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Straightforward Allocation
Mechanisms*

by

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I. Introduction

A mechanism for allocating resources among several individuals is called straightforward if each individual's optimal strategy depends only on his own preferences and not on the strategies that he expects other individuals to play. Most allocation mechanisms are not straightforward. The one, major exception known to economic theory is the competitive exchange economy that contains a large number of atomless consumers and a central price setting authority. The strategies available to individuals are to report an excess demand correspondence from prices to net trades. The central authority calculates a market clearing vector of prices and, based on the reported demand correspondences, assigns each individhal a final allocation of goods. Agents' optimal strategies are to act as price takers and report their true excess demand correspondences. Because the excess demand correspondence of an agent depends only on his initial endowment and his preferences, the mechanism is straightforward.[1]

The general question with which I am concerned here is why more mechanisms are not straightforward. The specific means by which I answer this question is identification of two conditions that guarantee that an allocation mechanism is not straightforward. An informal statement of the result that I derive is this. Within the classical economic environment where the consumption set is a

subset of R^n and preferences are strictly monotonic and continuous, two conditions are jointly sufficient to rule out the possibility of constructing a straightforward allocation mechanisms. The first condition is that the mechanism allows pairs of individuals to interact and affect each other. For example, such interaction takes place within a competitive economy where individuals are sufficiently few that each can affect prices and thereby affect the welfare of the other individuals in the economy. The second condition is that the class of orderings that are admissible as individuals' preferences be broad enough. In particular, if an individual's income level and his consumption pattern at that income level is not sufficient information to make an accurate prediction of how he will change his consumption pattern in response to an income increase, then the class of admissible orderings is broad enough. This breadth requirement, of course, is almost always satisfied within economic situations; different individuals who have identical incomes and who have chosen identical consumption bundles may, for specific commodities, have widely varying income elasticities of demand. The conclusion that I draw, which answers the general question I posed above, is that the two conditions for nonstraightforwardness are almost always satisfied. Therefore construction of a straightforward allocation mechanism within an economic environment is impossible except in the special circumstance where a large number of individuals, all of whom are economically small, exist.

This paper's theorem is a generalization of two theorems, the first by Hurwicz [9] and the second by Gibbard [4] and,

independently, by me [12,13]. Hurwicz's result, like this paper's
result, is within the context of the classical economic model.
Individuals have convex, monotonic, and continuous preferences
over an n-dimensional consumption set. They report their prefer-
ences to the central authority who calculates the allocation
each receives. The preference map that an individual reports need
not be his true preference map; the only requirement is that
he reports preferences that are convex, monotonic, and continu-
ous. This type of mechanism, Hurwicz showed, cannot always (a)
grant each individual the option to retain his original endowment,
(b) generate Pareto satisfactory outcomes, and (c) provide each
individual with an optimal, straightforward strategy.

Gibbard and I have shown a similar result in the context of
voting situations. A group of individuals must select one alter-
native from a set of three or more alternatives. Individuals have
preferences over the set of alternatives and no a priori restric-
tion, such as continuity or convexity, is placed on the order in
which an individual may rank the several alternatives. Each indi-
vidual casts a ballot that consists of a ranking (indifference is
permitted) of the alternatives. Individuals may or may not report
a ranking that agrees with their true preferences. The ballots
are counted by a voting procedure that selects one alternative as
the group's choice. The theorem states that no voting procedure
exists that is both nondictatorial and straightforward.

Together these two theorems suggest that construction of a straightforward allocation mechanism may be impossible within a wide variety of environments, but neither theorem adequately characterizes these environments. Hurwicz's result depends importantly on the requirement that the mechanism allow each individual the option to retain his original endowment and thus does not focus squarely on straightforwardness. Gibbard and I in our proofs depend on the restrictive assumption that preferences have no a priori structure such as convexity, monotonicity, and continuity. Thus our result, while focusing on straightforwardness, is not applicable to classical economic environments.

This paper consists of seven sections including this introduction. I formulate the model in Section 2 and state and prove the theorem in Section 3. Sections 4 and 5 each contain an application of the theorem. The example of Section 4 is a simple, explicitly specified, exchange economy while that of Section 5 is a monopoly market. In Section 6 I argue, in more depth than I have here in the introduction, my conclusion that construction of straightforward mechanisms is impossible within economic environments that are not composed of atomless agents. Moreoever I also argue that straightforwardness, if it were attainable, would be a desirable property. Finally, in the last section I show with three short examples that my two sufficient conditions for nonstraightforwardness are in a weak, informal sense also necessary conditions.

2. Formulation

Let $I = \{1,2,\ldots,\ell\}$ be the <u>group</u> of ℓ ($\ell \geq 2$) <u>individuals</u> who must allocate a quantity of resources among themselves. The consumption set of each individual i is C_i, either the m-dimensional non-negative orthant R_+^m, $m \geq 2$, or some m-dimensional subset of R_+^m. Over C_i each individual i has a <u>utility function</u> u_i that is monotonic, continuously differentiable, and is an element of some set U_i of <u>admissible utility functions</u>.[2] Every individual $i \in I$ has an n-dimensional compact <u>strategy space</u>: $S_i \subset R^n$ where $n \geq 1$.

The outcome function F is from the strategy space $S = \Pi_{i=1}^{\ell} S_i$ into the consumption sets $C = \Pi_{i=1}^{\ell} C_i$ of the individuals. The function F breaks into components:

(2.01)
$$F(s_1,\ldots,s_\ell)$$
$$= \{F_1(s_1,\ldots,s_\ell), F_2(s_1,\ldots,s_\ell), \ldots, F_\ell(s_1,\ldots,s_\ell)\}$$
$$= \{x_1,\ldots,x_\ell\}$$

where $s_i = (s_{i1},\ldots,s_{in}) \in S_i$ is individual i's n-dimensional strategy vector and $x_i = (x_{i1},\ldots,x_{im}) \in C_i$ is individual i's m-dimensional final allocation. Each component F_i, $i \in I$, of F itself breaks into components:

(2.02)
$$F_i(s_1,\ldots,s_\ell)$$
$$= \{F_{i1}(s_1,\ldots,s_\ell), F_{i2}(s_1,\ldots,s_\ell), \ldots, F_{im}(s_1,\ldots,s_\ell)\}$$
$$= (x_{i1},x_{i2},\ldots,x_{im})$$
$$= x_i$$

where x_{ij} is the allocation person i's allocation of good j.

The one assumption I make concerning F is this.   Let
$s_{)i(} = (s_1,\ldots,s_{i-1},s_{i+1},\ldots,s_\ell)$.   If for any $i \in I$, for some
$[s_{)i(},s_i'] \in S$, and for some $[s_{)i(},s_i''] \in S$

$$(2.03) \qquad F_i[s_{)i(},s_i'] = F_i[s_{)i(},s_i''],$$

then

$$(2.04) \qquad F_j[s_{)i(},s_i'] = F_j[s_{)i(},s'']$$

for all $j \in I$.   This assumption means that person i through a change
in his strategy can affect person j's final allocation x_j only
if his change in strategy also affects his own final allocation x_i.
The justification for this assumption is that if an allocation
function did not satisfy it, then the potential for coercion would
be unbounded.   I call this assumption the <u>nonzero cost assumption</u>.[3]

Competitive mechanisms, for example, always satisfy this
assumption because the only manner in which one person can affect
another person's allocation is to cause a change in price.   If a
person changes his strategy in such a manner that his final allo-
cation remains unchanged, then the original prices remain the market
clearing prices. Therefore, because no price change is caused by the
first person's change in strategy, no change in a second person's final
allocation is caused.   All other mechanisms of which I am able
to conceive also satisfy the nonzero cost assumption.   For example,
consider mechanisms that involve fixing levels of public goods'
production.   If one person changes his strategy in a manner that
leaves his allocation of public goods unchanged, then by definition

that change also leaves all other people's allocation of public goods unchanged, which is exactly the result that the nonzero cost assumption requires.

These basic definitions enable me to define the entity that is an allocation mechanism and the property that is straight-forwardness. An allocation mechanism Ω is the $3\ell+1$ tuple $\langle C_1,\ldots,C_\ell,U_1,\ldots,U_\ell,S_1,\ldots,S_\ell,F\rangle$. I define straightforwardness with respect to specific pairs of individuals. Let $\xi\in I$ and $\eta\in I$ be the indices of these two distinct individuals and let the strategies of all the other individuals be fixed at $\bar{s}_{)\xi,\eta(} = (\bar{s}_1,\ldots,\bar{s}_{\xi-1}, s_{\xi+1},\ldots,\bar{s}_{\eta-1},\bar{s}_{\eta+1},\ldots,\bar{s}_\ell) \in \prod_{\eta\neq i\neq\xi} S_i$. The allocation mechanism Ω is <u>straightforward</u> <u>for</u> ξ <u>and</u> η <u>at</u> $\bar{s}_{)\xi,\eta(}$ if and only if single-valued functions σ_ξ and σ_η, with domains U_ξ and U_η and ranges contained in S_ξ and S_η, exist such that, for all $u_\xi\in U_\xi$, $s_\xi\in S_\xi$, $u_\eta\in U_\eta$, and $s_\eta\in S_\eta$,

(2.05)
$$u_\xi\{F_\xi[\sigma_\xi(u_\xi),s_\eta,\bar{s}_{)\xi,\eta(}]\}$$

$$= \max_{s_\xi\in S_\xi} u_\xi\{F_\xi[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}]\}$$

and

(2.06)
$$u_\eta\{F_\eta[s_\xi,\sigma_\eta(u_\eta),\bar{s}_{)\xi,\eta(}]\}$$

$$= \max_{s_\eta\in S_\eta} u_\eta\{F_\eta[s_\xi,s_\eta,\bar{s}_{)\xi,\eta(}]\}.$$

The strategy $s_\xi = \sigma_\xi(u_\xi)$ is ξ's <u>straightforward</u> <u>strategy</u> <u>at</u>

$\overline{s})_{\xi,\eta(}$ and the strategy $s_\eta = \sigma_\eta(u_\eta)$ is person η's <u>straightforward</u> <u>strategy at</u> $\overline{s})_{\xi,\eta(}$. Thus person ξ's straightforward strategy at $\overline{s})_{\xi,\eta(}$ is a pure dominant strategy vis-a-vis person η's choice of strategy and similarly, η's straightforward strategy at $\overline{s})_{\xi,\eta(}$ is a pure dominant strategy vis-a-vis ξ's choice of strategy.

This particular concept of straightforwardness is closely related to, but less demanding than the concept of straightforwardness that Gibbard [4] formulated. His definition is that an allocation mechanism Ω is straightforward if and only if, for each person $i \in I$, a singlevalued function σ_i^* with domain U_i and range contained in S_i exists such that, for any $u_i \in U_i$ and any $s)_{i(} = [s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_\ell) \in \prod_{j \neq i} S_j$,

(2.07) $u_i \{ F_i [\sigma_i^*(u_i), s)_{i(}] \}$

$$= \max_{s_i \in S_i} u_i \{ F_i [s_i, s)_{i(}] \} \ .$$

The strategy $s_i = \sigma_i^*(u_i)$ is called person i's straightforward strategy. Thus each person's straightforward strategy is a pure dominant strategy vis-a-vis every one else's choice of strategy and, consequently, every ℓ-tuple of straightforward strategies is a Nash equilibrium. Evidently a mechanism that is straightforward is straightforward for any pair of people ξ and η at any point $\overline{s})_{\xi,\eta(}$. The converse, however, is not true. I use this weaker concept because it suffices and because, as I show in Section 5, it permits the analysis of situations that can not be analyzed using the stronger concept.

