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DYNAMIC MONOPOLY POWER WHEN SEARCH IS COSTLY

by

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Abstract

In this paper, a repeated search model is studied to determine whether the repeat nature of sales can be used to limit the monopoly power that firms enjoy when search is costly. To make matters interesting, we assume that production coasts exhibit decreasing returns to scale. After carefully characterizing the static game, we consider a dynamic game in which buyers and sellers are completely informed about their common history. Here, in any equilibrium that is stable against some randomness in buyers' search strategies, if buyers use symmetric strategies, then monopoly power is not limited. Next, we explore a dynamic game in which buyers and sellers have private histories. Punishment strategies are more difficult to implement here, and in fact monopoly power is unrestrained in any equilibrium in which buyers use strongly symmetric strategies. The full force of stability is not required. Our results give a dynamic equilibrium foundation to Diamond's theory of monopoly power, when production costs are rising with output.

I. INTRODUCTION

In the classical search model pioneered by Stigler (1961) and Rothschild (1973), buyers of commodities have poor information about the prices charged by different sellers. They must undertake a costly search process in order to acquire information about the price distribution and make a partially informed choice of supplier. These models have proved popular for at least three reasons. First, as originally argued by Stigler, they offer explanations for empirical anomalies like price dispersion. Second, they provide a simple maximizing framework within which markets clear slowly. The models are thus useful in understanding the nature of unemployment. Finally, and most fundamentally, the search models give an explanation of the persistence of monopoly power. This argument was made most forcefully by Diamond (1971). If it costs an existing buyer K to go and find the price charged by one more seller, then the current seller can set a price K above what he thinks his competitors are charging before he loses his customer. Since this is true for every firm at once, prices must rise to their monopoly level.

Diamond's arguments about monopoly are clearly fundamental and have been applied in a variety of extended models.[1] One extension to which the monopoly arguments do not obviously apply is the extension to a repeated search framework. Here, it would seem, strategic buyers could force down prices by threatening to respond to a high current price with a reduction in future patronage. As such, it is not clear that monopoly pricing would characterize

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[1] See, for example, Axell (1977), Butters (1977), Reinganum (1979), Burdett and Judd (1983), Rob (1985), Bagwell (1987), and Diamond (1987). This work suggests that a significant amount of monopoly power survives the possibility of heterogeneous buyers and sellers.

an environment in which sellers and buyers meet repeatedly. The formal analysis of this problem has not been previously developed, probably for two reasons. First, a particular equilibrium of a dynamic model is simply the repeated static equilibrium, and, in this narrow sense, repeated search does not necessarily rule out monopoly power. The second reason is that when repeated search does restrict monopoly power, the reasoning is both conceptually and technically straightforward. Firms that attempt to exercise monopoly power are punished by buyers, who simply refuse to trade with them in the future. In a strategic version of Diamond's model, where sellers select prices and buyers select sellers simultaneously, such a punishment threat is clearly credible. A seller threatened with a boycott expects that no buyers will see his price, and thus may set his price equal to (or higher than) the price of rivals, making credible his own punishment. This possibility is particularly sensible in the case of constant returns to scale, where pricing incentives are independent of scale. In any case, it is easily understood that strategic consumers can restrict monopoly power in a repeated search framework.

The purpose of this paper is to show that this simple reasoning is not always compelling, as such punishment strategies are not in fact credible when production costs exhibit decreasing returns to scale <u>and</u> buyers' search strategies involve some exogenous or endogenous randomness. Consequently, when returns to scale are decreasing, there is a strong sense in which the possibility of repeated interaction is insufficient to restrict monopoly power.

The argument runs as follows. Suppose a seller exercises monopoly power and a group of buyers decides to punish the firm by withdrawing their future patronage. Of course, this threat is credible only if buyers expect to pay at least as high of price at the seller's store as they would at any of their other options. Now, the deviating seller may be happy to set a higher price if he does not expect to get any customers. But, if he perceives a small chance that some buyers will return to his store, then the price the seller sets, and therefore the price buyers expect to pay, is determined by the returns to scale exhibited by the seller's cost function. If costs rise with output and the seller expects few buyers as consequence of his punishment, then he will set a low price, undermining the very credibility of his punishment.

The key issue is whether the seller perceives a chance that he will receive any buyers, following a deviation. This perception is of course appropriate if buyers' search strategies are affected by non-price random factors. It is quite natural to consider the role of such factors. A buyer might expect to find the commodity at a low price at one store, but he may nevertheless pick a different store which stocks an alternative item he desires, for example. Now we do not really wish to study these randomized models directly, since the equilibrium behavior of buyers and sellers might be sensitive to the particular randomization chosen. However, it does seem reasonable to restrict attention to equilibria which are stable against a slight impact of exogenous randomness.

Using this notion of stability, we carefully analyze a static game and find that all stable equilibria in which buyers use symmetric strategies are characterized by identical, positive profits for firms, when costs exhibit decreasing returns to scale. The role of stability is crucial here; a plethora of non-stable equilibria exist with asymmetric firm profits.

Next, we repeat the static game over two periods, assuming that sellers and buyers begin period two completely informed about their common history of period one play. This dynamic game allows for the punishment possibilities discussed above. Yet, in any stable equilibrium with symmetric buyer strategies, if costs exhibit decreasing returns to scale, then in any period a seller chooses his price to maximize his current period expected profit, disregarding completely the possibility that his future profit might be affected by his current price. Monopoly pricing occurs in each period.

