or

$$\Delta.\frac{N - 1}{2N} > \frac{(\Omega_1)^2}{(1 + \Omega_1)^3}. \qquad \text{(A.5)}$$

Using (A.4), (A.5) is verified if

$$\frac{(N - 1)^2}{(N)^3} > \frac{(\Omega_1)^2}{(1 + \Omega_1)^3}. \qquad \text{(A.6)}$$

As the R.H.S. of (A.6) is maximum for $\Omega_1 = 2$, (A.6) is verified for $N > 3$ and for $N = 3$, the derivatives of F (Ω_1) and G (Ω_1) are identical.

Now note that (A.4) is not necessarily satisfied when (12) is. Indeed, it can easily be checked that (A.4) is a stronger condition on μ than (12) if and only if

$$8.(N - 1)^2 > N^3,$$

which is happening for $2 < N < 6$.

Q.E.D.

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