Suppose an allocation mechanism $\Omega = \langle C_1, \ldots, C_\ell, U_1, \ldots, U_\ell,$
$S_1, \ldots, S_\ell, F \rangle$ is straightforward for persons ξ and η at $\bar{s}_{)\xi,\eta(}$.
A reference quintuple $V = \langle \bar{u}_\xi, \bar{u}_\eta, \bar{s}_{)\xi,\eta(}, S_\xi^*, S_\eta^* \rangle$ for Ω is constructed
as follows. Define functions f_ξ and f_η to be:

$$(2.08) \qquad f_\xi(s_\xi, s_\eta) = F_\xi[s_\xi, s_\eta, \bar{s}_{)\xi,\eta(}],$$

$$(2.09) \qquad f_\eta(s_\xi, s_\eta) = F_\eta[s_\xi, s_\eta, \bar{s}_{)\xi,\eta(}].$$

Pick utility functions $\bar{u}_\xi \in U_\xi$ and $\bar{u}_\eta \in U_\eta$ for individuals ξ and η.
They have straightforward strategies $\bar{s}_\xi = \sigma_\xi(\bar{u}_\xi)$ and $\bar{s}_\eta = \sigma_\eta(\bar{u}_\eta)$
respectively. Finally pick open neighborhoods $S_\xi^* \subset \text{int } S_\xi$ and $S_\eta^* \subset$
$\text{int } S_\eta$ that contain $\bar{s}_\xi$ and $\bar{s}_\eta$ respectively.[4]

My reason for picking the reference quintuple $V = \langle \bar{u}_\xi, \bar{u}_\eta,$
$\bar{s}_{)\xi,\eta(}, S_\xi^*, S_\eta^* \rangle$ is to make statement and proof of the theorem pos-
sible in terms of Ω's local properties. Assume that Ω is straight-
forward for persons ξ and η at $\bar{s}_{)\xi,\eta(}$. I show that if the outcome
function F satisfies certain properties in the region $S_\xi^* \times S_\eta^*$ sur-
rounding $(\bar{s}_\xi, \bar{s}_\eta, \bar{s}_{)\xi,\eta(})$ and if person η's class U_η of admissible
utility functions has sufficient variety, then, contrary to the
definition of straightforwardness, person ξ has an incentive
to change his strategy from $\bar{s}_\xi$ as person η's preferences (and
therefore strategy) vary from $\bar{u}_\eta$. Consequently the selection
of the reference quintuple V is guided by the question: does a V ex-
ist at which Ω locally satisfies the aforesaid conditions on F and U_η?

If so, then the assumption that Ω is straightforward for persons ξ and η at $\overline{s}_{)\xi,\eta(}$ is contradicted.

Formal statement of these requirements on F and U_η require several additional, preparatory defnitions. Let the functions $\overline{g}_\xi$ and $\overline{g}_\eta$ be:

$$(2.10) \qquad \overline{g}_\xi(s_\eta) = \{x_\xi | x_\xi = f_\xi(s_\xi, s_\eta), \quad s_\xi \in S_\xi\},$$

$$(2.11) \qquad \overline{g}_\eta(s_\xi) = \{x_\eta | x_\eta = f_\eta(s_\xi, s_\eta), \quad s_\eta \in S_\eta\} \ .$$

These functions are the feasible sets for ξ and η respectively, given the other's strategy. Since each person has a monotonic utility function he chooses only from the frontier of his feasible set. These frontiers, which may be regarded as offer curves, are:

$$(2.12) \qquad g_\xi(s_\eta) = \{x_\xi | x_\xi \in \overline{g}_\xi(s_\eta) \ \& \ [y_\xi \in \overline{g}_\xi(s_\eta)$$

$$\Rightarrow (y_\xi - x_\xi) \text{ is not strictly monotonic}]\},$$

$$(2.13) \qquad g_\eta(s_\xi) = \{x_\eta | x_\eta \in \overline{g}_\eta(s_\xi) \ \& \ [y_\eta \in \overline{g}_\eta(s_\xi)$$

$$\Rightarrow (y_\eta - x_\eta) \text{ is not strictly monotonic}]\}.$$

A vector $\theta \in R^m$ is strictly monotonic if and only if each of its components is positive. An allocation mechanism is called continuously differentiable at the reference quintuple V if and only if f_ξ and f_η are both continuously differentiable within the region $S^* = S_\xi^* \times S_\eta^*$.

Finally, for the last preliminary definition, suppose F is continuously differentiable at V. Let $A\{[0,1]\}$ and $B\{[0,1]\}$ be smooth, rectifiable, simple curves contained respectively in S_ξ^* and S_η^*.[5] As such they break into components: for all $p \in [0,1]$,

$$(2.14) \qquad A(p) = \{A_1(p),\ldots,A_n(p)\} = \{s_{\xi 1},\ldots,s_{\xi n}\} = s_\xi \in S_\xi^*,$$

$$(2.15) \qquad B(p) = \{B_1(p),\ldots,B_n(p)\} = \{s_{\eta 1},\ldots,s_{\eta n}\} = s_\eta \in S_\eta^*.$$

Define, for all $p \in (0,1)$:

$$(2.16) \qquad v_\xi[\bar{s}_\xi, B(p)] = \{ \sum_{j=1}^{n} \frac{\partial f_{\xi 1}}{\partial s_{\eta j}} \frac{\partial B_j}{\partial p} ,\ldots, \sum_{j=1}^{n} \frac{\partial f_{\xi m}}{\partial s_{\eta j}} \frac{\partial B_j}{\partial p} \};$$

$$(2.17) \qquad v_\eta[A(p), \bar{s}_\eta] = \{ \sum_{j=1}^{n} \frac{\partial f_{\eta 1}}{\partial s_{\xi j}} \frac{\partial A_j}{\partial p} ,\ldots, \sum_{j=1}^{n} \frac{\partial f_{\eta m}}{\partial s_{\xi j}} \frac{\partial A_j}{\partial p} \} .$$

The vector v_ξ is the velocity with which person ξ's allocation changes as person η's strategy varies along the curve B. The vector v_η has the same interpretation, _mutatis mutandis._

The requirement on F and the requirement on U_η, which in the presence of straightforwardness are inconsistent with each other, may now be stated. The requirement on F is that smooth, rectifiable, and simple curves $A\{[0,1]\} \subset S_\xi^*$ and $B\{[0,1]\} \subset S_\eta^*$ exist that are suitable at V. The two curves A and B are _suitable at_ V if and only if: (a) F is continuously differentiable at V; (b) (b) $A(\frac{1}{2}) = \sigma_\xi(\bar{u}_\xi) = \bar{s}_\xi$ and $B(\frac{1}{2}) = \sigma_\eta(\bar{u}_\eta) = \bar{s}_\eta$; and (c), for all $p \in [0,1]$,

$$(2.18) \qquad \frac{d\bar{u}_\eta}{dp} = \nabla \bar{u}_\eta \{ f_\eta [A(p), \bar{s}_\eta] \} \cdot v_\eta [A(p), \bar{s}_\eta] > 0$$

and

$$(2.19) \qquad \frac{d\bar{u}_\xi}{dp} = \nabla \bar{u}_\xi \{ f_\xi [\bar{s}_\xi, B(p)] \} \cdot v_\xi [\bar{s}_\xi, B(p)] > 0.$$

Inequality (2.18) states that if person ξ changes his strategy along the curve A, then each increment of change favorably affects person η. Inequality (2.19) reverses the players roles. Thus this requirement that suitable curves exist is the requirement that individuals ξ and η interact and affect each other both favorably and unfavorably. As such it plays the role that the non-dictatorship and citizens' sovereignty requirements play in the impossibility results that Arrow [2] and others have proven within social choice theory.

The second requirement is that the set U_η of admissible utility functions be broad at V with respect to the suitable curve A. Suppose F is continuously differentiable at V and suppose that the curve $A\{[0,1]\}$ is suitable with respect to V. Let $\bar{x}_\eta = f_\eta(\bar{s}_\xi, \bar{s}_\eta) = f_\eta[A(\tfrac{1}{2}), B(\tfrac{1}{2})] = f_\eta[\sigma_\xi(\bar{u}_\xi), \sigma_\eta(\bar{u}_\eta)]$. The set U_η is A-V broad if and only if a scalar $\mu > 0$ exists such that the following condition is satisfied for every $p'_\xi \in (\tfrac{1}{2}, 1)$. If $z_\eta = f_\eta[A(p'_\xi), \bar{s}_\eta]$ and if y_η is any allocation such that both

$$(2.20) \qquad y_\eta \in \{ g_\eta(\bar{s}_\xi) \cap f_\eta(\bar{s}_\xi, s_\eta^*) \}$$

and

$$(2.21) \qquad \frac{|y_\eta - \bar{x}_\eta|}{|z_\eta - \bar{x}_\eta|} < \mu,$$

then a $u'_\eta \in U_\eta$ exists such that

$$(2.22) \qquad f_\eta[\overline{s}_\xi, \sigma_\eta(u'_\eta)] = y_\eta$$

and

$$(2.23) \qquad f_\eta[A(p'_\xi), \sigma_\eta(u'_\eta)] = z_\eta .$$

Recall that $g_\eta(\overline{s}_\xi)$ is the offer curve that faces η when ξ plays strategy $\overline{s}_\xi$.

Figure 1 shows the type of situation that must always be constructible if U_η is A-V broad. The definition of σ_η implies that $\overline{x}_\eta$ and z_η are points of maximum utility for person η given that his true utility function is $\overline{u}_\eta$ and given that person ξ's strategy is, respectively, $\overline{s}_\xi$ and $A(p'_\xi)$. In other words, the indifference curves generated by $\overline{u}_\eta$ are tangent to $g_\eta(\overline{s}_\xi)$ at $\overline{x}_\eta$ and to $g_\eta[A(p'_\xi)]$ at z_η. Similarly the indifference curves generated by u'_η are tangent to $g_\eta(\overline{s}_\xi)$ at y_η and to $g_\eta[A(p'_\xi)]$ at z_η. Thus broadness requires that U_η contain enough variety of admissible preferences to insure that at points such as z_η several possible trajectories of allocations exist and cross.

The purpose of the requirement that y_η may be any point such that (2.20) and (2.21) are satisfied is to guarantee that the angle at which the trajectories through z_η may cross does not approach zero as z_η is chosen increasingly close to $\overline{x}_\eta$. In other words, (2.20) and (2.21) guarantee that no neighborhood of $\overline{x}_\eta$ exists in which only degenerate crossings of trajectories occur.

Intuitively the broadness requirement may be considered as a requirement on η's income elasticity of demand at z_η. On Figure 1 the shifting out of the offer curve from $g_\eta(\bar{s}_\xi)$ to $g_\eta[A(p')]$ is primarily an income increase for person η. If a u'_η does not exist such that the trajectories $f_\eta[A\{[0,1]\}, \sigma_\eta(\bar{u}_\eta)]$ and $f_\eta[A\{[0,1]\}, \sigma_\eta(u'_\eta)]$ cross at z_η, then, for all $u_\eta \in U_\eta$ such that $f_\eta[A(p'), \sigma_\eta(u_\eta)] = z_\eta$ person η's income elasticity of demand at z_η is a constant no matter what his preferences are. If on the other hand a u'_η does exist such that the two trajectories cross at z_η, then η's income elasticity at z_η varies depending on whether his preferences are $\bar{u}_\eta$ or u'_η.[6] Consequently, within economic contexts where a priori restrictions on individual's tastes that go beyond monotonicity and concavity of utility functions are difficult to justify, the requirement that U_η is A-V broad is quite weak.

