Monopoly with resale

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Abstract

This paper examines the intricacies associated with the design of revenue-maximizing mechanisms for a monopolist who expects her buyers to resell. We consider two cases: resale to a third party who does not participate in the primary market and inter-bidder resale, where the winner resells to the losers.

To influence the resale outcome, the monopolist must design an allocation rule and a disclosure policy that optimally fashion the beliefs of the participants in the secondary market. Our results show that the revenue-maximizing mechanism may require a stochastic selling procedure and a disclosure policy richer than the simple announcement of the decision to trade.

Keywords: information linkage between primary and secondary markets, optimal disclosure policy, stochastic allocations, mechanism design.

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1 Introduction

Durable goods are typically traded in both primary and secondary markets. Indeed, auctions for real estate, artwork and antiques are often followed by resale. The same is true for licenses, patents, Treasury bills, emission and spectrum rights. Similarly, IPOs and privatizations generate ownership structures which change over time as a consequence of active trading in secondary markets.

Resale may have different explanations. It may be a consequence of the fact that not all potential buyers participate in the primary market.[1] This can occur when a buyer values a good only if it is first sold to another buyer, such as in the case of an intermediate product that needs to be processed before it can be used by a final user.[2] Alternatively, participation only in secondary markets may be due to a change in the environment: At the time the government decides to sell spectrum rights, a company may not participate in the auction because it does not formally exist yet or because it attaches a low value to the rights. After a merger, a privatization, or a successful takeover, the company may develop interest in purchasing the rights and decide to buy them from the winner in the primary market.[3] Lastly, there can be legal or political impediments that prevent a monopolist from contracting with certain buyers, such as in the case of an auction in which the government is constrained to sell only to domestic firms.[4]

Resale may also be the result of misallocations in the primary market. As shown first in Myerson (1981), optimal auctions are typically inefficient when the distributions of the bidders' valuations are asymmetric. By committing to a policy that places the good in the hands of a buyer who does not value it the most, a seller can induce more aggressive bidding and raise a higher expected revenue. When resale can not be prohibited, bidders may thus attempt to correct misallocations in the auction by further trading in a secondary market.[5]

This paper considers the design of *optimal* mechanisms for a monopolist who expects her buyers to resell.[6] We analyze a simple game of incomplete information where a monopolist sells a durable good to a primary buyer who then resells in a secondary market. Trade in the resale game is the result of an ultimatum bargaining procedure in which players make take-it-or-leave-it offers with a probability distribution that reflects their relative bargaining abilities. Although stylized, the model illustrates the dependence of the resale surplus on the information disclosed in the primary market and is sufficiently tractable to allow for a complete characterization of the optimal allocation rule and disclosure policy from the monopolist's viewpoint.

The first part of the paper considers the case where resale is to a third party who does not participate in the primary market. We show that the revenue-maximizing mechanism has some interesting features.

First, the monopolist may find it optimal to adopt a *stochastic selling procedure*, for example, using lotteries and/or inducing the buyer to randomize over different contract offers. By selling

[1] Bikhchandani and Huang (1989), Haile (1999), and Milgrom (1987) consider auctions followed by resale where the set of bidders in the primary market does not include all potential buyers.

[2] In this case, the monopolist is likely to lack the necessary bargaining power to extract money from the third party without selling anything to her, as indicated in Milgrom (1987).

[3] Haile (2003) and Schwarz and Sonin (2001) consider models where bidders' valuations change over time.

[4] Although not considered in this paper, participation only in secondary markets may also be strategic, as indicated in McMillan (1994) and Jehiel and Moldovanu (1996).

[5] See also Gupta and Lebrun (1999) for an analysis of first-price asymmetric sealed bid auctions followed by resale, where trade in the secondary market is motivated by the inefficiency of the allocation in the primary market.

[6] Revenue-maximizing mechanisms without resale have been examined, among others, by Maskin and Riley (1984) and Myerson (1981).

with different probability to different types, the monopolist uses the decision to trade to signal the buyer's valuation to the third party so as to induce her to offer a higher price. Contrary to deterministic mechanisms, stochastic selling procedures permit the monopolist to change the beliefs of the participants in the secondary market without excluding those buyers with a lower willingness to pay. To illustrate, suppose the buyer has either a high or low valuation and assume the third party's prior beliefs are unfavorable to the buyer, in the sense that she is expected to offer a low price in the event she learns nothing from the outcome in the primary market. If the monopolist uses a deterministic mechanism that sells to both types with certainty, the third party offers a low price. If, on the other hand, she sells only to the high type, the third party offers a price equal to the high valuation, but again this leaves no surplus to the buyer. In contrast, with a stochastic mechanism, the monopolist can sell to the high type with certainty and to the low type with positive but sufficiently low probability to induce the third party to offer a high resale price, increasing the surplus the low type expects from resale and hence his willingness to pay in the primary market.

Second, the optimal mechanism may require the adoption of a *disclosure policy* richer than the simple announcement of the decision to trade. In the example above, the monopolist could disclose two signals, the first one with a higher probability when the buyer reports a high valuation, the second with a higher probability when he reports a low valuation. The advantage of disclosing additional information stems from the possibility of increasing the level of trade with the low type. In the limit, if the monopolist knew the buyer's valuation, she could sell with certainty to both types and use only a stochastic disclosure policy to control the beliefs of the third party in the secondary market.

Things are more complicated when the buyer's valuation is not known to the initial seller. In this case, disclosure increases the level of trade but does not permit sale to both types with certainty, since the combination of a certain allocation rule and a stochastic disclosure policy is not incentive compatible. Indeed, if trade were certain, the high type would always select the contract with the lowest price, irrespective of the associated disclosure policy. But then the low type would mimic the high type, paying the same price and inducing the monopolist to disclose the most favorable signal with a higher probability. The only way the monopolist can sort the buyer's types and at the same time disclose information to the third party is by making the high type pay a higher price than the low type, which is possible only if a higher price is associated with a contract that delivers the good with a higher probability.

We show how the optimal disclosure policy can be obtained as part of a direct mechanism in which the monopolist sends recommendations to the third party about the price to offer in the resale game. We then discuss how these recommendations can be implemented by disclosing the price the buyer pays in the primary market.

Finally, in the second part of the paper, we examine optimal auctions followed by inter-bidder resale. To the best of our knowledge, this problem has been examined only by Ausubel and Cramton (1999) and Zheng (2002). Ausubel and Cramton assume perfect resale markets and show that if all gains from trade are exhausted through resale, then it is strictly optimal for the monopolist to implement an efficient allocation directly in the primary market. The case of perfect resale markets is a benchmark, but abstracts from important elements of resale. First, when bidders trade under asymmetric information, misallocations are not necessarily corrected in secondary markets (Myerson and Satterthwaite (1983)). Second, and more important, efficiency in the secondary market is endogenous as it depends on the information revealed in the primary market which is optimally fashioned by the monopolist through the choice of her allocation rule and disclosure policy.

Zheng assumes it is always the winner in the primary market who makes the offer in the secondary market and suggests a mechanism that, under certain conditions on the distributions of the bidders' valuations, gives the monopolist the same expected revenue as a standard optimal auction where resale is prohibited. Instead of selling to the bidder with the highest virtual valuation, the monopolist sells to the bidder who is most likely to implement in the secondary market the same allocation as in a Myerson (1981) optimal auction .

This result however relies on the possibility of perfectly controlling the distribution of bargaining power in the secondary market through the allocation of the good in the primary market. However, that the original seller has the power to design the initial selling mechanism rarely implies that any future seller of the same good will also have the power to determine the resale outcome. In general, the distribution of bargaining power depends on the allocation of the good, but also on the individual characteristics of the players, such as their personal bargaining abilities. When this is the case, not only is it generically impossible to achieve Myerson's expected revenue, it may also be impossible to maximize revenue with a deterministic selling procedure.

Equilibria in English, first-price, and second-price sealed bid auctions followed by resale have been analyzed also by Haile (1999, 2003). His results illustrate how the option to resell creates endogenous valuations and induces signaling incentives that may reverse the revenue ranking obtained by assuming no resale. Our analysis builds on some of his insights, but differs from his in that we do not restrict the monopolist to use any specific format. Furthermore, the focus is on the design of the optimal informational linkage between primary and secondary markets and on the possibility of implementing it through the adoption of an appropriate disclosure policy.[7]

The rest of the paper is organized as follows. Section 2 examines resale to third parties. Section 3 extends the analysis to markets where the monopolist can contract with all potential buyers but can not prohibit the winner from reselling to the losers. Section 4 concludes.

2 Resale to third parties

Model set-up

Consider an environment where in the primary market a monopolistic *seller*, S, trades a durable good with a (representative) *buyer,* B. If B receives the good, he can either keep it for himself, or resell it to a (representative) *third party*, T, who participates only in the secondary market.[8]

We use $x \in \{0, 1\}$ to denote the decision of whether to trade in the primary market. When $x = 1$, B obtains the good from S, whereas when $x = 0$, S retains the good. Similarly, $x^r = 1$ when in the resale market B sells to T and $x^r = 0$ otherwise. An allocation in the primary market $(x, t) \in \{0, 1\} \times \mathbb{R}$ consists of the decision to trade along with a monetary transfer $t \in \mathbb{R}$ from B to S. Similarly, a resale outcome (x^r, t^r) consists of the decision to trade along with a transfer $t^r \in \mathbb{R}$ from T to B.

[7] Another strand of the literature related to this paper considers bidding in auctions followed by aftermarket interactions. The seminal work is Jehiel and Moldovanu (2000). See also Goeree (2003) and Das Varma (2003). Katzman and Rhodes-Kropf (2002) and Zhong (2002) examine the effect of different bid announcement policies on revenue in standard auctions followed by Bertrand and Cournot competition.

[8] We adopt the convention of using masculine pronouns for B and feminine pronouns for S and T.

All players are assumed to have quasi-linear preferences, $u_S = t$, $u_B = \theta_B x(1-x^r) - t + t^r$, and $u_T = \theta_T x x^r - t^r$, where θ_i denotes the value $i \in \{B,T\}$ attaches to the good. The valuations (θ_B, θ_T) satisfy the following conditions:

A1: For $i \in \{B,T\}$, $\Theta_i = \{\bar{\theta}_i, \underline{\theta}_i\}$ with $\Delta\theta_i := \bar{\theta}_i - \underline{\theta}_i \geq 0$, $\underline{\theta}_i > 0$, and $\Pr(\bar{\theta}_i) = p_i$.
A2: For any $(\theta_B, \theta_T) \in \Theta_B \times \Theta_T$, $\Pr(\theta_B, \theta_T) = \Pr(\theta_B) \cdot \Pr(\theta_T)$.
A3: B is the only player who knows θ_B and T is the only player who knows θ_T.
A4: $\underline{\theta}_B \leq \underline{\theta}_T \leq \bar{\theta}_B \leq \bar{\theta}_T$.[9,10]

Secondary Market. The resale outcome is assumed to be the result of a random ultimatum bargaining game: With probability λ_B, B makes a take-it-or-leave-it offer to T, whereas with the complementary probability $\lambda_T = 1 - \lambda_B$, T makes a take-it-or-leave-it offer to B. We restrict these offers to simple prices $t^r \in \mathbb{R}$. From the perspective of B and T, simple prices are clearly optimal. What is more, as discussed in the online Appendix, S does not gain by recommending more complex resale mechanisms to B and T.

Primary Market. In this section we assume that S cannot sell to T, nor can she elicit any information or extract any money from T.[11] We also assume that S cannot contract directly upon the actions of B and T in the resale game (i.e. their price offers and acceptance decisions), nor can she design the resale game, for example by assigning bargaining power to one of the two players. If this were the case, S could also control the final allocation and the analysis of the constraints imposed by resale would be uninteresting.

S can however influence the behavior of T in the resale game through the design of her allocation rule and by disclosing information about the buyer's valuation. Formally, S offers B a mechanism $\psi : \mathcal{M} \to \mathbb{R} \times \triangle(\{0,1\} \times \Sigma)$ such that, when B reports a message $m \in \mathcal{M}$, he pays an expected transfer $t(m) \in \mathbb{R}$ and with probability $\psi(\sigma|m) := \Pr(x = 1, \tilde{\sigma} = \sigma|m)$ trade occurs and some information $\sigma \in \Sigma$ is disclosed to T.[12] Any information $\tilde{\sigma}$ ultimately results in a price offer from T. Following Myerson (1982), we can thus replicate the outcome induced by any indirect mechanism $(\psi, \mathcal{M}, \Sigma)$ through a direct mechanism $\phi : \Theta_B \to \mathbb{R} \times \triangle(\{0,1\} \times Z)$ in which B must report his type to S and then with probability $\phi(z|\theta_B)$ trade occurs and a recommendation $z = (t^r(\bar{\theta}_T), t^r(\underline{\theta}_T)) \in Z = \mathbb{R}^2$ is sent to T about the price to offer in the resale game as a function of θ_T.[13,14] Note that

[9] For brevity, we limit attention here to the case of overlapping supports (assumption A4). The results for the case $\underline{\theta}_B \leq \bar{\theta}_B \leq \underline{\theta}_T \leq \bar{\theta}_T$ are very similar. In all other cases, the characterization of the optimal mechanism is less interesting since the resale outcome does not depend on the beliefs that B or T have about the other player's valuation.

[10] Assuming Θ_B and Θ_T are binary sets permits to derive the optimal mechanism through linear programming. As discussed in the concluding section, extending the analysis to continuous distributions poses nontrivial technical problems.

[11] These assumptions are relaxed in Section 3.

[12] Since all players have quasi-linear preferences, restricting attention to mechanisms $\psi : \mathcal{M} \to \mathbb{R} \times \triangle(\{0,1\} \times \Sigma)$ instead of mechanisms $\psi : \mathcal{M} \to \triangle(\mathbb{R} \times \{0,1\} \times \Sigma)$ is without loss. Indeed, $t(m)$ can always be read as the expected transfer from B to S.

[13] Equivalently, to allow for recommendations for T that are type-dependent, we could have set $\Theta_B \times \Theta_T$ as the domain of ϕ, let $Z = \mathbb{R}$, and imposed restrictions on the mapping $\phi : \Theta_B \times \Theta_T \to \mathbb{R} \times \Delta(\{0,1\} \times \mathbb{R})$ to capture the impossibility for S to condition the terms of trade with B on T's private information. However, given the restriction that S cannot elicit information from T, we find our notation more intuitive.

[14] Also note that there is no need to specify a recommendation for B, since there is no information about θ_T that S can send to B. Similarly, there is no need to send acceptance recommendations to B and T, for the only sequentially rational recommendations are to accept any offer that is higher than θ_B (for the buyer) and below θ_T (for the third party).

since these recommendations are sent with a probability that depends on θ_B, they are also signals of the buyer's valuation. Furthermore, since recommendations will ultimately be implemented by disclosing some information to T, in what follows, we will refer to the mapping $d : \Theta_B \to \Delta(Z)$ induced by ϕ both as a *recommendation*, or equivalently, as a *disclosure policy.* We will then say that the mechanism ϕ discloses information when it assigns positive measure to more than one signal/recommendation.[15]

Timing.

- At $\tau = 1$, S publicly announces her mechanism ϕ. If B refuses to participate, the game ends and all players get their reservation payoffs which are equal to zero.[16] If B accepts, he reports θ_B, pays an expected transfer $t(\theta_B)$ and with probability $\phi(z|\theta_B)$ receives the good and a recommendation $z \in Z$ is sent to T. Although T can observe ϕ, she does not observe the announcement θ_B, nor the transfer t.[17]
- At $\tau = 2$, if $x = 1$, bargaining between B and T takes place according to the simple procedure described above. Otherwise, the game is over.

Revenue-maximizing mechanism

We first examine how the outcome in the secondary market is influenced by the mechanism adopted in the primary market. Next, we derive the monopolist's optimal mechanism.

The resale outcome.

