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EQUITY FOUNDATIONS OF WELFARE ECONOMICS

by

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I. INTRODUCTION

As indicated by its title, this essay is concerned with the analysis
of the problem of distributive equity in the context of modern welfare
economics. As is well known, the conceptual problems in this area are
considerable. My purpose accordingly is the relatively modest one of
attempting to clarify some of the issues involved and to seek to establish
minimal areas of agreement insofar as questions of basic methodology are
involved. As my objective is to promote methodological agreement rather
than controversy the specific equity notions advanced are accordingly
of as mild a nature as possible. In particular, the horizontal equity
axiom presented in Section III should be regarded as summarizing the sug-
gested areas of agreement. The equity criteria discussed in Section V,
on the other hand, should be viewed as an illustration of the types of
complete equity criteria with which the approach presented here is con-
sistent. These criteria are suggested as being interesting, but agreement
with either of them is neither sought nor implied.

The plan of the paper is as follows. In Section II some basic fea-
tures of modern Paretian welfare economics are recalled and the need for
additional equity considerations explained. Section III then presents

a minimal horizontal equity (or equal treatment of equals) requirement which it is argued is consistent with the traditional concern of welfare economics. In Section IV the implications of this horizontal equity requirement are examined and its relation to the social welfare function approach explained. Section V discusses the vertical equity implications of two equity criteria advanced in recent literature each of which is consistent with both the Pareto-efficiency criterion and the horizontal equity requirement advanced here. Section VI concludes the essay with a summary of the main points.

I would like to emphasize at the outset that the equity problem is approached here within the utopian framework of first-best Paretian welfare economics. This reflects my belief that the all-important second-best issues of the equity-efficiency tradeoff cannot be satisfactorily understood so long as a clear conception of its first-best aspects is lacking. Actually, a major point raised below is that the most fundamental equity-efficiency tradeoff can be viewed as already arising, paradoxically, in first-best types of settings in which according to conventional terminology it is absent by definition.

II. MODERN WELFARE ECONOMICS

Ever since the apparition in 1938 of Bergson's classic reformulation
of welfare economics [5], social criteria used in the appraisal of resource
allocation (and their underlying value judgements) have been understood as
involving restrictions on the class of normatively admissible social wel-
fare functions. By a social welfare function is meant an (ethical) ordering
of social states. Importantly, it is an ordinal concept. In its indivi-
dualistic version, the only restrictions imposed by Bergson [5], Lange [10]
and Samuelson [20] on what became to be known as an individualistic (Bergson-
Samuelson) social welfare function is that the social ordering ought to
depend on, only on and positively on individual preferences. As a conse-
quence any ordering of social states satisfying the Pareto criterion quali-
fies as an admissible individualistic Bergson-Samuelson social welfare
function.

While satisfaction of the Pareto criterion is the only a priori re-
striction on Bergson-Samuelson individualistic social welfare functions, it
is clear that in any given function there will be found additional ethical
content. In particular, some underlying ordinal interpersonal comparisons
of welfare must be implicit in any given function. It should be understood
therefore that by choosing not to impose additional restrictions, the
originators of modern welfare economics deliberately intended not to rule
out any possible interpersonal value judgements. Arrow's celebrated
impossibility theorem [1], on the other hand, makes clear the kind of
interpersonal comparisons that are effectively ruled out as logically im-
possible.

Since the welfare economic content of Arrow's impossibility theorem seems to be still widely misunderstood, an explanation of the last assertion is doubtless called for. If all that is required from a Bergson-Samuelson individualistic social welfare function is that it be an ordering of social states satisfying the Pareto-criterion then it is obvious that Arrow cannot have proved the impossibility of such a function. In this sense Bergson [6-8], Little [11], and Samuelson [22-24], are perfectly justified in their conviction that such is not the case. What is debatable, I think, is their insistence that while an epochal contribution to mathematical politics, Arrow's theorem has little or no bearing on welfare economics as traditionally conceived. As I have been unable to find in the literature an interpretation of Arrow's theorem in line with my understanding of it, let me slightly elaborate on this last point.