3. The Theorem and Proof

Theorem. Consider an allocation mechanism $\Omega = \langle C_1,\ldots,C_\ell,\ U_1,\ldots,U_\ell,\ S_1,\ldots,S_\ell,\ F \rangle$ and suppose that it is straightforward for persons ξ and η at $\bar{s}_{)\xi,\eta(}$. Pick a reference quintuple $V = \langle \bar{u}_\xi, \bar{u}_\eta, \bar{s}_{)\xi,\eta(}, S_\xi^*, S_\eta^* \rangle$. If F is continuously differentiable at V, then the existence of curves A and B that are suitable with respect to V is inconsistent with U_η being A-V broad.

Proof. Step 1. Suppose that an allocation mechanism Ω is straightforward and continuously differentiable at some reference quintuple $V = \langle \bar{u}_\xi, \bar{u}_\eta, \bar{s}_{)\xi,\eta(}, S_\xi^*, S_\eta^* \rangle$. For convenience let $\xi=1$ and $\eta=2$. Suppose also that A and B are two curves that are suitable with respect to V. Finally suppose that, contrary to the theorem, U_2 is A-V broad. I shall show that this hypothesis leads to a contradiction.

Step 2. Define $u_1^*(p_1,p_2) = \bar{u}_1\{f_1[A(p_1),B(p_2)]\}$ over the domain $P^2 = [0,1]\times[0,1]$. Three properties are of interest:

(3.01) u_1^* is continuously differentiable over P^2,

(3.02) $\dfrac{\partial u_1^*(\frac{1}{2},p_2)}{\partial P_1} = 0$ per all $p_2 \in (0,1)$;

(3.03) $\dfrac{\partial u_1^*(\frac{1}{2},p_2)}{\partial P_2} > 0$ for all $p_2 \in (0,1)$.

Property (3.01) is a consequence of Ω being continuously differentiable at V and of curves A and B being smooth. Property (3.02) is a consequence of Ω being straightforward, of $A(\frac{1}{2}) = \bar{s}_1 = \sigma_1(\bar{u}_1)$ being person one's straightforward strategy, and of u_1^* being continuously differentiable. Property (3.03) is a consequence of curve B being suitable -- see inequality (2.19).

Because u_1^* is continuously differentiable on P^2, it has a differential h at $\bar{p} = (\bar{p}_1, \bar{p}_2) = (\frac{1}{2}, \frac{1}{2})$:

$$(3.04) \quad h(\bar{p}, p) = \nabla u_1^*(\bar{p}) \cdot p$$

where $p \in P^2$. The definition of the differential states that, for every $\varepsilon > 0$, a neighborhood of $\bar{p}$, denoted by $N(\varepsilon, \bar{p}) \subset P^2$, exists such that[7]

$$(3.05) \quad |u_1^*(p) - u_1^*(\bar{p}) - \nabla u_1^*(\bar{p}) \cdot (p - \bar{p})| < \varepsilon |p - \bar{p}|$$

for all $p = (p_1, p_2) \in N(\varepsilon, \bar{p})$.

Step 3. Pick an $\varepsilon > 0$. Let $N(\varepsilon, \bar{p})$ be the neighborhood such that (3.05) is satisfied. Pick points $p^z = (p_1^z, \frac{1}{2}) \in N(\varepsilon, \bar{p})$ and $p^y = (\frac{1}{2}, p_2^y) \in N(\varepsilon, \bar{p})$ such that, for some admissible utility function $u_2' \in U_2$,

$$(3.06) \quad p_1^z > \frac{1}{2} \quad \text{and} \quad p_2^y < \frac{1}{2};$$

(3.07) $z_2 = f_2 [A(p_1^z), \bar{s}_2]$ and $y_2 = f_2 [\bar{s}_1, B(p_2^y)]$

(3.08) $\dfrac{|y_2 - \bar{x}_2|}{|z_2 - \bar{x}_2|} > \frac{1}{2}\mu$

(3.09) $z_2 = f_2 [A_1(p_1^z), \sigma_2(u_2')]$ and $y_2 = f_2 [A(\frac{1}{2}), \sigma_2(u_2')]$.

where μ is the positive scalar constant that appears in the defi-
nition of A-V broad classes of preferences and where
$\bar{x}_2 = f_2 (\bar{s}_1, \bar{s}_2)$. If p_ξ' is allowed to represent p_1^z, then Figure 1
shows the configuration these points have within person two's
consumption set.

The existence of such a set of points is not immediately
clear. Nevertheless, they do exist and can be found by the
following algorithm. Pick a point $_{(1)}p^z = (_{(1)}p_1^z, \frac{1}{2}) \in N(\epsilon, \bar{p})$ such
that $\frac{1}{2} < _{(1)}p_1^z < 1$. This determines $_{(1)}z_2 = f_2 [A(_{(1)}p^z), \bar{s}_2]$.
Pick a point $_{(1)}y_2 \in f_2 [\bar{s}_1, B\{(0, \frac{1}{2})\}]$ such that

(3.10) $\mu > \dfrac{|_{(1)}y_2 - \bar{x}_2|}{|_{(1)}z_2 - x_2|} > \frac{1}{2}\mu$.

This point $_{(1)}y_2$ will not exist if

(3.11) $\frac{1}{2}\mu > \dfrac{|f_2 [\bar{s}_1, B(0)] - \bar{x}_2|}{|_{(1)}z_2 - \bar{x}_2|}$.

In that event pick a point $_{(2)}p^z = (_{(2)}p_1^z, \frac{1}{2})$ such that
$\frac{1}{2} < _{(2)}p_1^z < _{(1)}p_1^z$. If $_{(2)}p_1^z$ is made close enough to $\frac{1}{2}$,

then a $_{(2)}y_2$ will exist such that

(3.12) $\quad \mu > \dfrac{|_{(2)}y_2 - \overline{x}_2|}{|_{(2)}z_2 - \overline{x}_2|} > \frac{1}{2}\mu$

where $_{(2)}z_2 = f_2[A(_{(1)}p^z), \overline{s}_2]$. This is because $f_2[A(p_1), \overline{s}_2]$ is continuous. Therefore, as p_1 approaches $\frac{1}{2}$ from above,

(3.13) $\quad |z_2 - \overline{x}_2| = |f_2[A(p_1), \overline{s}_2] - \overline{x}_2|$

is a quantity that approaches zero from above.

Requirements (3.06) through (3.08) are therefore satisfied; the only remaining requirement to check is (3.09). It is satisfied because requirement (3.08) and the assumption that U_2 is A-V broad together imply a $u_2' \in U_2$ exists such that:

(3.14) $\quad z_2 = f_2[A(p_1^z), \sigma_2(u_2')]$

(3.15) $\quad y_2 = f_2[\overline{s}_1, \sigma_2(u_2')]$

Therefore associated with each $\epsilon > 0$ are points $p^z, p^y \in N(\epsilon, \overline{p})$ that satisfy (3.06)-(3.09).

This algorithm for finding the points (z_2, y_2) makes clear the existence of the following infinite sequence. Given the two points $_{(1)}p^z$, $_{(1)}p^y \in N(\epsilon, \overline{p})$ satisfying (3.06)-(3.09),

an infinite sequence of paired points

$$(3.16) \quad \{ [_{(1)}p^z, _{(2)}p^y], \; [_{(2)}p^z, _{(2)}p^y], \dots, [_{(n)}p^z, _{(n)}p^y], \dots \}$$

exists such that each pair $[_{(n)}p^z, _{(n)}p^y]$ satisfies (3.06)-(3.09), $_{(n)}p_1^z$ converges to $\frac{1}{2}$ from above, and $_{(n)}p_2^y$ converges to $\frac{1}{2}$ from below. Note that the construction of this sequence, together with (3.02) and (3.03), implies that, for all n:

$$(3.17) \quad \nabla u_1^*(\bar{p}) \cdot [_{(n)}p^z - \bar{p}] = \frac{\partial u_1^*(\bar{p})}{\partial P_1} \cdot (_{(n)}p_1^z - \tfrac{1}{2}) = 0;$$

$$(3.18) \quad \nabla u_1^*(\bar{p}) \cdot [_{(n)}p^y - \bar{p}] = \frac{\partial u_1^*(\bar{p})}{\partial P_2} \cdot (_{(n)}p_2^y - \tfrac{1}{2}) < 0.$$

The sequence (3.16), the equality (3.17), and the inequality (3.18) play key roles later in the proof.

Step 4. Because $f_2[A(p_1), B(p_2)]$ is continuously differentiable on P^2 it satisfies a Lipschitz condition at $\bar{p} = (\tfrac{1}{2}, \tfrac{1}{2})$: for some scalar $M > 0$ and some neighborhood $N(\bar{p})$ of $\bar{p}$

$$(3.19) \quad |f_2[A(p_1), B(p_2)] - \bar{x}_2| < M|p - \bar{p}|$$

for all $p = (p_1, p_2) \in N(\bar{p})$. Recall, from (3.06) through (3.09),

that

(3.20) $y_2 = f_2[\bar{s}_1, B(p_2^y)]$ and $z_2 = f_2[A(p_1^z), \bar{s}_2]$

(3.21) $p_2^y < \frac{1}{2}$

(3.22) $|y_2 - \bar{x}_2| > \frac{1}{2} \mu |z_2 - \bar{x}_2|$.

Assume that $p^y = (\frac{1}{2}, p_2^y) \in N(\bar{p})$. Substitution into (3.19) gives

(3.23) $|y_2 - \bar{x}_2| < M|p^y - \bar{p}| = M(\frac{1}{2} - p_2^y)$.

Finally substitution of (3.23) into (3.22) gives

(3.24) $2 \frac{M}{\mu} (\frac{1}{2} - p_2^y) > |z_2 - \bar{x}_2|$.

This becomes, after simplification,

(3.25) $0 < p_2^y < \frac{1}{2} \{ 1 - \frac{\mu}{M} |f_2[A(p_1^z), \bar{s}_2] - \bar{x}_2| \}$,

for all $p^y = (\frac{1}{2}, p_2^y) \in N(\bar{p})$. This is a useful upper bound on p_2^y that is used in step 7.

   Step 5. The assumption that B is a suitable curve, which means it satisfies (2.19), together with my choice of p_2^y such that it is less than $\frac{1}{2}$ implies that

(3.26) $\bar{u}_1\{f_1[\bar{s}_1, \bar{s}_2]\} > \bar{u}_1\{f_1[\bar{s}_1, B(p_2^y)]\}$.

Because Ω is straightforward for person one

(3.27) $\quad \bar{u}_1\{f_1[\bar{s}_1,\sigma_2(u_2')]\} \geq \bar{u}_1\{f_1[A(p_1^Z), \sigma_2(u_2')]\}.$

From (3.07) and (3.08)

(3.28) $\quad z_2 = f_2[A(p_1^Z),\bar{s}_2] = f_2[A(p_1^Z), \sigma_2(u_2')]$

and

(3.29) $\quad y_2 = f_2[\bar{s}_1,B(p_2^y)] = f_2[\bar{s}_1, \sigma_2(u_2')].$

The nonzero cost assumption on F therefore implies that

(3.30) $\quad f_1[A(p_1^Z), \bar{s}_2] = f_1[A(p_1^Z), \sigma_2(u_2')]$

and

(3.31) $\quad f_1[\bar{s}_1,B(p_2^y)] = f_1[\bar{s}_1, \sigma_2(u_2')].$

Substitution of (3.30) and (3.31) into (3.27) and comparison with (3.26) gives

(3.32) $\quad \bar{u}_1\{f_1[\bar{s}_1,\bar{s}_2]\} > \bar{u}_1\{f_1[\bar{s}_1,B(p_2^y)]\} \geq \bar{u}_1\{f_1[A(p_1^Z),\bar{s}_2]\}.$

Restated this is:

$$(3.33) \quad u_1^*(\overline{p}) > u_1^*(p^y) \geq u_1^*(p^z).$$

This inequality is used in step 6.