Finally, we analyze a two period game in which buyers and sellers observe only their respective private histories. In such an environment, punishment strategies are difficult to implement, because only those buyers who initially visit a particular firm have the opportunity to observe a deviation by that firm. Focusing on equilibria in which buyer strategies are strongly symmetric, we show that firms with decreasing returns to scale must necessarily receive buyers in period two who "cross over" from other firms. This crossing over occurs as part of a mixed strategy for buyers, and so the second period of the dynamic game is characterized by an endogenous form of randomness. Accordingly, in any strongly symmetric equilibrium when costs exhibit decreasing returns to scale, firms again monopoly price in each period, disregarding completely threats associated with the future. An exogenous notion of stability is not required for monopoly pricing, when history is private and correspondingly punishment is difficult.

The paper is organized in five sections. We begin in section two with an extended discussion of the static version of Diamond's search game. Our notion of stability is introduced here. The static model is also interesting, because it involves finite numbers of buyers and sellers. This case is unusual in the literature, and therefore the arguments may be somewhat novel. Next, in section three, we display the various equilibrium possibilities for the dynamic game with common history. Stability is then employed to select a unique equilibrium. Section four contains an analogous study of the private history model, though the full force of stability is no longer required. We conclude in section five by exploring various extensions of the model.

II. THE STATIC MODEL

We begin with consideration of a one time search game. The model is closest in spirit to Diamond's. There are assumed to be a finite number J of sellers and a finite number I of buyers, with I and J exceeding one. Each seller produces the same homogenous output according to a common differentiable cost function C(q). Buyers search out one and only one seller during the day and then buy a quantity q(p) that depends on the price charged at the firm that they visit. Buyers are not informed about the prices charged by each of the firms at the beginning of the day, and they can only acquire information about the prices that are actually charged by visiting the firm and getting a price quote.[2]

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[2] This strict interpretation of costly search, that buyers can acquire only one price quotation in a given period, is somewhat unusual in the search literature. As we will see, nothing is gained for the purposes of this paper in making the search technology more complex, and indeed much is lost in that the notation becomes very complicated. Hence it seems sensible to adopt this restrictive approach.

The search problem is modelled as an extensive form game. The first player to move will be nature. For every buyer i and seller j, nature assigns i to j with some small probability ϵ. After each of the sellers has been given a chance to capture buyers in this way, those buyers who have not been assigned are allowed to choose the respective sellers that they intend to visit. The idea behind this portion of the tree is to capture the notion the there will be exogenous random forces that impinge on buyers' search decisions.[3] For example, some of buyer i's other activities may bring i to exactly j's location at the point where the commodity is required. Alternative, i may require some other commodity that can only be acquired at j's store. There could be any host of random factors that induce i to buy from j's store even when i believes that somewhat lower prices could be acquired by going elsewhere. For simplicity, we have assumed that nature's assignment probability is independent of i and j. In general, one should imagine some random factor affecting the search cost function faced by a particular buyer. These random factors might make it considerably cheaper for buyer i to go to one particular store.

Now in this simpler version, nature assigns the buyer arbitrarily with probability ϵJ. With probability $1-\epsilon J$, the buyer is free to choose a store according to his beliefs about the prices being offered at various stores. At this set of nodes in the game tree, buyer i chooses a search strategy $s_i \epsilon S^J$, where $S^J = \{x \epsilon \mathbb{R}^J_+ : \Sigma^J_{j=1} x_j = 1\}$. In this s_{ij} is interpreted as the probability that buyer i selects seller j conditional on his being allowed to choose. Recalling that each buyer gets to select one store per period, we see that i's

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[3] Nature can be thought of as inducing trembles into the buyers strategies. See Selten (1975)

complete search strategy for a particular period is characterized by the vector s_i.

At the beginning of each period, every store selects a price to post on its output for that period. Once posted, this price is fixed throughout the period. Naturally the price will be posted in accordance with the seller's belief about the number of buyers who will visit his store.

Thus the search game is a game of incomplete information. Sellers do not know buyer's search choices when they post their price, and buyers do not know seller's prices at the point where they must make their initial search decision. It may be simplest to think of this game as a simultaneous move game between buyers and sellers which is preceded by some moves of nature. However, since sellers do not observe nature's assignment, it is inessential whether sellers are thought to move before or after nature.

Associated with the buyers common demand curve q(p), is an indirect utility function u(p). It is convenient to assume that q is differentiable, and that both q and u are decreasing in price. A seller who receives n buyers will then get profits

$$v(p,n) = q(p)pn-C[q(p)n]$$

when he charges a price p, independent of the identity of the buyers who actually visit the firm. Buyers who select firm j and pay p will receive utility u(p). These payoffs, along with the tree described earlier, will be referred to as the game $G(\epsilon)$. Our main interest will of course be in the game G(0), which in the case of constant costs is essentially Diamond's version of the search problem. One way to view the discussion here is as an attempt to understand the way that more general assumptions about production costs will affect Diamond's analysis.