An Arrow social welfare function is an ordering of social states as a function of individual preferences. A Bergson-Samuelson social welfare function is an ordering of social states which depends on individual preferences but is not a function of them. Both types of social welfare functions satisfy the Pareto-criterion; in case of the Bergson-Samuelson function this is the only restriction on the link between individual and social preferences. In Arrow's case, so called interprofile conditions are also specified since the domain of an Arrow function precisely consists of preference profiles. The most controversial of Arrow's conditions, that of independence of irrelevant alternatives, is such an interprofile condition. While the rationality (consistency) content of this condition is widely understood, it seems that its implicit ethical content has been overlooked. I will try to argue now

that it is precisely from the correct ethical interpretation of this condi-
tion that the contribution of Arrow's theorem to welfare economics can best
be appreciated.

Recall that the entire framework is couched in terms of ordinal pre-
ferences. Interpersonal welfare comparisons, which are implicit in any
ordering of social states, therefore, have to be of an ordinal nature as
well. In other words, such comparisons can be based only on the ranking
properties of individual preferences. What the·condition of independence
of irrelevant alternatives means in this context is the following:

Given that society committed itself to a certain method of·
interpersonal welfare comparison when ranking any two alternatives in a
certain way under a given profile of preferences, then in any other profile
of preferences in which all individuals rank the above two alternatives in
the same way as they happened to rank them in the original profile, society
shall be committed to apply the same method of interpersonal welfare com-
parison (i.e. shall be committed to rank the two alternatives in the same
way as in the original profile.)

Thus the condition of independence of irrelevant alternatives can be
viewed as a requirement that (ordinally speaking) interpersonal welfare com-
parisons should not be made in an arbitrary way (i.e. should depend
only on the relevant ordinal properties of individual rankings). Viewed
in this way, Arrow's theorem effectively states that there can exist no
(nondictatorial) Bergson-Samuelson individualistic social welfare function
that will not involve interpersonal welfare comparisons that are arbitrary
from an ordinalist viewpoint. This way of stating Arrow's theorem, I believe,

serves to indicate its relevance to welfare economics .

Coming back now to Bergson-Samuelson individualistic social welfare functions (B-S ISWF in the sequel), the preceding discussion should make it clear that while their logical existence is not in doubt, their restrictive power might be. For no norming of social states satisfying the Pareto-criterion can be disqualified from serving as the B-S ISWF for some conceivable society. As a consequence the a priori restrictive content of the B-S ISWF approach insofar as social optima are concerned is equivalent to that directly induced by the Pareto-efficiency criterion. In other words, all that is known a priori is that the social welfare optimum (the ultimate object of study) ought to be Pareto-efficient. In particular, it is impossible to deduce from the B-S ISWF approach any a priori restriction of the set of Pareto-efficient allocations on grounds of distributive equity. To be sure, any particular B-S ISWF will embody distributional judgments but the question of real interest is whether one could advance mild equity requirements that ought to apply under all circumstances. The central purpose of what is to follow is precisely the advancement of such.

But since for reasons to be explained later on, the imposition of seemingly plausible equity restrictions on B-S ISWF's may not be conducive to the kind of outcomes that one has in mind when imposing them, the equity axiom to be formulated in the next section will be couched directly in terms of final outcomes. Since our present concern is with Paretian welfare economics, the equity axiom can be looked upon as directly adding an additional restriction on the set of allocations considered to be normatively admissible in addition to the requirement that the allocation be Pareto-efficient. In terms of the B-S ISWF the restriction can thus be thought

of as being addressed directly to its set of admissible optima (subject to the standard economic constraints).

The question of what kind of B-S ISWF's will always satisfy both the Pareto and equity axioms turns out to have a rather interesting answer. As will become clear, this answer could not be expected to emerge were one to impose equity restrictions on the B-S ISWF to begin with rather than imposing them directly on the set of normatively acceptable outcomes.

III A HORIZONTAL EQUITY AXIOM

From the viewpoint of welfare economics the problem of equity is concerned with setting limits to the extent that different individuals might be treated differently by the socioeconomic system. Individuals that are equal in every relevant respect ought to be treated equally (Musgrave's [13] horizontal equity requirement); individuals that are not equal ought or might be treated differently to an extent which is the precise object of the analysis (the problem of vertical equity). While it seems intuitively clear that the problem of vertical equity poses more difficult questions than that of horizontal equity, on closer inspection it is by no means clear what the standard of equality (of individuals) ought to be in the first place (see [14]). The norm for equality of circumstances, in other words, might depend on the particular conceptual framework in which the analysis of equity is to take place.