Step 6. Relation (3.05) applies to both $p^z \in N(\epsilon,\overline{p})$ and $p^y \in N(\epsilon,\overline{p})$:

$$(3.34) \quad |u_1^*(p^y) - u_1^*(\overline{p}) - \nabla u_1^*(\overline{p}) \cdot (p^y-\overline{p})| < \epsilon|p^y-\overline{p}|;$$

$$(3.35) \quad |u_1^*(p^z) - u_1^*(\overline{p}) - \nabla u_1^*(\overline{p}) \cdot (p^z-\overline{p})| < \epsilon|p^z-\overline{p}|.$$

Substitution of (3.17) into (3.35) gives

$$(3.36) \quad |u_1^*(p^z) - u_1^*(\overline{p})| < \epsilon|p^z-\overline{p}|.$$

The inequality (3.33) implies that (3.36) may be written as

$$(3.37) \quad |u_1^*(\overline{p}) - u_1^*(p^y)| < \epsilon|p^z-\overline{p}|.$$

Inequalities (3.34) and (3.37) may be added:

$$(3.38) \quad |u_1^*(p^y)-u_1^*(\overline{p}) - \nabla u_1^*(\overline{p}) \cdot (p^y-\overline{p})|+|u_1^*(\overline{p}) - u_1^*(p^y)|$$

$$< \epsilon \ \{|p^y-\overline{p}|+|p^z-\overline{p}|\}.$$

The triangle inequality simplifies the left hand side:

$$(3.39) \quad |\nabla u_1^*(\overline{p}) \cdot (p^y-\overline{p})| < \epsilon \ \{|p^y-\overline{p}| + |p^z-\overline{p}|\}.$$

Relation (3.18) and the definitions of the points $\bar{p}$, p^y, and p^z allows further simplification:

$$(3.40) \quad \frac{\partial u_1^*(\bar{p})}{\partial P_2} \; (\tfrac{1}{2} - p_2^y) \; < \; \epsilon\{(\tfrac{1}{2} - p_2^y) + (p_1^z - \tfrac{1}{2})\}$$

or, rewriting in a particular manner,

$$(3.41) \quad \frac{\partial u_1^*(\bar{p})}{\partial P_2} \; < \; \epsilon\{1 + \frac{p_1^z - \tfrac{1}{2}}{\tfrac{1}{2} - p_2^y}\}.$$

Inequality (3.41) is true for any $(p^z, p^y) \in N(\bar{p}, \epsilon) \times N(\bar{p}, \epsilon)$ pair that satisfies (3.06) through (3.09). Finally from (3.03) I know that

$$(3.42) \quad \frac{\partial u_1^*(\bar{p})}{\partial P_2} \; > \; 0.$$

The last step of the proof is to show that (3.41) leads to a contradiction of (3.42).

   Step 7. Recall from step 3 that, given the arbitrarily chosen $\epsilon > 0$, an infinite sequence of pairs $\{_{(n)}p^z, _{(n)}p^y\}$ exists such that each pair satisfies (3.06) through (3.09), $_{(n)}p_1^z$ approaches $\tfrac{1}{2}$ from above, and $_{(n)}p_2^y$ approaches $\tfrac{1}{2}$ from below. Since I may choose ϵ arbitrarily close to zero, if I can show that in equation (3.41) the term

$$(3.43) \quad \frac{_{(n)}p_1^z - \tfrac{1}{2}}{\tfrac{1}{2} - _{(n)}p_2^y}$$

does not explode as n goes to infinity, then I have succeeded in contradicting (3.42). The calculation that (3.43), in fact, does not explode is this:

$$(3.44) \qquad 0 < \frac{(n)^{p_1^z - \frac{1}{2}}}{\frac{1}{2} - (n)^{p_2^y}}$$

$$< \frac{(n)^{p_1^z} - \frac{1}{2}}{\frac{1}{2} - \frac{1}{2}\{1 - \frac{\mu}{M}|f_2[A((n)^{p_1^z}),\bar{s}_2] - \bar{x}_2|\}}$$

$$= \frac{2M}{\mu} \left\{ \frac{(n)^{p_1^z - \frac{1}{2}}}{|f_2[A((n)^{p_1^z}),\bar{s}_2] - \bar{x}_2|} \right\}$$

where the second line is a substitution of (3.25). That substitution is valid provided that n is picked large enough that $(n)^{p^y} = (\frac{1}{2}, (n)^{p_2^y}) \in N(\bar{p})$. The last line of (3.44) is of the indeterminate form $0/0$ as n goes to infinity (or equivalently as p_1^z goes to $\frac{1}{2}$ from above). When evaluated by L'Hopital's rule the result is:

$$(3.45) \qquad \lim_{n \to \infty} \frac{2M}{\mu} \left\{ \frac{(n)^{p_1^z - \frac{1}{2}}}{|f_2[A((n)^{p_1^z}),\bar{s}_2] - x_2|} \right\}$$

$$= \frac{2M}{\mu|v_2[A(\frac{1}{2}),\bar{s}_2]|} \ .$$

Because the curve A is suitable, $|v_2[A(\tfrac{1}{2}),\bar{s}_2]|\neq 0$. If it did
equal zero, then (2.17) would be violated and A would not be
a suitable curve. Consequently, because M, μ, and $|v_2[A(\tfrac{1}{2}),\bar{s}_2]|$
are positive, finite numbers, the limit (3.45) is finite.

Therefore, based on (3.42), (3.44) and (3.45),
I can rewrite relation (3.41) as:

$$(3.46) \quad 0 < \frac{\partial u_1^*(\bar{p})}{\partial p_2} < \lim_{n\to\infty} \; \epsilon\{1 + \frac{(n)^{p_1^{z+\frac{1}{2}}}}{\frac{1}{2}(n)^{p_2^y}}\}$$

$$< \epsilon\{1 + \frac{2M}{\mu|v_2[A(\tfrac{1}{2}),\bar{s}_2]|}\}.$$

The right hand side of (3.46) can be made arbitrarily close
to zero because, in step 3, ϵ may be selected arbitrarily close
to zero and because the value of the term

$$(3.47) \quad \{1 + \frac{2M}{\mu|v_2[A(\tfrac{1}{2}),\bar{s}_2)|}\}$$

is not dependent on the choice of ϵ. This creates the contradiction
I have been seeking: $\partial u_1^*(\bar{p})/\partial p_2$ can not be a constant that
is simultaneously greater than zero and arbitrarily close to zero.
Therefore the proof is complete.

4.   Nonstraightforwardness of a Simple Exchange Economy.

In this section I illustrate the theorem by showing that a simple competitive exchange economy is not straightforward.[8]   The group consists of two people who are trading two goods.   Both begin with initial endowments of one unit of good x and one unit of good y.   Each individual has preferences that can be described by a utility function that is a member of the class of utility functions that Stone [14] used in his econometric studies of demand.

Specifically I define U_i, i $\in$ {1,2}, to consist of every utility function of the form

(4.01) $u_i(x_i,y_i) = \alpha_i \log_e(x_i-a_i) + (1-\alpha_i)\log_e(y_i-b_i)$

where the parameters are subject to the bounds

(4.02) $.4 \leq \alpha_i \leq .6,$

$.0 \leq a_i \leq .3,$

$.0 \leq b_i \leq .3.$

The pair (x_i,y_i) is person i's allocation of goods x and y.
For convenience,   let U_i be represented by the set of triples (α_i,a_i,b_i) that satisfy (4.02).   The interpretation of a_i is that it is the minimum quantity of good x that person i needs for survival.   Note that if both $a_i = 0$ and $b_i = 0$, then u_i has the familiar Cobb-Douglas form.

Person i has true preferences $\bar{u}_i \in U_i$; these preferences are fully described by a triple $(\bar{\alpha}_i,\bar{a}_i,\bar{b}_i)$ that satisfies the bounds of

(4.02), i.e. $(\alpha_i, a_i, b_i) \in U_i$. Let the consumption sets be $C_i = [.4, 1.6]$ x [.4, 1.6]. Given the bounds of (4.02), the utility functions $\overline{u}_i \in U_i$ are defined for every $(x_i, y_i) \in C_i$.

The strategy s_i for each person is to report an admissible utility function $u_i \in U_i$. He reports it by specifying a triple (α_i, a_i, b_i) that satisfies (4.02). Thus $S_i = [0.4, 0.6]$ x $[0.0, 0.3]$ x $[0.0, 0.3]$. He may state whatever parameters he calculates are to his advantage; they need not coincide with his true values $(\overline{\alpha}_i, \overline{a}_i, \overline{b}_i)$.

The outcome function F picks the reallocation of goods among the two individuals that is a competitive equilibrium with respect to the utility functions they reported. To be specific, let, for i=1,2, (α_i, a_i, b_i) be the parameters that are reported and $(\overline{\alpha}_i, \overline{a}_i, \overline{b}_i)$ be the true parameters. Let good x have price p and let good y be the numeraire. Recall that the initial endowments for each are one unit of x and one unit of y. Taking the price p as given, utility maximization implies the following demand curves:

(4.03) $x_i = \alpha_i + a_i(1-\alpha_i) + \dfrac{\alpha_i(1-b_i)}{p}$

(4.04) $y_i = (1-\alpha_i)[1 + (1-a_i)p] + \alpha_i b_i$

for $i \in \{1, 2\}$. From these the market clearing price is derived as a function of the strategies $s_1 = (\alpha_1, a_1, b_1)$ and $s_2 = (\alpha_2, a_2, b_2)$; it is:

$$(4.05) \qquad p = \frac{\alpha_1 (1-b_1) + \alpha_2 (1-b_2)}{(1-\alpha_1)(1-a_1) + (1-\alpha_2)(1-a_2)} \quad .$$

Substitution of (4.05) into (4.03) and (4.04) therefore gives the components of the outcome function:

$$(4.06) \qquad F_{i1}(s_1, s_2) = x_i = \alpha_i + a_i(1-\alpha_i)$$

$$+ \ \alpha_i(1-b_i) \ \frac{(1-\alpha_1)(1-a_1) + (1-\alpha_2)(1-a_2)}{\alpha_1(1-b_1) + \alpha_2(1-b_2)}$$

$$(4.07) \qquad F_{i2}(s_1, s_2) = y_i = (1-\alpha_i) \left[1 + \right.$$

$$\left. (1-a_i) \ \frac{\alpha_1(1-b_1) + \alpha_2(1-b_2)}{(1-\alpha_1)(1-a_1) + (1-\alpha_2)(1-a_2)} \right] + \alpha_i \, b_i$$

where $i \in \{1,2\}$.

