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Continuous and Hypograph Convergence of Utilities

by

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ABSTRACT

The compact, metrizable topology of hypograph convergence is shown to provide a useful notion of similarity for utility functions, in the sense that "similar agents behave similarly in similar (wealth-price) situations." This observation applies to utility functions with possibly different domains. The concept of continuous convergence has been extended to this setting, and it is seen that this notion corresponds to a stronger topology than that of hypograph convergence. Various properties of uniform convergence on compacta, on classes of functions having a common domain, are extended to classes for which the domains may differ by giving those classes the topology of continuous convergence. In particular an important subset of the class of preference relations, equipped with the topology of closed convergence, is shown to be homeomorphic to a corresponding subset of the utility functions. A dominated convergence theorem is also proven for measurable maps into this space of utility functions, i.e., for von Neumann-Morgenstern utility functions.