The equilibria of these games are straightforward. A strategy for firm j is just a price, $p_j \epsilon \mathbb{R}_+$, while a strategy for buyer i is just a vector $s_i \epsilon S^J$. If a buyer is assigned by nature in $G[\epsilon]$ to firm j, his payoff is just $u(p_j)$, while if a buyer gets to choose the firm that he will patronize, his payoff is equal to

$$U[p,s_i] = \Sigma_{j=1}^{J} s_{ij} u(p_j)$$

Given the set of buyers search strategies $s=[s_1, \ldots s_I]$, the probability that each of the buyers in the set K selects firm j in the game $G[\epsilon]$ is

$$Q_{j\epsilon}^{K}(s)=\Pi_{k\epsilon K}[\epsilon+(1-\epsilon)s_{kj}]\Pi_{k\notin K}[1-\epsilon-(1-\epsilon)s_{kj}]$$

From this formula the firm's payoff can be written in one of two ways. Letting P(I) be the power set of I (i.e., the set of all subsets of I plus the null set), we have

$$V_{j\epsilon}[P_j,s]=\sum_{K\epsilon P(I)} Q_{j\epsilon}^{K}(s)\ v\ (P_j,|K|)$$

where |K| represents the cardinality of the set K. Alternately we can define for $G[\epsilon]$ the probability that firm j gets n buyers as

$$a_{j\epsilon}^{n}(s)=\sum_{K\epsilon P(I):|K|=n} Q_{j\epsilon}^{K}(s)$$

and write the payoff function as

$$V_{j\epsilon}[p_j,s]\equiv\sum_{n=0}^{I} a_{j\epsilon}^{n}(s)v(P_j,n)$$

An <u>equilibrium for the game G[ϵ]</u> can now be defined as a pair of strategies p* and s* satisfying

$$U[p^*,s_i^*]\geq U[p^*,s_i] \text{ for all } s_i\epsilon S^J$$

and

$$V_{j\epsilon}[p_j^*,s^*]\geq V_{j\epsilon}[p_j,s^*] \text{ for all } p_j\epsilon \mathbb{R}_+ \text{ and } j\epsilon J$$

EQUILIBRIA OF G[0]

Our focus is on the game G[0]. Many of its properties are immediate. For example, it is clear that all firms that acquire buyers in equilibrium must charge the same price. This is because the buyers know the firm's pricing strategies in the usual game theoretic sense, and because the utility that a buyer gets at any particular firm does not depend on the number of other buyers who visit. Price dispersion is therefore not a property of the game G[0], except in the sense that non-active firms may charge high prices. Nevertheless, the game G[0] can admit a tremendous variety of equilibria, as we will see below.

To begin, it is useful to consider the case in which firms have constant costs of production. Let c^o be the production cost and let p^o be the price that maximizes $[p-c^o]q(p)$. As all buyers are identical in this model, any firm acting as a monopolist would charge the price p^o no matter how many buyers he expected at his firm. We can thus refer to p^o as the monopoly price. Now, since buyers cannot condition their strategies on prices, it must be that p^o is charged by every firm receiving a positive flow of buyers in equilibrium.[4] This result is essentially Diamond's result with strategic consumers. Notice, however, that the distribution of buyers over sellers is not unique. There are equilibria in which all sellers receive buyers and in which only strict subsets of sellers attract buyers. The latter equilibria are characterized by active sellers charging p^o and non-active sellers setting a price in excess of p^o.

Before proceeding, it is important to note that the price at which transactions occur is not unique when firms have different constant cost technologies. To see why, suppose that all but one of the firms have common cost c^o while there is a single firm with cost $c^{oo}>c^o$. Then the monopoly price of this higher cost firm should strictly exceed the monopoly price of the other firms. There is clearly an equilibrium where each of the lower cost firms sells at its appropriate monopoly price. However, there is another equilibrium where the high cost firm sells to the entire market at its monopoly price, and each of the lower cost firms sets a price that is strictly higher than the high cost firm's monopoly price. Thus, there are two potential equilibrium prices for this model. The trouble with the latter equilibrium is that it is unstable in a very strong sense. If we move from this (appropriately modified since cost functions differ) version of G[0] to a

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[4] As Diamond has argued, this result continues to hold when a buyer can visit more than one store per period, so long as each visit is costly.

game G $[\epsilon]$, where ϵ could be arbitrarily small, then it will no longer be a best reply for the low cost firms to set very high prices. If there is the smallest chance a buyer will come, they will set prices below the high cost firm and the equilibrium will break down. The difficulty is that some of the equilibria of G[0] are not close in any sense to any of the equilibria in $G[\epsilon]$. We have a failure of lower semicontinuity in the equilibrium correspondence.

With non constant productions cost, we get a variety of equilibria for very different reasons. Assume that the production costs are such that the price that any firm will charge when it gets n buyers for sure is an increasing function of n. Now it is possible to allocate the buyers over the sellers in a variety of different ways to create different equilibria. For example, all the buyers could choose a search strategy that sends them with probability 1 to one particular firm. This firm would charge

$$\mathrm{argmax}_p v(p,1)$$

and all the other firms would be happy to charge high prices, knowing that they will not be receiving any buyers anyway. On the other hand, the buyers could be spread evenly over the firms, assuming I is an integer multiple of J, so that each firm charges

$$\mathrm{argmax}_p v(p,I/J)$$

Every firm is active in this event, so we do not have to worry about setting higher prices for non active firms. As a final example, consider the buyer strategy sij=1/J, for all i and j. Here, buyers adopt a symmetric and completely mixed strategy, and sellers each choose price to maximize

$$\sum_{n=0}^{I} \binom{I}{n} (1/J)^n (1-1/J)^{I-n} v(p,n)$$

This assignment will be considered in more detail later. As these examples illustrate, a wide variety of equilibrium behavior is possible Moreover, the allocation of buyers over sellers as well as the market price are non-unique when costs are not constant.