First, consider the price $t^r(\theta_T)$ offered by T. Given the mechanism ϕ, a recommendation $z = (t^r(\bar{\theta}_T), t^r(\underline{\theta}_T))$ is incentive-compatible if T finds it optimal to obey to the recommendation instead of offering a different price. Let

$$\Pr(\bar{\theta}_B|z) := \frac{\phi(z|\bar{\theta}_B)p_B}{\phi(z|\bar{\theta}_B)p_B + \phi(z|\underline{\theta}_B)(1-p_B)}$$

denote T's posterior beliefs about the value B attaches to the good, given the mechanism ϕ and the recommendation $z \in Z$. Since the supports Θ_B and Θ_T overlap, a recommendation z is incentive-compatible if and only if $t^r(\bar{\theta}_T) \in \Theta_B$ and $t^r(\underline{\theta}_T) = \underline{\theta}_B$ whenever $\Pr(\bar{\theta}_B|z) < 1$.[18] We can thus simplify the notation and describe a recommendation simply by the price that S recommends to $\bar{\theta}_T$.

[15] We are assuming that S can commit to any mechanism of her choosing. Without commitment, S can still fashion the informational linkage with the secondary market, but this has to be done entirely through a stochastic allocation rule, as discussed in the online Appendix.

[16] One could assume the distribution of bargaining power in the primary market to be also stochastic so that with probability λ_S, S designs the mechanism, whereas with the complementary probability, it is B. Our analysis starts from the point where nature selects S. We thank the editor for suggesting this interpretation.

[17] Whether T can observe x is irrelevant, since she always makes her offer contingent on the event that $x = 1$. Indeed, trade between B and T is possible only if B received the good from S in the primary market. In what follows, we thus consider the decision to trade as the the minimal information disclosed by S.

[18] When $\Pr(\bar{\theta}_B|z) = 1$, any $t^r(\underline{\theta}_T) < \bar{\theta}_B$ is incentive-compatible.

We use $\bar{z}$ and $\underline{z}$ to denote the recommendations to offer $t^r(\bar{\theta}_T) = \bar{\theta}_B$ and $t^r(\bar{\theta}_T) = \underline{\theta}_B$, respectively. To be incentive-compatible, $\bar{z}$ and $\underline{z}$, must satisfy the following constraints

$$\Pr(\bar{\theta}_B|\bar{z}) \quad \geq \quad \Delta\theta_B/[\bar{\theta}_T - \underline{\theta}_B] \tag{1}$$

$$\Pr(\bar{\theta}_B|\underline{z}) \quad \leq \quad \Delta\theta_B/[\bar{\theta}_T - \underline{\theta}_B] \tag{2}$$

Next, consider the price $t^r(\theta_B)$ asked by B. Clearly, $t^r(\bar{\theta}_B) = \bar{\theta}_T$, whereas $t^r(\underline{\theta}_B) = \bar{\theta}_T$ if $p_T \geq [\underline{\theta}_T - \underline{\theta}_B]/[\bar{\theta}_T - \underline{\theta}_B]$ and $t^r(\underline{\theta}_B) = \underline{\theta}_T$ otherwise.[19]

Denoting by $r(\theta_B|z)$ and $s(\theta_B)$ the surplus B expects from resale, respectively when it is T and B who makes the offer in the resale game, we have that $\Delta s := [s(\bar{\theta}_B) - s(\underline{\theta}_B)] \leq 0$ and $\Delta r(z) := [r(\bar{\theta}_B|z) - r(\underline{\theta}_B|z)] \leq 0$ for any $z \in \{\bar{z}, \underline{z}\}$. Resale not only increases the value B attaches to the good from θ_B to $\theta_B + \lambda_B s_B(\theta_B) + \lambda_T r_B(\theta_B|z)$, but since it is more valuable for a low-valuation buyer than a high-valuation one, it also reduces the differences between types. As we show next, this affects the monopolist's ability to extract surplus as well as the structure of the optimal mechanism.

Optimal mechanism.

Taking into account how T's posterior beliefs depend on the mechanism adopted in the primary market and letting $J := [p_B(\bar{\theta}_T - \bar{\theta}_B)]/[(1-p_B)\Delta\theta_B]$, the monopolist's problem consists in choosing a mechanism ϕ^* that solves the following (linear) program

$$\mathcal{P}_S : \begin{cases} \max\ \mathbb{E}_{\theta_B}\left[t\left(\theta_B\right)\right] \\ \text{subject to} \\ U(\theta_B) := \sum\limits_{z\in\{\underline{z},\bar{z}\}} \phi(z|\theta_B)\{\theta_B + \lambda_B s\left(\theta_B\right) + \lambda_T r(\theta_B|z)\} - t(\theta_B) \geq 0,\ \forall \theta_B \in \Theta_B & IR\left(\theta_B\right) \\ U(\theta_B) \geq \sum\limits_{z\in\{\underline{z},\bar{z}\}} \phi(z|\hat{\theta}_B)\{\theta_B + \lambda_B s\left(\theta_B\right) + \lambda_T r(\theta_B|z)\} - t(\hat{\theta}_B),\ \forall (\theta_B, \hat{\theta}_B) \in \Theta_B^2 & IC\left(\theta_B\right) \\ \phi\left(\bar{z}|\underline{\theta}_B\right) \leq J\phi\left(\bar{z}|\bar{\theta}_B\right) & IC(\bar{z}) \\ \phi\left(\underline{z}|\underline{\theta}_B\right) \geq J\phi\left(\underline{z}|\bar{\theta}_B\right) & IC(\underline{z}) \\ \phi\left(z|\theta_B\right) \geq 0 \text{ for any } \theta_B \in \Theta_B \text{ and } z \in Z,\ \text{with } \sum\limits_{z\in\{\underline{z},\bar{z}\}} \phi\left(z|\theta_B\right) \leq 1 & (\mathcal{F}) \end{cases}$$

The constraints $IR(\theta_B)$ and $IC(\theta_B)$ are resale-augmented incentive-compatibility and participation constraints and guarantee that B finds it optimal to participate and reveal his type. The incentive-compatibility constraints $IC(\bar{z})$ and $IC(\underline{z})$ are obtained from (1) and (2) and guarantee that T finds it optimal to follow S's recommendations. Finally, $(\mathcal{F})$ are standard feasibility constraints that guarantee that all probabilities are well defined.

Now, let

$$V(\underline{\theta}_B|z) := \underline{\theta}_B - \tfrac{p_B}{1-p_B}\Delta\theta_B + \lambda_B[s(\underline{\theta}_B) - \tfrac{p_B}{1-p_B}\Delta s] + \lambda_T[r(\underline{\theta}_B|z) - \tfrac{p_B}{1-p_B}\Delta r(z)] \tag{3}$$

[19] Assuming $\bar{\theta}_B$ asks $t^r(\bar{\theta}_B) = \bar{\theta}_T$ when he believes with probability one that $\theta_T = \underline{\theta}_T$ and that $\underline{\theta}_B$ asks $t^r(\underline{\theta}_B) = \bar{\theta}_T$ when indifferent between $t^r(\underline{\theta}_B) = \bar{\theta}_T$ and $t^r(\underline{\theta}_B) = \bar{\theta}_T$ has no effect on any of the results.

$$K := [J(\Delta\theta_B + \lambda_B \Delta s)]/[J(\Delta\theta_B + \lambda_B \Delta s) + (1-J)\lambda_T p_T \Delta\theta_B]$$

and consider the following two parameters' regions

R_1 : $J < 1$ and either $V(\underline{\theta}_B|\underline{z}) \leq 0 < V(\underline{\theta}_B|\bar{z})$, or $V(\underline{\theta}_B|\underline{z}) \in (0, K\ V(\underline{\theta}_B|\bar{z}))$ and $K = J$;

R_2 : $V(\underline{\theta}_B|\underline{z}) \in (0, K\ V(\underline{\theta}_B|\bar{z}))$ and $K \in (J, 1)$.

Proposition 1 (Optimal mechanism) (i) Suppose R_1 holds. Then, the monopolist sells with probability less than one to the low type and always recommends $\bar{z}$: $\phi^*\left(\bar{z}|\bar{\theta}_B\right) = 1$, $\phi^*\left(\bar{z}|\underline{\theta}_B\right) = J$ and $\phi^*\left(0|\underline{\theta}_B\right) = 1 - J$.

(ii) When instead R_2 holds, the monopolist sells with probability less than one to the low type and recommends both $\bar{z}$ and $\underline{z}$ with positive probability: $\phi^*\left(\bar{z}|\bar{\theta}_B\right) = 1$, $\phi^*\left(\bar{z}|\underline{\theta}_B\right) = J$, $\phi^*\left(\underline{z}|\underline{\theta}_B\right) = 1 - J/K$, and $\phi^*\left(0|\underline{\theta}_B\right) = J/K - J$.

In all other cases, the monopolist sends only one recommendation and either sells to both types with certainty, or excludes completely the low type.

Proof. Using the expressions for $U(\bar{\theta}_B)$ and $U(\underline{\theta}_B)$, the constraints $IC(\bar{\theta}_B)$ and $IC(\underline{\theta}_B)$ in $\mathcal{P}_S$ can be rewritten as

$$U(\underline{\theta}_B) + \sum_{z \in \{\underline{z}, \bar{z}\}} \phi\left(z|\underline{\theta}_B\right)\left[\Delta\theta_B + \lambda_B \Delta s + \lambda_T \Delta r(z)\right] \leq U(\bar{\theta}_B) \leq$$

$$U(\underline{\theta}_B) + \sum_{z \in \{\underline{z}, \bar{z}\}} \phi\left(z|\bar{\theta}_B\right)\left[\Delta\theta_B + \lambda_B \Delta s + \lambda_T \Delta r(z)\right]$$

As it is standard, at the optimum, $IR(\underline{\theta}_B)$ and $IC(\bar{\theta}_B)$ necessarily bind, since otherwise S could reduce both $U(\underline{\theta}_B)$ and $U(\bar{\theta}_B)$ by the same amount increasing her payoff. It follows that

$$U^*(\underline{\theta}_B) = 0 \text{ and } U^*(\bar{\theta}_B) = \sum_{z \in \{\underline{z}, \bar{z}\}} \phi^*\left(z|\underline{\theta}_B\right)\left[\Delta\theta_B + \lambda_B \Delta s + \lambda_T \Delta r(z)\right].$$

Furthermore, since $\Delta\theta_B + \lambda_B \Delta s + \lambda_T \Delta r(z) \geq 0$ for any $z \in \{\underline{z}, \bar{z}\}$, when $IC(\bar{\theta}_B)$ and $IR(\underline{\theta}_B)$ are satisfied, so is $IR(\bar{\theta}_B)$.[20] Substituting $t(\underline{\theta}_B)$ and $t(\bar{\theta}_B)$ from $IR(\underline{\theta}_B)$ and $IC(\bar{\theta}_B)$ into $\mathcal{P}_S$, and using $\Delta r(\bar{z}) = -p_T \Delta\theta_B$, $\Delta r(\underline{z}) = 0$ and (3), the problem of the monopolist reduces to the choice of a mechanism ϕ^* that solves the following program

$$\mathcal{P}'_S : \begin{cases} \max\ p_B[\bar{\theta}_B + \lambda_B s(\bar{\theta}_B)][\sum_{z \in \{\underline{z}, \bar{z}\}} \phi\left(z|\bar{\theta}_B\right)] + (1-p_B)[\sum_{z \in \{\underline{z}, \bar{z}\}} V(\underline{\theta}_B|z)\phi\left(z|\underline{\theta}_B\right)] \\ \text{subject to } IC(\bar{z}),\ IC(\underline{z}),\ (\mathcal{F}) \text{ and} \\ \sum_{z \in \{\underline{z}, \bar{z}\}} \phi\left(z|\bar{\theta}_B\right)[\Delta\theta_B + \lambda_B \Delta s] + \phi\left(\bar{z}|\bar{\theta}_B\right)\lambda_T p_T \Delta\theta_B \geq \\ \sum_{z \in \{\underline{z}, \bar{z}\}} \phi\left(z|\underline{\theta}_B\right)[\Delta\theta_B + \lambda_B \Delta s] + \phi\left(\bar{z}|\underline{\theta}_B\right)\lambda_T p_T \Delta\theta_B \qquad IC(\underline{\theta}_B) \end{cases}$$

Note that $V(\underline{\theta}_B|z)$ are the standard Myerson (1981) virtual valuations – $M(\underline{\theta}_B) := \underline{\theta}_B - \frac{p_B}{1-p_B}\Delta\theta_B$ – augmented by the resale surplus $\lambda_B s(\underline{\theta}_B) + \lambda_T r(\underline{\theta}_B|z)$ discounted by the effect of resale on

[20]Note that $r(\underline{\theta}_B|\bar{z}) = p_T \Delta\theta_B$, $r(\underline{\theta}_B|\underline{z}) = 0$, and $r(\bar{\theta}_B|z) = 0$ for any $z \in \{\underline{z}, \bar{z}\}$, implying that $\Delta r(z) \in \{-p_T \Delta\theta_B, 0\}$. Similarly, $s(\bar{\theta}_B) = p_T(\bar{\theta}_T - \bar{\theta}_B)$ and $s(\underline{\theta}_B) = p_T(\bar{\theta}_T - \underline{\theta}_B)$ if $t^r(\underline{\theta}_B) = \bar{\theta}_T$ and $s(\underline{\theta}_B) = \underline{\theta}_T - \underline{\theta}_B$ if $t^r(\underline{\theta}_B) = \underline{\theta}_T$, so that $\Delta s \in (-\Delta\theta_B, -p_T \Delta\theta_B]$.

the informational rent for the high type. We proceed ignoring $IC(\underline{z})$ since it never binds at the optimum.

Favorable beliefs. When $J \geq 1$, $\bar{\theta}_T$ offers a high price in the event she learns nothing from the outcome in the primary market. This is clearly the most favorable case for the monopolist. Since $V(\underline{\theta}_B|\bar{z}) > V(\underline{\theta}_B|\underline{z})$, at the optimum, $\phi^*(\bar{z}|\theta_B) = 1$ for any θ_B if $V(\underline{\theta}_B|\bar{z}) \geq 0$ and $\phi^*(\bar{z}|\bar{\theta}_B) = 1 = \phi^*(0|\underline{\theta}_B) = 1$ otherwise.

Unfavorable beliefs ($J < 1$). When $V(\underline{\theta}_B|\bar{z}) \leq 0$, the rent S must leave to $\bar{\theta}_B$ when she sells to $\underline{\theta}_B$ is so high that it is optimal to exclude the low type and set $\phi^*(0|\underline{\theta}_B) = 1$ and $\phi^*(\bar{z}|\bar{\theta}_B) = 1$. Next, assume $V(\underline{\theta}_B|\bar{z}) > 0$. The solution then depends on the value of $V(\underline{\theta}_B|\underline{z})$. When $V(\underline{\theta}_B|\underline{z}) \leq 0$, it is clearly optimal to set $\phi^*(\underline{z}|\underline{\theta}_B) = 0$ in which case the optimal mechanism is the one described in (i).[21] When, instead, $V(\underline{\theta}_B|\underline{z}) > 0$, ignoring $IC(\underline{\theta}_B)$, the solution would be $\phi(\bar{z}|\bar{\theta}_B) = 1$, $\phi(\bar{z}|\underline{\theta}_B) = J$ and $\phi(\underline{z}|\underline{\theta}_B) = 1 - J$, which however violates $IC(\underline{\theta}_B)$. Hence, $IC(\underline{\theta}_B)$, must bind. Given any $\phi(\bar{z}|\bar{\theta}_B) \in [0,1]$, it is optimal to set $\phi(\underline{z}|\bar{\theta}_B) = 1 - \phi(\bar{z}|\bar{\theta}_B)$. Indeed, U_S is increasing in $\phi(\underline{z}|\bar{\theta}_B)$ and a larger $\phi(\underline{z}|\bar{\theta}_B)$ also helps relaxing $IC(\underline{\theta}_B)$ and hence permits the monopolist to increase $\phi(\bar{z}|\underline{\theta}_B)$. At the optimum, constraint $IC(\bar{z})$ must also bind. If not, S could increase $\phi(\bar{z}|\underline{\theta}_B)$, possibly reducing $\phi(\underline{z}|\underline{\theta}_B)$, relaxing $IC(\underline{\theta}_B)$ and enhancing U_S. Combining $IC(\bar{z})$ and $IC(\underline{\theta}_B)$ gives an upper bound on $\phi(\underline{z}|\underline{\theta}_B)$ represented by $\phi(\underline{z}|\underline{\theta}_B) = 1 - (J/K)\phi(\bar{z}|\bar{\theta}_B)$. Note that since $K \in [J, 1)$, $\phi(\underline{z}|\underline{\theta}_B) < 1 - J\phi(\bar{z}|\bar{\theta}_B) = 1 - \phi(\bar{z}|\underline{\theta}_B)$. The monopolist thus faces a trade-off between selling with certainty to both types, inducing T to offer a low resale price (i.e. setting $\phi(\underline{z}|\underline{\theta}_B) = \phi(\underline{z}|\bar{\theta}_B) = 1$), or sustaining a higher resale price but at the cost of not being able to sell with probability one to the low type (i.e. setting $\phi(\bar{z}|\bar{\theta}_B) = \phi(\bar{z}|\underline{\theta}_B) = J$ and $\phi(\underline{z}|\underline{\theta}_B) = 1 - J/K$). When $V(\underline{\theta}_B|\underline{z}) \geq K\ V(\underline{\theta}_B|\bar{z})$, S finds it optimal to favor trade over a higher resale price and at the optimum $\phi^*(\underline{z}|\theta_B) = 1$ for any θ_B. When instead $V(\underline{\theta}_B|\underline{z}) \in (0, K\ V(\underline{\theta}_B|\bar{z}))$, the optimal mechanism is either the one in (ii) if $K > J$, or that in (i) if $K = J$, that is if $p_T = 1$. Q.E.D.

