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A THEORETICAL APPROACH TO THE DECISION TO STOP DELIBERATING OVER LEGISLATIVE ALTERNATIVES*

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INTRODUCTION

Since the publication of Arrow's (1951) results on the impossibility of generating a well-behaved social welfare function from transitive individual preference orderings, there has been considerable scholarly interest in the predictability of collective decision-making. In the special case of majority rule, it was supposed that "real world" distributions of preference orderings would frequently lead to the existence of alternatives that could not be defeated in pairwise majority votes, thus avoiding the spectre of Condorcet cycles. On the contrary, such majority rule equilibria or core points have been shown to exist only when exceedingly restrictive symmetry assumptions are made about the distribution of preferences (Plott, 1967; Sloss, 1973). Moreover, Plott and Levine (1978) have shown experimentally that by manipulating the agenda it is possible to effect any outcome among the alternatives, even when an equilibrium exists. And, Hoffman and Plott (1978) found experimentally that by changing the voting procedure it is possible to change the probability that a 5-person, majority rule committee will adopt the equilibrium outcome.

When equilbria do not exist, there is no generally accepted prediction of the outcome. In fact, recent results suggest that any point in the alternative space might theoretically be the outcome of a majority rule voting process (McKelvey, 1976; Schofield, 1979). Yet, experimental results suggest that outcomes fall into possibly predictable distributions even when equilibria do not exist, (Fiorina and Plott, 1978; McKelvey, Ordeshook, and Winer, 1978).

Thus, both when equilibria exist and when they do not, the theoretical and experimental results run somewhat counter to one another. This observation suggests the need for an alternative set of theoretical models which can be verified experimentally. A recent paper by Ferejohn, Fiorina and Packel (1978) provides a step in that direction. They recognize that, even when equilibria do not exist, transactions costs and political realities should lead people to choose one particular solution in finite time. The end result of the model developed in their paper and generalized in Packel (1978) is a probability measure over the set of alternatives, henceforth referred to as the <u>stochastic solution</u>. Thus, the attempt to single out a restrictive set of alternatives as <u>the</u> solution is abandoned. Instead, a nondeterministic approach assigns a numerical value to each (measurable) subset of alternatives, representing the probability that the final outcome will belong to the subset. Since the stopping model we propose builds upon the stochastic solution approach, we briefly summarize the definition and main properties of the stochastic solution.

An alternative representing the initial status quo is selected randomly. The probability, at each stage, of moving from the current status quo to a new status quo is proportional to the number of minimal winning coalitions preferring the new alternative to the current one. A Markov chain model thus emerges for describing and computing the stage-by-stage status quo probabilities. Final probabilities and the stochastic solution are determined by studying the limiting behavior of the Markov process. This is tantamount, loosely speaking, to the imposition of a stopping rule which terminates with equal probability at any given stage. The stochastic model predicts that, if a strong core or equilibrium exists, the process will lead to that equilibrium with probability 1. If an equilibrium does not exist, a final probability distribution over outcomes exists without qualification in the finite alternative case and under reasonable assumptions in the spatial alternative case. Simulations of the model on Fiorina and Plott's (1978) 5-person majority rule spatial committees without equilibria yield results similar to the experimental results obtained by Fiorina and Plott (1978). Simulations on Fiorina and Plott's (1978) and Hoffman and Plott's (1978) committees with equilibria yield results similar to the experimental results if the simulation process is stopped arbitrarily after the same number of steps as were taken by the experimental subjects.

The correspondence between the stochastic model of Ferejohn, Fiorina, and Packel (1978) and the experimental results of Fiorina and Plott (1978) and Hoffman and Plott (1978) suggests that modelling a committee decision process as a Markov process may not be unreasonably simplistic, despite the myopia assumption implicit in the use of a Markov model. What is missing from their model, however, is an explicit formulation of the way the process stops. Why, for instance, might subjects in an equilibrium experiment stop and adopt a nonequilibrium outcome, when they would have eventually reached the equilibrium if they had continued?

The model developed in this paper is an adaptation of the stochastic solution model to a committee decision process over finite alternatives with an endogenous stopping rule. The model assumes that the decision process itself entails costs which must be borne by the committee members. Real world examples of decision costs might be the opportunity cost of time or the potential for losing friends or future allies if an individual were to exercise his or her full power to keep the committee from reaching a decision.

To begin with we assume myopic individual decision rules. An individual votes to move from a current alternative x to alternative y if and only if the utility of y minus the cost of an added stage in the process exceeds the utility of x. The process will proceed to the next stage if some winning coalition profers to move from x to some other y. If no such coalition and alternative exist, the process stops. The decision rule is myopic because each committee member only considers the next stage in deciding how to vote. The resultant utility and cost based model is no longer strictly Markovian (the Markov matrix may change at each stage), but analysis proceeds much as it did for the stochastic solution, which now becomes the special case of zero costs.