This large set of equilibria for G[0] admits a simple characterization. First, recall that buyers are behaving optimally only if all sellers who receive buyers with positive probability charge the same price. Second, notice that sellers are optimizing only if this common price maximizes the expected profit of each of the firms who charges it, given the probability distribution over number buyers that is generated by the buyers' search strategies. We thus have:

LEMMA 1 A set of search strategies $s^* = \{s_1^*, \ldots, s_I^*\}$ and prices $p^* = \{p_1^*, \ldots, p_J^*\}$ is an equilibrium for the search game G[0] if and only if

i) $p_k^* = p^o$ for each k such that $s_{ik}^* > 0$ for some i

ii) $p_k^* \geq p^o$ for each k such that $s_{ik}^* = 0$ for all i

iii) p^o maximizes[5]

$$\sum_{n=0}^{I} a_j^n(s^*) \, v(p,n)$$

for each j=1,...,J.

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[5] In this and all subsequent work, we write $a_{j0}^n = a_j^n$ to save notation.

STABLE EQUILIBRIA OF G[0]

Lemma 1 pertains to the entire set of equilibria for the game G[0]. However, as mentioned above, our fundamental interest is in equilibria of G[0] which are "close" to equilibria of $G[\epsilon]$, for small ϵ.

DEFINITION 1: An equilibrium [p*,s*] of G[0] is said to be stable if and only if there exists some sequence $\{p(\epsilon), s(\epsilon)\}$ such that $\{p(\epsilon), s(\epsilon)\}$ is an equilibrium for $G[\epsilon]$ and $\lim_{\epsilon \to 0} \{p(\epsilon), s(\epsilon)\} = \{p^*, s^*\}$.

Thus, stable equilibria of G[0] are simply equilibria which are limit points of equilibria of $G[\epsilon]$.

With some extra assumptions, we will see that the set of stable equilibria of G[0] is easily characterized. Henceforth, it is assumed that:

Assumption 1:

i) $v(p,n)$ is strictly concave in p for all n.

ii) for every a_j^n with $a_j^o \neq 1$, there exists p^o such that $\sum_{n=0}^{I} a_j^n v(p^o, n) > 0$.

iii) $v(p,0)=0$.

The implication of Assumption 1 is that the profit maximizing price is always unique, with positive maximized profits, for any probability distribution over the number of visiting sellers that satisfies $a_j^o \neq 1$. Observe that Assumption 1, part ii, will hold if demand is sufficiently high relative to cost. The following lemma will prove useful in characterizing stable equilibria of G[0].

LEMMA 2 In any equilibrium of the game $G[\epsilon]$ there can be at most two distinct prices.

PROOF Consider an equilibrium $\{p_1^o, \ldots, p_J^o\}$ consisting of an array containing more than one price. Without loss of generality, let p_1^o be the lowest price. Consider any firm j whose price exceeds p_1^o. Buyers left free to choose will select such a firm j with probability 0 since they can assure themselves a strictly higher utility or lower price by going to firm 1. The probability that any buyer i selects j must therefore be ϵ. Hence for j's price to be optimal, it must maximize

$$\sum_{k=0}^{I} \binom{I}{k} \epsilon^k (1-\epsilon)^{I-k} v(p,k)$$

Since Assumption 1 assures that the solution to this maximization is unique, all firms charging prices strictly above the minimum must charge this price, and the result follows. QED

With this lemma established, we can now state our first result for stable equilibria of G [0].

<u>**Proposition 1**</u>: Let $\{p^*, s^*\}$ be a stable equilibrium for $G[0]$. Then $p_j^* = p_m^1 \equiv \text{argmax}_p\ v(p,1)$ whenever $s_{ij}^* = 0$ for all $i=1,\ldots,I$.

<u>**Proof**</u>: Suppose that $\lim_{\epsilon \to o} s_{ij}(\epsilon) = 0$ for all i for some firm j. The price that this firm j charges must maximize

$$\sum_{K \epsilon P(I)} \Pi_{k \epsilon K} [\epsilon + (1-\epsilon) s_{kj}(\epsilon)] \Pi_{k \notin K} [1-\epsilon-(1-\epsilon) s_{kj}(\epsilon)]\ v(p,|K|)$$

Now consider the probability that exactly one buyer visits. This is

$$\sum_{i=1}^{I} [\epsilon + (1-\epsilon) s_{ij}(\epsilon)] \Pi_{k \neq i} [1-\epsilon-(1-\epsilon) s_{kj}(\epsilon)] \equiv Q_1(\epsilon)$$

For any firm p_j^* must maximize

$$\sum_{K\epsilon P(I)}\{\Pi_{k\epsilon K}[\epsilon+(1-\epsilon)s_{kj}(\epsilon)]\Pi_{k\notin K}[1-\epsilon-(1-\epsilon)s_{kj}(\epsilon)]\}v(p,|K|)/Q(\epsilon)$$