The following is then a direct implication of Proposition 1.

Corollary 1 (Suboptimality of deterministic allocations and trivial disclosure policies) In the presence of resale, a monopolist may need to adopt a stochastic selling procedure and a disclosure policy richer than the simple announcement of the decision to trade.

It is well known that when a seller faces a single buyer, the optimal selling mechanism consists in setting a fixed price. The resale market makes it possible for the initial seller to extract surplus indirectly from the third party through the buyer. When T has some bargaining power (i.e. $\lambda_T > 0$), the seller's mechanism design problem thus involves two agents, not one. To induce T to offer a high resale price, S may then find it optimal to randomize over the decision to trade with the low type. This makes the decision to trade more indicative of the high-value buyer. It follows that the classic fixed price result may fail when there is a resale market. What is more, S may find it optimal to disclose information in addition to the decision to trade. The advantage of a richer disclosure policy stems from the possibility of increasing the level of trade with the low type without affecting the probability T offers a high resale price.

With quasi-linear preferences, this additional information can simply be the price paid in the primary market. For example, S could offer a menu of two contracts: the first one is such that

[21] Note that if S were constrained to use a deterministic selling procedure, the maximal revenue would be $p_B[\bar{\theta}_B + \lambda_B s(\bar{\theta}_B)]$ since $V(\underline{\theta}_B|\underline{z}) < 0$ implies that S prefers to exclude $\underline{\theta}_B$ rather than leaving a rent to the high type. In contrast, with a stochastic procedure she can obtain $U_S^* = p_B[\bar{\theta}_B + \lambda_B s(\bar{\theta}_B)] + (1 - p_B)JV(\underline{\theta}_B|\underline{z})$.

the good is delivered with certainty at a price $t_H = t^*(\bar{\theta}_B)$, the second with probability $\delta = [1 - J/K]/[1 - J]$ at a price $t_L = [t^*(\underline{\theta}_B) - Jt^*(\bar{\theta}_B)]/[1 - J]$, where $t^*(\underline{\theta}_B)$ and $t^*(\bar{\theta}_B)$ are the optimal transfers in the direct mechanism of Proposition 1. As we show in the online appendix, this menu is designed so as to induce the high type to pay t_H and the low type to randomize choosing t_H with probability J and t_L with probability $1 - J$. Note that this particular implementation, which combines lotteries with mixed strategies, has the property that S fully discloses to T all the information she learns from the buyer. In general, however, it may be difficult to rely on mixed strategies to conceal some information. When this is the case, S can still implement the optimal mechanism using the price to signal the buyer's valuation, but she may need to conceal the choice of the contract. The following is an example. S could offer B a menu of two contracts. The first one delivers the good with certainty at a price t_H. The second uses a lottery to determine both the decision to trade and the price. The lottery is such that with probability J the buyer receives the good and pays t_H, with probability $1-J/K$ he receives the good and pays t_L, and with probability $J/K - J$, the monopolist retains the good and B pays nothing. The prices t_H and t_L serve the same role as the recommendations $\bar{z}$ and $\underline{z}$ in the direct mechanism and solve $t_H = t^*(\bar{\theta}_B)$ and $Jt_H + [1 - J/K]t_L = t^*(\underline{\theta}_B)$. In equilibrium, the high type chooses the first contract, while the low type the second. In this case, the choice of the contract is a perfect signal of the buyer's valuation and must not be disclosed. Finally, an alternative implementation which also uses prices as signals, but does not require the latter to be stochastic, is such that S never discloses the price paid by the high type, whereas she discloses the price paid by the low type with probability $1 - J/K$. Not disclosing the price then leads to the same outcome as sending the recommendation $\bar{z}$, whereas disclosing $t^*(\underline{\theta}_B)$ is perfectly informative of the low type and plays the same role as sending the recommendation $\underline{z}$.

We conclude that

Corollary 2 (Price disclosures) When the direct mechanism of Proposition 1 can not be implemented announcing only the decision to trade, it suffices to disclose the price to create the optimal informational linkage with the secondary market.

Finally, consider the effect of resale on revenue. This depends on whether the secondary buyer is a third party, as in the current setting, or a bidder who did not win in the primary market. This second possibility is examined in Section 3. In what follows, we briefly comment on the effect of resale to third parties in markets with possibly many buyers.[22]

The option to resell increases the value a buyer assigns to winning the good. Furthermore, resale reduces the difference between high and low valuation types and hence the rents S must leave to a buyer to induce truthful information revelation. It follows that the resale-augmented virtual valuations are higher than the corresponding Myerson virtual valuations for auctions without resale. Nevertheless, this alone does not imply that resale is revenue-enhancing. Indeed, the monopolist may not be able to implement the same allocations as in the absence of resale (note that the simple monotonicity condition for standard mechanisms does not guarantee that $IC(\underline{\theta}_B)$ is satisfied when bidders can resell). However, through a policy that discloses only the identity of the winner, the monopolist can always implement exactly the same allocation rule as in a Myerson optimal auction

[22] The extension to multiple buyers that can resell only to a third party who does not participate in the primary market leads to results similar to those examined here. This extension is considered in the online Appendix.

without resale. Clearly, this policy need not be optimal, as discussed in Corollary 1, but it implies that resale to third parties is always revenue-enhancing.

3 Inter-bidder resale

Next, we consider an environment where the monopolist can contract with both B and T, but cannot prohibit inter-bidder resale. Bargaining in the resale game takes place according to the same procedure as described in the previous section.

Let the identity of the winner be denoted by $h \in \{B, T\}$. A direct mechanism (with an embedded recommendation/disclosure policy) is now a mapping $\phi : \Theta_T \times \Theta_B \to \mathbb{R}^2 \times \Delta(\{B, T\} \times \mathbf{Z})$ such that when B and T report $\boldsymbol{\theta} = (\theta_T, \theta_B)$, they pay $t_B(\theta_B)$ and $t_T(\theta_T)$ and with probability $\phi(i, \mathbf{z}|\boldsymbol{\theta}) := \Pr(h = i, \tilde{\mathbf{z}} = \mathbf{z}|\boldsymbol{\theta})$ the good is assigned to bidder $h \in \{B, T\}$ and recommendations $\mathbf{z} := (z_B, z_T) \in \mathbf{Z} := (Z_B \times Z_T) = \mathbb{R}^2$ are sent to B and T specifying the price to offer/ask in the resale game.[23] As usual, these recommendations are private, in the sense that B observes only $z_B \in Z_B$ and T only $z_T \in Z_T$. Note that the monopolist can now influence not only the price offered by a resale-buyer, but also the price asked by a resale-seller. Furthermore, she can make a bidder pay for the surplus he (or she) expects from resale even without trading with him (her). Finally, note that, as in the previous section, S does not need to send acceptance recommendations.

Given a true type profile $\boldsymbol{\theta}$, let $s_i(\boldsymbol{\theta}|h, t^r)$ denote the surplus that player $i \in \{B, T\}$ obtains in the secondary market when he or she offers (asks) t^r and player $h \in \{B, T\}$ is awarded the good in the primary market. Similarly, $r_i(\boldsymbol{\theta}|h, t^r)$ denotes the surplus for bidder i when j offers (asks) t^r with $j \neq i$.

An optimal auction followed by inter-bidder resale maximizes

$$U_S = \mathbb{E}_{\boldsymbol{\theta}}[\textstyle\sum_{i=B,T} t_i(\theta_i)]$$

subject to the following individual-rationality and incentive-compatibility constraints for $i \in \{B, T\}$:

$$\begin{aligned} U(\theta_i) := \mathbb{E}_{\theta_j}\{ &\sum_{h\in\{B,T\}} \sum_{\mathbf{z}\in\mathbf{Z}} [\theta_i \mathbb{I}_{h=i} + \lambda_i s_i(\theta_i, \theta_j | h, z_i) \\ &+ \lambda_j r_i(\theta_i, \theta_j | h, z_j)] \phi(h, \mathbf{z}|\theta_i, \theta_j)\} - t_i(\theta_i) \geq 0 \qquad \forall \theta_i \in \Theta_i \end{aligned} \tag{4}$$

$$\begin{aligned} U(\theta_i) \geq \mathbb{E}_{\theta_j}\{ &\sum_{h\in\{B,T\}} \sum_{\mathbf{z}\in\mathbf{Z}} [\theta_i \mathbb{I}_{h=i} + \lambda_i s_i(\theta_i, \theta_j | h, t^r(\theta_i, \hat{\theta}_i, h, z_i)) \\ &+ \lambda_j r_i(\theta_i, \theta_j | h, z_j)] \phi(h, \mathbf{z}|\hat{\theta}_i, \theta_j)\} - t_i(\hat{\theta}_i) \qquad \forall (\theta_i, \hat{\theta}_i) \in \Theta_i^2 \text{ and } \forall\ t^r : \Theta_i^2 \times \{B, T\} \times Z_i \to \mathbb{R}, \end{aligned} \tag{5}$$

where $\mathbb{I}_{h=i} = 1$ if $h = i$ and zero otherwise. Following Myerson (1982), (5) controls for two types of incentives. It guarantees that, conditional on reporting θ_i truthfully to S, in the resale game bidder i prefers to obey to the monopolist's recommendation and offer (ask) $t^r = z_i$ rather than any other price $t^r \neq z_i$. It also implies that revealing θ_i is sequentially rational. Note that the optimal resale price $t^r(\theta_i, \hat{\theta}_i, h, z_i)$ is a function of bidder i's true type θ_i, bidder i's report in the primary market $\hat{\theta}_i$, the identity of the winner h, and the recommendation z_i.

[23] Since all players have quasi-linear preferences, we can restrict attention to mechanisms in which t_i are deterministic and depend only on θ_i, for $i \in \{B, T\}$. Formally, for any mechanism $\phi : \Theta_B \times \Theta_T \to \Delta(\mathbb{R}^2 \times \{B, T\} \times \mathbb{R}^2)$, there exists a mechanism $\phi : \Theta_B \times \Theta_T \to \mathbb{R}^2 \times \Delta(\{B, T\} \times \mathbb{R}^2)$ in which $t_i(\boldsymbol{\theta}) = t_i(\theta_i)$ for any $\boldsymbol{\theta} \in \Theta_B \times \Theta_T$ that is payoff-equivalent for all players.

In what follows, instead of describing the solution to the above program for all possible parameter configurations, we find it more interesting to discuss directly the effect of inter-bidder resale on the structure of the optimal allocation rule.[24]

Proposition 2 (Inter-bidder resale) Suppose the monopolist cannot prohibit inter-bidder resale. Then, it is generically impossible to maximize revenue with a deterministic selling procedure.

Proof. See the Appendix.

As in the case where resale is to a third party, the monopolist uses the identity of the winner to signal the bidders' valuations. However, a difference is that now the monopolist may find it optimal to influence beliefs both on and off the equilibrium path. Suppose, for example, that $\lambda_T = 1$ and $J < 1$, in which case $\bar{\theta}_T$ is expected to offer a low price in the event she loses the auction without learning any information about θ_B. By inducing $\bar{\theta}_T$ to offer $t^r = \bar{\theta}_B$ instead of $t^r = \underline{\theta}_B$ out of equilibrium, that is, after announcing $\theta_T = \underline{\theta}_T$, the monopolist can reduce the informational rent she must leave to $\bar{\theta}_T$ and extract more revenue. For example, S could sell to B when the two bidders report $(\underline{\theta}_T, \bar{\theta}_B)$ and to T when they report $(\underline{\theta}_T, \underline{\theta}_B)$, so that losing the auction when announcing $\theta_T = \underline{\theta}_T$ becomes a perfect signal of B having a high valuation. However, when selling to T in state $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$ is dominated (in terms of rents for both bidders) by selling to B, the monopolist can do better by assigning the good to B with probability J and to T with the complementary probability.[25] Once again, the advantage of stochastic procedures stems from the possibility of manipulating beliefs (on and off equilibrium) and at the same time implementing more profitable allocations.

Note that when both bidders are expected to influence the resale price, it is also generically impossible to create the desired informational linkage with the secondary market disclosing only the identity of the winner. In fact, even if a certain allocation rule induces the right beliefs for one bidder, it typically fails to induce the desired beliefs for the other. When this is the case, S may gain by disclosing more information, such as the winning price, or more generally a statistic of the bids submitted in the auction.

Finally, consider the effect of resale on revenue. Clearly, when the monopolist can contract with all potential buyers, the revenue in any auction followed by resale is never higher than in a Myerson optimal auction where resale is prohibited. The latter is a mechanism that assigns the good to the bidder with the highest virtual valuation $M(\theta_i)$, provided that $\max_{i \in B,T} \{M(\theta_i)\} \geq u_s = 0$, where $M(\bar{\theta}_i) = \bar{\theta}_i$ and $M(\underline{\theta}_i) := \underline{\theta}_i - \frac{p_B}{1-p_B}\Delta\theta_i$. For example, when $M(\underline{\theta}_B) > \max\{M(\underline{\theta}_T), 0\}$, the monopolist sells to T when the latter has a high valuation and to B otherwise with an expected revenue equal to $\mathbb{E}_{\boldsymbol{\theta}}[\max\{M(\theta_T), M(\theta_B), 0\} = p_T\bar{\theta}_T + (1-p_T)\underline{\theta}_B$. Now, suppose S can not prohibit resale, but can control the distribution of bargaining power in the secondary market through the allocation of the good in the primary market. Precisely, suppose it is always the winner who sets the price in the resale bargaining game (cfr Zheng, 2002). The impossibility of prohibiting resale then does not hurt the monopolist. Indeed, S can simply sell to B at a price $p_T\bar{\theta}_T + (1-p_T)\underline{\theta}_B$ and use the latter as a middleman to extract surplus from T in the secondary market. Since in this case B learns nothing about the value T attaches to the good, he asks a price $t^r(\theta_B) = \bar{\theta}_T$ independently

[24] Despite the fact that the program is linear and that Θ_B and Θ_T are binary sets, the number of controls and constraints in the program for the optimal mechanism is significantly high.

[25] Selling to B with probability higher than J would induce $\bar{\theta}_T$ to reduce her offer from $t^r = \bar{\theta}_B$ to $t^r = \underline{\theta}_B$ which, as discussed above, is not optimal.

of his type.[26] Through resale, S thus implements the same final allocation and obtains exactly the same expected revenue as in a Myerson optimal auction.

When instead the distribution of bargaining power in the resale game is a function of the bidders' personal characteristics, such as their bargaining abilities, and $\lambda_T > 0$, any mechanism in which S sells to B with positive probability must necessarily leave some rent to $\bar{\theta}_T$. This implies a loss of revenue for the monopolist, as formally proved in the online Appendix.