The very first question that ought to be considered therefore is what are the relevant welfare dimensions according to which we will decide whether two individuals are equal or not. Since our concern is with welfare economics, I will consider it as axiomatic that the relevant yardstick for equality relates to preferences.

From the viewpoint of Samuelson's "ethical observer" it seems clear from the very definition of a B-S ISWF that the class of acceptable value judgements in modern welfare economics is understood to be confined to those that can be imbedded into social orderings of states without making an appeal to individual characteristics other than individual "needs" as expressed by individual preferences. In particular, as made clear in Rawls [19], innate abilities and inherited wealth (as opposed to acquired skills under, hard to define, conditions of fair equality of opportunity) are resultant upon random natural contingencies which are utterly irrelevant from a moral viewpoint and seem to have been so regarded by the welfare economics tradition. From the viewpoint of that tradition therefore, any two individuals who are identical preferencewise ought to be

considered as being equal from the welfare economic standpoint. While other
yardsticks for equality are doubtless possible, it should be agreed that from
the viewpoint of the methodology of modern welfare economics the only individual
characteristic considered in that school to be of nomative significance relates
to individual preferences. As I consider this point to be of fundamental impor-
tance, some further elaboration in support of the implied basic methodological
premise might be called for.

As has been noted in the discussion of the previous section reliance on -
and sole reliance on - individual preferences in evaluating social states
is the basic methodological premise of individualistic (Paretian) welfare
economics. Perusal of Bergson [5-8], Lange[10], Samuelson [20-24], Arrow
[1-4] and de Graaff [9] should convince the reader that the central aim of
modern welfare economics has indeed been to define and achieve a social
optimum derived from (and only from) individual preferences. In other words
the only arguments entering B-S ISWF's are (consumption) variables that
affect individual preferences. Importantly, productivity variables (indivi-
dual skills, innate abilities and the like) are entirely relegated to the
constraints side of the social welfare optimization problem; they do not
directly enter into the evaluation of social states. As explained in the
next section, this practice actually leads to possible implications of the
social optimization problem that on reflection are likely to be found un-
acceptable. This issue, and the way in which the equity axiom to be
stated below circumvents it, will be discussed in some detail later on.
From the viewpoint of our immediate concern, it suffices to note that for
the reasons given here equal treatment properties of the good society had

best be formulated in terms of individual preferences.

We are, therefore, led to formulate the following horizontal equity or equal treatment axiom:

HORIZONTAL EQUITY AXIOM:

Any two individuals who are identical preferencewise ought to be treated equally. In light of the ordinal nature of the analysis, equal treatment as understood here means placing any two such individuals on the same indifference level.

Thus horizontal equity as understood here requires that any two individuals that are identical preferencewise be assigned consumption bundles that lie on the same indifference surface (visualized to be drawn in the same commodity space with respect to the same origin). Note that the (inescapable) interpersonal comparison implicit in the axiom is summarized by the value judgement that individuals who have identical ordinal preferences enjoy so to speak the same "welfare level" when placed on the same level of indifference (i.e. on the same indifference surface). In the next section it will be explained why the natural requirement that the B-S ISWF be symmetric in its individual arguments does not ensure horizontal equity in the present sense. This actually is precisely the reason why the horizontal equity axiom advanced here is not formulated in terms of direct restrictions on the class of admissible B-S ISWF's. The implications of the axiom as stated here for the admissible B-S ISWF's are discussed in the next section.

In order to avoid the possible misunderstanding that individuals who are identical preferencewise ought to be treated identically regardless of their individual "needs" let me briefly attempt to clarify the important

question of needs in the context of welfare economics. When we think in
terms of individuals having different needs, we usually associate such
differences in the different physical or mental makeup (including health
problems) of different individuals. Regarding the mental aspects, these
could be expected to be directly reflected in the individual's preferences; note
that the case of mental disturbances, where we may not be willing to respect
the individual's overt preferences, is beyond the scope of individualistic
welfare economics. In case of physical differences, including health pro-
blems, then again the differences can be expected to be captured by differ-
ences in preferences, via differences in individual consumption sets.
Noting that for preferences to be identical, it is of course required that
both the consumption sets over which preferences are defined and the rankings
over these sets be identical it follows that different needs will in all
likelihood be reflected in different preferences. Since only individuals
with identical preferences are to be treated equally by the above horizontal
equity requirement, we see that the problem of differential needs in the
present formulation is quite naturally relegated to the difficult area of
vertical equity where it, of course, belongs. In other words, while no
solution to this difficult issue is attempted here, it is important to note
that the present framework is in principle amenable to its investigation.