Now this expression is jointly continuous in ϵ and in the s_i, whence by the maximum theorem $p_j(\epsilon)$ is a continuous function of ϵ, where $p_j(\epsilon)$ is the price that maximizes the above function for any given value of ϵ. It follows that p_j^* must maximize

$$\lim_{\epsilon->0} \sum_{K\epsilon P(I)} \{\Pi_{k\epsilon K}[\epsilon+(1-\epsilon)s_{kj}(\epsilon)]\Pi_{k\notin K}[1-\epsilon-(1-\epsilon)s_{kj}(\epsilon)\} \; v(p,|K|)/Q_1(\epsilon)$$

Since the probability that two buyers visit is of smaller order than the probability that one buyer visits, this limit is equal to v(p,1) and the result is proved. QED

This proposition makes it easy to rule out stability in certain instances. If there are multiple prices, the highest prices must all be p_m^1, the monopoly price for the firm expecting one buyer. If all the prices are the same, but some sellers get no buyers, then all prices must be p_m^1. To say more, we will need to restrict attention to a somewhat smaller class of equilibrium.

<u>DEFINITION 2</u> An equilibrium for G[0] is said to be <u>allocation-symmetric</u> if $a_j^n(s)=a_k^n(s)$ for all k and j such that $a_k^0\neq 1$.

Thus, in any allocation-symmetric equilibrium, buyers are distributed symmetrically over sellers who might receive buyers. This is a very specialized definition of symmetry in two ways. First, it does not require that the buyers use symmetric strategies. Consider the pure strategy equilibrium. Symmetry is strategies will be considered further below. Secondly, it does not literally require symmetric outcomes for sellers, since some sellers might get no buyers.

Now we have the following set of sufficient conditions for stability.

PROPOSITION 2 Let $\{p^*,s^*\}$ be an allocation-symmetric equilibrium for G[0]. Then $\{p^*,s^*\}$ is a stable equilibrium of G[0] if either

i) $p_j^*=p_k^*$ for all j and k and $s_{ik}^*>0$ for at least one i, for every k=1,...J

or

ii) $p_j^*=p_m^1$ for $p_j^*>\min_k p_k^*$, and $s_{ik}^*>0$ for at least one i, for every k such that $p_k^*=\min_j p_j^*$[6].

PROOF: This merely requires the construction of the appropriate sequences. If (i) holds define $p(\epsilon)$ to equal

$$\operatorname{argmax}_p \sum_{K\epsilon P(I)} \Pi_{k\epsilon K}[\epsilon+(1-\epsilon)s_{kj}^*]\Pi_{k\notin K}[1-\epsilon-(1-\epsilon)s_{kj}^*]\ v(p,|K|)$$

Since $\{p^*,s^*\}$ is an allocation-symmetric equilibrium, and nature's assignment rule is also symmetric, it is straightforward to see that the strategies $\{p(\epsilon),s^*\}$ constitute an equilibrium for $G[\epsilon]$, and that this strategy converges appropriately. If (ii) holds, define $p(\epsilon)$ as above for each k such that $p_k^*=\min_j p_j^*$. For each of the other sellers define $p'(\epsilon)$ to equal

$$\operatorname{argmax}_p \sum_{k=0}^{I} \binom{I}{k} \epsilon^k (1-\epsilon)^{I-k}\ v(P,k)$$

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[6] Allocation-symmetry does not imply stability in general. For example, there exist allocation-symmetric equilibria in which sellers not receiving buyers set price above p_m^1. By proposition 1, such an equilibrium can not be stable.

Now since these sequences converge to $\min_j p_j^*$ and p_m^1 respectively, we will have $p'(\epsilon)>p(\epsilon)$ for all ϵ small enough, and the strategies constructed by using these two prices along with the original search strategies will constitute an equilibrium for $G[\epsilon]$. QED

STABLE SYMMETRIC EQUILIBRIA OF G[0] UNDER DECREASING RETURNS TO SCALE

Having established some general properties of stable equilibria of G[0], we turn now to our topic of interest and explore the effects of returns to scale on the stability of equilibria. Our results here are not completely general; however, we are able to make extremely strong predictions when buyers adopt symmetric strategies. Specifically, in the remainder of this section, we will argue that all firms must be active and make identical positive profit in any stable equilibrium of G[0], when buyers use symmetric strategies and returns to scale are decreasing. As we show in the next section, this result is particularly important when placed in the context of a repeated game, since it suggests that buyers have limited credibility in threatening future punishment in response to high current prices.

To begin, we offer the following definition.

DEFINITION 3 A symmetric equilibrium for the game G[0] is a set of equilibrium strategies $\{p^*,s^*\}$ such that $s_{ij}^*=s_{kj}^*\equiv s_j^*$, for every buyer pair $\{i,k\}$ and for every seller j.

Notice that this definition requires symmetry in search strategies only, and does not require that all firms set the same price. Moreover, there do exist symmetric equilibria in which some firms receive no visitors and charge high prices. Symmetry does not itself rule out punishment.

We must also define a notion of scale economies.

DEFINITION 4 A cost function C(q) is said to exhibit decreasing returns to scale over some interval if the marginal cost function is monotonically increasing over that interval.

We now have:

LEMMA 3 Let $p_m^n \equiv \text{argmax}_p\ v(p,n)$. The sequence p_m^n is monotonically increasing in n if C(q) exhibits decreasing returns to scale.