4 Concluding remarks

When buyers anticipate the possibility of resale, their willingness to pay incorporates the surplus they expect from the secondary market. The outcome in the resale game is also endogenous as it depends on the information disclosed in the primary market. Starting from these observations, we have proposed a tractable model that illustrates the intricacies associated with the design of optimal mechanisms for a monopolist who expects her buyers to resell. The main insight is that it may be impossible to maximize revenue with a deterministic selling procedure and a disclosure policy that announces only the decision to trade. This result has been derived assuming finite valuations (binary). Extending the analysis to continuous distributions represents an interesting line for future research. The difficulty with the continuum stems from the fact that the program for the optimal mechanism is no longer linear and from the fact that the set of incentive-compatible price recommendations is often difficult to characterize without imposing ad hoc restrictions. A similar difficulty arises in the literature on dynamic contracting where a principal needs to control the beliefs of his future selves; although a complete characterization is available in the two-type case (Laffont and Tirole 1988, 1990), the generalization to the continuum poses nontrivial problems.

Finally, a last remark concerns the foundations for resale. In this paper, we have assumed resale occurs as a result of (i) the impossibility of contracting with all potential buyers, and (ii) the possibility that the bidders correct misallocations in the primary market by trading in the secondary market. Allowing resale to be a consequence of changes in valuations is also likely to deliver interesting insights for the design of optimal mechanisms.

Appendix

Proof of Proposition 2. Let

$$Z_i(\hat{\theta}_i, h) = \{z_i \in Z_i : \phi(h, z_i, z_j|\hat{\theta}_i, \theta_j) > 0 \text{ for some } (z_j, \theta_j) \in Z_j \times \Theta_j\}$$

denote the set of recommendations that S sends to bidder i when the latter reports $\hat{\theta}_i$ and bidder $h \in \{B, T\}$ receives the good. For any $z_i \in Z_i(\hat{\theta}_i, h)$, let $\Pr(\theta_j|\hat{\theta}_i, h, z_i)$ denote the posterior beliefs of bidder i about θ_j when i announces $\hat{\theta}_i$ in the primary market, bidder h is awarded the good and i receives a recommendation $z_i \in Z_i(\hat{\theta}_i, h)$, with $i, j, h \in \{B, T\}$ and $j \neq i$. Finally, for any $(\theta_i, \hat{\theta}_i) \in \Theta_i^2$, $h \in \{B, T\}$ and $z_i \in Z_i(\hat{\theta}_i, h)$, let

$$T^r(\theta_i, \hat{\theta}_i, h, z_i) = \arg\max_{t^r \in \mathbb{R}} \{ \Pr(\bar{\theta}_j|\hat{\theta}_i, h, z_i) s_i(\theta_i, \bar{\theta}_j|h, t^r) + \Pr(\underline{\theta}_j|\hat{\theta}_i, h, z_i) s_i(\theta_i, \underline{\theta}_j|h, t^r)\} \quad (6)$$

denote the set of optimal resale prices for bidder i.

[26] Indeed, $M(\underline{\theta}_T) \leq M(\underline{\theta}_B)$ implies $p_T \geq [\underline{\theta}_T - \underline{\theta}_B]/[\bar{\theta}_T - \underline{\theta}_B]$.

Using (6), the incentive-compatibility constraints (5) can be decomposed into the following constraints

$$z_i \in T^r(\theta_i, \theta_i, h, z_i) \quad \forall (\theta_i, h, z_i) \in \Theta_i \times \{B, T\} \times Z_i,\ z_i \in Z_i(\theta_i, h) \tag{7}$$

$$\begin{aligned} U(\theta_i) \geq \mathbb{E}_{\theta_j}\{ & \sum_{h \in \{B,T\}} \sum_{\mathbf{z} \in \mathbf{Z}} [\theta_i \mathbb{I}_{h=i} + \lambda_i s_i(\theta_i, \theta_j | h, t^r(\theta_i, \hat{\theta}_i, h, z_i)) \\ & + \lambda_j r_i(\theta_i, \theta_j | h, z_j)]\phi(h, \mathbf{z} | \hat{\theta}_i, \theta_j)\} - t_i(\hat{\theta}_i) \text{ for any } \theta_i \text{ and } \hat{\theta}_i \neq \theta_i, \\ & \text{with } t^r(\theta_i, \hat{\theta}_i, h, z_i) \in T^r(\theta_i, \hat{\theta}_i, h, z_i) \text{ for any } (h, z_i) \in \{B, T\} \times Z_i \text{ s.t. } z_i \in Z_i(\hat{\theta}_i, h). \end{aligned} \tag{8}$$

The constraints in (7) guarantee that, conditional on reporting θ_i truthfully in the primary market, in the resale game bidder i prefers to follow the recommendation z_i instead of offering/asking a price $t^r \neq z_i$. The constraints in (8) in turn guarantee that it is indeed optimal for i to report his (her) type truthfully.

An optimal auction ϕ^* thus maximizes $U_S = \mathbb{E}_{\boldsymbol{\theta}} \left[\sum_{i=B,T} t_i(\theta_i) \right]$ subject to (4), (7) and (8).

Following arguments similar to those in standard mechanism design, it is easy to verify that at the optimum the individual-rationality constraint (4) must bind for $\underline{\theta}_i$ and the incentive-compatibility constraint (8) for $\bar{\theta}_i$, for $i = B, T$. This implies that the monopolist's objective function can be rewritten as the expected sum of the bidders' *resale augmented virtual valuations,*

$$U_S = \mathbb{E}_{\boldsymbol{\theta}}\{ \sum_{h \in \{B,T\}} \sum_{\mathbf{z} \in \mathbf{Z}} [\sum_i V_i(\boldsymbol{\theta} | h, \mathbf{z})] \phi(h, \mathbf{z} | \boldsymbol{\theta})\} \tag{9}$$

where

$$\begin{aligned} V_i(\bar{\theta}_i, \theta_j | h, \mathbf{z}) &:= M(\bar{\theta}_i)\mathbb{I}_{h=i} + \lambda_i s_i(\bar{\theta}_i, \theta_j | h, z_i) + \lambda_j r_i(\bar{\theta}_i, \theta_j | h, z_j) \\ V_i(\underline{\theta}_i, \theta_j | h, \mathbf{z}) &:= M(\underline{\theta}_i)\mathbb{I}_{h=i} + \lambda_i \{ s_i(\underline{\theta}_i, \theta_j | h, z_i) - \tfrac{p_i}{1-p_i} \Delta s_i(\theta_j | h, z_i) \} \\ & \quad + \lambda_j \{ r_i(\underline{\theta}_i, \theta_j | h, z_j) - \tfrac{p_i}{1-p_i} \Delta r_i(\theta_j | h, z_j) \} \end{aligned}$$

and

$$\begin{aligned} \Delta s_i(\theta_j | h, z_i) &:= s_i(\bar{\theta}_i, \theta_j | h, t^r(\bar{\theta}_i, \underline{\theta}_i, h, z_i)) - s_i(\underline{\theta}_i, \theta_j | h, z_i) \\ \Delta r_i(\theta_j | h, z_j) &:= r_i(\bar{\theta}_i, \theta_j | h, z_j) - r_i(\underline{\theta}_i, \theta_j | h, z_j) \end{aligned}$$

The monopolist's problem thus consists in selecting a mechanism ϕ^* that maximizes (9) subject to (7) and the following incentive-compatibility constraints for $\underline{\theta}_i$, $i = B, T$,

$$\begin{aligned} & \mathbb{E}_{\theta_j}\{ \sum_{h \in \{B,T\}} \sum_{\mathbf{z} \in \mathbf{Z}} [\Delta\theta_i \mathbb{I}_{h=i} + \lambda_i \Delta s_i(\theta_j | h, z_i) + \lambda_j \Delta r_i(\theta_j | h, z_j)] \phi(h, \mathbf{z} | \underline{\theta}_i, \theta_j)\} \leq \\ & \mathbb{E}_{\theta_j}\{ \sum_{h \in \{B,T\}} \sum_{\mathbf{z} \in \mathbf{Z}} [\Delta\theta_i \mathbb{I}_{h=i} + \lambda_i (s_i(\bar{\theta}_i, \theta_j | h, z_i) - s_i(\underline{\theta}_i, \theta_j | h, t^r(\underline{\theta}_i, \bar{\theta}_i, h, z_i))) \\ & \quad + \lambda_j \Delta r_i(\theta_j | h, z_j)] \phi(h, \mathbf{z} | \bar{\theta}_i, \theta_j)\} \\ & \text{with } t^r(\underline{\theta}_i, \bar{\theta}_i, h, z_i)) \in T^r(\underline{\theta}_i, \bar{\theta}_i, h, z_i) \text{ for any } (h, z_i) \in \{B, T\} \times Z_i \text{ s.t. } z_i \in Z_i(\bar{\theta}_i, h). \end{aligned}$$

To prove the claim in Proposition 2, it suffices to consider the case $\lambda_T = 1$, $J < 1$, $M(\underline{\theta}_T) > 0$ and $p_T > p_B$.[27] In this case, S does not need to send any recommendation to B. Furthermore,

[27] See the online Appendix for a complete characterization of the optimal mechanism.

since $\underline{\theta}_B \leq \underline{\theta}_T \leq \bar{\theta}_B \leq \bar{\theta}_T$, when $h = T$, the only incentive-compatible recommendation for $\bar{\theta}_T$ is to ask $t^r(\bar{\theta}_T) \geq \bar{\theta}_T$ and for $\underline{\theta}_T$ to ask $t^r(\underline{\theta}_T) = \bar{\theta}_B$ if $\Pr(\bar{\theta}_B|\cdot) > 0$ and any $t^r \geq \underline{\theta}_T$ otherwise. Similarly, when $h = B$, the only incentive-compatible recommendation for $\underline{\theta}_T$ is to offer a price $t^r(\underline{\theta}_T) = \underline{\theta}_B$ if $\Pr(\underline{\theta}_B|\cdot) > 0$ and any price $t^r(\underline{\theta}_T) \leq \underline{\theta}_T$ otherwise. We will assume that $\underline{\theta}_T$ always asks $t^r(\underline{\theta}_T) = \bar{\theta}_B$ when $h = T$ and offers $t^r(\underline{\theta}_T) = \underline{\theta}_B$ when $h = B$.[28] Since these are the same prices that T offers in the absence of any explicit recommendation, to save on notation, we will drop z from the mapping ϕ when $h = T$, or $h = B$ and $\theta_T = \underline{\theta}_T$. When instead $h = B$ and $\theta_T = \bar{\theta}_T$, we will denote by $\bar{z}$ and $\underline{z}$ the recommendations to offer $t^r(\bar{\theta}_T) = \bar{\theta}_B$ and $t^r(\bar{\theta}_T) = \underline{\theta}_B$, respectively. For these recommendations to be incentive compatible, the mechanism ϕ must satisfy

$$\phi(B, \underline{z}|\bar{\theta}_T, \underline{\theta}_B) \geq J\phi(B, \underline{z}|\bar{\theta}_T, \bar{\theta}_B) \tag{10}$$

$$\phi(B, \bar{z}|\bar{\theta}_T, \underline{\theta}_B) \leq J\phi(B, \bar{z}|\bar{\theta}_T, \bar{\theta}_B) \tag{11}$$

Next, let Φ_1 denote the set of mechanisms such that $\bar{\theta}_T$ finds it (weakly) optimal to offer a high resale price $t^r(\bar{\theta}_T) = \bar{\theta}_B$ off-equilibrium, that is, after reporting $\hat{\theta}_T = \underline{\theta}_T$ in the primary market. A mechanism $\phi \in \Phi_1$ only if

$$\phi(B|\underline{\theta}_T, \underline{\theta}_B) \leq J\phi(B|\underline{\theta}_T, \bar{\theta}_B). \tag{12}$$

Letting $\mathbb{I}_{\phi\in\Phi_1} = 1$ if $\phi \in \Phi_1$ and zero otherwise, and substituting for the values of $s_T(\cdot)$ and $r_B(\cdot)$, the problem for the monopolist reduces to the choice of a mechanism ϕ^* that maximizes[29]

$$\begin{aligned}
U_S = {} & p_T p_B\{\bar{\theta}_T\phi(T|\bar{\theta}_T, \bar{\theta}_B) + \bar{\theta}_T\phi(B, \bar{z}|\bar{\theta}_T, \bar{\theta}_B) + \bar{\theta}_B\phi(B, \underline{z}|\bar{\theta}_T, \bar{\theta}_B)\} \\
& + p_T(1-p_B)\{\bar{\theta}_T\phi(T|\bar{\theta}_T, \underline{\theta}_B) + \bar{\theta}_T\phi(B, \bar{z}|\bar{\theta}_T, \underline{\theta}_B) + (\bar{\theta}_T - \tfrac{p_B}{1-p_B}\Delta\theta_B)\phi(B, \underline{z}|\bar{\theta}_T, \underline{\theta}_B)\} \\
& + (1-p_T)p_B\{[\bar{\theta}_B - \tfrac{p_T}{1-p_T}\left(\bar{\theta}_T - \bar{\theta}_B\right)][\phi(T|\underline{\theta}_T, \bar{\theta}_B) + \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \bar{\theta}_B)] + \bar{\theta}_B[1 - \mathbb{I}_{\phi\in\Phi_1}]\phi(B|\underline{\theta}_T, \bar{\theta}_B)\} \\
& + (1-p_T)(1-p_B)\{M(\underline{\theta}_T)\phi(T|\underline{\theta}_T, \underline{\theta}_B) + \mathbb{I}_{\phi\in\Phi_1}[M(\underline{\theta}_T) + (\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B})\Delta\theta_B]\phi(B|\underline{\theta}_T, \underline{\theta}_B) \\
& + [1 - \mathbb{I}_{\phi\in\Phi_1}](M(\underline{\theta}_T) - \tfrac{p_B}{1-p_B}\Delta\theta_B)\phi(B|\underline{\theta}_T, \underline{\theta}_B)\}
\end{aligned}$$

subject to (11), (10) and the following incentive-compatibility constraints, respectively for $\underline{\theta}_B$ and $\underline{\theta}_T$

$$p_T[\phi\left(B, \underline{z}|\bar{\theta}_T, \bar{\theta}_B\right) - \phi\left(B, \underline{z}|\bar{\theta}_T, \underline{\theta}_B\right)] + (1-p_T)[\phi\left(B|\underline{\theta}_T, \bar{\theta}_B\right) - \phi\left(B|\underline{\theta}_T, \underline{\theta}_B\right)] \geq 0 \tag{13}$$

$$\begin{aligned}
& p_B\left(\bar{\theta}_T - \bar{\theta}_B\right)[\phi(T|\bar{\theta}_T, \bar{\theta}_B) + \phi(B, \bar{z}|\bar{\theta}_T, \bar{\theta}_B) - \phi(T|\underline{\theta}_T, \bar{\theta}_B) - \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \bar{\theta}_B)] \\
& + (1-p_B)\Delta\theta_T[\phi(T|\bar{\theta}_T, \underline{\theta}_B) + \phi(B, \underline{z}|\bar{\theta}_T, \underline{\theta}_B) - \phi(T|\underline{\theta}_T, \underline{\theta}_B) - [1 - \mathbb{I}_{\phi\in\Phi_1}]\phi(B|\underline{\theta}_T, \underline{\theta}_B)] \\
& + (1-p_B)(\Delta\theta_T - \Delta\theta_B)[\phi(B, \bar{z}|\bar{\theta}_T, \underline{\theta}_B) - \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \underline{\theta}_B)] \geq 0.
\end{aligned} \tag{14}$$

Note that the controls $\phi(\cdot|\boldsymbol{\theta})$ associated with the states $\boldsymbol{\theta} = \left(\bar{\theta}_T, \bar{\theta}_B\right)$ and $\boldsymbol{\theta} = \left(\bar{\theta}_T, \underline{\theta}_B\right)$ are linked to the controls associated with the other two states $\boldsymbol{\theta} = \left(\underline{\theta}_T, \bar{\theta}_B\right)$, $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$ only through the constraints (13) and (14). In the following, we ignore (13) and (14) since they do not bind at the

[28] Clearly, S does not have any incentive to recommend a different price.

[29] Assuming that $\bar{\theta}_T$ offers a high price off-equilibrium when she is indifferent between $t^r = \bar{\theta}_B$ and $t^r = \underline{\theta}_B$ is without loss of generality. Indeed, when $\phi(B|\underline{\theta}_T, \underline{\theta}_B) = J\phi(B|\underline{\theta}_T, \bar{\theta}_B)$, both U_S and (14) are insensitive to whether $\bar{\theta}_T$ offers a low or a high resale price.

optimum. This implies that the optimal mechanism can be obtained by first choosing the controls for the two states $\boldsymbol{\theta} = (\bar{\theta}_T, \bar{\theta}_B)$ and $\boldsymbol{\theta} = (\bar{\theta}_T, \underline{\theta}_B)$ that maximize U_S under the constraints (10) and (11) and then choosing the controls for the other two states $\boldsymbol{\theta} = (\underline{\theta}_T, \bar{\theta}_B)$ and $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$.