To sum up the present section, I believe that the equal treatment
axiom presented here will be found acceptable from the viewpoint of the
traditional concern of welfare economics. What might seem surprising is
that any significant implications can be drawn from insistence on its fulfill-
ment. I therefore turn in the next section to an examination of the force

of this mild horizontal equity axiom insofar as its restrictive power (regarding the set of normatively admissible allocations) is concerned. Because the subject matter of this essay is Paretian welfare economics, it really is the joint fulfillment of the horizontal equity and Pareto criteria that will be of interest in our subsequent discussion.

IV. SOME IMPLICATIONS OF THE HORIZONTAL EQUITY AXIOM

In order to illustrate the power of the mild horizontal equity axiom advanced in the previous section, I will consider here a number of cases of gradually increasing interest. In order to sharpen the issues, most of the discussion will be addressed to the case where all individuals have identical ordinal preferences as the intuition of the results seems clearest in such a setting. But of course, the results presented below should more generally be thought of as applying to any two individuals who are identical preferencewise (without implying anything about individuals who are not; this vertical equity problem is the subject matter of the next section). For the sake of simplicity, the discussion will be confined to the case where the only consumption commodities present in the economy are of the pure private goods variety. While public goods and certain other forms of consumption externalities could easily be accommodated, nothing of any additional interest would be learned and so their treatment is omitted. Let me also point out that since the horizontal equity axiom is an equal treat-ment (of equals) axiom I will at times use these two terms interchangeably.

It might be appropriate at this point to note that the general problem considered here is that of achieving equity and efficiency in what may be referred to as an utopian "first-best" type of setting where the only economic constraints are of the resource scarcity variety. The important "second-best" (first-feasible-best" might be more appropriate) incentive effects that are bound to arise at the implementation stage (due to the infeasi-bility of costless lump-sum taxation) are thus entirely ignored. The justification for this utopian setting lies in my conviction that it is

impossible to intelligently analyze the celebrated equity-efficiency tradeoff arising in more realistic (second-best) settings if one does not have as a guiding norm a clear conception of the kind of good society that one would like to see realized in Utopia. In other words, since second-best solutions are best thought of as being compromising departures from some ideal unattainable first-best norm, we must have a clear notion of the nature of the (an) utopian solution in the first place. I cannot refrain from mentioning at this point that regarding the question of the equity-efficiency tradeoff it will be argued in the next section that the most fundamental such tradeoff actually already is found in first-best types of settings.

Keeping in mind that the analysis is explicitly intended to rely solely on the ordinal properties of individual preferences, let us move then to consider the implications of the horizontal equity axiom in a number of cases of interest. Note that whenever mention will be made of "utilities" in what follows, it should be understood that all the utilities are "calibrated" in the same way, i.e. identical utility representations will be used for all individuals since those have identical preferences.

(1) Consider first the purest version of the distribution problem in which there is a fixed one-dimensional income (commodity) to be divided among a number of individuals. Under the standard assumption that each individual has selfish and strictly monotonic preferences over the income line, it is true, by definition, that all the individuals have identical ordinal preferences. This simple truth implies, without any further assumptions, that fulfillment of the horizontal equity axiom requires in this classic case that income be distributed in a perfectly equalitarian way. Since

every income distribution is Pareto-efficient in the present case, the
equalitarian norm can be satisfied at no efficiency cost.

Since the equal treatment axiom refers to individuals with identical
preferences, we will assume from now on that all individuals have identical
ordinal preferences (an assumption that, as we saw, would have been super-
fluous in the preceding case).

(2) A moment's reflection will convince the reader that the perfectly
equalitarian result carries over to the case where, while still in a manna-
from-heaven type of world in which production is nonexistent, there is more
than one fixed pure private good to be divided among the individuals. For
when all the individuals have identical (continuous and monotone) ordinal
preferences, the equalitarian distribution of society's fixed aggregate
resources (i.e. assigning to every individual the average resource bundle)
is Pareto-efficient and clearly satisfies the horizontal equity axiom.
Note that under the additional standard assumption of strict convexity of
preferences this equalitarian allocation will actually be the only one
satisfying both the horizontal equity axiom and the Pareto-efficiency criter-
ion.

