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ON THE DEGREE OF RIVALRY FOR MAXIMUM INNOVATIVE ACTIVITY

by

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INTRODUCTION

Fisher and Temin [4] have recently argued that published empirical studies of the relationship between firm size and innovative activity do not test the Schumpeterian hypothesis. Their contention is challenged by Scherer [19] on grounds of both interpretation of the theory and evaluation of the empirical investigations. This dispute suggests that empirical tests relating monopoly power and innovation might also deserve closer scrutiny.

According to Schumpeter, it is the combined absence of perfect price competition and presence of nonprice competition that stimulates innovation

> ...in capitalist reality as distinguished from its textbook picture, it is not that kind of competition (price) which counts but the competition from the new commodity, the new technology, the new source of supply... It is hardly necessary to point out that competition of the kind we now have in mind acts not only when in being but also when it is an ever present threat. The businessman feels himself to be in a competitive situation even if he is alone in his field...

[21, p. 84-5]. Villard [23] refers to this situation as "competitive oligopoly".

To determine the empirical content of Schumpeter's contention, Scherer tested the hypothesis that inventive activity increases with the four-firm concentration ratio, using data on 56 industry groups and employment of technical engineers and scientists as a measure of inventive activity. He concludes that

> "When the four-firm concentration ratio exceeds 50 to 55 percent, additional market power is probably not conducive to more vigorous technological efforts..."

[17, p. 530]. Essentially the same conclusion is drawn by Kelly [9] in a similar analysis of 1950 data on 181 firms in six industry groups. Mansfield's [10] intensive investigation of innovative behavior in the iron and steel, petroleum refining, and bituminous coal industries during 1919-38 and 1939-58 led him to suggest that break-up of the five largest firms in the petroleum and coal industries would result in greater innovative activity. Williamson [24] analyzing the data generated by Mansfield finds that a 5-30 percent market share for the four largest firms is "optimal" from the standpoint of innovative activity.

These tentative conclusions appear to be paralleled in studies of innovative activity and firm size as summarized by Markham [12] and Hamberg [6]. To the extent that firm size and monopoly power are correlated, these conclusions are reinforcing. Moreover, these findings have prompted speculation by Schmookler [20] and Phillips [14] that there might exist an "optimal" degree of rivalry for motivation of innovative activity between the extremes of monopoly and perfect competition.

The above studies link innovative activity with monopoly power, or some absence of price competition, but not directly with the presence of nonprice competition. That connection is suggested by Stigler [22] in a study of the decline in labor requirement per unit output in twenty-nine industries for the period 1899-1937. More recently, Comanor [3] studied the level of research activity measured by R & D employees in twenty-one industries during 1955 and 1960 and concluded that

> "industrial research spending appears strongest in industries where some measure of technical entry barriers exists, so that rapid imitation is impeded, but also where entry has not been effectively foreclosed."

Finally, the bridge between concentration ratios and nonprice competition in the form of innovative activity is supported in the empirical study by Grabowski and Baxter [5]. They find a significant negative relationship between eight-firm concentration ratios and the coefficient of variation of research intensity among firms in 29 industries during 1954.

Our intention in this paper is to address the reported empirical finding that the rate of innovative activity increases with the intensity of rivalry up to a point, peaks, and declines thereafter with further increase in the competitiveness of the industry. We will show that the possibility of an intermediate "optimal" degree of technological rivalry for innovation can be deduced via comparative statics analysis of two related models of the profit maximizing firm. The parameter on which we focus, the intensity of technological rivalry, is not, however, a proxy for any of the usual notions of market structure, nor can it be directly identified with any of the independent variables employed in the empirical studies cited. Both models deal with the firm's choice of a development rate for an innovation under uncertainty regarding the similar decision of rivals. Focus on the development rate is inspired by the notion that resource allocation affects the speed with which physical laws are discovered and not their creation; see Arrow [1]. The viewpoint of these models is similar to those of Barzel [2], Phillips [15], Scherer [18], Roberts and Holdren [16], and more distantly related to those of Mansfield [11] and Olivera [13].