When $\boldsymbol{\theta} = (\bar{\theta}_T, \bar{\theta}_B)$ and $\boldsymbol{\theta} = (\bar{\theta}_T, \underline{\theta}_B)$, it is always optimal to set $\phi^* \left(T|\bar{\theta}_T, \bar{\theta}_B\right) = \phi^* \left(T|\bar{\theta}_T, \underline{\theta}_B\right) = 1$. Next, consider the other two states $\boldsymbol{\theta} = (\underline{\theta}_T, \bar{\theta}_B)$ and $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$. Note that necessarily $\phi^* \in \Phi_1$ since otherwise S could reduce $\phi(B|\underline{\theta}_T, \underline{\theta}_B)$ and increase $\phi(T|\underline{\theta}_T, \underline{\theta}_B)$ increasing U_S. Furthermore, since $M(\underline{\theta}_T) > 0$ and $p_T > p_B$, at the optimum (12) must necessarily bind. Substituting $\phi(B|\underline{\theta}_T, \underline{\theta}_B) = J\phi(B|\underline{\theta}_T, \bar{\theta}_B)$, we then have that U_S is strictly increasing in $\phi(B|\underline{\theta}_T, \bar{\theta}_B)$. We conclude that any optimal mechanism must satisfy $\phi^*(B|\underline{\theta}_T, \bar{\theta}_B) = 1$, $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = J$ and $\phi^* \left(T|\underline{\theta}_T, \underline{\theta}_B\right) = 1-J$, implying that the allocation rule is *necessarily stochastic* when $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$.[30] Q.E.D.

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Monopoly with Resale

Online Appendix

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1 Restriction to price offers in the resale ultimatum bargaining game

In the model set up, we assume that in the resale ultimatum bargaining game B and T are restricted to simple take-it-or-leave-it price offers. In this appendix, we prove that, not only are price offers sequentially optimal for B and T, but S does not gain from recommending more complex mechanisms.[1] We first characterize the allocations (for the primary and the secondary market) that maximize the monopolist's revenue under minimal sequential rationality constraints for B and T and then show that these allocations can also be sustained in the game where offers in the resale bargaining game are restricted to simple prices.

For simplicity, assume $\lambda_T = 1$ and consider the case where resale is to a third party (the proof for $\lambda_T \in [0, 1]$ and inter-bidder resale follows similar arguments).

Suppose that at $\tau = 2$, T can choose from a topological space of feasible resale mechanisms Π^r. A resale mechanism $\pi = (\mathcal{M}^r, \alpha) \in \Pi^r$ consists of a set of messages $\mathcal{M}^r$ for player B along with a measurable mapping $\alpha : \mathcal{M}^r \to \mathbb{R}\times\Delta(\{0, 1\})$ that assigns to each message $m^r \in \mathcal{M}^r$ a lottery over the decision to trade and an expected payment from T to B. Let Υ denote the set of resale mechanisms that consist of simple take-it-or-leave-it price offers.[2] Without confusion, an element of Υ can be denoted simply by the price t^r. Finally, let Ξ denote the set of direct resale mechanisms $\xi : \Theta_B \mapsto \mathbb{R}\times\Delta(\{0, 1\})$. Since any mechanism in Ξ has the same message space, to save on notation, an element of Ξ will be denoted simply by the mapping ξ.

Now, consider the monopolist. Let Ψ represent a topological space of feasible mechanisms for S. A mechanism $\psi = (\mathcal{M}, \mathcal{R}, \beta) \in \Psi$ consists of a set of messages $\mathcal{M}$ for B, a set of signals/recommendations $\mathcal{R}$ that S can send to T and a measurable mapping $\beta : \mathcal{M} \mapsto \mathbb{R}\times\Delta(\{0, 1\} \times \mathcal{R})$ that assigns to each message $m \in \mathcal{M}$ an expected transfer $t(m) \in \mathbb{R}$ from B to S and a joint lottery $\delta(m) \in \Delta(\{0, 1\} \times \mathcal{R})$ over the decision to trade and the recommendations $\mathcal{R}$. Now, let $\tilde{\Phi}$ denote the set of direct mechanisms $\tilde{\phi} : \Theta_B \mapsto \mathbb{R}\times\Delta(\{0, 1\} \times \tilde{Z})$ in which the recommendations $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T)) \in \tilde{Z} = \Xi^2$ that S sends to T consists in a pair of direct resale mechanisms, respectively for $\overline{\theta}_T$ and $\underline{\theta}_T$. Finally, let Φ denote the set of direct mechanisms $\phi : \Theta_B \mapsto \mathbb{R}\times\Delta(\{0, 1\} \times Z)$ in which S recommends simple take-it-or-leave-it price offers $z = (t^r(\overline{\theta}_T), t^r(\underline{\theta}_T)) \in Z = \Upsilon^2$. In the following, we will denote by $\tilde{\phi} \in \tilde{\Phi}$ an element of $\tilde{\Phi}$ and by $\phi \in \Phi$ an element of Φ.

[1] This does not mean that a stochastic ultimatum bargaining game where B and T are randomly selected to design the resale mechanism is the most favorable resale procedure from the perspective of the initial seller. For example, in the case of inter-bidder resale, if S could choose the resale game, she could simply prohibit any future transaction between B and T and then implement a Myerson optimal auction in the primary market. Alternatively, she could dictate that it is always the resale-seller who makes the offer in the secondary market (as discussed in the paper, sometimes this also allows the monopolist to extract the Myerson revenue – see Zheng (2002)).

[2] Formally, a take-it-or-leave-it price offer is a mechanism $(\mathcal{M}^r, \alpha)$, with $\mathcal{M}^r = \{yes, no\}$, such that, when B chooses the message $m = yes$, the good is transferred to T and B receives a payment t^r, whereas when he chooses $m = no$, he keeps the good and receives no money from T.

Now, let U_S^* represent the highest equilibrium payoff for the monopolist in the restricted game where $\Pi^r = \Upsilon$ and $\Psi = \Phi$, that is in the game where T can only make take-it-or-leave-it price offers and S can only offer direct mechanisms $\phi \in \Phi$, as assumed in the model set up. Similarly, let $\tilde{U}_S^*$ denote the highest equilibrium payoff in an unrestricted game where $\Pi^r \supseteq \Xi \cup \Upsilon$ and $\Psi \supseteq \tilde{\Phi} \cup \Phi$.

Claim A1. *S does not gain from recommending that T and B offer mechanisms in the ultimatum bargaining game more complex than simple price offers*: $\tilde{U}_S^* = U_S^*$.

Proof. We prove the result in three steps. Step 1 shows that in the unrestricted game, $\tilde{U}_S^*$ can be sustained by an equilibrium where S offers a mechanism $\tilde{\phi}^* \in \tilde{\Phi}$ and T follows the monopolist's recommendations. Step 2 characterizes the allocations induced by $\tilde{\phi}^*$. Finally, step 3 shows that these allocations can also be sustained by an equilibrium in the restricted game where $\Pi^r = \Upsilon$ and $\Psi = \Phi$.

Step 1. Given any direct mechanism $\tilde{\phi} \in \tilde{\Phi}$, let $\tilde{Z}(\tilde{\phi})$ denote the set of recommendations $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T))$ in the support of $\tilde{\phi}$ and $\Xi(\tilde{\phi})$ the set of all direct resale mechanisms recommended by $\tilde{\phi}$. Formally, $\Xi(\tilde{\phi}) = \{\xi \in \Xi : \exists\ \tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T)) \in \tilde{Z}(\tilde{\phi}) \text{ s.t. } \xi = \xi(\overline{\theta}_T) \text{ or } \xi = \xi(\underline{\theta}_T)\}$.

Now, consider a mechanism $\tilde{\phi}^*$ with the following properties:

(i) B finds it optimal to participate and truthfully report his type in $\tilde{\phi}^*$ as well as in any resale mechanism $\xi \in \Xi(\tilde{\phi}^*)$;

(ii) given any $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T)) \in \tilde{Z}(\tilde{\phi}^*)$, the direct mechanism $\xi(\theta_T)$ is optimal for θ_T – any other mechanism $\xi \in \Xi$ that is individually-rational and incentive-compatible for B leads to a lower payoff for θ_T (formally, $\xi(\theta_T)$ is a solution to the program $P_T(r, \theta_T)$ described below with $r = \tilde{z}$).

(iii) $\tilde{\phi}^*$ is optimal for S – any other $\tilde{\phi}$ that strictly dominates $\tilde{\phi}^*$ necessarily violates (i) or (ii).

In the sequel, we prove the following two results. First, for any mechanism $\tilde{\phi}^*$ that satisfies (i)-(iii), there exists *an* equilibrium in the unrestricted game where $\Pi^r \supseteq \Xi \cup \Upsilon$ and $\Psi \supseteq \tilde{\Phi} \cup \Phi$ that supports $\tilde{\phi}^*$. That is, we can construct a (sequentially rational) strategy for B that specifies a complete plan of action for any pair of mechanisms $(\psi, \pi) \in \Psi \times \Pi^r$ – that is, a pair of function $\sigma_B : \Theta_B \times \Psi \to \Delta(\mathcal{M})$ and $\sigma_B : \Theta_B \times \Psi \times \mathcal{M} \times \mathbb{R} \times \{0,1\} \times \mathcal{R} \times \Pi \to \Delta(\mathcal{M}^r)$ – and a (sequentially rational) strategy for T that specifies a reaction to any upstream mechanism ψ – that is, a function $\sigma : \Theta_T \times \Psi \times \mathcal{R} \to \Delta(\Pi^r)$ – such that: (a) S finds it optimal to offer $\tilde{\phi}^*$; (b) T finds it optimal to obey to the recommendations $\tilde{z} \in \tilde{Z}(\tilde{\phi}^*)$; and (c) B finds it optimal to participate and truthfully report his type in $\tilde{\phi}^*$ as well as in any resale mechanism $\xi \in \Xi(\tilde{\phi}^*)$. Second, the monopolist's payoff in the equilibrium supporting $\tilde{\phi}^*$ is higher than in any other equilibrium, i.e. it yields $\tilde{U}_S^*$.[3]

To prove these claims, take any equilibrium of the unrestricted game. Given any upstream

[3] As we show below, $\tilde{\phi}^*$ identifies a profile of allocations – probabilities of trade and transfers for each state (θ_B, θ_T) – that maximize the monopolist's payoff under minimal sequential rationality constraints for B and T.

mechanism $\psi = (\mathcal{M}, \mathcal{R}, \beta) \in \Psi$, let

$$\mathcal{R}(\sigma_B) := \{r \in \mathcal{R} : \exists\ m \in Supp[\sigma_B(\theta_B, \psi)] \text{ s.t. } r \in Supp[\delta(m)] \text{ for some } \theta_B \in \Theta_B\}$$

denote the set of recommendations that, given the buyer's strategy at $\tau = 1$, are sent with positive probability to T. For any recommendation $r \in \mathcal{R}(\sigma_B)$, the reaction $\sigma_T(\theta_T, \psi, r) \in \Delta(\Pi^r)$ is sequentially rational for θ_T if and only if, given the buyer's strategy at $\tau = 2$, it leads to a pair of probability of trade $\{x^r(\overline{\theta}_B), x^r(\underline{\theta}_B)\} \in [0,1]^2$ and a pair of expected transfers $\{t^r(\overline{\theta}_B), t^r(\underline{\theta}_B)\} \in \mathbb{R}^2$ – that solve the following program:[4]

$$\mathcal{P}_T(r, \theta_T) : \begin{cases} \max\limits_{x^r(\cdot), t^r(\cdot)} \sum\limits_{\theta_B} [\theta_T x^r(\theta_B) - t^r(\theta_B)] \Pr(\theta_B | r; \psi) \\ \text{s.t. for any } (\theta_B, \widehat{\theta}_B) \in \Theta_B^2 \\ t^r(\theta_B) - \theta_B x^r(\theta_B) \geq 0 \qquad (IR_B(\theta_B)) \\ t^r(\theta_B) - \theta_B x^r(\theta_B) \geq t^r(\widehat{\theta}_B) - \theta_B x^r(\widehat{\theta}_B) \qquad (IC_B(\theta_B)) \end{cases}$$

where $\Pr(\theta_B | r; \psi)$ is computed using Bayes' rule and the buyer's strategy at $\tau = 1$. Indeed, if the allocations induced by $\sigma_T(\theta_T, \psi, r)$ do not solve $\mathcal{P}_T(r, \theta_T)$, then, θ_T has a profitable deviation that consists in offering a direct mechanism $\xi \in \Xi$ which solves the above program.[5]

We conclude that, given any pair of (sequentially rational) strategies σ_B and σ_T, an upstream mechanism for the monopolist $\psi = (\mathcal{M}, \mathcal{R}, \beta) \in \Psi$ (no matter whether it is on or off the equilibrium path) ultimately leads to a mapping $f : \Theta_B \mapsto \mathbb{R} \times \Delta(\{0,1\} \times \mathcal{R})$ that assigns to each θ_B an expected transfer $t(\theta_B) \in \mathbb{R}$ from B to S and a joint lottery $\delta(\theta_B) \in \Delta(\{0,1\} \times \mathcal{R})$, with the following properties:

(A) type $\theta_B \in \Theta_B$ prefers the outcome $f(\theta_B) = (t(\theta_B), \delta(\theta_B))$ to the outcome $f(\hat{\theta}_B) = (t(\hat{\theta}_B), \delta(\hat{\theta}_B))$ that can be obtained by mimicking the behavior of type $\hat{\theta}_B \neq \theta_B$.

(B) for any $r \in \mathcal{R}(\sigma_B)$, the allocations induced by the reaction of θ_T solve $\mathcal{P}_T(r, \theta_T)$.

Consider the following transformation of $\psi = (\mathcal{M}, \mathcal{R}, \beta)$ into a direct mechanism $\tilde{\phi} \in \tilde{\Phi}$ (using σ_B and σ_T). For any $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T)) \in \tilde{Z} = \Xi^2$, let $\tilde{\mathcal{R}}(\tilde{z}) \subseteq \mathcal{R}(\sigma_B)$ denote the set of recommendations that, given σ_B and σ_T, ultimately lead to the same allocations as those specified in the pair of direct resale mechanisms $(\xi(\overline{\theta}_T), \xi(\underline{\theta}_T))$. Now, take a direct mechanism $\tilde{\phi} : \Theta_B \mapsto \mathbb{R} \times \Delta(\{0,1\} \times \tilde{Z})$ that assigns to each $\theta_B \in \Theta_B$ the same expected transfer as ψ, and a lottery over $\{0,1\} \times \tilde{Z}$ such

[4] Note that, even if T faces an "informed principal" mechanism design problem, since both B and T have quasi-linear preferences, private values and finite types, T never gains from hiding her private information to B – see Maskin and Tirole (1990).