(3) Turning now to the more interesting case where production is possible
and where labor-leisure preference margins have to be allowed for, note
first that when individuals are identical abilitywise the equalitarian
result still applies and for essentially the same reasons as above. This
follows from the fact that the fundamental symmetry property of the alloca-
tion problem that we had so far is retained when we continue to assume that
individuals also are identical preferencewise. However, when individuals
differ in their productive abilities the fundamental symmetry is destroyed.

Allocations in which all individuals are assigned identical consumption (cum-leisure) bundles are in general Pareto-inefficient due to individual differences in the labor (leisure) transformation margin. It can easily be shown that in cases like the present one, there does always exist an allocation satisfying both the Pareto and the horizontal equity axioms. The difference between the nature of the solution in this case as opposed to the previous ones is that while "utilities" (appropriately calibrated in the same way for all individuals) at the said allocation are equal, consumption-cum-leisure bundles will generally differ across individuals. The asymmetry (in general) in the "socially optimal" distribution of consumption-cum-leisure bundles in this case reflects of course the asymmetry with which individual labor inputs will enter into the aggregate production possibility frontier of the economy under the present circumstances.

A closer look at what is likely to happen under symmetric B-S ISWF's in cases such as the one under consideration will, I believe, help to gain insight into the nature of the equity problem in relation to the B-S ISWF approach. The reason I focus on B-S ISWF's that are symmetric in their individualistic arguments is that such a symmetry requirement is plausible since we deal with cases where individuals have identical preferences. Yet, what I basically want to demonstrate is that this in no way ensures satisfaction of the equal treatment (horizontal equity) axiom.

Consider then the problem of maximizing a B-S ISWF which is symmetric in its individual arguments subject to a constraints set which is not. The result in general will be that individual utilities at the social optimum

will differ, violating the equal treatment axiom. As an illustration of this phenomenon, consider the interesting modern literature on optimal taxation pioneered by Mirrlees [12]. The basic structure of the optimal taxation model involves individuals who are identical preferencewise but who differ productivitywise due to differences in innate abilities. Under the utilitarian (Benthamite) sum-of-utilities social welfare function and a linear technology, utility at the first-best optimum is a function of ability (see Mirrlees [12] p. 201 for a specific example). Upon reflection, it is clear that the (Nash) product-of-utilities social welfare function or for this matter any smooth symmetric social welfare function will gener- ally imply an assignment of "utilities" at the social welfare maximum which will depend on individual "abilities." While certainly present in the "second-best" type of optimum inherent in the problem of optimal income taxation usually considered in that literature, I wish to emphasize that this phenomenon occurs, even more clearly, in the "first-best" types of problems of interest here where the B-S ISWF is maximized subject to the production constraints only (involving as it were a problem of optimal costless lump-sum taxation). Only under the Rawlsian maximin B-S ISWF, is it always true that utilities at the (first-best) optimum are equalized.

In other words, symmetric B-S ISWF's do not guarantee that individuals with identical preferences will be treated equally (preferencewise) when they happen to differ productivitywise, i.e. symmetric B-S ISWF's do not ensure fulfillment of the mild equal treatment axiom suggested here. True, under symmetric B-S ISWF's it will always be the case that indivi- duals who are identical productivitywise (in addition to their being

so preferencewise) will enjoy precisely the same treatment at the optimum.
Thus the equal treatment axiom implicit in the modern optimal taxation
literature might be expressed as invoking equal treatment of individuals
who are equal in every possible respect (preferences and productivities
in the present context). While I certainly have no quarrel with the ethi-
cal necessity of this condition (which is of course implied by the hori-
zontal equity axiom advocated here), I wish to reiterate my conviction that
from the viewpoint of modern welfare economics its ethical sufficiency is
very much in question. For, the sole concern of Paretian welfare economics
has always been to somehow amalgamate individual preferences (and preferences
only) in a way conducive to normatively appealing social choices. The only
arguments in B-S ISWF's are individual (utility-yielding) consumption items;
productivities are allowed to enter only on the constraints side of the
welfare optimization problem. From the viewpoint of this tradition therefore,
I believe my present standpoint to be justified. However, it turns out that
B-S ISWF's social optima will in general depend (positively or negatively) on
productivities. Thus an implicit evaluation of social states according to
productivities is implied, a value judgement that was not intended (and arises
from the fact that the shape of the utility possibility frontier is as impor-
tant in determining social optima as that of the B-S ISWF) and that on reflec-
tion will, I believe, be found questionable.