Equations (4.06) and (4.07) complete the description of this allocation mechanism. Now I shall apply the theorem and show that the mechanism is not straightforward between persons one and two. Suppose that it is straightforward. Therefore, for $i \in \{1,2\}$, a function σ_i exists such that person i's optimal strategy is $\tilde{s}_i = (\tilde{\alpha}_i, \tilde{a}_i, \tilde{b}_i) = \sigma_i(\bar{\alpha}_i, \bar{a}_i, \bar{b}_i)$. Suppose further that the true utility function $\bar{u}_1 = (\bar{\alpha}_1, \bar{a}_1, \bar{b}_1) \in$ int U_1 and $\bar{u}_2 = (\bar{\alpha}_2, \bar{a}_2, \bar{b}_2) \in$ int U_2 exist such that (a) trade is potentially mutually beneficial[9] and (b) $\tilde{s}_1 = \sigma_1(\bar{u}_1) \in$ int S_1 and $\tilde{s}_2 = \sigma_2(\bar{u}_2) \in$ int S_2. Let the reference quintuple be $V = \langle \bar{u}_1, \bar{u}_2, \emptyset, S_1, S_2 \rangle$. The third component of V is the null set because the group consists of only two people. Setting S_1^* and S_2^* as identical to S_1 and S_2 respectively causes no problem because F is continuously differentiable over its entire domain.

The first step in applying the theorem is to ascertain that suitable curves A and B exist. That they do is seen by noting two facts. First each individual can affect the price p either positively or negatively by appropriately changing his strategy from his straightforward strategy $\tilde{s}_i$. This follows from (4.05) and the assumption that $\tilde{s}_i \in \text{int } S_i$. Second, a change in price generally leads to either a positive or negative change in their utility. This follows by direct substitution of the demand curves (4.03) and (4.04) into the utility function (4.01) and differentiating with respect to p, e.g. for person one

$$(4.08) \quad \bar{u}_1(s_1,p) = \bar{\alpha}_1 \, \log_e\{\alpha_1 + a_1(1-a_1) + \frac{\alpha_1(1-b_1)}{p} - \bar{a}_1\}$$

$$+ (1-\bar{\alpha}_1) \, \log_e\{(1-\alpha_1)[1+(1-a_1)p] + \alpha_1 b_1 - \bar{b}_1\}$$

$$= \bar{\alpha}_1 \, \log_e(w-\bar{a}_1) + (1-\bar{\alpha}_1) \, \log_e(z-\bar{b}_1)$$

and

$$(4.09) \quad \frac{d\bar{u}_1}{dp} = -\frac{\bar{\alpha}_1}{(w-\bar{a}_1)} \, \frac{\alpha_1(1-b_1)}{p^2} + \frac{(1-\bar{\alpha}_1)}{(z-\bar{b}_1)} \, (1-\alpha_1)(1-a_1) \; .$$

Thus except at the unique value of p such the right hand side of (4.09) equals zero any change in price leads to a change in person one's utility. Consequently suitable curves A and B generally exist at $V = \langle \bar{u}_1, \bar{u}_2, \emptyset, S_1, S_2 \rangle$.

The second step in applying the theorem is to ascertain that the set U_2 of admissible utility functions is A-V broad. Figure 2

diagrams the problem. Let point $\bar{x}_2$ be person two's optimal alloca-
tion given that his utility function is $\bar{u}_2$ and person one's strategy
is $\tilde{s}_1$. If person one changes his strategy to $A(p')$, then let person
two's new optimal point be z_2. Therefore

(4.10) $\bar{x}_2 = f_2[A(\tfrac{1}{2}), \sigma_2(\bar{u}_2)] = f_2[\tilde{s}_1, \tilde{s}_2]$

and

(4.11) $z_2 = f_2[A(p'), \sigma_2(\bar{u}_2)]$

because the definition of straightforwardness states that person
i's straightforward strategy $\sigma_2(\cdot)$ always results in selection of
that feasible allocation which person i most prefers. Reference
back to the definition and accompanying diagram, Figure 1, shows
that A-V broadness essentially requires that a utility function
$u_2' = (\alpha_2', a_2', b_2') \in U_2$ exist such that z_2 remains optimal, x_2 becomes
suboptimal, and w_2 becomes optimal. If such a u_2' exists,
then, for the same reasons (4.10) and (4.11) are true, $z_2 =$
$f_2[A(p'), \sigma_2(\alpha_2', a_2', b_2')]$ and $w_2 = f_2[\tilde{s}_1, \sigma_2(\alpha_2', a_2', b_2')]$. Therefore
existence of u_2' implies that the trajectories that $\sigma_2(\bar{u}_2)$ and
$\sigma_1(u_2')$ generate as s_1 varies from $A(\tfrac{1}{2})$ to $A(1)$ cross at z_2.

Thus it is sufficient to find a triple $u_2' = (\alpha_2', a_2', b_2') \in U_2$
such that, first, the marginal rate of substitution for u_2' (denoted
by MRS') at z_2 equals the marginal rate of substitution for $\bar{u}_2$
(denoted by $\overline{MRS}$) at z_2 and, second, the MRS' at $\bar{x}_2$ does not equal
the $\overline{MRS}$ at x_2. This is easily done because U_2 is a three parameter
family of utility functions. Specifically let $\bar{M}_z$ be the $\overline{MRS}$ at z_2

and let $\overline{M}_x$ be the $\overline{MRS}$ at $\overline{x}_2$. Appropriate differentiation of (4.01) gives the $\overline{MRS}$ at any point (x_2, y_2):

$$(4.12) \qquad \overline{MRS} = - \frac{\overline{\alpha}_2 (y_2 - \overline{b}_2)}{(1 - \overline{\alpha}_2)(x_2 - \overline{a}_2)} \quad .$$

Let $z_2 = (x_z, y_z)$ and $\overline{x}_2 = (\overline{x}_x, \overline{y}_x)$. Consequently $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2)$ satisfy the equations

$$(4.13) \qquad \overline{M}_z = - \frac{\overline{\alpha}_2 (y_z - \overline{b}_2)}{(1 - \overline{\alpha}_2)(x_z - \overline{a}_2)} \quad ,$$

$$(4.14) \qquad \overline{M}_x = - \frac{\overline{\alpha}_2 (\overline{y}_x - \overline{b}_2)}{(1 - \overline{\alpha}_2)(\overline{x}_x - \overline{a}_2)} \quad .$$

Let $M_z' = \overline{M}_z$ and pick a M_x' such that $M_x' \neq \overline{M}_x$. If a $u_2' \in U_2$ exists such that its MRS' at z_2 is M_z' and its MRS' at $\overline{x}$ is M_x', then the requirements of A-V broadness are met. The parameter for such a u_2' can be found by picking a triple $(\alpha_2', a_2', b_2') \in U_2$ that satisfies the equations

$$(4.15) \qquad M_z' = - \frac{\alpha_2'(y_z - b_2')}{(1 - \alpha_2')(x_z - a_2')}$$

$$(4.16) \qquad M_x' = - \frac{\alpha_2'(y_x - b_2')}{(1 - \alpha_2')(x_x - a_2')} \quad .$$

Such a triple exists and is contained in U_2 provided that the differ-
ence between M'_x and $\overline{M}_x$ is made small. This follows from the con-
tinuity of the equations and the existence of $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2) \in$ int U_2
that satisfy equations (4.13) and (4.14). In other words, if
$\overline{M}_x$ and M'_x are picked so that their difference is small, then a
perturbation of the point $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2)$ exists such that (4.15) and
(4.16) are satisfied. Therefore U_2 is A-V broad.

This creates a contradiction. Because suitable curves A
and B exist at V and because U_2 is A-V broad, the mechanism, with
one qualification, can not be straightforward. The qualification
stems from the dependence of this demonstration on the existence
of straightforward strategies that lie within the interior of
each person's strategy space S_i. Therefore the possibility re-
mains that σ_i, $i \in \{1,2\}$, exist and always pick strategies that
lie on the frontier of S_i.

5. Straightforwardness and Nonstraightforwardness within Monopoly
   Markets

A monopolist who sells to a continuum of atomless consumers
acts straightforwardly towards each individual consumer in the
following sense. He analyses the market demand curve and his cost
curves and sets his price such that marginal cost equals marginal
revenue. He does not need to know, in order to act optimally,
what any particular consumer's preferences are. Only the aggre-
gate quantity that he expects to sell is significant. Indi-
vidual consumers also act straightforwardly towards the monopolist.
Each acts as a price taker, calculates his demand curve, and, when
the monopolist announces his price, buys the quantity specified
on his demand curve. The consumer does not modify his behavior
depending on the cost functions of the monopolist; the only
information a consumer needs is the price and his own preferences.
Behavior, however, is no longer straightforward if consumers are
not atomless. The monopolist needs to know each consumer's true
demand curve in order to determine his optimal strategy and
consumers need to know the monopolist's costs in order to determine
their optimal strategies.

My purpose in this section is twofold. First, I show that
the straightforward behavior that exists between the monopolist
and each atomless consumer is consistent with the theorem.
Second, I show that if consumers are not atomless, then the theorem
applies and the behavior between each consumer and the monopolist can
not be straightforward. Throughout this discussion the definition

of straightforwardness that I use is the narrow definition (see Section 2) of straightforwardness between a pair of individuals -- the monopolist and a specific consumer -- where the strategies of all other individuals are held fixed. The monopolist's behavior is not globally straightforward in Gibbard's sense; he must alter his strategy (the price) in response to the preferences held by groups of individuals because, even in the atomless case, groups of individuals may affect the market demand curve.

Consider first the case of a monopolized market with a continuum of atomless consumers. Let the consumer be person ξ and let the monopolist be person η. The monopolist's consumption set is all possible price-quantity pairs; his class of admissible utility functions consists of all possible profit functions that are consistent with the usual assumptions of production theory; his strategy space consists of all nonnegative prices. The consumer's consumption set consists of all price-quantity pairs that his income allows him to afford; his class of admissible preferences over the consumption set are all weak orderings that can be derived from the usual assumptions of consumer behavior; his strategy space is the set of all demand curves that are consistent with the simple maximization of admissible utility functions. Let the reference quintuple V be chosen such that (a) the monopolist's straightforward strategy, as shown in Figure 3, is to set the price P and (b) the consumer's straightforward strategy, as shown in Figure 4, is to report demand curve DD.

Given this model, how does it relate to the theorem? The answer is that a suitable curve A does not exist. In other words, the consumer through a change in his strategy is unable to affect the monopolist's utility level and therefore the theorem's hypothesis is not satisfied. Suppose, as shown on Figure 4, the consumer changes his strategy from reporting the demand curve $DD = A(\frac{1}{2})$ to reporting the demand curve $D''D'' = A(1)$. This increases the quantity he purchases from the monopolist, but, because consumers are assumed atomless, does not perceptibly shift the monopolist's optimal point on Figure 3 outward from B. Every consumer would have to change his reported demand curve from DD to $D''D''$ in order to shift the monopolist's market demand cruve from dd to $d''d''$. The individual consumer can not affect, as the definition of suitability requires, the utility level of the monopolist. Consequently the straightforwardness of this market is consistent with the theorem.