[5] To guarantee that, whenever indifferent, B participates and truthfully reveals his type, T may need to increase the transfers $t^r(\bar{\theta}_B)$ and $t^r(\underline{\theta}_B)$, that solve $\mathcal{P}_T(r, \theta_T)$, respectively by ε and δ. However, with quasilinear payoffs, ε and δ can be set arbitrarily close to zero.

that[6]

$$\tilde{\phi}(\tilde{z}|\theta_B) = \sum_{r \in \tilde{\mathcal{R}}(\tilde{z})} \psi(r|\theta_B)$$

where $\psi(r|\theta_B) := \Pr(x = 1, r' = r|\theta_B; \psi)$ and $\tilde{\phi}(\tilde{z}|\theta_B) := \Pr(x = 1, \tilde{z}' = \tilde{z}|\theta_B; \tilde{\phi})$. The mechanism $\tilde{\phi}$ constructed this way maps Θ_B into the same final outcomes – probability of trade and expected payments – as the mechanism ψ. Furthermore, given $\tilde{\phi}$, T has the correct incentives to follow the monopolist's recommendations. To see this, note that when the supports Θ_B and Θ_T overlap, a recommendation $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T)) \in \tilde{Z}(\tilde{\phi})$, is incentive-compatible if and only if $\xi(\underline{\theta}_T)$ is such that

$$x^r(\underline{\theta}_B) = 1, \; x^r(\overline{\theta}_B) = 0, \; t^r(\underline{\theta}_B) = \underline{\theta}_B, \; t^r(\overline{\theta}_B) = 0, \qquad (SR(\tilde{z}, \underline{\theta}_T))$$

and $\xi(\overline{\theta}_T)$ is such that

$$\begin{array}{l} x^r(\underline{\theta}_B) = 1; \; x^r(\overline{\theta}_B) = \begin{cases} 1 \text{ if } \Pr(\overline{\theta}_B|\tilde{z}) > \Delta\theta_B/[\overline{\theta}_T - \underline{\theta}_B] \\ 0 \text{ if } \Pr(\overline{\theta}_B|\tilde{z}) < \Delta\theta_B/[\overline{\theta}_T - \underline{\theta}_B] \\ \text{any } \eta \in [0,1] \text{ if } \Pr(\overline{\theta}_B|\tilde{z}) = \Delta\theta_B/[\overline{\theta}_T - \underline{\theta}_B] \end{cases} \\ t^r(\overline{\theta}_B) = x^r(\overline{\theta}_B)\overline{\theta}_B; \; t^r(\underline{\theta}_B) = \underline{\theta}_B + x^r(\overline{\theta}_B)\Delta\theta_B \end{array} \qquad (SR(\tilde{z}, \overline{\theta}_T))$$

Hence, recommendations may differ only with respect to what S recommends to $\overline{\theta}_T$. Now, take a recommendation $\tilde{z} \in \tilde{Z}(\tilde{\phi})$ such that $\xi(\overline{\theta}_T) = (x^r(\overline{\theta}_B) = x^r(\underline{\theta}_B) = 1; t^r(\overline{\theta}_B) = t^r(\underline{\theta}_B) = \overline{\theta}_B)$. Then, for any $r \in \tilde{\mathcal{R}}(\tilde{z})$,

$$\Pr(\overline{\theta}_B|r; \psi) = \frac{\psi(r|\overline{\theta}_B)\Pr(\overline{\theta}_B)}{\psi(r|\overline{\theta}_B)\Pr(\overline{\theta}_B) + \psi(r|\underline{\theta}_B)\Pr(\underline{\theta}_B)} \geq \Delta\theta_B/[\overline{\theta}_T - \underline{\theta}_B].$$

Since, given $\tilde{\phi}$, T's posterior beliefs when she receives the recommendation $\tilde{z}$ are given by

$$\Pr(\overline{\theta}_B|\tilde{z}; \tilde{\phi}) = \frac{\tilde{\phi}(\tilde{z}|\overline{\theta}_B)\Pr(\overline{\theta}_B)}{\tilde{\phi}(\tilde{z}|\overline{\theta}_B)\Pr(\overline{\theta}_B) + \tilde{\phi}(\tilde{z}|\underline{\theta}_B)\Pr(\underline{\theta}_B)} = \frac{\sum_{r \in \tilde{\mathcal{R}}(\tilde{z})} \psi(r|\overline{\theta}_B)\Pr(\overline{\theta}_B)}{\sum_{r \in \tilde{\mathcal{R}}(\tilde{z})} \psi(r|\overline{\theta}_B)\Pr(\overline{\theta}_B) + \sum_{r \in \tilde{\mathcal{R}}(\tilde{z})} \psi(r|\underline{\theta}_B)\Pr(\underline{\theta}_B)},$$

then $\Pr(\overline{\theta}_B|\tilde{z}; \tilde{\phi}) \geq \Delta\theta_B/[\overline{\theta}_T - \underline{\theta}_B]$, which implies that $\tilde{z}$ is indeed incentive-compatible. The same result can be established for any $\tilde{z} \in \tilde{Z}(\tilde{\phi})$.

We conclude that for any mechanism ψ, there exists a mechanism $\tilde{\phi}$ satisfying (i) and (ii) that is payoff-equivalent for all players. >From (iii), it is then immediate that in the unrestricted game, there exists an equilibrium sustaining $\tilde{\phi}^*$. Furthermore, the monopolist's payoff in such an equilibrium is necessarily (weakly) higher than in any other equilibrium of the unrestricted game.

Step 2. Now, let $r(\theta_B; \xi) := t^r(\theta_B) - \xi(1|\theta_B)\theta_B$ denote the resale surplus that θ_B obtains when T offers a direct resale mechanism ξ, and $r(\theta_B|\tilde{z}) := p_T r(\theta_B; \xi(\overline{\theta}_T)) + (1 - p_T) r(\theta_B; \xi(\underline{\theta}_T))$ the

[6] For simplicity, we assume $\mathcal{R}(\tilde{z})$ is a finite set. If not, then let $\tilde{\phi}(\tilde{z}|\theta_B) = \int_{r \in \mathcal{R}(\tilde{z})} d\delta(r|\theta_B)$, where $\delta(r|\theta_B)$ denotes the probability measure of recommendation r induced by the buyer's strategy at $\tau = 1$.

expected surplus given the recommendation $\tilde{z} = (\xi(\overline{\theta}_T), \xi(\underline{\theta}_T))$. The mechanism $\tilde{\phi}^*$ satisfies (i)-(iii) if and only if it is a solution to the following program

$$\widetilde{\mathcal{P}}_S : \begin{cases} \max_{\tilde{\phi}\in\tilde{\Phi}} \; \mathbb{E}_{\theta_B}\left[t\left(\theta_B\right)\right] \\ \text{s.t.– for any } (\theta_B, \widehat{\theta}_B) \in \Theta_B^2 \text{ –} \\ \quad U(\theta_B) := \sum_{\tilde{z}\in\tilde{Z}} \tilde{\phi}(\tilde{z}|\theta_B)\left\{\theta_B + r(\theta_B|\tilde{z})\right\} - t(\theta_B) \geq 0 \\ \quad U(\theta_B) \geq \sum_{\tilde{z}\in\tilde{Z}} \tilde{\phi}(\tilde{z}|\hat{\theta}_B)\left\{\theta_B + r(\theta_B|\tilde{z})\right\} - t(\hat{\theta}_B) \\ \text{for any } \tilde{z} \in \tilde{Z}(\tilde{\phi}) \text{ and any } \theta_T \in \Theta_T, \; \xi(\theta_T) \text{ satisfies } SR(\tilde{z}, \theta_T)) \\ \tilde{\phi}\left(\tilde{z}|\theta_B\right) \geq 0 \text{ with } \sum_{z\in Z} \tilde{\phi}\left(\tilde{z}|\theta_B\right) \leq 1 \text{ for any } \theta_B \in \Theta_B \qquad (\mathcal{F}) \end{cases}$$

Step 3. Note that S never gains from using a mechanism $\tilde{\phi}$ that recommends a $\xi(\overline{\theta}_T)$ in which $x^r(\overline{\theta}_B) \in (0,1)$. Indeed, for any such mechanism, there exists another mechanism $\tilde{\phi}'$ in which S recommends only $\xi(\overline{\theta}_T)$ such that either $x^r(\overline{\theta}_B) = 0$, or $x^r(\overline{\theta}_B) = 1$, which leads to a higher payoff. It follows that S sends only two possible incentive-compatible recommendations: the first one is for both $\overline{\theta}_T$ and $\underline{\theta}_T$ to trade only with $\underline{\theta}_B$ at a price $t^r = \underline{\theta}_B$; the second is for $\underline{\theta}_T$ to trade only with $\underline{\theta}_B$ at a price $t^r = \underline{\theta}_B$ and for $\overline{\theta}_T$ to trade with both types at a price $t^r = \overline{\theta}_B$. But these are exactly the same resale outcomes that can be implemented recommending simple take-it-or-leave-it price offers. It is then immediate that the solution to $\widetilde{\mathcal{P}}_S$ leads exactly to the same revenue as the solution to $\mathcal{P}_S$ in the main text. We conclude that $\tilde{U}_S^*$ can also be achieved in the game where $\Pi^r = \Upsilon$ and $\Psi = \Phi$. Q.E.D.

2 Implementation of the optimal mechanism of Proposition 1 with price disclosures

Claim A2. *When the direct mechanism of Proposition 1 can not be implemented announcing only the decision to trade, it suffices to disclose the price to implement the optimal informational linkage with the secondary market.*

Proof. The implementations in which S discloses the price but keeps the choice of the contract secret, or discloses the contract with probability less than one, are immediate. In what follows, we prove that S could also fully disclose the choice of the contract by inducing B to play a mixed strategy.

Suppose S offers a menu of two price-lottery pairs. The menu is such that B receives the good with certainty if he pays $t_H = t^*(\overline{\theta}_B)$ and with probability $\delta = [1 - J/K]/[1 - J]$ if he pays $t_L = \delta\left[\underline{\theta}_B + \lambda_B s(\underline{\theta}_B)\right]$, where $t^*(\overline{\theta}_B)$ is the price $\overline{\theta}_B$ pays in the direct mechanism of Proposition 1.

We want to show that it is an equilibrium for the high type to pay t_H and for the low type to randomize over t_H and t_L with probability respectively equal to J and $1-J$. Given this strategy, $\overline{\theta}_T$ offers $t^r = \overline{\theta}_B$ when she observes t_H and $t^r = \underline{\theta}_B$ when she observes t_L, that is t_H and t_L serve the same role as $\bar{z}$ and $\underline{z}$ in the direct mechanism. For the low type to be indifferent between t_H and t_L it must be that

$$\underline{\theta}_B + \lambda_B s(\underline{\theta}_B) + \lambda_T p_T \Delta\theta_B - t_H = \delta\left[\underline{\theta}_B + \lambda_B s(\underline{\theta}_B)\right] - t_L. \qquad (1)$$

Since $t_H = t^*(\overline{\theta}_B)$, the left hand side in (1) is also equal to the payoff $\underline{\theta}_B$ obtains by announcing $\theta_B = \overline{\theta}_B$ in the direct mechanism, which is equal to zero since $IC(\underline{\theta}_B)$ and $IR(\underline{\theta}_B)$ bind in the optimal mechanism. As a consequence, $t_L = \delta\left[\underline{\theta}_B + \lambda_B s(\underline{\theta}_B)\right]$.

Next, we prove that the high type is also indifferent between t_H and t_L, that is $\overline{\theta}_B - t_H = \delta\left[\overline{\theta}_B + \lambda_B s(\overline{\theta}_B)\right] - t_L$. Using the values of δ and t_L, the previous equality is equivalent to

$$\overline{\theta}_B - t_H - \left[\Delta\theta_B + \lambda_B \Delta s - \lambda_T p_T \Delta\theta_B\right] = 0$$

which holds true since $t_H = t^*(\overline{\theta}_B)$ and in the optimal mechanism both $IR(\underline{\theta}_B)$ and $IC(\overline{\theta}_B)$ are binding, which implies that $0 = \overline{\theta}_B - t_H - \left[\Delta\theta_B + \lambda_B \Delta s - \lambda_T p_T \Delta\theta_B\right]$.

Since this mechanism gives B the same payoff and induces the same distribution over x and Z as the optimal direct mechanism, it must also give S the same expected revenue. Q.E.D.

3 Resale to third parties with multiple bidders in the primary market

Claim A3. *A monopolist always benefits from the existence of a secondary market when she is not able to contract with all potential buyers and resale can only be to a third party who does not participate in the primary market.*

Proof. Assume there are $N \geq 2$ potential buyers in the primary market. At the end of the auction, the winner may keep the good for himself or resell it to T in the secondary market, in which case the bargaining game is exactly as in the single-bidder case with λ_i denoting the relative bargaining power of bidder i with respect to T. Continue to assume A1-A4 hold for each bidder and let $\boldsymbol{\theta}_B := (\theta_1, \theta_2, ..., \theta_N) \in \boldsymbol{\Theta}_B := \prod_{i=1}^N \Theta_i$ denote a profile of independent private values. Following the same steps as for the single bidder case, one can show that, conditional on bidder i winning the auction, S needs to send only two recommendations: $\underline{z}^i$ must induce $\overline{\theta}_T$ to offer $t^r(\overline{\theta}_T) = \underline{\theta}_i$ and $\bar{z}^i$ to offer $t^r(\overline{\theta}_T) = \overline{\theta}_i$. Let $\phi\left(z^i|\boldsymbol{\theta}_B\right)$ denote the probability the good is assigned to bidder i and a recommendation $z^i \in \{\bar{z}^i, \underline{z}^i\}$ is sent to T when the bidders report $\boldsymbol{\theta}_B$. Also, let

$$V(\theta_i|z^i) := \overline{\theta}_i + \lambda_i s_i(\overline{\theta}_i)$$

$$V_i(\underline{\theta}_i|z^i) := \underline{\theta}_i - \tfrac{p_i}{1-p_i}\Delta\theta_i + \lambda_i\{s_i(\underline{\theta}_i) - \tfrac{p_i}{1-p_i}\Delta s_i\} + (1-\lambda_i)\{r_i(\underline{\theta}_i|z^i) - \tfrac{p_i}{1-p_i}\Delta r_i(z^i)\}$$

denote the resale-augmented virtual valuations of bidder i. Following the same steps as for the single bidder case, we can show that an optimal auction ϕ^* maximizes

$$\mathbb{E}_{\boldsymbol{\theta}_B}[\sum_{i=1}^{N} \sum_{z^i\in\{\bar{z}^i,\underline{z}^i\}} V(\theta_i|z^i)\phi\left(z^i|\boldsymbol{\theta}_B\right)]$$

subject to

$$\begin{aligned}\mathbb{E}_{\boldsymbol{\theta}_{-i}}\left\{\sum_{z^i\in\{\bar{z}^i,\underline{z}^i\}}\phi\left(z^i|\overline{\theta}_i,\boldsymbol{\theta}_{-i}\right)\left[\Delta\theta_i+\lambda_i\Delta s_i+(1-\lambda_i)\Delta r_i(z^i)\right]\right\} \geq \\ \mathbb{E}_{\boldsymbol{\theta}_{-i}}\left\{\sum_{z^i\in\{\bar{z}^i,\underline{z}^i\}}\phi\left(z^i|\underline{\theta}_i,\boldsymbol{\theta}_{-i}\right)\left[\Delta\theta_i+\lambda_i\Delta s_i+(1-\lambda_i)\Delta r_i(z^i)\right]\right\}\end{aligned} \qquad (IC(\underline{\theta}_i))$$

$$\Pr(\overline{\theta}_i|\underline{z}^i) \leq \tfrac{\Delta\theta_i}{\overline{\theta}_T-\underline{\theta}_i}, \qquad (IC(\underline{z}^i))$$

$$\Pr(\overline{\theta}_i|\bar{z}^i) \geq \tfrac{\Delta\theta_i}{\overline{\theta}_T-\underline{\theta}_i}, \qquad (IC(\bar{z}^i))$$

for $i \in \{1,2,...,N\}$ and $\boldsymbol{\theta}_{-i} := (\theta_1, \theta_2, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N)$.

To prove the claim, we compare the expected revenue associated with the solution to the above program with the revenue S could achieve in a Myerson optimal auction without resale. Recall that for any type profile $\boldsymbol{\theta}_B$, a Myerson auction consists in assigning the good to the bidder with the highest virtual valuation, $M(\theta_i)$, provided that $\max_i\{M(\theta_i)\} \geq 0$, and in withholding the good otherwise. The expected revenue of a Myerson optimal auction is thus $\mathbb{E}_{\boldsymbol{\theta}_B}[\max\{0,\ M(\theta_1),...,M(\theta_N)\}]$, where $M(\overline{\theta}_i) := \overline{\theta}_i$ and $M(\underline{\theta}_i) := \underline{\theta}_i - \frac{p_i}{1-p_i}\Delta\theta_i$, for each $i = 1,...,N$.

The proof is in two steps. The first step proves that for any $\theta_i \in \Theta_i$ and z^i, the resale-augmented virtual valuations are higher than the corresponding Myerson virtual valuations; that is, $V(\theta_i|z^i) \geq M(\theta_i)$. This follows directly from the fact that $s(\theta_i) \geq 0$, $r_i(z^i) \geq 0$, $\Delta s_i \leq 0$ and $\Delta r_i(z^i) \leq 0$, for any $z^i \in \{\bar{z}^i, \underline{z}^i\}$ and any i.