PROOF The first order condition is

$$q'(p)p + q(p) - C'[q(p)n]q'(p) = 0$$

Implicitly differentiating this condition and exploiting the concavity of v(p,n), it is straightforward to verify that the lemma holds. QED

Lemma 3 establishes that a firm expecting exactly n buyers will charge a higher price than a firm expecting exactly n-1 buyers, when marginal cost is rising over the relevant region. This result accords naturally with simple economic reasoning.

Now, in a symmetric equilibrium, firm j will be visited by any buyer i with probability s_j^*. Thus, firm j picks its price to maximize

$$\sum_{n=0}^{I} \binom{I}{n} (s_j^*)^n\ (1-s_j^*)^{I-n}\ v(P,n)$$

which is expected profit under the binomial distribution implied by symmetric buyer strategies. For every $s\epsilon(o,1]$ it is therefore useful to define $p_m(s)$ as

$$P_m(s) \equiv \text{argmax}_p \sum_{n=0}^{I} \binom{I}{n}\ s^n\ (1-s)^{I-n}\ v(P,n)$$

and to investigate the possible monotonicity of the sequence $p_m(s)$.

Many observations are immediate. For example, $p_m(1)$ is clearly p_m^I. Also, it is straightforward to see that, in the case of constant returns to scale, $p_m(s)$ is independent of s. Reasoning as in Lemma 3, it seems natural to conjecture that $p_m(s)$ is increasing in s when costs exhibit decreasing returns to scale. Though we believe the relationship to be true quite generally, it is difficult to verify, except in particularly tractable examples.[7]

<u>EXAMPLE</u> General demand, quadratic costs

Suppose $C(qn) = bqn + \alpha(qn)^2$. The first order condition is then

$$\sum_{n=0}^{I} \binom{I}{n} s^n(1-s)^{I-n} \{Pq'(P)+q(P) - (b+2\alpha nq(P))q'(P)\}n=0$$

Using

$$sI=\sum_{n=0}^{I} \binom{I}{n} s^n (1-s)^{I-n}n$$

and

$$sI[sI+1-s]=\sum_{n=0}^{I} \binom{I}{n} s^n (1-s)^{I-n}n^2,$$

which come by differentiation of the binomial equation, the condition simplifies to

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[7] A sufficient condition would seem to be that $v_{pn}(p,n)=v_p(p,n)/n-nq'qc''>0$. Then, shifting more weight to higher n's would surely require a raise in price. But, for $p>p_m^n$, $v_p(p,n)<0$ and so $v_{pn}(p,n)<0$ is possible. Notice that a very convex cost function would suffice.

$$Pq'(p) + q(p) - bq'(p) - 2\alpha q'(p)q(p)\ (sI + 1-s) = 0$$

Implicitly differentiating and exploiting the concavity of v(p,n) in p, it is straightforward to show that

$$\frac{dPm(s)}{ds} > 0$$

if and only if I>1 and α>0. Thus, $p_m(s)$ is increasing in s exactly when costs exhibit decreasing returns to scale.

Rather than explore other particular structures, we instead propose to define a (possibly) stronger notion of decreasing returns to scale directly in terms of the sequence $p_m(s)$.

DEFINITION 5 v(p,n) exhibits stochastic decreasing returns to scale if C(q) exhibits decreasing returns to scale and the sequence $p_m(s)$ is monotonically increasing in s, for $s\epsilon(0,1]$.

Thus, when stochastic decreasing returns to scale exist, if two firms receive buyers in a symmetric equilibrium of G[0], then the firms will charge the same price if and only if buyers adopt the same search strategies for the two firms. Since all active firms must charge the same price, it follow that all active firms face the same distribution of buyers in a symmetric equilibrium. Consequently, under stochastic decreasing returns to scale, in any symmetric equilibrium of G[0], all active firms earn the same expected profit.

Symmetry does not eliminate the possibility of inactive firms. However, if in addition one focuses on stable equilibria of G[0], then it must be that all firms are active. Intuitively, if firm j were inactive in G[ϵ], with ϵ small, then by Proposition 1 j would charge p_m^1. But, under stochastic decreasing returns to scale, active firms must price higher than p_m^1, so it cannot be that j receives no visitors in a stable equilibrium of G[0]. Thus, if returns to scale are stochastically decreasing, then in any stable, symmetric equilibrium of G[0], all firms are active and <u>all firms</u> earn the same expected profit. This result is now stated as Proposition 3, which is followed by a formal proof.

<u>PROPOSITION 3</u> Suppose v(p,n) exhibits stochastic decreasing returns to scale. If {p*,s*} is a stable, symmetric equilibrium of G[0], then for every firm j, $p_j^*=p_m(1/J)$, $s_j^*=1/J$, and thus

$$V_j[p_j^*,s^*]=\sum_{n=0}^{I}\binom{I}{n}(1/J)^n\,(1-1/J)^{I-n}\,v(P_m(1/J),n)$$

<u>PROOF</u> Let {p*,s*} be a stable, symmetric equilibrium of G[0], and let $\{p(\epsilon),s(\epsilon)\}$ be a convergent sequence of equilibrium strategies. Suppose to the contrary that $p_j^*>p_k^*$, with $s_k^*>0=s_j^*$. By Proposition 1, it must then be that $p_m^1=p_j^*>p_k^*=p_m(s_k^*)$.