(4) It should not escape the reader's attention that the Rawlsian
maximin criterion is of course a B-S ISWF that is always consistent with
the perfectly egalitarian results derived in the simple cases (1) and (2)
above. Maybe more interesting is the fact that this social welfare criterion
is the only one that is always consistent with both the Pareto and equal
treatment axioms in the more interesting case (3) where "utilities" are

not perfectly transferable among individuals (i.e. where due to inherent individual asymmetries in the production and/or transfer of some commodities the utility possibility frontier of the economy is not symmetric among individuals when their identical preferences are calibrated in identical manner). This would seem to suggest that while application of the maximin criterion is in no way dictated by the above discussion for (the important) cases where individuals do not possess identical ordinal preferences it is however true that the preceding discussion suggests that whenever two individuals do have identical preferences, then, over any two such individuals, the social welfare hand should mitigate claims by means of the maximin criterion. This in fact is the extent to which the horizontal equity axiom has restrictive power.

Also, it might be of interest to remark that in the context of Arrow's interesting model of equality in public expenditure [3], since preferences there are assumed to be identical the present approach directly implies the maximin result (as opposed to the Benthamite one analyzed in [3]) of equalization of utilities and hence strong input progressivity. Actually, Arrow's model in [3] can properly be viewed as a generalization of the structure of Mirrlees's model [12] discussed above. It should also be noted that Phelps's [18] application of the maximin criterion to the optimal taxation problem is consistent with the approach suggested here. (when taxation is of the lump-sum variety).

V. REMARKS ON THE PROBLEM OF VERTICAL EQUITY

The discussion so far was confined to the horizontal equity problem of treating equals equally. As I have tried to make it clear, "equals" from the viewpoint of the tradition of modern welfare economics can only be taken to mean equality in terms of individuals' ordinal preferences. While the vertical equity problem of how to treat individuals who differ in their preferences is admittedly a more controversial subject, I believe it to be of interest to briefly discuss the vertical equity properties of two equity criteria advanced in recent literature. As the two criteria to be briefly discussed are consistent with both the Pareto-criterion and the equal-treatment (horizontal equity) axiom presented above, their implied vertical equity properties assume particular significance from the viewpoint of assessing their overall normative significance.

(1) The first equity concept I wish to discuss is that of egalitarian-equivalence. An allocation (list of feasible consumption bundles, one for each individual) is said to be egalitarian-equivalent if its underlying (ordinal) welfare distribution could have been generated by an egalitarian economy (see [17] where the concept is introduced and its properties formally analyzed). An egalitarian economy is defined as one in which all individuals are assigned identical consumption bundles. Clearly, the underlying norm is that of perfect egalitarianism. That is to say, equality is valued here as such and departures from perfect equality ought to be limited to an extent dictated by the definition of egalitarian-equivalence. The question is under what conditions will departures from perfect equality of bundles be called for under the present equity concept.

Observe first that two individuals who are identical preferencewise will always find themselves on identical indifference surfaces at any egalitarian-equivalent allocation; this concept thus is consistent with the equal treatment axiom. Also, it has been established in [17] that under even weaker than the standard assumptions on the economic environment egalitarian-equivalent allocations that are Pareto-efficient can always be found. Thus the concept is also consistent with the Pareto-criterion. In order to get a feeling for its vertical equity implications, it is enough to consider the case in which no production can take place. If all the individuals are identical preferencewise, then (under strict convexity of preferences) the only egalitarian-equivalent allocation that is Pareto-efficient is the perfectly egalitarian allocation in which each individual is assigned the same consumption bundle. But, of course, no vertical equity problem arises in such a setting in the first place.