Costs are found to affect the decision to stop the process in the following ways. First, if an individual voter's marginal cost is always less than the smallest absolute difference between the voter's utilities over distinct alternatives, then the cost function does not affect that voter's decision at any stage of the process. At the other extreme, if a blocking coalition of voters have marginal cost functions which eventually exceed the largest difference between their respective utilities, the process must stop in finite time and may very well stop short of an equilibrium. Additional results are obtained which relate cost functions and the decision to stop and it is shown that our stochastic procedure will converge for a broad class of individual cost functions.

FINITE ALTERNATIVE STOCHASTIC MODEL WITH ENDOGENOUS STOPPING RULE

We begin with the following definitions and notation:

$X = \{1,2,\ldots,j,\ldots,J\}$ is the set of alternatives.

$V = \{1,2,\ldots,i,\ldots,N\}$ is the set of voters.

$k = 1,2,\ldots$ denotes the various stages in the process.

c_i^k = total cost incurred by voter i after k stages.

$\dot{c}_i^k = c_i^k - c_i^{k-1}$ = marginal cost incurred by i from stage k-1 to stage k.

U_{ij} = utility to voter i if the group chooses outcome j (ignoring costs)

$U_{ij}^k = U_{ij} - c_i^k$ = utility to voter i if the group chooses outcome j after k stages.

Though the various experiments referred to and the example of the next section all use absolute majority rule for voting, we generally allow voting rules from the class of simple games [see Packel (1978) for extension to a larger class]. This merely requires specification of a collection W of winning coalitions (subsets of V). Given W, we can define

$M = \{m \in W \mid \text{no proper subset of } m \text{ is in } W\}$, the minimal winning coalitions

$B = \{b \subseteq V \mid b \cap m \neq \Phi \ \forall m \in M\}$, the blocking coalitions.

Having specified the game-theoretic structure, we can now define the key components of the stochastic model. For all alternatives $j,h \in X$ and each stage k, define

$A_{jh}^k = \left|\{m \in M\colon U_{ij}^{k-1} < U_{ih}^k \ \forall i \in m\}\right|$, the number of minimal winning coalitions preferring h at stage k to j at stage k-1.

$$P_{jh}^k = \begin{cases} A_{jh}^k / \sum_{\ell=1}^{J} A_{j\ell}^k & \text{if } \sum_{\ell=1}^{J} A_{j\ell}^k > 0 \\ \delta_{jh} & \text{otherwise,} \end{cases}$$

where $\delta_{jh} = 1$ if $j = h$ and 0 otherwise.

The J x J matrix $P_k = \left(P_{jh}^k\right)$ is a stochastic matrix whose (j,h) entry is the transition probability of going to alternative h at stage k, given that alternative j is the status quo at stage k-1. The product matrix $P^{(T)} = \prod_{k=1}^{T} P_k$ provides transition probabilities from the start of the process through stage T. The "final" transition probabilities are given by $P^{(\infty)} = \lim_{T\to\infty} P^{(T)}$ if the process is aperiodic and by

$P^{(\infty)} = \lim_{T\to\infty} \frac{1}{T+1} \sum_{k=0}^{T} P^{(k)}$ in general. If marginal costs are nondecreasing for each voter, the matrices P_k must eventually agree for k large enough, and the above limits will exist in accordance with the standard theory of finite Markov chains. See, for example, Kemeny and Snell (1960). We do not consider more general cost functions, but we conjecture the existence of $P^{(\infty)}$ whenever marginal costs are nonnegative.

Now let $Q(0) = (q_1,\ldots,q_J)$ be a new vector whose j^{th} entry is the probability of starting the process at alternative j. Then $Q = Q(0)P^{(\infty)}$ provides the desired limiting probability distribution on the outcomes, the end product of our stochastic model

Results for the stochastic solution with no costs (Ferejohn, Fiorina, and Packel, 1978; Packel, 1978) show that Q will be independent of the starting distribution if and only if the process has a single ergodic set. Furthermore, if a strong equilibrium exists, the process will converge to this equilibrium with probability 1. Generally, however, the stochastic solution for costless processes has no built in mechanism for stopping after finitely many stages.

The results below consider ways that costs may or may not impose an endogenous stopping rule on the process.

Theorem 1. If $\dot{c}_i^k < \text{MIN}\left\{\left|U_{ij} - U_{ih}\right| : j,h \in X, j \neq h\right\}$, then costs do not affect the action of voter i at stage k.

Proof.

$$\left(U_{ij}^{k-1} - U_{ih}^{k}\right) \Big/ \left(U_{ij} - U_{ih}\right) = \left(U_{ij} - c_i^{k-1} - (U_{ih} - c_i^k)\right) \Big/ \left(U_{ij} - U_{ih}\right)$$

$$= \left(U_{ij} - U_{ih} + \dot{c}_i^k\right) \Big/ \left(U_{ij} - U_{ih}\right)$$

$$> 0 \quad [\text{since } \dot{c}_i^k < \left|U_{ij} - U_{ih}\right|].$$

Thus, $U_{ij}^{k-1} - U_{ih}^{k}$ and $U_{ij} - U_{ih}$ always have the same sign and costs cannot alter i's preferences at stage k. Q.E.D.