Consider now the second case where the assumption that a continuum of atomless consumers exists is replaced with the polar assumption that a bilateral monopoly exists. In this situation the theorem's requirements are met and, consequently, bilateral monopoly is not straightforward. The analysis is this. Again let the consumer be person ξ and the monopolist be person η. Assume that both have straightforward strategies. A suitable curve A may be constructed as follows. Let the consumer shift his strategy, as shown on Figure 4, from reporting demand curve $D'D' = A(0)$ to reporting $DD = A(\frac{1}{2})$ to reporting $D''D'' = A(1)$. These shifts, since the market is a bilateral monopoly, are shifts in the monopolist's

market demand curve as shown on Figure 3. Because they are outward
shifts and because, by definition, the monopolist's straightforward
strategy always results in attainment of the feasible point that
is optimal, these shifts in the demand curve result in the
monopolist's equilibrium moving in sequence from a low profit level
at point A to a high profit level at point C. The requirement that
a suitable curve A exists is thus met. A suitable curve B may be
similarly constructed. If on Figure 3 the monopolist changes his
strategy from $P'' = B(0)$ to $P = B(\frac{1}{2})$ to $P' = B(1)$, then the consumer's
utility continuously increases, which is precisely the requirement
for curve B. The last requirement of the theorem is that the
monopolist have a set of admissible preferences that is A-V broad.
That it is A-V broad is easily seen by reference to Figure 5 and
by reference back to the definition of broadness and its accompanying
diagram, Figure 1. The crossing of the marginal cost curves cc
and CC in Figure 5 is all that is necessary to produce the crossing
at z of the trajectories of the monopolist's most preferred points.
Consequently the theorem's requirements are met and bilateral
monopoly is not straightforward.

6. The Desirability and Impossibility of Straightforwardness

If an allocation mechanism is nonstraightforward, then it should be analyzed as a game of incomplete information. To see this, recall the situation of the monopolist and the consumer in the previous section's bilateral monopoly illustration. If each knows the other's preferences, then the problem is a two-person, nonzero sum, complete information game for which the core is an appropriate solution concept. This, however, is an understatement of the actual problem because the model with which this paper is concerned and which gives rise to the problem of nonstraightforwardness implicitly assumes that each individual only uncertainly knows the true preferences of the other individuals. In the bilateral monopoly illustration the monopolist is uncertain what the consumer's true, price-taking demand curve is and the consumer is uncertain what the monopolist's marginal cost curve is. Consequently it is an incomplete information game: individuals are uncertain concerning other individuals payoff functions.

This allows me to state, very briefly, my argument in favor of straightforwardness. Economic allocation mechanisms, as shown above, are games of incomplete information. Games of incomplete information have undesirable properties. The only known way in which the problems that are attendant to games of incomplete information may be solved is to redesign the game to be straightforward. Consequently straightforwardward allocation mechanisms are desirable.

The second and third steps in this argument need elabora-
tion. If the allocation mechanism is not straightforward, then
it is a nontrivial game of incomplete information which gives
rise to a host of problems. These problems are only dimly under-
stood because game theory does not have fully satisfactory concepts
of what rational behavior consists of within incomplete information
games and what types of equilibriums may result.[11] Since even
in simple, theoretical examples the analysis is very difficult,
actual players of complicated, real world, incomplete information
games may justifiably act in accordance with any of a wide variety
of rules of thumb. Consequently the outcomes and optimality of
incomplete information games is very difficult to predict or evaluate.

Nevertheless, despite the problems within the theory, the
nature of certain phenomena within games of incomplete
information are clear. First is the problem of the infinite
regress of expectations. Returning to the bilateral monopoly example,
the monopolist and the consumer each wants to play the strategy that
is optimal given the strategy the other is likely to play. In order
to calculate what the consumer is likely to play, the monopolist
needs knowledge of what the consumer's true preferences are and of
what the consumer's expectations concerning his own strategy are. For
the consumer to form reasonable expectations concerning the monopo-
list's expected strategy he needs knowledge of what the monopolist's
true preferences are and of what the monoplist's expectations con-
cerning his, the consumer's, choice of strategy is. Therefore the

monopolist, in order to choose his strategy, needs to know the
consumer's true preferences, the consumer's perception of his
own true preferences, etc. The result is an infinite regress.[12]

That each person's choice of strategy depends on uncertain
estimates of other people's true preferences makes possible a
type of strategic manipulation that is not available within
games of complete information. Individual one, for example,
may have an incentive to mislead individual two concerning what
his, one's, preferences truly are.[13] The reason is that if
individual two falls for the ploy, then individual two bases his
choice of strategy on incorrect information and, if individual one
has been clever, may be led to choose a strategy that is more
favorable to individual one than the strategy that he would
choose if individual one had not mislead him. An example of this
is the negotiations between the monopolist and consumer within
bilateral monopoly. If the consumer proposes that the monopolist
sell at a particular price, the monopolist is likely to respond,
perhaps untruthfully, that the price is too low to meet his costs
and therefore must be raised. Symmetrically the consumer tries
to convince the monopolist that the product has low value to him.
If he succeeds, then the monopolist will be willing to sell at a
price just above cost in order to make at least some profit. This
process has the danger that if one or both individuals push their
misrepresentations too stubbornly, then either no trade or a
drastically inefficient trade may take place. Thus, to summarize,
nonstraightforward allocation mechanisms are nontrivial games of
incomplete information   that appear to consume substantial resources

in bargaining and produce final allocations that are quite unpre-
dictable and perhaps objectively unsatisfactory.

These problems that are associated with games of incomplete
information evaporate when the allocation mechanism is straightfor-
ward. The fact that individuals are playing a game of incomplete
information is irrelevant if the allocation mechanism is straight-
forward. The players have no need for the information they lack
because they can select their optimal strategies on the basis of
their preferences and the outcome function alone.

Therefore, if they exist, straightforward allocation
mechanisms are desirable. Unfortunately within economic environ-
ments they do not exist except in the case of an infinite number
of atomless agents. This assertion -- that straightforward alloca-
tion mechanisms do not exist within economic environments --
follows directly from this paper's theorem and identification
of two, essentially universal characteristics of economic alloca-
tion mechanisms. The first characteristic, which reflects the
variability of people's tastes, is that an individual's preferences
may be any one of a wide variety of admissible orderings. The second
characteristic is that within an economy with a market sector the
demand of each individual feeds back through the price system and
affects all other individuals. These two characteristics are exactly
the two properties that the requirements of broadness and suitability
formalize. Consequently, according to the theorem, the only means
by which economic allocation mechanisms can be made straightforward
is to enforce the requirement that individuals be permitted only

negligible economic power. This, of course, is precisely the means
by which the perfectly competitive model with a large number of atomless
consumers achieves straightforwardness.

7. Comments on the Necessity of the Suitability and Broadness
    Requirements

This section consists of three examples whose point is
this.[14] The theorem shows that two conditions -- suitability
and broadness -- are sufficient to guarantee that a resource
allocation mechanism is not straightforward. These conditions are
not necessary for a mechanism to be nonstraightforward. Yet in
an informal, weak sense they do contain an element of necessity.
Mechanisms do exist that are straightforward and that satisfy only
one or the other of the suitability and broadness conditions.
Consequently neither broadness nor suitability can completely
be discarded and still retain a valid theorem.

The first example is a mechanism that proves, by counter-
example, that the theorem does not specify necessary as well as
sufficient conditions for nonstraightforwardness. This mechanism,
which does not satisfy the broadness requirement, is derived from
the exchange economy model presented in Section 4 by fixing the
parameters $a_1, a_2, b_1, b_2, \bar{a}_1, \bar{a}_2, \bar{b}_1, \bar{b}_2$ equal to zero. Under this
restriction only Cobb-Douglas utility functions are admissible
as individual's true preferences and as their reported preferences.
This class can not satisfy the broadness assumption because a
change in the one parameter α_i causes a change in the marginal
rate of substitution at every point in the consumption set.[15]

Consequently the necessary crossing of trajectories that is shown on Figure 1 is impossible to obtain. Inspection of the outcome function's components (4.06) through (4.07) shows that suitable curves A and B exist.

That this modified mechanism is not straightforward is demonstrated by maximizing each person's utility function and the other person's choice of strategy. For person one this results in a quadratic equation:

$$(7.01) \quad (1-\bar{\alpha}_1-\alpha_2)\alpha_1^2 + (\alpha_2-\alpha_2^2+2\bar{\alpha}_1\,\alpha_2)\,\alpha_1 + \bar{\alpha}_1\,a_2^2 - 2\bar{\alpha}_1\,\alpha_2 = 0.$$

Person one's optimal strategy is the root α_1 of this equation that satisfies the second order conditions for a maximum. Clearly its value depends not only on the value of a $\bar{\alpha}_1$, person one's true perferences, but also on α_2, person two's reported preferences. Therefore this mechanism is not straightforward, despite the lack of broadness within the sets of admissible utility functions.

My second example is a mechanism Hurwicz [10] discovered. Its set of admissible utility functions does not satisfy the broadness condition, it admits suitable curves A and B, and it is straightforward. Its existence shows that some version of the broadness condition is necessary to guarantee nonstraightforward-ness. The example is this. Three individuals, $i \in \{1,2,3\}$, are jointly deciding on allocation of one public good y and one private good x. Each individual begins with an initial endowment of w_i

units of the private good and has a utility function

$$(7.02) \quad u_i(x_i,y_i) = x_i + 2\theta_i \, y^{\frac{1}{2}}$$

where x_i is person i's private good allocation and θ_i is a positive parameter. Each person's true preferences are characterized by a parameter value $\bar\theta_i$ and each person's strategy is to report a value for the parameter θ_i. The outcome functions are the "basic quadratic" of Groves and Ledyard [5]:

$$(7.03) \quad F_i[\theta_1,\theta_2,\theta_3] = x_i = w_i - \theta_i^2 - 2\theta_j \, \theta_k$$

$$F_y[\theta_1,\theta_2,\theta_3] = y = (\theta_1 + \theta_2 + \theta_3)^2$$

where $i \neq j \neq k \neq i$. Person i's utility, given the strategies θ_j and θ_k of the other two people, is therefore

$$(7.04) \quad u_i = w_i - \theta_i^2 - 2\theta_j \, \theta_k + 2\bar\theta_i \, (\theta_1 + \theta_2 + \theta_3).$$

Maximization with respect to person i's reported preferences θ_i gives

$$(7.05) \quad \frac{du_i}{d\theta_i} = - 2\theta_i + 2\bar\theta_i = 0$$

or

$$(7.06) \quad \theta_i = \bar\theta_i,$$

i.e. the mechanism is straightforward because the optimal strategy of person i is to report a parameter value that is identical to his true parameter.[16] Inspection of (7.04) shows that suitable curves A and B exist. The broadness requirement is violated

for the same reason that it is violated by the family of Cobb-Douglas utility functions: a change in the single parameter changes the marginal rate of substitution at every point within the consumption set.

My third example, which I already discussed from one point of view in Section 5, is the monoplist who sells to a continuum of atomless consumers. My purpose for reintroducing it here is that it is a straightforward mechanism for which the broadness requirement is satisfied.[17] Therefore it demonstrates that some version of the suitability condition is necessary to guarantee that a mechanism is not straightforward. By itself broadness of the set of admissible utility functions is not sufficient.

Nevertheless, in addition to the theorem, one general result does suggest itself. If a mechanism with a continuously differentiable outcome function does not admit suitable curves, then it is straightforward between pairs of individuals. An informal constructive proof of this assertion is as follows. Suppose, as in the monopoly example where the consumer is person ξ and the monopolist is person η, a suitable curve A can not be constructed, i.e. person ξ can not affect person η. Therefore person η's choice of strategy is straightforward -- he can ignore ξ because ξ can not affect him. Person η, however, may affect person ξ. Therefore, generally, person ξ's choice of strategy must be dependent on person η's choice of strategy. This, however, poses no problem. Because η's choice of strategy is not influenced by

ξ's choice of strategy, person ξ's optimal strategy is a list of actions whose implementation is conditional on η's choice of strategy. The consumer's demand curve in the monopoly example is an illustration of this approach. Such a list is a straightforward strategy because person ξ can optimally prepare it without access to any information concerning person η's true preferences. Thus if suitable curves A and B do not exist, the mechanism is straightforward between the pair of individuals in question.