The second step proves that there exists a recommendation policy that allows to implement Myerson allocation rule with resale. Conditional on i winning the auction, suppose S sends only one recommendation $z^i \in \{\bar{z}^i, \underline{z}^i\}$, independently of whether i announces a low or a high type. The particular recommendation S sends to T depends on the posterior beliefs that are generated by the Myerson allocation rule; that is, S recommends $z^i = \underline{z}^i$ if $\Pr(\overline{\theta}_i|i) \leq \frac{\Delta\theta_i}{\overline{\theta}_T-\underline{\theta}_i}$, and $z^i = \bar{z}^i$ otherwise, where $\Pr(\overline{\theta}_i|i)$ denotes the probability that $\theta_i = \overline{\theta}_i$ given that bidder i wins the auction. Given this policy, which can be trivially implemented disclosing only the identity of the winner, $IC(\underline{z}^i) - IC(\bar{z}^i)$ are clearly satisfied. Furthermore, since Myerson allocation rule is monotonic – i.e. $\mathbb{E}_{\boldsymbol{\theta}_{-i}}\left\{\phi\left(z^i|\overline{\theta}_i,\boldsymbol{\theta}_{-i}\right)\right\} \geq \mathbb{E}_{\boldsymbol{\theta}_{-i}}\left\{\phi\left(z^i|\underline{\theta}_i,\boldsymbol{\theta}_{-i}\right)\right\}$, constraints $IC(\underline{\theta}_i)$ are also satisfied for each i. It follows that Myerson allocation rule remains implementable also in the presence of resale. It is then immediate that the optimal mechanism ϕ^* must satisfy

$$\mathbb{E}_{\boldsymbol{\theta}_B}[\sum_{i=1}^{N} \sum_{z^i\in\{\bar{z}^i,\underline{z}^i\}} V(\theta_i|z^i)\phi^*\left(z^i|\boldsymbol{\theta}_B\right)] \geq \mathbb{E}_{\boldsymbol{\theta}_B}[\max\{0,\ M(\theta_1),...,M(\theta_N)\}],$$

which proves the result. Q.E.D.

4 Resale to third parties: collusion in the primary market

When S lacks of the commitment not to collude with B, the only credible information that can be disclosed to the secondary market is the decision to trade. Furthermore, the possibility for S to make ϕ public has no strategic effect so that ϕ must be a best response to the strategy T is expected to follow in the secondary market. The optimal mechanism can be designed by looking at the value of the (collusion proof) resale-augmented virtual valuations

$$\begin{aligned} V(\overline{\theta}_B|\gamma) &:= \overline{\theta}_B + \lambda_B s(\overline{\theta}_B), \\ V(\underline{\theta}_B|\gamma) &:= \underline{\theta}_B - \tfrac{p_B}{1-p_B}\Delta\theta_B + \lambda_B \left[s(\underline{\theta}_B) - \tfrac{p_B}{1-p_B}\Delta s \right] + \lambda_T \gamma \left[\Delta\theta_B + \tfrac{p_B}{1-p_B}\Delta\theta_B \right], \end{aligned}$$

where $\gamma \in [0, p_T]$ is the probability T is expected to offer a high price in the resale game. The seller's optimal (collusion-proof) mechanism then maximizes $U_S := \mathbb{E}_{\theta_B}[V(\theta_B|\gamma)\phi(\theta_B)]$ under the monotonicity condition $\phi(\overline{\theta}_B) \geq \phi(\underline{\theta}_B)$, where $\phi(\theta_B)$ denotes the probability of trade when B reports θ_B.

To see how the informational linkage with the secondary market can be fashioned through a stochastic allocation rule, assume T's prior beliefs are unfavorable ($J < 1$) and $V(\underline{\theta}_B|0) < 0 < V(\underline{\theta}_B|p_T)$. In the unique equilibrium, S sells to $\underline{\theta}_B$ with probability J and to $\overline{\theta}_B$ with certainty. $\overline{\theta}_T$ is then indifferent between offering a high and a low price and randomizes offering $t^r(\overline{\theta}_T) = \overline{\theta}_B$ with probability $\gamma^* \in (0,1)$ and $t^r(\overline{\theta}_T) = \underline{\theta}_B$ with probability $1-\gamma^*$, where γ^* solves $V(\underline{\theta}_B|p_T\gamma^*) = 0$ and hence makes S indifferent between selling to the low type and retaining the good.

5 Extended proof of Proposition 2: Optimal auctions with inter-bidder resale

The reduced program is in the Appendix of the paper (proof of Proposition 2). Here we derive a complete characterization of the optimal mechanism in the two polar cases where $\lambda_T = 1$ and $\lambda_B = 1$.

T has all bargaining power (i.e. $\lambda_T = 1$).

In this case, S does not need to disclose any information to B. Therefore, we eliminate z_B from the mechanism ϕ. Furthermore, since $\underline{\theta}_B \leq \underline{\theta}_T \leq \overline{\theta}_B \leq \overline{\theta}_T$, when $h = T$, the only incentive-compatible recommendation for $\overline{\theta}_T$ is to ask $t^r(\overline{\theta}_T) \geq \overline{\theta}_T$ and for $\underline{\theta}_T$ to ask $t^r(\underline{\theta}_T) = \overline{\theta}_B$ if $\Pr(\overline{\theta}_B|\cdot) > 0$ and any $t^r \geq \underline{\theta}_T$ otherwise. Similarly, when $h = B$, the only incentive-compatible recommendation for $\underline{\theta}_T$ is to offer a price $t^r(\underline{\theta}_T) = \underline{\theta}_B$ if $\Pr(\underline{\theta}_B|\cdot) > 0$ and any price $t^r(\underline{\theta}_T) \le \underline{\theta}_T$ otherwise. Without loss, we will assume that $\underline{\theta}_T$ always asks $t^r(\underline{\theta}_T) = \overline{\theta}_B$ when $h = T$ and offers $t^r(\underline{\theta}_T) = \underline{\theta}_B$ when $h = B$.[7] Since these are the same prices that T offers in the absence of any explicit recommendation, to save on notation, we will drop z_T from the mapping ϕ when $h = T$, or $h = B$ and $\theta_T = \underline{\theta}_T$. When instead $h = B$ and $\theta_T = \overline{\theta}_T$, we will denote by $\bar{z}$ and $\underline{z}$ the recommendations (for $\overline{\theta}_T$) to offer $t^r(\overline{\theta}_T) = \overline{\theta}_B$ and $t^r(\overline{\theta}_T) = \underline{\theta}_B$, respectively. For these recommendations to be incentive compatible, the mechanism ϕ must satisfy

$$\phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B) \ge J\phi(B, \underline{z}|\overline{\theta}_T, \overline{\theta}_B) \qquad \widetilde{IC}(\underline{z}, \overline{\theta}_T)$$
$$\phi(B, \bar{z}|\overline{\theta}_T, \underline{\theta}_B) \le J\phi(B, \bar{z}|\overline{\theta}_T, \overline{\theta}_B) \qquad \widetilde{IC}(\bar{z}, \overline{\theta}_T)$$

where $J := \frac{p_B(\overline{\theta}_T - \overline{\theta}_B)}{(1-p_B)\Delta\theta_B}$. The maximal revenue for the monopolist can be derived by partitioning the set of direct mechanisms Φ into two classes. The first one, which we denote by Φ_1, is such that $\overline{\theta}_T$ finds it (weakly) optimal to offer a high resale price $t^r(\overline{\theta}_T) = \overline{\theta}_B$ off-equilibrium, after reporting $\hat{\theta}_T = \underline{\theta}_T$. The second is such that $\overline{\theta}_T$ strictly prefers to offer $t^r(\overline{\theta}_T) = \underline{\theta}_B$. For a mechanism ϕ to belong to Φ_1 it must be that

$$\phi(B|\underline{\theta}_T, \underline{\theta}_B) \le J\phi(B|\underline{\theta}_T, \overline{\theta}_B) \qquad (C_1)$$

Letting $\mathbb{I}_{\phi\in\Phi_1} = 1$ if $\phi \in \Phi_1$ and zero otherwise, and substituting for the values of $s_T(\cdot)$ and $r_B(\cdot)$, the problem for the monopolist reduces to the choice of a mechanism ϕ^* that maximizes[8]

$$\begin{aligned}
U_S = {} & p_T p_B\{\overline{\theta}_T\phi(T|\overline{\theta}_T, \overline{\theta}_B) + \overline{\theta}_T\phi(B, \bar{z}|\overline{\theta}_T, \overline{\theta}_B) + \overline{\theta}_B\phi(B, \underline{z}|\overline{\theta}_T, \overline{\theta}_B)\} \\
& + p_T(1-p_B)\{\overline{\theta}_T\phi(T|\overline{\theta}_T, \underline{\theta}_B) + \overline{\theta}_T\phi(B, \bar{z}|\overline{\theta}_T, \underline{\theta}_B) + (\overline{\theta}_T - \tfrac{p_B}{1-p_B}\Delta\theta_B)\phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B)\} \\
& + (1-p_T)p_B\{[\overline{\theta}_B - \tfrac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right)][\phi(T|\underline{\theta}_T, \overline{\theta}_B) + \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \overline{\theta}_B)] + \overline{\theta}_B[1 - \mathbb{I}_{\phi\in\Phi_1}]\phi(B|\underline{\theta}_T, \overline{\theta}_B)\} \\
& + (1-p_T)(1-p_B)\{M(\underline{\theta}_T)\phi(T|\underline{\theta}_T, \underline{\theta}_B) + \mathbb{I}_{\phi\in\Phi_1}[M(\underline{\theta}_T) + (\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B})\Delta\theta_B]\phi(B|\underline{\theta}_T, \underline{\theta}_B) \\
& + [1 - \mathbb{I}_{\phi\in\Phi_1}](M(\underline{\theta}_T) - \tfrac{p_B}{1-p_B}\Delta\theta_B)\phi(B|\underline{\theta}_T, \underline{\theta}_B)\}
\end{aligned}$$

subject to $\widetilde{IC}(\bar{z}, \overline{\theta}_T)$, $\widetilde{IC}(\underline{z}, \overline{\theta}_T)$ and

$$p_T[\phi\left(B, \underline{z}|\overline{\theta}_T, \overline{\theta}_B\right) - \phi\left(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B\right)] + (1-p_T)[\phi\left(B|\underline{\theta}_T, \overline{\theta}_B\right) - \phi\left(B|\underline{\theta}_T, \underline{\theta}_B\right)] \ge 0 \qquad (\widetilde{IC}(\underline{\theta}_B))$$

$$\begin{aligned}
& p_B\left(\overline{\theta}_T - \overline{\theta}_B\right)[\phi(T|\overline{\theta}_T, \overline{\theta}_B) + \phi(B, \bar{z}|\overline{\theta}_T, \overline{\theta}_B) - \phi(T|\underline{\theta}_T, \overline{\theta}_B) - \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \overline{\theta}_B)] \\
& + (1-p_B)\Delta\theta_T[\phi(T|\overline{\theta}_T, \underline{\theta}_B) + \phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B) - \phi(T|\underline{\theta}_T, \underline{\theta}_B) - [1 - \mathbb{I}_{\phi\in\Phi_1}]\phi(B|\underline{\theta}_T, \underline{\theta}_B)] \\
& + (1-p_B)(\Delta\theta_T - \Delta\theta_B)[\phi(B, \bar{z}|\overline{\theta}_T, \underline{\theta}_B) - \mathbb{I}_{\phi\in\Phi_1}\phi(B|\underline{\theta}_T, \underline{\theta}_B)] \ge 0.
\end{aligned}$$
$$(\widetilde{IC}(\underline{\theta}_T))$$

[7] Clearly, S has no incentive to recommend a different price.

[8] Assuming that $\bar{\theta}_T$ offers a high price off-equilibrium when she is indifferent between $t^r = \bar{\theta}_B$ and $t^r = \underline{\theta}_B$ is without loss of generality. Indeed, when $\phi(B|\underline{\theta}_T, \underline{\theta}_B) = J\phi(B|\underline{\theta}_T, \bar{\theta}_B)$, the program for the optimal mechanism is the same no matter whether $\bar{\theta}_T$ offers a low or a high resale price.

Note that the controls $\phi(\cdot|\boldsymbol{\theta})$ associated with the states $\boldsymbol{\theta} = \left(\overline{\theta}_T, \overline{\theta}_B\right)$ and $\boldsymbol{\theta} = \left(\overline{\theta}_T, \underline{\theta}_B\right)$ are linked to the controls associated with the other two states $\boldsymbol{\theta} = \left(\underline{\theta}_T, \overline{\theta}_B\right)$, $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$ only through the constraints $\widetilde{IC}(\underline{\theta}_T)$ and $\widetilde{IC}(\underline{\theta}_B)$. In what follows, we disregard $\widetilde{IC}(\underline{\theta}_T)$ since it never binds at the optimum. Also note that it is always optimal to set $\phi^*(T|\boldsymbol{\theta}) = 1$ for $\boldsymbol{\theta} = (\overline{\theta}_T, \overline{\theta}_B)$ and $\boldsymbol{\theta} = (\overline{\theta}_T, \underline{\theta}_B)$. Indeed, this maximizes U_S and it helps relaxing $\widetilde{IC}(\underline{\theta}_B)$.[9]

To derive the optimal mechanism, it thus suffices to consider the monopolist's payoff in the other two sates $\boldsymbol{\theta} = (\underline{\theta}_T, \overline{\theta}_B)$ and $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$.

- Consider first $J \geq 1$.

  1. Suppose $\phi^* \notin \Phi_1$. Then S could reduce $\phi(B|\underline{\theta}_T, \underline{\theta}_B)$ and increase $\phi(T|\underline{\theta}_T, \underline{\theta}_B)$ enhancing her payoff. The optimal mechanism thus necessarily belongs to Φ_1.
  2. When $\overline{\theta}_B - \frac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right) \geq 0$, $\phi^*\left(B|\underline{\theta}_T, \overline{\theta}_B\right) = 1$ is clearly optimal. In this case constraints (C_1) and $\widetilde{IC}(\underline{\theta}_B)$ are always satisfied. As for $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$, if

  $$M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B \geq \max\{0;\ M(\underline{\theta}_T)\},$$

  then $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = 1$ in which case the revenue is $U_S = (1-p_B)\underline{\theta}_T + p_T\Delta\theta_B + p_B\underline{\theta}_B$. If instead

  $$M(\underline{\theta}_T) > \max\left\{0;\ M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B\right\},$$

  then $\phi^*(T|\underline{\theta}_T, \underline{\theta}_B) = 1$ and the revenue is $(1-p_B)\underline{\theta}_T + p_B\overline{\theta}_B$. Finally, if

  $$\max\left\{M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B;\ M(\underline{\theta}_T)\right\} < 0,$$

  then S retains the good when $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$ and the revenue is $U_S = p_T(1-p_B)\overline{\theta}_T + p_B\overline{\theta}_B$.[10]

  Next, assume $\overline{\theta}_B - \frac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right) < 0$. In this case $\widetilde{IC}(\underline{\theta}_B)$ necessarily binds, i.e.

  $$\phi^*\left(B|\underline{\theta}_T, \overline{\theta}_B\right) = \phi^*(B|\underline{\theta}_T, \underline{\theta}_B),$$

  and hence (C_1) is always satisfied. Furthermore, since $M(\underline{\theta}_T) \leq \overline{\theta}_B - \frac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right) < 0$, S never sells to T when the latter reports a low valuation, i.e. when $\boldsymbol{\theta} = (\underline{\theta}_T, \overline{\theta}_B)$ or $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$. At the optimum $\phi^*\left(B|\underline{\theta}_T, \overline{\theta}_B\right) = \phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = 1$ if

  $$p_B\left[\overline{\theta}_B - \tfrac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right)\right] + (1-p_B)\left[M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B\right] \geq 0$$

[9] Note that $\phi(T|\boldsymbol{\theta}) = 1$ is payoff equivalent to $\phi(B, \underline{z}|\boldsymbol{\theta}) = 1$ for $\boldsymbol{\theta} = (\overline{\theta}_T, \overline{\theta}_B)$, and $\boldsymbol{\theta} = (\overline{\theta}_T, \underline{\theta}_B)$. Nevertheless, selling to T in these two states is more effective in relaxing $\widetilde{IC}(\underline{\theta}_T)$ than selling to B. This also implies that when $\widetilde{IC}(\underline{\theta}_T)$ does not bind, the optimal allocation rule need not be unique.