Consider therefore the case in which at least two individuals differ preferencewise. The perfectly egalitarian allocation (which of course satisfies both the egalitarian equivalence criterion and the horizontal equity axiom) will in general cease to be Pareto-efficient in such a case. Now the only departures from perfect egalitarianism justifiable in terms of the egalitarian-equivalence criterion are those leading to allocations whose underlying ordinal welfare distribution are consistent with some perfectly egalitarian economy. This limits the extent of vertical differentiation to those unequal allocations that are considered, individual by individual, to be preferencewise indifferent to the perfectly equal allocation in some (hypothetical) economy. In other words, vertical differentiation is

limited by the requirement that there be a "good" economy ("perfectly egalitarian" under the present norm) such that each individual will be indifferent between living at the unequal actual allocation and living in the good (egalitarian) reference economy. Note, of course, that the need for vertical differentiation (i.e. the need for departures from perfect equality) in this case arises solely due to the fact that Pareto-efficiency is also assumed to be one of society's normative objectives and the perfectly egalitarian distribution of resources while clearly satisfying the present equity desiderata is (in general) Pareto-inefficient when individuals are not identical preferencewise.

In terms of the oft-mentioned equity-efficiency tradeoff it might be useful at this point to note that if the norm for an equitable society is perfect egalitarianism of consumption bundles (as is implicit in the egalitarian-equivalence criterion) then an inescapable equity-efficiency tradeoff immediately arises in its most basic form since perfect egalitarianism is in general incompatible with Pareto-efficiency. The point to be noted is that this tradeoff arises even in the absence of the much discussed equity-efficiency tradeoff arising from the adverse ("second-best") incentive effects of "real world" distortionary (redistributive) taxation on the labor-leisure and/or consumption-saving margins. In other words, an equity-efficiency tradeoff already arises in the kind of ("first-best") timeless pure exchange economy considered here (in which costless lump-sum taxation is implicitly assumed) from the very moment that the equity concept is defined. The particular equity-efficiency tradeoff that already arises in "utopias" of the kind dealt with here is in my mind the purest expression

of the kind of difficult conceptual obstacles encountered by those engaged in the search of the good society. Be that as it may, the vertical equity departures from perfect egalitarianism discussed above are entirely the result of this kind of equity-efficiency tradeoff.

Summing up the properties of the egalitarian equivalence criterion, it satisfies the equal treatment (horizontal equity) axiom, is consistent with Pareto-efficiency and its vertical equity implications are regulated by reference to the perfectly egalitarian yardstick underlying its very definition.

(2) The second concept I wish to mention is related to the interesting literature on fairness motivated by Foley's definition of fair (envy-free) allocations. An allocation is said to be fair if no individual in the economy prefers anyone else's consumption bundle over his own. While the fairness criterion clearly satisfies the equal treatment axiom, it was shown in [16] that it is inconsistent with the Pareto-criterion. In a sense this brings us back to the kind of equity-efficiency tradeoff discussed in the previous section. There, starting from a basic desire for perfect egalitarianism, we were immediately confronted with its general incompatibility with Pareto-efficiency. By moving to the concept of egalitarian-equivalence, we saw that one can still satisfy to a certain extent one's own intuitive wish for equality (if one has such) without having to give up Pareto-efficiency.

Likewise, if one's basic intuition about equity is that of fairness, then fairness ought to be the yardstick according to which one will want to evaluate the welfare implications of any allocation of consumption bundles. If fairness itself is incompatible with Pareto-efficiency, one

might be willing to weaken the requirement in the following way. At any
allocation, it can be asked whether or not the underlying ordinal welfare
distribution could have been generated by a fair economy. If the answer
is yes, it means that there exists a "good" ("fair" under the present norma-
tive benchmark) society with the special property that each individual
would be indifferent between living in it and living in the actual society
at the particular allocation under consideration, satisfying hence (to a
certain extent) one's wish for fairness.

These considerations motivated me to advance in [15] the concept of
fair-equivalent allocations. An allocation is said to be fair-equivalent
if its underlying ordinal welfare distribution could have been generated
by a fair allocation (in some reference economy). As explained in [15],
the criterion of fairness-equivalence satisfies the horizontal equity
axiom and is consistent with the Pareto-efficiency criterion. The question
of interest here relates to its vertical equity implications. In light of
our earlier discussion of the egalitarian equivalence criterion, it will
suffice to note that differential treatment of individuals who are not
identical preferencewise is limited by the requirement that there should
exist a (hypothetical) assignment of bundles that is fair (envy-free)
and individualwise indifferent to the allocation under consideration. It
should be observed that when all the individuals are identical preference-
wise, then a fair and Pareto-efficient allocation (at which all individuals
are assigned bundles lying on identical indifference surfaces) does of
course always exist but no problem of vertical equity arises in this case in
the first place.