Both models have three basic components, namely

(a) the rewards for successful development of a new or improved product or process,

(b) the development function relating monetary spending to accumulation of effort toward project completion

(c) the potential of rivals seeking a similar innovation.

In each instance, the firm selects the development period (introduction date) that maximizes expected present value of the project.

The available benefits from the innovation are assumed known. The models differ in the assumption about the appropriability of rewards by the innovator. In the first instance, imitators can receive some rewards and they do diminish the rewards to the innovator. In the second model, however, complete patent protection is awarded so imitation has no value.

The development function is known with a high degree of certainty. That is, there is no important uncertainty about the feasibility of development by any desired date through accumulation of effective development effort by expenditure of money. There is an inverse convex relationship between the length of the development period and the minimum total development cost. Thus faster development is more than proportionately more costly. The assumed time-cost trade-off is supported by the empirical studies of Scherer [18] and Mansfield [11] who also discuss in some detail the underlying economic and technical bases of the relationship.

In the first model, development is assumed to be contractual, so that plans for development by a certain date will not be altered by rival innovation. The cost is the same whether the firm is innovator or imitator, and depends solely on the planned completion date. In the second model, since patent protection is complete, development effort will cease upon project completion or rival innovation and preemption, whichever occurs first.

Rival introduction of a similar product or process may reduce (first model) or eliminate (second model) the possible returns to the given firm for its development efforts.

Rivals and their plans are neither known to the firm nor are they ignored. We suppose the firm recognizes the threat of other innovators through a single subjective probability distribution over the introduction date by a rival. The intensity of the threat is measured by a parameter of the probability distribution over the time of rival introduction. The instantaneous probability of rival introduction at any moment, given no prior rival appearance, is presumed constant. The larger that constant conditional probability of rival appearance, the sooner the expected time of rival appearance, and the greater the intensity of rivalry. Mansfield [10] employs a very similar notion of rivalry to determine an optimal structure for the steel industry.

The term "rate of innovation" is shorthand for a multidimensional phenomenon. One is interested in both the size or importance of innovations and the speed with which they appear, or their number. In this paper we focus upon the speed of development of a single new or improved product or process. Its size is held fixed. The firm's expected profit maximizing development period is shown to depend on the intensity of rivalry. The longer that period, the lower the innovation rate may be said to be. Conversely, the degree of rivalry that renders the development period minimal may be said to maximize the rate of innovation.

While each model has been previously developed and studied, the specific relationship between the firm's choice of development period and the intensity of rivalry was not pursued before. In this paper we perform that analysis, showing in each model that there are precisely two possible relationships. For projects that are very good in the sense of having large benefits to the innovator, the development period chosen is decreasing with increasing rivalry up to a point; as rivalry becomes still more intense, the development period chosen lengthens. The second possible pattern occurs for projects that are less profitable; for such innovations, the firm's development period is increasing with the degree of rivalry throughout.

Since the rate of development determines the number of innovations introduced during an interval of time, we can relate the degree of rivalry to innovative activity. Thus, we demonstrate the existence of a degree of rivalry for maximum innovation, and show its dependence on the structure of rewards to the innovator. The fact that the same qualitative conclusions are obtained under alternative assumptions about the appropriability of rewards and the contractual nature of the development agreement suggests a theoretical robustness of these results.

MODEL I - THE FRAMEWORK

We posit a firm seeking the introduction date (development period) T that maximizes the expected present value of an innovation. Uncertainty arises from absence of knowledge regarding the intended introduction date v of a similar innovation by any one of many rivals. The firm's beliefs about the introduction date of rivals, viewed as a composite, are summarized in terms of a probability distribution $F(v)$, the probability of rival introduction by time v.

The rewards from the innovation to the firm depend on whether it is the innovator or a follower, and on whether it is the sole producer of the innovation. That is, the return to the firm at time $t \geq T$ from introduction of the improvement at time $T > 0$ depends on the rivals' introduction date v, as follows:

$$(1) \quad \begin{cases} e^{gt}P_o & \text{if } T \leq t < v \quad \text{(firm innovates, is monopolist)} \\ e^{gt}P_1 & \text{if } T < v \leq t \quad \text{(firm innovates, has imitators)} \\ e^{gt}P_2 & \text{if } v < T \leq t \quad \text{(firm imitates)} \end{cases}$$

where

$$0 \leq P_1 \leq P_o, \; 0 \leq P_2 \leq P_o$$

The constant g may be positive, reflecting a growing market, or negative, indicating a shrinking market (perhaps from obsolescence).