[10] Again, the solution may not be unique, as $\phi(T|\boldsymbol{\theta}) = 1$ is payoff equivalent to $\phi(B|\boldsymbol{\theta}) = 1$ for $\boldsymbol{\theta} = (\underline{\theta}_T, \overline{\theta}_B)$. For example, if $M(\underline{\theta}_T) > \max\left\{0; M(\underline{\theta}_T) + \left(\frac{p_T}{1-p_T} - \frac{p_B}{1-p_B}\right)\Delta\theta_B\right\}$, then $\phi^*\left(T|\underline{\theta}_T, \overline{\theta}_B\right) = 1$ is also optimal.

and $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = \phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = 0$ otherwise. In the first case, the revenue is $U_S = (1-p_B)\underline{\theta}_T + p_T\Delta\theta_B + p_B\underline{\theta}_B$, whereas in the second $U_S = p_T\overline{\theta}_T$.

- Suppose now $J < 1$.

  1. In this case $\widetilde{IC}(\underline{\theta}_B)$ can be neglected as it never binds at the optimum. Furthermore, the optimal mechanism necessarily belongs to Φ_1. The argument is the same as for $J \geq 1$.
  2. Assume now $\overline{\theta}_B - \frac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right) \geq 0$. Then $\phi^*(B|\underline{\theta}_T, \overline{\theta}_B) = 1$. As for $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$, if

$$M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B \geq \max\{0,\ M(\underline{\theta}_T)\}$$

then (C_1) binds and thus $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = J$. If in addition $M(\underline{\theta}_T) \geq 0$, then $\phi^*(T|\underline{\theta}_T, \underline{\theta}_B) = 1 - J$; otherwise, $\phi^*(T|\underline{\theta}_T, \underline{\theta}_B) = 0$. In the former case, the expected revenue is $\frac{p_B}{1-p_B}\left[\overline{\theta}_T(p_T - p_B) + (1-p_T)\overline{\theta}_B\right] + \underline{\theta}_T(1-p_B)$, whereas in the latter $(1-p_B)\underline{\theta}_T + p_T\Delta\theta_B + p_B\underline{\theta}_B$. If, on the contrary,

$$M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B < \max\{0,\ M(\underline{\theta}_T)\},$$

then necessarily $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = 0$. As for $\phi(T|\underline{\theta}_T, \underline{\theta}_B)$, at the optimum $\phi^*(T|\underline{\theta}_T, \underline{\theta}_B) = 1$ when $M(\underline{\theta}_T) \geq 0$, whereas $\phi^*(T|\underline{\theta}_T, \underline{\theta}_B) = 0$ when $M(\underline{\theta}_T) < 0$. The revenue is equal to $(1-p_B)\underline{\theta}_T + p_B\overline{\theta}_B$ in the first case and $(1-p_B)\underline{\theta}_T + p_T\Delta\theta_B + p_B\underline{\theta}_B$ in the second. Next, consider $\overline{\theta}_B - \frac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right) < 0$. At the optimum, (C_1) necessarily binds. It follows that $\phi^*\left(B|\underline{\theta}_T, \overline{\theta}_B\right) = 1$ and $\phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = J$ if

$$p_B\left[\overline{\theta}_B - \tfrac{p_T}{1-p_T}\left(\overline{\theta}_T - \overline{\theta}_B\right)\right] + (1-p_B)J\left[M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B\right] > 0, \tag{2}$$

whereas $\phi^*\left(B|\underline{\theta}_T, \overline{\theta}_B\right) = \phi^*(B|\underline{\theta}_T, \underline{\theta}_B) = 0$ when (2) is reversed . In either case, S never sells to T when the latter reports a low valuation, that is, $\phi^*(T|\boldsymbol{\theta}) = 0$ when $\boldsymbol{\theta} = (\underline{\theta}_T, \overline{\theta}_B)$ and $\boldsymbol{\theta} = (\underline{\theta}_T, \underline{\theta}_B)$. The revenue is

$$p_T\overline{\theta}_T + p_B(\overline{\theta}_B - p_T\overline{\theta}_T) + (1-p_T)(1-p_B)\left[M(\underline{\theta}_T) + \left(\tfrac{p_T}{1-p_T} - \tfrac{p_B}{1-p_B}\right)\Delta\theta_B\right]J$$

in the former case, and $p_T\overline{\theta}_T$ in the latter.

B has all bargaining power (i.e. $\lambda_B = 1$).

In this case, S does not need to disclose any information to T. Therefore, we eliminate z_T from the mechanism ϕ. Furthermore, since $\underline{\theta}_B \leq \underline{\theta}_T \leq \overline{\theta}_B \leq \overline{\theta}_T$, when $h = T$, the only incentive-compatible recommendation is for $\underline{\theta}_B$ to offer $t^r(\underline{\theta}_B) \leq \underline{\theta}_B$ and for $\overline{\theta}_B$ to offer $t^r(\overline{\theta}_B) = \underline{\theta}_T$ if $\Pr(\underline{\theta}_T|\cdot) > 0$ and any $t^r \leq \overline{\theta}_B$ otherwise. Similarly, when $h = B$, the only incentive-compatible recommendation for $\overline{\theta}_B$ is to ask a price $t^r(\overline{\theta}_B) = \overline{\theta}_T$ if $\Pr(\overline{\theta}_T|\cdot) > 0$ and any price $t^r \geq \overline{\theta}_B$ otherwise. Without loss of generality, we will assume that $\overline{\theta}_B$ always asks $t^r(\overline{\theta}_B) = \overline{\theta}_T$ when $h = B$ and offers $t^r(\overline{\theta}_B) = \underline{\theta}_T$ when $h = T$. Since these are the same prices that B offers in the absence of any explicit recommendation, to save on notation, we will drop z_B from the mapping ϕ when $h = T$, or $h = B$ and $\theta_B = \overline{\theta}_B$. When instead $h = B$ and $\theta_B = \underline{\theta}_B$, we will denote by $\bar{z}$ and $\underline{z}$ the recommendations (for $\underline{\theta}_B$) to ask $t^r(\underline{\theta}_B) = \overline{\theta}_T$ and $t^r(\underline{\theta}_B) = \underline{\theta}_T$, respectively. For these recommendations to be incentive compatible, the mechanism ϕ must satisfy

$$\phi(B, \underline{z}|\underline{\theta}_T, \underline{\theta}_B) \geq Q\phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B) \qquad \widetilde{IC}(\underline{z}, \underline{\theta}_B)$$
$$\phi(B, \bar{z}|\underline{\theta}_T, \underline{\theta}_B) \leq Q\phi(B, \bar{z}|\overline{\theta}_T, \underline{\theta}_B) \qquad \widetilde{IC}(\bar{z}, \underline{\theta}_B)$$

where $Q := \frac{p_T \Delta\theta_T}{(1-p_T)(\underline{\theta}_T - \underline{\theta}_B)}$. Let Φ_1 denote the set of mechanisms such that $\underline{\theta}_B$ finds it (weakly) optimal to ask a high resale price $t^r(\underline{\theta}_B) = \overline{\theta}_T$ off-equilibrium, after reporting $\hat{\theta}_B = \overline{\theta}_B$. A mechanism $\phi \in \Phi_1$ only if

$$\phi(B|\underline{\theta}_T, \overline{\theta}_B) \leq Q\phi(B|\overline{\theta}_T, \overline{\theta}_B) \qquad (C_1)$$

Letting $\mathbb{I}_{\phi\in\Phi_1} = 1$ if $\phi \in \Phi_1$ and zero otherwise, and substituting for the values of $s_T(\cdot)$ and $r_B(\cdot)$, the program for the monopolist reduces to the choice of a mechanism that maximizes[11]

$$\begin{aligned} U_S = \; & p_T p_B \left\{\overline{\theta}_T \left[\phi(T|\overline{\theta}_T, \overline{\theta}_B) + \phi(B|\overline{\theta}_T, \overline{\theta}_B)\right]\right\} \\ & + p_T (1-p_B) \left\{\overline{\theta}_T \left[\phi(T|\overline{\theta}_T, \underline{\theta}_B) + \phi(B, \bar{z}|\overline{\theta}_T, \underline{\theta}_B)\right] \right. \\ & \left. + \left[\overline{\theta}_T - \tfrac{p_B}{1-p_B}\Delta\theta_T\right] \phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B)\right\} \\ & + (1-p_T)\, p_B \left\{\left[\overline{\theta}_B - \tfrac{p_T}{1-p_T}\Delta\theta_T\right] \phi(T|\underline{\theta}_T, \overline{\theta}_B) + \overline{\theta}_B \phi(B|\underline{\theta}_T, \overline{\theta}_B)\right\} \\ & + (1-p_T)(1-p_B) \left\{M(\underline{\theta}_B)\phi(B, \bar{z}|\underline{\theta}_T, \underline{\theta}_B) \right. \\ & \left. + \left[M(\underline{\theta}_T) - \tfrac{p_B}{1-p_B}(\overline{\theta}_B - \underline{\theta}_T)\right] \left[\phi(T|\underline{\theta}_T, \underline{\theta}_B) + \phi(B, \underline{z}|\underline{\theta}_T, \underline{\theta}_B)\right]\right\}, \end{aligned}$$

subject to $\widetilde{IC}(\bar{z}, \underline{\theta}_B)$, $\widetilde{IC}(\underline{z}, \underline{\theta}_B)$ and

$$\begin{aligned} & p_B \left[\phi(T|\overline{\theta}_T, \overline{\theta}_B) - \phi(T|\underline{\theta}_T, \overline{\theta}_B)\right] + (1-p_B)\left[\phi(T|\overline{\theta}_T, \underline{\theta}_B) - \phi(T|\underline{\theta}_T, \underline{\theta}_B)\right] \\ & + (1-p_B)\left[\phi(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B) - \phi(B, \underline{z}|\underline{\theta}_T, \underline{\theta}_B)\right] \geq 0, \end{aligned} \qquad (\widetilde{IC}(\underline{\theta}_T))$$

$$\begin{aligned} & p_T \Delta\theta_T \left[(1 - \mathbb{I}_{\phi\in\Phi_1})\phi\left(B|\overline{\theta}_T, \overline{\theta}_B\right) - \phi\left(B, \underline{z}|\overline{\theta}_T, \underline{\theta}_B\right)\right] \\ & + (1-p_T)\Delta\theta_B \left[\mathbb{I}_{\phi\in\Phi_1}\phi\left(B|\underline{\theta}_T, \overline{\theta}_B\right) - \phi\left(B, \bar{z}|\underline{\theta}_T, \underline{\theta}_B\right)\right] \\ & + (1-p_T)\left(\overline{\theta}_B - \underline{\theta}_T\right)\left[\phi\left(T|\underline{\theta}_T, \overline{\theta}_B\right) - \phi\left(T|\underline{\theta}_T, \underline{\theta}_B\right)\right] \\ & + (1-p_T)\left(\overline{\theta}_B - \underline{\theta}_T\right)\left[(1 - \mathbb{I}_{\phi\in\Phi_1})\phi\left(B|\underline{\theta}_T, \overline{\theta}_B\right) - \phi\left(B, \underline{z}|\underline{\theta}_T, \underline{\theta}_B\right)\right] \geq 0. \end{aligned} \qquad (\widetilde{IC}(\underline{\theta}_B))$$

[11] Assuming that $\underline{\theta}_B$ asks a high price off-equilibrium when she is indifferent between $t^r = \bar{\theta}_T$ and $t^r = \underline{\theta}_T$ is without loss of generality. Indeed, when $\phi(B|\underline{\theta}_T, \overline{\theta}_B) = Q\phi(B|\overline{\theta}_T, \overline{\theta}_B)$, the program for the optimal mechanism is the same no matter whether $\underline{\theta}_B$ asks a low or a high resale price.

- Assume first $\max\left\{M(\underline{\theta}_T)-\frac{p_B}{1-p_B}(\overline{\theta}_B-\underline{\theta}_T);M(\underline{\theta}_B)\right\}\geq 0.$

  1. If $M(\underline{\theta}_T)-\frac{p_B}{1-p_B}(\overline{\theta}_B-\underline{\theta}_T)\leq M(\underline{\theta}_B)$, then $Q\geq 1$. In this case the mechanism $\phi^*(B|\boldsymbol{\theta})=1$ for $\boldsymbol{\theta}=(\underline{\theta}_T,\overline{\theta}_B)$ and $\boldsymbol{\theta}=(\overline{\theta}_T,\overline{\theta}_B)$, and $\phi^*(B,\bar{z}|\boldsymbol{\theta})=1$ for $\boldsymbol{\theta}=(\underline{\theta}_T,\underline{\theta}_B)$ and $\boldsymbol{\theta}=(\overline{\theta}_T,\underline{\theta}_B)$ maximizes U_S and satisfies all constraints. For any θ_B, B always asks $t^r(\theta_B)=\overline{\theta}_T$ and thus trade occurs in the secondary market if and only if T has a high valuation. In this case, the final allocation and the expected revenue coincide with that in the Myerson optimal auction if $M(\underline{\theta}_T)\leq M(\underline{\theta}_B)$. If, on the contrary, $M(\underline{\theta}_T)>M(\underline{\theta}_B)$, then in state $\boldsymbol{\theta}=(\underline{\theta}_T,\underline{\theta}_B)$, B retains the good, contrary to what prescribed by the Myerson allocation rule. This in turn induces a loss of expected revenue equal to $(1-p_T)(1-p_B)\left[M(\underline{\theta}_T)-M(\underline{\theta}_B)\right]$.
  2. If $M(\underline{\theta}_T)-\frac{p_B}{1-p_B}(\overline{\theta}_B-\underline{\theta}_T)>M(\underline{\theta}_B)$, the following mechanism $\phi^*\notin\Phi_1$ maximizes U_S and satisfies all constraints

  $$\phi^*(T|\overline{\theta}_T,\underline{\theta}_B)=\phi^*(T|\underline{\theta}_T,\underline{\theta}_B)=\phi^*(T|\overline{\theta}_T,\overline{\theta}_B)=\phi^*(B|\underline{\theta}_T,\overline{\theta}_B)=1.$$

  Trade does not occur in the secondary market, the final allocation is exactly as in Myerson, but the expected revenue is just $p_Tp_B\overline{\theta}_T+(1-p_Tp_B)\,\underline{\theta}_T$ instead of

  $$\mathbb{E}_{\boldsymbol{\theta}}\left[\max\{0,\ M(\theta_T),\ M(\theta_B)\}\right]=p_B[p_T\overline{\theta}_T+(1-p_T)\overline{\theta}_B]+(1-p_B)\underline{\theta}_T.$$

- Assume now $\max\left\{M(\underline{\theta}_T)-\frac{p_B}{1-p_B}(\overline{\theta}_B-\underline{\theta}_T);M(\underline{\theta}_B)\right\}<0$. In this case, S finds it optimal to retain the good when $\boldsymbol{\theta}=(\underline{\theta}_T,\underline{\theta}_B)$. As for the other states, the following mechanism maximizes U_S and satisfies all constraints

  $$\phi^*(T|\overline{\theta}_T,\overline{\theta}_B)=\phi^*(T|\overline{\theta}_T,\underline{\theta}_B)=\phi^*(B|\underline{\theta}_T,\overline{\theta}_B)=1.$$

  The monopolist's expected revenue is $p_T\overline{\theta}_T+(1-p_T)\,p_B\overline{\theta}_B$, trade occurs in the primary market if and only if at least one of the two bidders has a high valuation, and no offers are made in the resale game. If $M(\underline{\theta}_T)\leq 0$, the expected revenue is the same as in the Myerson auction. On the contrary, if $M(\underline{\theta}_T)>0>M(\underline{\theta}_B)$, S incurs a loss equal to $(1-p_T)\,(1-p_B)\,M(\underline{\theta}_T)$.

We conclude that when $\lambda_B=1$, the impossibility to prohibit resale results in a loss of expected revenue for the monopolist if and only if $M(\underline{\theta}_T)>\max\{0,M(\underline{\theta}_B)\}$. ■

References

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