Footnotes

1. Roberts and Postlewaite [11] have discussed in detail the incentive properties of large exchange economies.

2. A monotonic utility function u_i has the property that if x_i, $y_i \in C_i$ is a pair of allocations such that $x_i > y_i$, then $u_i(x_i) > u_i(y_i)$. The notation $x_i > y_i$ means that every component of x_i is strictly greater than every component of y_i.

3. This assumption shares with Arrow's independence of irrelevant alternatives assumption a common justification. Arrow [2,p.109] stated in support of that assumption: "The essential point ... is the application of Leibniz's principle of the identity of indiscernables. Only observable differences can be used as a basis for explanation." This principle applies equally well to my nonzero cost assumption.

4. The notation "int S_η" represents the interior of the set S_η.

5. A curve $A\{[0,1]\}$ is smooth if its components A_i are differentiable, it is rectifiable if it has finite length, and it is simple if it has no closed loops, i.e. a smooth, rectifiable, simple curve is well behaved.

6. Groves and Ledyard [6, Section 10] have observed that if the income elasticity of demand is not constant as preferences vary, then no allocation mechanism exists that is straightforward and satisfies the Lindahl-Samuelson optimality condition.

7. See Apostol [1,p.107] for the definition of a differential.

8. That this mechanism is not straightforward can also be determined by direct calculation.

9. I want to rule out the possibility that the two individuals' initial endowments are on the contract curve implied by their true utility functions.

10. That F is continuously differentiable and that $(\bar{s}_1, \tilde{s}_1, \bar{s}_2, \tilde{s}_2)$ $\in$ int $[S_1 \times S_1 \times S_2 \times S_2]$ assures that first order conditions are satisfied at person two's optimal points. Satisfaction of first order conditions in turn implies that the optimal points are tangencies.

11. Harsanyi [8] has developed the most complete theory of such games.

12. Harsanyi [7] has discussed in detail the possibility of an infinite regress arising in this manner.

13. Within the context of voting Blin and I [3] have discussed in detail the possibilities of profitably misleading others concerning one's true preferences.

14. One interesting and important example of a straightforward mechanism that I do not discuss here is majority rule with single-peaked preferences. Analysis of it shows that it satisfies neither the suitability nor broadness requirements.

15. This assertion may be checked by differentiating equations (4.10) with respect to α.

16. Hurwicz [10] shows that this mechanism, in addition to being straightforward, gives optimal outcomes and that $u_i(x_i,y) = x_i + 2\theta_i \, y^{\frac{1}{2}}$ is the only class of utility functions that is consistent with a three person, one public good, and one private good economy being straightforward.

17. A dictatorial allocation mechanism is a second example of a straightforward mechanism that satisfies the broadness re-requirement, but not to suitability requirement. A dictatorial mechanism is one where a single individual has the power to specify fully the final allocation.

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Figure 1
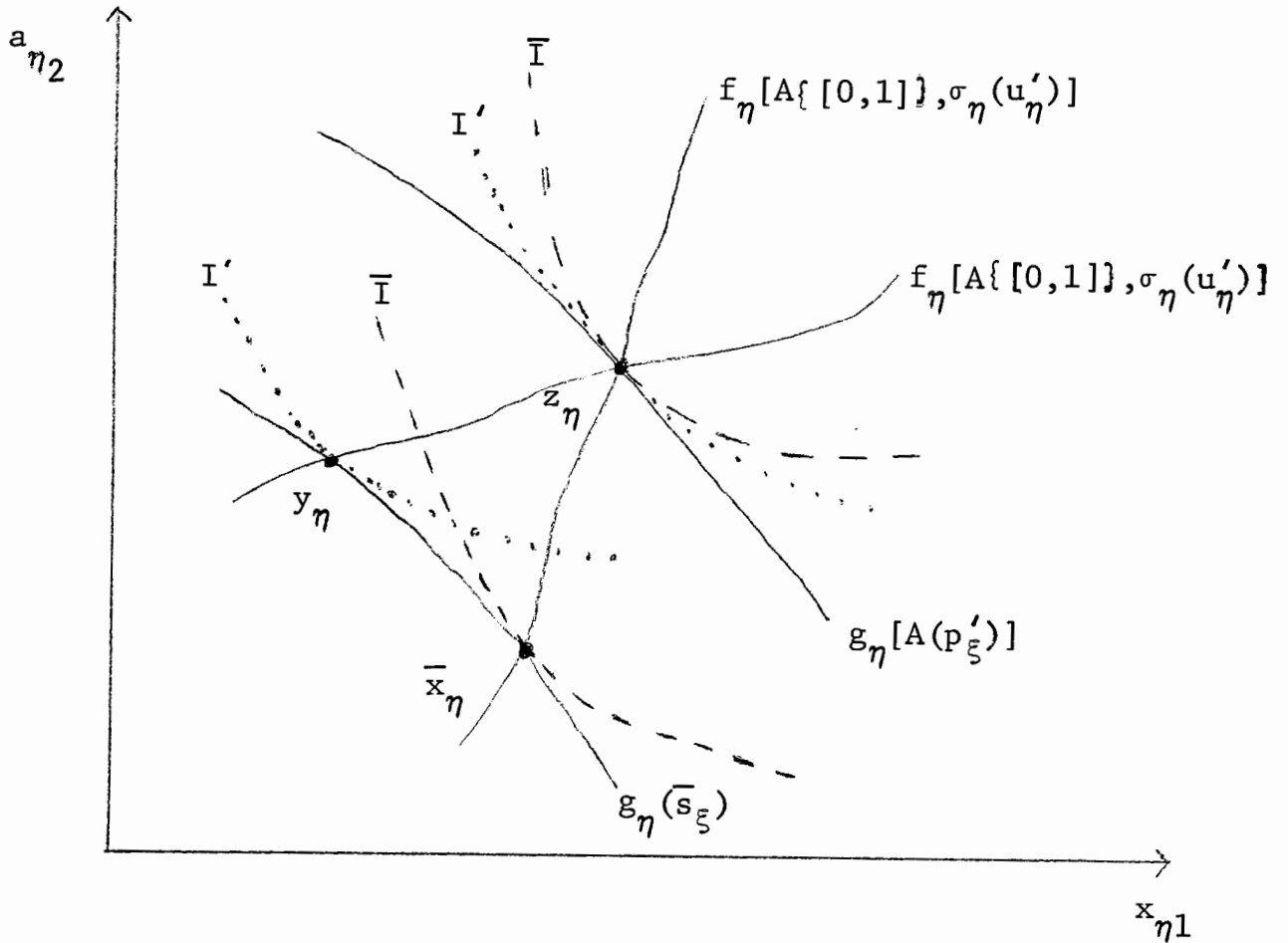
Explanation. The curve $f_\eta[A\{[0,1]\},\sigma_\eta(\bar{u}_\eta)]$ is the trajectory person η's allocation follows if he plays his straightforward strategy $\bar{s}_\eta = \sigma_\eta(\bar{u}_\eta)$ and person ξ varies his strategy along the suitable curve $A\{[0,1]\} \subset S_\xi^*$. The dashed curves $\bar{I}$ represent two of the indifference curves that $\bar{u}_\eta$ generates. The curve $f_\eta[A\{0,1]\},\sigma_\eta(u_\eta')$ has a similar interpretation as a trajectory except that person η is playing that strategy which is straightforward for utility function u_η'. The dotted curves I' represent the indifference curves generated by u_η'. The curves $g_\eta(\bar{s}_\xi)$ and $g_\eta[A(p_\xi')]$ represent the offer curves facing η when ξ's strategy is, respectively, $\bar{s}_\xi$ and $A(p_\xi')$.
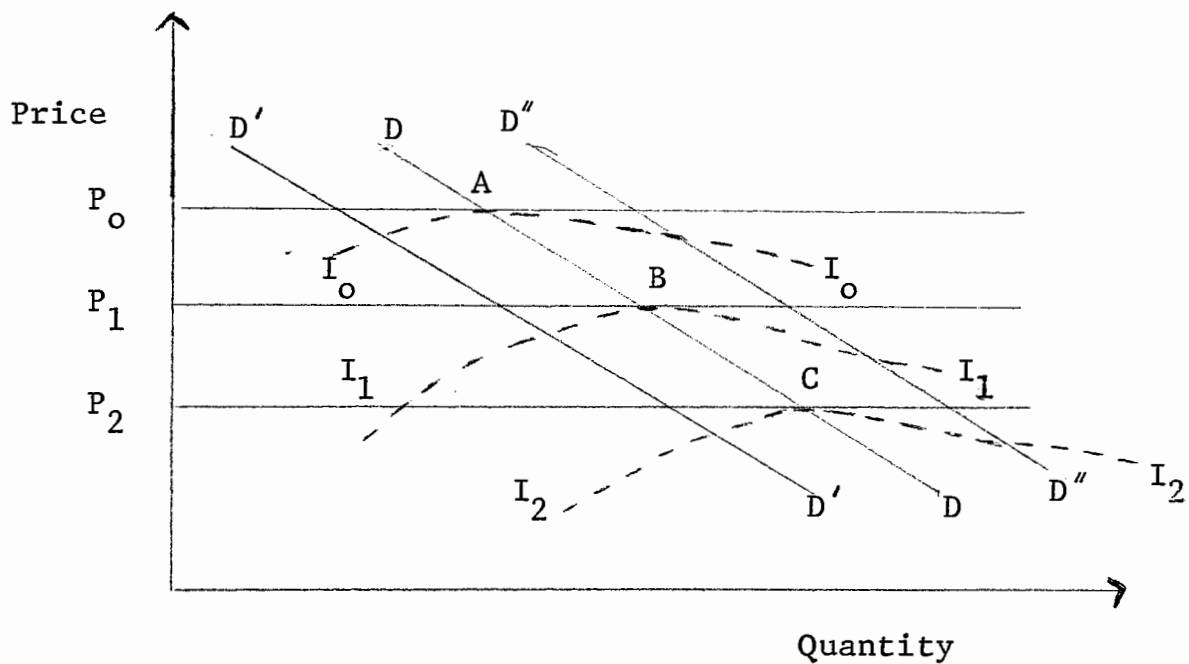
Figure 2
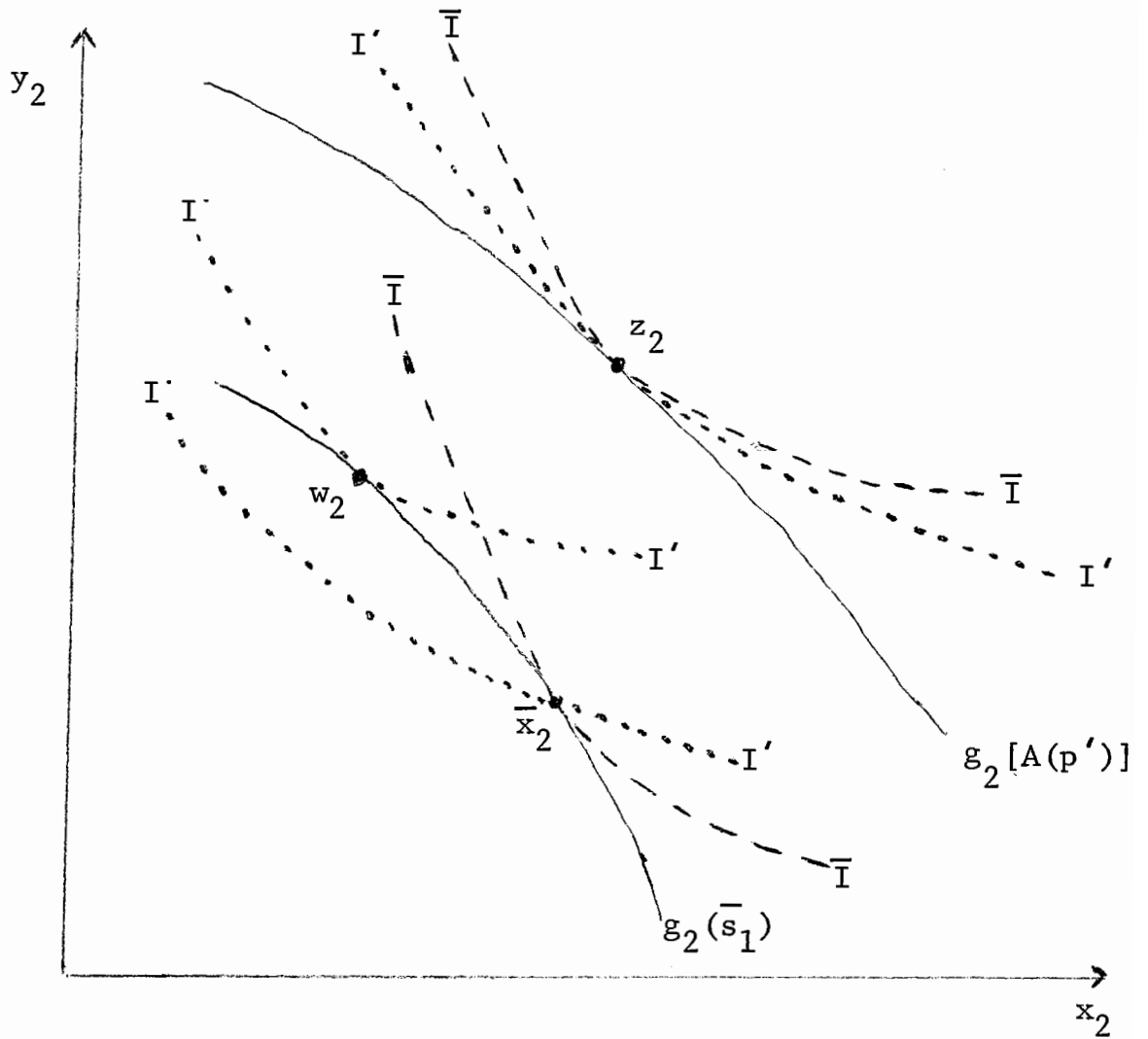
Explanation. The dashed lines $\overline{I}\ \overline{I}$ represent indifference curves generated by $(\overline{\alpha}_2, \overline{a}_2, \overline{b}_2)$. The dotted lines $I'I'$ represent indifference curves generated by (α'_2, a'_2, b'_2).