The reward to the firm as monopolist includes all quasi-rents, from whatever source, attributable to the innovation. They could result from the firm's own use of the innovation, from royalties, or both. The innovator's reward after rival entry is the best the firm thinks it can obtain once its monopoly control is lost. The best return the firm thinks it will be able to get as follower may be greater or less than this.

Successful completion of the project development requires a cumulative effort A to which monetary expenditure $y(t)$ at time t contributes in accordance with the relation

$$(2)\quad \int_0^T y^a(t)dt = A,\ 0 < a < 1,\quad y(t) \geq 0$$

This may be viewed as a production relation whose inputs are the monetary expenditures and whose output is the completion date T. The more rapidly money is spent, the sooner project completion, but faster spending hastens completion at a diminishing rate since the technical parameter a is less than one.

The minimum present value of the cost of completing development by time T is found [7] as

$$(3)\quad \begin{aligned} C(T) &= \min \int_0^T e^{-rt} y(t)dt \quad \text{subject to} \quad (2) \\ &= A(Anr)^{1/n}(e^{nrT}-1)^{-1/n} \quad \text{where} \quad n = a/(1-a) \end{aligned}$$

This development cost function has the properties that extending the development period reduces total cost, $C'(T) < 0$, and that $C''(T) > 0$, where primes indicate differentiation. In the present model, only these two properties are employed and no use is made of the specific functional form of $C(T)$ suggested above.

While rival introduction will diminish the rewards available to the firm whose project development is still underway, the remaining benefits are assumed sufficient to warrant completion. Development is assumed done under contract, so the appearance of a rival will not lead to modification of the development schedule. These and other restrictions on the rewards and on the behavior of $C(T)$ will be made more specific shortly.

The firm's objective can now be posed formally as

$$(4) \quad \max_{T} \int_{T}^{\infty} e^{-(r-g)t}[P_o(1-F(t)) + P_1(F(t)-F(T)) + P_2F(T)]dt \; - C(T)$$

where the discount rate $r > g$ to insure convergence of the integral. The integral gives the present expected value of rewards from innovation at time T; recall (1) and note that $1-F(t)$ is the probability of no rival entry by time t, $F(t) - F(T)$ is the probability that the firm is the innovator and a rival will have appeared in the time interval between T and t, and $F(T)$ is the probability of rival entry by time T so the firm is not first.

The specific supposition about rival behavior F is that the conditional (i.e. given no rival appearance to date) probability density of rival innovation at any moment is a constant: $F'(t)/(1 - F(t)) = h$. It turns out, as a consequence, that the expected introduction time of some rival is $1/h$. If the firm is the innovator, rivals may alter their development rate; the conditional probability of rival imitation at any moment after T is supposed equal to a constant, k. (The constant k could be equal to h or different.) The continuous function F that accommodates these suppositions has the explicit form

$$(5) \quad F(v) = \begin{cases} 1 - e^{-hv} & \text{if } 0 \leq v < T \text{ (rival innovation)} \\ 1 - e^{(k-h)T-kv} & \text{if } v \geq T \quad \text{(rival imitation)} \end{cases}$$

Substitution of (5) into (4) and integration with respect to t yields the objective

$$(6) \quad \max_{T} Be^{-(r-g+h)T} + e^{-(r-g)T}P_2/(r-g) - C(T)$$

where

$$(7) \quad B \equiv (P_o - P_1)/(r-g+k) + (P_1 - P_2)/(r-g) \geq 0$$

The first term in (6) can be interpreted [7] as the excess of expected rewards from innovation over the rewards from imitation and is assumed non-negative. (We permit $P_1 \gtrless P_2$ however.) The second term in (6) is the reward from imitation.