Note that an egalitarian allocation is, of course, both egalitarian-
equivalent and fair (also, a fair allocation is, of course, fair-equivalent
as well). In those special cases where egalitarian allocations are Pareto-

efficient, there is no need to appeal to either egalitarian-equivalence or to fairness equivalence (and the problem of vertical equity effectively disappears). It is for the important general case where the conditions of the previous sentence are not fulfilled (and where fair and Pareto-efficient allocations fail to exist) that the problem of vertical equity becomes all important and where recourse to either of the "equivalence" criteria may be called for.

It is also worthwhile to note that, clearly, an egalitarian-equivalent allocation is, in particular, fair-equivalent. The converse, however, is generally untrue. Thus the egalitarian-equivalence criterion is more restrictive, a desirable property in itself. But if one's view of the good society is that of an envy-free society, the fairness-equivalence criterion will seem more basic. Thus, it seems best to let each reader decide for himself which criterion (if any) best represents his own personal values. But let me stress once more that from the viewpoint of the present discussion, the important properties of both criteria are that each is consistent with the minimal normative desirata expressed by the equal treatment (horizontal equity) axiom and by the Pareto-efficiency criterion.

VI CONCLUDING REMARKS

The following summarizes the main points made in the present essay:

(1) In a brief review of the modern welfare economics of Bergson and Samuelson, it was recalled that the _only_ normative desideratum that will _always_ be satisfied under this approach is Pareto-efficiency. In light of ongoing interpretive disagreement, it was also attempted to clarify the ethical implications of Arrow's impossibility theorem by restating it in what I believe to be a suitable welfare economic translation.

(2) Given that the only _a priori_ restriction imposed by the Bergson-Samuelson approach on the set of social optima is that those be Pareto-efficient it was argued that additional mild equity restrictions capable of commanding wide acceptance are called for. In particular, it was suggested that, as a minimum, one might wish to impose the horizontal equity restriction that equals ought to be treated equally.

(3) It was then argued that from the viewpoint of the traditional concern of modern welfare economics such a horizontal equity requirement had best be formulated in terms of equal treatment of individuals who are identical preferencewise. It was, therefore, suggested that a minimal horizontal equity axiom, stating that whenever individuals have identical ordinal preferences they be assigned bundles placing them at the same indifference level, be satisfied.

(4) The immediate implication of such an equal treatment axiom for the classic problem of distributing a fixed aggregate income was shown to be that such income ought to be divided perfectly equally. More generally,

it was shown that whenever two individuals are identical preferencewise, the only B-S ISWF's that guarantee that the equal treatment axiom is always satisfied are those that are of the maximin variety over the preferences of the two individuals concerned. In particular, it was noted that imposition of the natural symmetry requirement on B-S ISWF's will not in general satisfy the equal treatment (horizontal equity) axiom.

(5) In the same vein, it was argued that in the stylized model of the modern literature on optimal taxation where all the individuals are identical preferencewise, the maximin criterion is called for. Any other B-S ISWF will, in such a setting, run into possible inconsistencies with the above equal treatment property. The same remark applies to Arrow's model of equality of public expenditure. When individual preferences differ, there is no presumption in favor of the maximin criterion(and of course no natural way to apply it).

(6) Regarding the general case where individuals may differ preferencewise, it was explained that the recently advanced criteria of egalitarian-equivalence and fairness - equivalence satisfy both the Pareto and the equal treatment desiderata. Their respective vertical equity implications were discussed in terms of their different underlying norms regarding the basic conception of the good society.

(7) Finally let me stress again that the analysis presented here is of the (utopian) first-best variety. As explained in the text the important and difficult (real-world) second-best aspects of the equity-efficiency problem can be fruitfully approached only if a clear conception of the first-best norm is available. What the second-best implications of the approach presented here are is I think a relevant and challenging question. I hope that future research will successfully address itself to this important problem.

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