Now, since the equilibrium is stable, there must exist ϵ, with $0<\epsilon<s_k^*$, such that $p_j(\epsilon)>p_k(\epsilon)$, which implies that $p_j(\epsilon)=p_m(\epsilon)$. But then, by stochastic decreasing returns to scale, $p_j(\epsilon)=p_m(\epsilon)<p_m(s_k^*)$. Since ϵ can be arbitrarily close to 0, stability implies $p_j^*<p_m(s_k^*)=p_k^*$, a contradiction.

Suppose next that $p_j^* = p_k^*$ for all pairs (j,k) but that $s_j^* = 0$. Then, by Proposition 1, $p_j^*=p_k^*=p_m^1$. For all active firms k, it follows that $p_k^*=p_m(s_k^*)=p_m^1$. But, from the concavity of v(p,n) and the monotonicity of p_m^n implied by decreasing scale economies, it is clear that

$$\frac{\partial v(P^1_m,n)}{\partial P} = \begin{cases} 0, \text{ if } n=1 \\ f(n)>0, \text{ if } n>1 \end{cases}$$

Thus, since I>1, p^1_m certainly does not satisfy

$$\sum_{n=0}^{I} \binom{I}{n} (s^*_k)^n(1-s^*_k)^{I-n} \frac{\partial v(P,n)}{\partial ,P} = 0$$

Yet this equation defines $p_m(s^*_k)$. $p^1_m=p_m(s^*_k)$ is thus contradictory.

It follows that, for every pair (j,k), $p^*_j=p^*_k$ and $s^*_j=s^*_k>0$. Hence, $Js^*_j=1$ or $s^*_j=1/J$. $p^*_j=p_m$ (1/J) must then be the equilibrium price. QED

Proposition 3 is important, as it establishes conditions under which expected profit is independent of firm identity and the choice of equilibrium. If attention is restricted to stable, symmetric equilibria when v(p,n) exhibits stochastic decreasing returns to scale, then profit is uniquely defined. This result will, of course, have very strong implications for the possibility of punishment in a repeated game.[8]

We conclude this section with the following corollary, which states that the conditions of Proposition 3 are sufficient for stability.

COROLLARY 1 Suppose {p*,s*} is a symmetric equilibrium of G[0], in which, for every pair (j,k), $p^*_j=p^*_k$ and $s^*_j=s^*_k>0$. Then {p*,s*} is a stable equilibrium of G[0].

PROOF The proof follows directly from Proposition 2 and the observation that symmetry in strategies implies symmetry in allocation. QED

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[8] In particular, it rules out punishments in finite horizon games based upon the existence of multiple equilibria, as discussed by Benoit-Krishna (1985).

3. THE DYNAMIC MODEL WITH COMMON HISTORY

We now extend the analysis into a two period model. The game proceeds as follows. At the beginning of the first period, sellers simultaneously select prices that will be posted on their output for the period. Without knowing the prices that sellers have set, buyers then select sellers with whom to trade. To keep the discussion concise, we will ignore any random assignment by nature in the first period. At the close of the first period, we assume that all first period selections are commonly observed. Thus, all second period strategies draw off a common history.[9] The game $G[\epsilon]$ is then played in period two, with the modification that strategies condition on history. The extended two period game that arises with this extensive form will be referred to as $G'[\epsilon]$.

The assumption of common history is made for three reasons. First, it presumably approximates a market in which word-of-mouth communication is very efficient, or in which trade journals publish past pricing records. Second, it reduces the notational complexity required to illustrate our basic point. Finally, and most importantly, the assumption makes <u>easier</u> the implementation of buyer punishment strategies. When buyers have a common history, <u>all</u> buyers can potentially punish a particular seller who priced high in period one by refusing to visit that seller in period two. The assumption thus "stacks the odds" against monopoly pricing by making punishment easy. Hence, if

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[9] For the proposition that we establish, it is not important that buyer choices are observed. We adopt this assumption solely for notational simplicity.

Diamond's conclusion survives in the common history setting, then it is a very strong conclusion indeed.

We turn now to a formal definition of the game $G'[\epsilon]$. A seller's first period strategy consists of a point in $\mathbb{R}_+$, while a buyer's first period selection is again a probability distribution over the set of firm indices. We use p and s to represent these respective strategies. At the end of the first period, all players observe all first period choices. Let $\{p',s'\}$ represent this history of choice, where primes are used to denote the possibility of disequilibrium selections. Independent of $\{p',s'\}$, nature then assigns buyer i to seller j with probability ϵ. Observing $\{p',s'\}$but not nature's assignment, firms and buyers simultaneously make their respective second period selections. Firms each choose a second period price from $\mathbb{R}_+$, and we represent these choices with the vector σ. Those buyers not assigned by nature select a second period distribution over firms, which we write as the vector t. Strategies for the game $G'[\epsilon]$ can thus be summarized as $\{s,t\} \equiv \{(s_1,t_1),\ldots,(s_I,t_I)\}$ and $\{p,\sigma\} \equiv \{(p_1,\sigma_1),\ldots(p_J,\sigma_J)\}$.