We suppose that the present value of development cost shrinks to zero as the completion is postponed indefinitely

$$(8) \quad \lim_{T\to\infty} C(T) = 0$$

and that the cost of instantaneous innovation is so high that immediate entry is precluded:

$$(9) \quad C(0) > (P_o - P_1)/(r-g+k) + P_1/(r-g)$$

(The specific cost function in (3) satisfies (8) and (9).) The rewards to the firm from imitation are assumed sufficiently large relative to the development cost that

$$(10) \quad e^{-(r-g)t}P_2/(r-g) > C(t) \quad \text{for some } t, \quad 0 < t < \infty$$

Finally, we suppose that the maximand in (6) has exactly one regular local maximum, the global maximum, i.e. it is strictly quasiconcave in T.

Under our suppositions, the solution to (6) will involve introduction at a positive but finite time. To see this, let $V(T)$ denote the maximand in (6) and note from (9) that $V(0) < 0$ while from (8) that $\lim_{T\to\infty} V(T) = 0$. But from (7) and (10), there is a finite t such that $V(t) > 0$. Since this positive finite introduction time yields a higher (positive) value than does either immediate entry or indefinite postponement, the optimal introduction time T^* must be positive, finite, and render $V(T^*) > 0$.

The conditions satisfied by the introduction date T^* are obtained on differentiating $V(T)$:

$$(11)\quad -(r-g+h)e^{-(r-g+h)T}B - e^{-(r-g)T}P_2 - C'(T) = 0$$

$$(12)\quad (r-g+h)^2e^{-(r-g+h)T}B + (r-g)e^{-(r-g)T}P_2 - C''(T) < 0$$

Conditions (11) and (12) together define a relationship between the optimal introduction date T^* and the parameters of the problem:

$$(13)\quad T^* = T^*(g,r,h,k,P_o,P_1,P_2) \quad .$$

Comparative statics analysis can be performed to sign the partial derivatives of the function in (13) and thereby generate testable hypotheses regarding the influence of various factors upon the rate of innovation. Following this procedure in [7], we established that introduction is spurred by a higher reward for being first and is retarded by more rapid imitation; that is, we showed that $\partial T^*/\partial P_o < 0$, $\partial T^*/\partial P_1 < 0$, and $\partial T^*/\partial k > 0$. Fuller discussion of the underlying suppositions of this model and its implications appear in [7].

MODEL I - RIVALRY AND SPEED OF INNOVATION

Our objective is the qualitative behavior of the firm's development period T^* as the intensity of rivalry h to be innovator is varied. Thus we will examine the sign of $\partial T^*/\partial h$.

To see that h is appropriately interpreted as a measure of the intensity of rivalry for innovation, review the upper portion of (5). In the limiting case of $h = 0$, rivals will not innovate and the firm is certain to be innovator. The probability of rival innovation by any date $v < T$ is increasing with the parameter h, approaching unity as h tends to infinity. To put it another way, the expected date of rival innovation, $1/h$, is inversely proportional to h; thus the more intense the rivalry, the sooner rival innovation is expected. (The lower portion of (5) pertains to imitation and is not of direct relevance to the discussion of rivalry for innovation. Its influence is through the expected rewards for innovation.)

We begin by noting that

$$(14)\quad \text{sign } (\partial T^*/\partial h) = \text{sign } [(r-g+h)T^* - 1],$$

a result obtained by partial differentiation of (11). It is not apparent from (14) what sign $\partial T^*/\partial h$ takes. However, we can develop a construction useful to that determination.

For given values of r and g, the equation

$$(15)\quad (r-g+h)T^o = 1 \quad \text{or } T^o = 1/(r-g+h)$$

defines a locus of points (h, T^o) satisfying it. For any given value of h, there are corresponding values of T^* determined from (13) and of T^o determined by (15). It follows from (14) and (15) that $\partial T^*/\partial h \gtrless 0$ according as $T^* \gtrless T^o$, for given h. This means that, viewed as functions of h, $T^*(h)$ is falling, stationary, or rising when it lies respectively below, on, or above $T^o(h)$. But since dT^o/dh is negative, a stationary point $h = h^*$ of $T^*(h)$ must be a minimum.