Corollary. If the hypothesis of Theorem 1 holds for all voters i and all stages k, then costs will not affect the stochastic solution and no endogenous stopping rule is imposed.

At the other extreme to the above results, we now consider restrictions for which costs must induce stopping and can alter the final distribution Q.

Theorem 2. If $\dot{c}_i^k \geq \text{MAX}\left\{\left|U_{ij} - U_{ih}\right| : j,h \in X\right\}$, then voter i will prefer the status quo during stage k.

Proof. $U_{ij}^{k-1} - U_{ih}^{k} = U_{ij} - U_{ih} + \dot{c}_i^k \geq 0$ for all $j,h \epsilon X$. Thus voter i will prefer the current status quo at stage k. Q.E.D.

Corollary. If the hypothesis of Theorem 2 is satisfied at some stage k for all voters i in some blocking coalition b, then the process will stop at or prior to stage k. This will be the case for some k if, in particular, $\left\{c_i^k\right\}_{k=1}^{\infty}$ is nondecreasing and unbounded $\forall i \epsilon b$.

The extreme situation for stopping would occur at stage 1 if $\dot{c}_i^1 \geq \text{MAX}\{\left|U_{ij} - U_{ih}\right| : j,h \epsilon X\}$ for all voters i in some blocking coalition b. The process would never get started, and the result would simply be the starting distribution Q(0). Thus, even if an equilibrium were to exist, this provides a rather extreme example of how the model could stop short of the core with positive probability. Some thought about the model suggests a variety of less contrived situations in which this might occur.

AN EXAMPLE

This section considers a finite alternative majority rule game with a strong equilibrium. Costs are assumed to grow quadratically and the limit distribution assigns a probability less than 1 to the equilibrium outcome.

Let $X = \{1,2,3,4,5,6,7\}$

$V = \{1,2,3\}$

$M = \{12,13,23\}$

Payoff Matrix $[U_{ij}] =$

		Voters 1	2	3	
	1	35	20	15	
	2	30	10	30	
	3	20	35	20	
Alternatives	4	25	30	35	←Equilibrium: alternative 4
	5	15	15	45	
	6	10	25	40	
	7	5	40	25	

$c_i^k = k^2 \; \forall i$, so $\dot{c}_i^k = 2k-1$

$Q(0) = (1/7, 1/7, 1/7, 1/7, 1/7, 1/7, 1/7)$

Applying the model developed in the previous section, we find that $P_1 = P_2$, $P_3 = P_4 = P_5$, and $P_k = I$ (identity matrix) for all $k \geq 6$. Thus $P^{(\infty)} = P^{(5)} = P_1^2 P_3^3$, and the specific form of $P^{(\infty)}$ is given by:

		Alternatives 1	2	3	4	5	6	7
	1	0	11/48	0	34/48	0	3/48	0
	2	0	11/48	0	34/48	0	3/48	0
	3	0	1/12	0	3/4	0	1/6	0
Alternatives	4	0	0	0	1	0	0	0
	5	0	7/36	0	29/36	0	0	0
	6	0	2/9	0	7/9	0	0	0
	7	0	1/16	0	37/48	0	1/6	0

The final distribution over alternatives, $Q = Q(0)P^{(\infty)}$, is given by

Alternatives	Probabilities
1	0
2	7/48
3	0
4	265/336
5	0
6	22/336
7	0

Thus, the equilibrium will be chosen with probability 265/336 and alternatives 2 and 6 will be chosen with probabilities 7/48 and 22/336 respectively. This illustrates the corollary to Theorem 2 and shows that selection of the equilibrium outcome, while still a fairly likely event is by no means a certainty.

CONCLUDING REMARKS

The model developed in this paper provides a possible theoretical explanation for the experimental observation that committees with equilibrium generating preference orderings do not necessarily choose equilibrium outcomes. The model takes a rather specific form, assuming that all status quo changing minimal winning coalitions form with equal likelihood at each stage. We note, however, that various other coalition formation mechanisms can be treated in the Markov manner we propose; and the same theoretical and qualitative results will emerge.

Extension of the model to the case of spatial alternatives (say a compact subset of R^m) can be achieved with continuous state Markov process methods. For the no cost case see Ferejohn, Fiorina, and Packel (1978) or Packel (1978). With the addition of individual costs at each stage, our results about stopping and the phenomenon of stopping short of the equilibrium will again emerge.

The myopic nature of our model provides another area for possible investigation. Perhaps voters, by looking ahead and anticipating future costs, will be more inclined to unearth an equilibrium alternative than our model indicates. Confirmation of the qualitative and quantitative results we have obtained and the feasibility of possible refinements require the design of committee experiments with explicit instead of implicit cost functions.

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