Figure 3
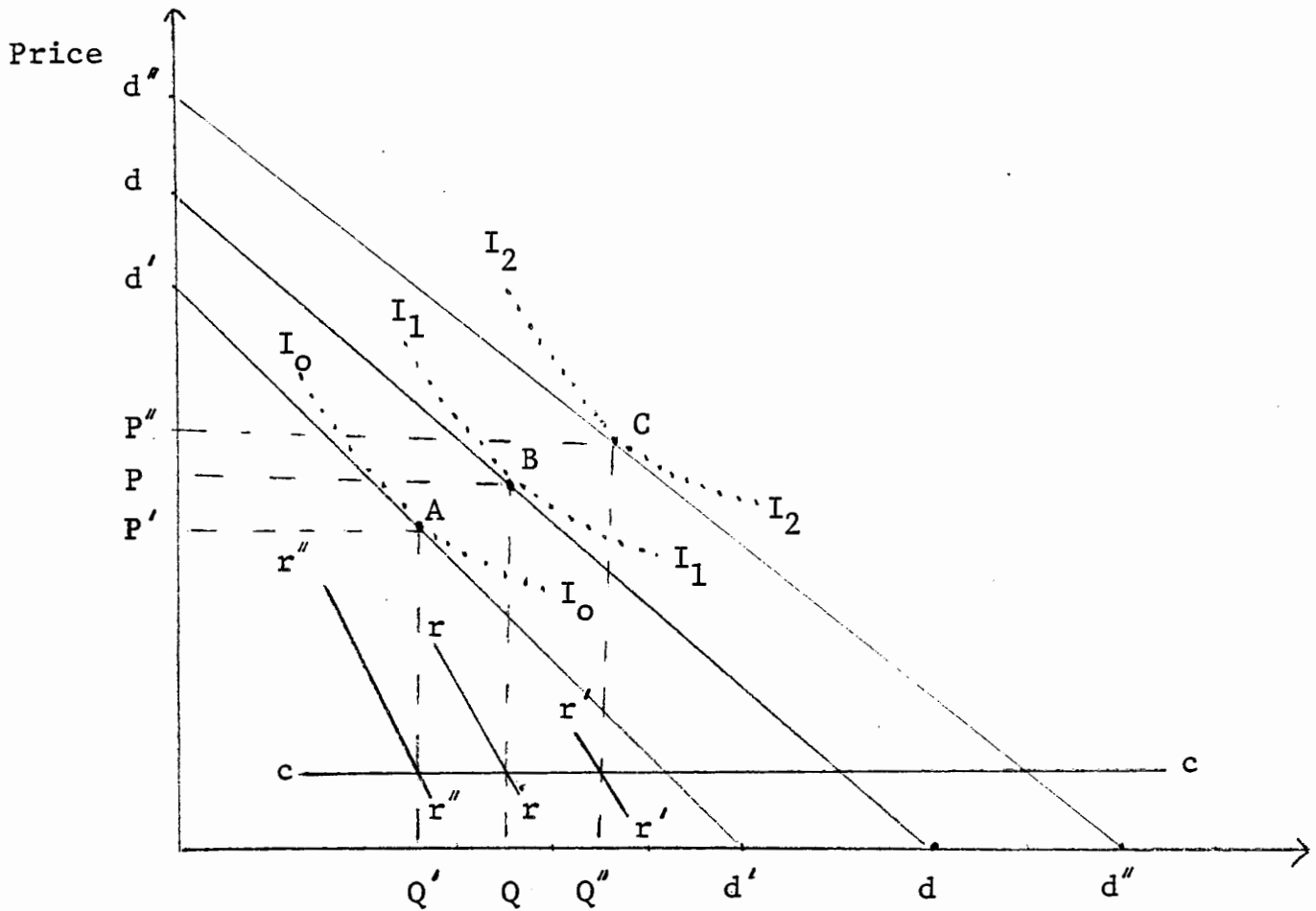
Explanation. Lines $r''r''$, rr, and $r'r'$ are the marginal revenue curves associated with the market demand curves $d''d''$, dd, and $d'd'$ respectively. Line cc is the firm's marginal cost curve. The dotted curves I_0I_0, I_1I_1, and I_2I_2 are the monopolist's indifference (isoprofit) curves.

Figure 4

Explanation. The dotted lines I_oI_o, I_1I_1 and I_2I_2 are the consumers indifference curves among price-quantity pairs. I_oI_o represents lower satisfaction than I_1I_1, etc.

Figure 5
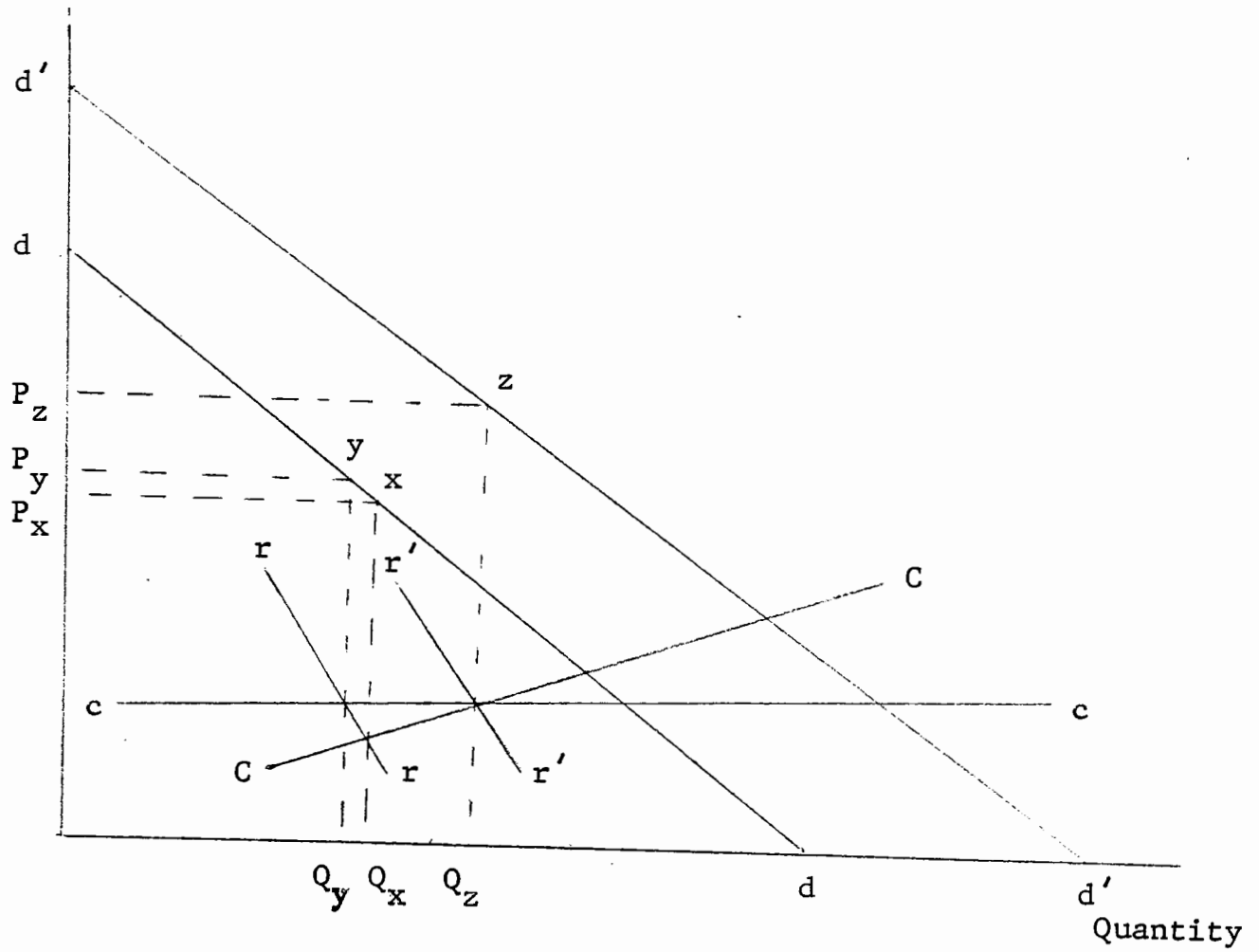
Explanation. Lines rr and r′r′ are marginal revenue curves for demand curves dd and d′d′ respectively. Lines CC and cc are marginal cost curves.