Since $T^o \to \infty$ as $r-g+h \to 0$ while T^* is finite for all $h > -(r-g)$, it follows that either there is an h^* such that $T^o(h^*) = T^*(h^*)$ or else $T^o(h) > T^*(h)$ for all $h > -(r-g)$. Combining this observation with those of the preceding paragraph, we conclude that there are at most three possible behavior patterns for the function $T^*(h)$, namely

(i) T^* has a stationary point $h^* \leq 0$ so that

$$T^*(h) > T^o(h),\ h > 0$$

and T^* is an increasing function of h for all $h > 0$.

(ii) T^* has a stationary point $h^* > 0$

so that $T^*(h) \gtrless T^o(h)$ as $h \gtrless h^*$

In this case, T^* is decreasing for $0 \leq h < h^*$, attains a minimum at h^*, and is increasing for $h > h^*$.

(iii) $T^*(h) < T^o(h),\ h \geq 0$

so that T^* is a decreasing function of h for all $h \geq 0$.

We will now show that case (iii) is impossible. Then the conditions under which cases (i) and (ii) each will obtain will be indicated.

To show that case (iii) cannot occur, we recall that the maximand $V(T;h)$ of (6) is continuous with $V(0;h) < 0$. Consequently there is a fixed number $T_1 > 0$ such that $V(T;h) < 0$ for $0 \leq T \leq T_1$. The optimal introduction time $T^*(h)$ renders V positive, so $T^*(h) > T_1 > 0$. Hence

(16) $(r-g+h)T^*(h) > (r-g+h)T_1$ for all $h \geq 0$

But the right side of (16) becomes arbitrarily large as h does, so the left side must also. It follows that

$$(r-g+h)T^*(h) > 1 \text{ for all } h \text{ sufficiently large}$$

which in turn implies, using (14), that

(17) $\partial T^*(h)/\partial h > 0$ for all sufficiently large h.

This eliminates case (iii).

There are now two cases to consider. In case (i), $\partial T^*/\partial h$ is positive for all $h > 0$. The development period is increasing with the intensity of rivalry, and no rivalry at all ($h = 0$) yields the shortest development period and thereby maximum inventive activity. On the other hand, if case (ii) obtains, then the development period is decreasing with increasing rivalry up to h^*, and then increasing as rivalry becomes still more intense with values of h greater than h^*. Maximum inventive activity occurs at a degree of rivalry intermediate between competition and monopoly.

To provide an intuitive explanation for the possible nonmonotonicity of $T^*(h)$, case (ii), we begin with the polar case in which the firm views itself completely free of rivals, $h = 0$. In that case the firm chooses an introduction date that balances the marginal cost saving of postponement against the marginal loss of delay, the latter reflecting the lower value of more distant rewards relative to nearer ones. If the threat of prior rival introduction now appears, i.e. h becomes positive, the firm faces an additional consequence of postponement, preemption of a quasi-rent. This threat will, in the case being considered, spur it to hasten its introduction date and thereby incur higher development cost. As h continues to grow, the probability of preemption grows as does the cost of defending against it. Moreover, the firm realizes that if despite its accelerated development it winds up second, it will sacrifice a portion of the additional development cost as well as the quasi-rent. Eventually, beyond $h = h^*$, the firm will attempt to hedge against the possible combined loss by prolonging its development period.

The next questions of interest, of course, are the circumstances under which case (ii) will obtain so there in an intermediate "optimal" degree of rivalry, $h^* > 0$, and how that h^* varies with changes in the relevant parameters. It can be shown that h^* varies directly with P_o, P_1, and g and inversely with k (see Appendix (A7), (A9)). The project is more attractive the higher the reward stream to the innovator (P_o, P_1), the more rapidly the potential market for the innovation is growing (g), and the longer the expected period of monopoly before the appearance of followers $(1/k)$. Thus, it follows that h^* is larger and is more likely to be positive, and the innovation-maximizing degree of rivalry to be intermediate, the better the project.

If $h^* > 0$, then (see (A8)) h^* varies inversely with P_2 which means the higher the returns to the firm as follower, the lower the "optimal" degree of rivalry. Finally, it turns out that the minimum innovation time $T^*(h^*)$ varies in opposite fashion from h^* for all parameters considered ((A11) - (A13)).

MODEL II - THE FRAMEWORK

In this model, a patent is awarded the (earliest) innovator. The patent confers exclusive right to the rewards from the innovation for L years; the rewards are the equivalent of P dollars per unit time. At the expiration of the patent, rivalry is sufficient to eliminate economic profits. The firm's subjective assessment of the probability of a rival claim to the patent by time t is given by

$$(18)\quad F(t) = 1 - e^{-ht}$$

The project development function (2) is assumed. Actual development expenditure is uncertain, however, since development will cease if a rival claims the patent before our firm does. In this model, we assume sunk costs are sunk, and there is no benefit to pursuing the development once the patent is awarded to another. Thus spending will continue only so long as no one has introduced the new product or process. With the probability of rival introduction by time t given in (18), the firm's objective is to find a spending plan and development date to

$$(19)\quad \underset{y(t),T}{\text{maximize}}\ (1-F(T)) \int_T^{T+L} e^{-rt} P\, dt - \int_0^T e^{-rt} y(t)(1-F(t))dt$$

subject to (2)

The first term in (19) represents the present value of being the innovator multiplied by its probability while the second term represents expected spending upon development, with discounting at rate r. This problem can be solved stepwise by finding the optimal expenditure plan y for any fixed development period T and then finding the optimal introduction date T^*. It is shown in [8] that

$$(20)\quad T^{*} = -\,(n(r+h))^{-1}\ln[1 - z(n(r+h))^{1-a}]$$

if and only if

$$(21)\quad z(n(r+h))^{1-a} < 1$$

where

$$(22)\quad n \equiv a/(1-a)\ , \qquad z \equiv A[r/P(1-e^{-rL})]^{a}$$

If (21) is violated, then the project is not undertaken (since the argument of the log function in (20) would then be negative). It is shown in [8] that the planned development period will be longer the smaller the value of the rewards P, the greater the required development effort A, the shorter the patent life L, and the higher the discount rate r; also additional implications of the model are drawn out there.

MODEL II - RIVALRY AND SPEED OF INNOVATION

As in Model I, we are especially interested in how the firm's choice of development period T^* varies with the intensity of rivalry h. To facilitate the algebra, define

$$(23) \quad w \equiv 1 - z(n(r+h))^{1-a}$$

and observe from (21), (22) and the definition of the parameter a that

$$(24) \quad 0 < w < 1$$

for any project undertaken. Differentiation of T^* in (20) with respect to h, using definition (23), yields

$$(25) \quad \partial T^*/\partial h = n^{-1}(r+h)^{-2}[\ln w + (1-a)(1-w)/w] \quad .$$

In view of (24), the expression in square brackets is the sum of a negative term and a positive term. If T^* has a stationary point with respect to h, it occurs at a point at which the square bracketed term vanishes. Our next task, therefore, is to investigate the existence of such a point.

Define

$$(26) \quad f(w) = \ln w + (1-a)(1-w)/w, \quad 0 < w < 1$$

We seek a root w^* of the equation

$$(27) \quad f(w) = 0$$

satisfying $0 < w^* < 1$. If there is such a value of w^*, then there is a corresponding value of $h = h^*$, given through (23), namely

(28) $h^* = n^{-1}[(1-w^*)/z]^{1/(1-a)} - r$

at which T^* attains a stationary point.

Now, we may compute from (26) the derivative

(29) $f'(w) = [w - (1-a)]/w^2$.

Thus $f(w)$ is strictly decreasing in the interval $0 < w < 1-a$, attains a minimum at $w = 1-a$, and is strictly increasing in the interval $1-a < w < 1$. The minimum value of f is

(30) $f(1-a) = \ln(1-a) + a < 0$

the negativity of which is established by Taylor series expansion of $\ln(1-a)$ about the point $a = 0$. Next we observe that

(31) $\lim_{w\to 0} f(w) = \lim_{w\to 0} [w \ln w + (1-a)(1-w)]/w = \infty$

using l' Hospital's rule to evaluate the limit of $(w \ln w)$. Finally,

(32) $\lim_{w\to 1} f(w) = 0$

Thus, from (29) - (32), $f(w)$ is positive for very small positive values of w, decreasing in the interval $(0, 1-a)$, negative at its minimum at $w = 1-a$, and increasing toward zero in the interval $(1-a,1)$. Combining these observations with the continuity of $f(w)$ in the open unit interval, and the intermediate value theorem, it follows that there exists a unique root w^* of (27) and it lies in the interval $0 < w^* < 1-a$. Note that $f'(w^*) < 0$. The corresponding value of h^* at which (25) is zero is given by (28).

To show that the stationary point of T^* just established is a minimum, evaluate the second partial derivative of (20) with respect to h at h^*

$$(33) \quad \partial^2 T^*(h^*)/\partial h^2 = -(1-a)(1-w^*)f'(w^*)/n(r+h^*)^3 > 0$$

where positivity follows from $f'(w^*) < 0$. Thus T^* does achieve a minimum with respect to h at h^*.

It appears from (28) that the minimizing h^* might be negative. If it is, then the firm's choice of development period is increasing with the degree of rivalry for all $h > 0$; in this case, the assurance of monopoly $(h = 0)$ will result in maximal innovative activity. On the other hand, a positive h^* corresponds to a degree of rivalry that maximizes innovation and lies between the extremes of perfect competition and monopoly.

There remain the two related questions of (i) the conditions under which the innovation maximizing degree of rivalry is positive and (ii) if positive, the direction of change of that degree of rivalry with parametric variation. From (28), $\partial h^*/\partial z < 0$; it follows that the smaller z the more likely an intermediate market structure will be optimal. Recalling definition (22), z may be interpreted as a cost/benefit ratio since the numerator represents the effort required for development and the denominator is the capitalized value of the patent. This interpretation of z suggests the conclusion that an intermediate market structure leads to the greatest innovative activity for those innovations whose benefits are high relative to the difficulty of producing them.

Using (28) and (22) it is easy to show that h^* varies directly with the magnitude P and length L of the rewards stream and inversely with the required effort A and the interest rate r. The minimum development period $T^{**} = T^*(h^*)$ varies directly with z and hence inversely with P and L directly with A.

SUMMARY AND CONCLUSIONS

We have demonstrated analytically the possible existence of an intermediate intensity of technological rivalry that is most stimulating for innovative activity. In addition, the critical parameters relating intensity of rivalry to innovative activity were identified and their impact exhibited. We were able to show that intermediate rivalry was more likely than monopoly to be optimal the higher the expected quasi-rent to the innovator.

Our formalization should also allow for incorporation and analysis of factors other than interfirm rivalry that affect the pace of innovative activity. For example, the presence of a rich technological base in an industry could be interpreted as leading firms to view rival introduction as more probable than in a technologically poor industry. Likewise, differences in firm size or access to capital markets might give rise to differences in subjective assessment of rival introduction. Large firms might, for instance, underestimate the actual probability of rival introduction.

We should point out the opportunities and desirability for further research along these lines. A rather obvious question regards the preservation of our results under less restrictive assumptions on the firm's probability distribution over rival introduction and the other specified functional relationships. To capture the interactive effects of individual firm decisions, a general equilibrium analysis of the models presented here would be useful. To further bridge the gap between theory and reality a dynamic version of the models presented here that captures the reverse influence of innovation on interfirm rivalry and subsequent incentives to innovate is needed. It has been suggested to us that the analysis might also apply to planning for any race involving rivalry. Thus, T. Schelling cites the land (gold,oil) rush in the west and the search for the North Pole as examples, to which G. Becker adds the quest for the Nobel prize (in economics perhaps).

Finally, precisely because the intensity of technological rivalry cannot be identified with any conventional measure of market structure, two added lines of investigation are suggested. First is the task of developing an empirical measure of the intensity of technological rivalry. Second is the question of the relationship between conventional measures of market structure, such as concentration, and the degree of technological rivalry. It is possible the new measure of technological rivalry will prove useful for a broader range of policy analysis queries.

APPENDIX

To see how the innovation-maximizing degree of rivalry h^* and the minimum development period $T^{**} = T^*(h^*)$ in Model I depend on the underlying parameters of the reward structure, we use the relations (11), (12), and (15) obeyed by h^*, T^{**}. Suppressing stars, the pair of equations (11) and (15) is equivalent to the pair

(A1) $-(r - g+h)B/e - e^{-(r-g)T}P_2 - C'(T) = 0$

(15) $(r-g+h)T = 1$

Differentiating totally, we obtain the system of equations obeyed by infinitesimal changes. Since P_o, P_1, and k appear only through B (see (7)) and since B is monotonic in each of them, we can investigate their incremental effect on h^*, T^{**} through study of the marginal effects of B. Hence,

(A2) $-(B/e)dh + [(r-g)e^{-(r-g)T}P_2 - C''(T)]dT$

$$= [(r - g+h)/e]dB + [-(r-g+h)/(r-g)e + e^{-(r-g)T}]dP_2$$

$$+ [-B/e + ((r - g+h)/e)\partial B/\partial g + Te^{-(r-g)T}P_2]dg$$

(A3) $Tdh + (r - g+h)dT = T\,dg$

with the partial derivatives of B noted for reference as

(A4) $\partial B/\partial P_o = 1/(r - g+k) > 0 \qquad \partial B/\partial P_1 = k/(r-g)(r - g+k) > 0$

$$\partial B/\partial P_2 = -1/(r-g) < 0 \qquad \partial B/\partial k = -(P_o - P_1)/(r - g+k)^2 < 0$$

$$\partial B/\partial g = (P_o - P_1)/(r - g+k)^2 + (P_1 - P_2)/(r-g)^2 > 0$$

The determinant of the coefficients on the left of the equation system (A2) - (A3) is

$$\text{(A5)}\quad D \equiv -(r - g+h)B/e - T(r-g)e^{-(r-g)T}P_2 + TC''(T) > 0$$

where positivity follows from using (15) in (12). Using Cramer's rule, the partial derivatives of h^* are found to be

$$\text{(A6)}\quad D\partial h^*/\partial B = (r - g+h)^2/e > 0$$

$$\text{(A7)}\quad D\partial h^*/\partial g = D + e^{-(r-g)T}P_2 + ((r - g+h)^2/e)\ \partial B/\partial g > 0$$

$$\text{(A8)}\quad D\ \partial h^*/\partial P_2 = (r - g+h)[e^{-(r-g)T} - (r - g+h)/(r-g)e] < 0 \quad \text{for } h > 0$$

To verify the negativity of (A8), rewrite the right side using (15) as

$$\frac{(r - g+h)^2}{(r-g)}\left[\frac{r-g}{r - g+h}\ e^{-(r-g)/(r - g+h)} - 1/e\right]$$

The sign of (A8) is the sign of the square-bracketed term; that term is zero for $h = 0$ and is decreasing with h for h positive, as may be verified by differentiating and collecting terms. Combining (A6) and (A4) yields

$$\text{(A9)}\quad \partial h^*/\partial P_o > 0\ ,\partial h^*/\partial P_1 > 0\ ,\ \partial h^*/\partial k < 0$$

The effects of parametric variation upon T^{**} are given as

$$\text{(A10)}\quad D\ \partial T^{**}/\partial B = -\ 1/e < 0$$

$$\text{(A11)}\quad D\ \partial T^{**}/\partial g = -\ T^2e^{-(r-g)T}P_2 - e^{-1}\partial B/\partial g < 0$$

$$\text{(A12)}\quad D\,\partial T^{**}/\partial P_2 = T[(r - g+h)/(r-g)e - e^{-(r-g)T}] > 0 \quad \text{for } h > 0$$

The sign of (A12) follows on comparing with (A8). Combining (A4) and (A10) gives

$$\text{(A13)}\quad \partial T^{**}/\partial P_o < 0 \quad , \ \partial T^{**}/\partial P_1 < 0 \quad , \ \partial T^{**}/\partial k > 0$$

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