To define an equilibrium for the game $G'[\epsilon]$, we begin by considering the second period of the game. A firm j has a payoff of form

$$\sum_{n=0}^{I} a^n_{j\epsilon} v(r,n)$$

where r is the price j charges in period two and $a^n_{j\epsilon}$ is the probability that exactly n buyers visit j. Letting $t_{ij}\ [p',s']$ be the probability that consumer i visits firm j following history $\{p',s'\}$, it follows that

$$a^n_{j\epsilon}[p',s',t] \equiv \sum_{B:|B|=n} \prod_{k\epsilon B} [\epsilon+(1-\epsilon)t_{kj}(p',s')] \prod_{k \notin B} [1-\epsilon-(1-\epsilon)t_{kj}(p',s')]$$

Firm j's second period payoff is thus

$$V_{j\epsilon}^{2}[r,p',s',t] \equiv \sum_{n=0}^{I} a_{j\epsilon}^{n}[p',s',t']\ v(r,n)$$

A seller's second period strategy is then a <u>best reply</u> to other strategies if and only if

$$V_{j\epsilon}^{2}[\sigma_j(p',s'),p',s',t] \geq V_{j\epsilon}^{2}[r,p',s',t]$$

for all $r \geq 0$ and for each history (p',s').

Consider next buyers' second period outcomes. If a buyer i is assigned by nature to firm j, his payoff is just $u(\sigma_j)$, whereas if the buyer gets to choose, his payoff is

$$U^2[t_i,p',s',\sigma] \equiv \sum_{j=1}^{I} t_{ij}[p',s']u(\sigma_j)$$

Buyer i's second period strategy is then a <u>best reply</u> if and only if

$$U^2[t_i(p',s'),p',s',\sigma] \geq U^2[\tau,p',s',\sigma]$$

for all $\tau \epsilon S^J$ and for each history (p',s').

Consider now the first period. Given the buyer search strategy s, the probability that each of the buyers in the set K select firm in period one is

$$Q_j^k[s] \equiv \Pi_{k \epsilon K} s_{kj} \Pi_{k \epsilon \{I-k\}} [1-s_{kj}]$$

Firm j's expected game payoff can now be written as

$$V^1_{j\epsilon}[p_j',p,s,\sigma,t] \equiv \sum_{K\epsilon P(I)} Q^K_j[s]v(p_j',|K|) + \delta\ V^2_{j\epsilon}[\sigma_j(p',s),p',s,t]$$

where δ is a discount factor. Likewise, buyer i's expected game payoff is

$$U^1[s_i',p,s,\sigma,t] \equiv \sum_{j=1}^{J} s_{ij}' \left\{ u(p_j) + \delta \sum_{k=1}^{J} t_{ik}[p,s']\ U^2[t_i(p,s'),p,s',\sigma] \right\}$$

With these definitions in place, we say that the set of strategies $\{p^*,s^*,\sigma^*,t^*\}$ is an <u>equilibrium</u> for the repeated search game $G'[\epsilon]$ if and only if second period strategies are best replies and

$$V^1_{j\epsilon}[p_j^*,p^*,s^*,\sigma^*,t^*] \geq V^1_{j\epsilon}[p',p^*,s^*,\sigma^*,t^*]$$

for all $p'\epsilon\mathbb{R}_+$ and for every $j=1,\ldots,J$; and

$$U^1[s_i^*,p^*,s^*,\sigma^*,t^*] \geq U^1[s_i',p^*,\sigma^*,t^*]$$

for all $s_i'\epsilon S^J$ and for every $i=1,\ldots,I$.

EQUILIBRIA OF G'[0]

As before, our fundamental interest lies in the game G'[0]. We note first that, in either period one or period two, all firms who might receive buyers must charge the same price. Otherwise, buyers' strategies would be suboptimal. Not surprisingly, there are nevertheless an immense number of equilibria in G'[0].

The immensity of the equilibrium set, and in particular the variety of equilibrium prices, is perhaps best illustrated with the following example. Let buyers adopt a completely mixed and symmetric strategy by setting $s^*_{ij}=1/J$ for every i and j. Suppose further that buyers adopt the following symmetric second period rule:

$$t^*_{ij}\ (p,s) = \begin{cases} 1/(J-|K|) \text{ for all } j\epsilon\ J-K \\ 0 \text{ for all } j\epsilon K, \end{cases}$$

where $K \equiv \{j\epsilon J | p_j \neq \underline{p}\}$. In other words, if firms in J-K charge $\underline{p}$ in period one while firms in K do not, then buyers mix evenly over the firms in J-K in period two. Against such a strategy, firms clearly have an incentive to select $\underline{p}$ in period one and p_m (1/(J-|K|)) in period two.

Two questions remain. First, can the strategy t^*_{ij} (p,s) be a best reply? In the game G[0], the answer is clearly "yes." To see why, suppose $|K| \epsilon$ (0,J) firms "cheated" in period one and charged a price different from $\underline{p}$. Since all buyers observe the deviations, these firms expect no visitors in period two and are in fact willing to set a very high second period price. But given this pricing strategy, t^*_{ij} (p,s) is indeed a best reply.

Second, what restriction must be placed on $\underline{p}$ for there to exist an equilibrium for G'[0] in which $p_j^* = \underline{p}$ for all j and $\sigma_j^*(p^*, s^*) = p_m(1/J)$? Given the above buyer strategies, the condition is straightforward. Any $\underline{p}$ such that j would rather set $\underline{p}$ today and be active tomorrow at the price $p_m(1/J)$ as opposed to maximizing at the price $p_m(1/J)$ and being inactive tomorrow is supportable as an equilibrium for G'[0]. That is, $\underline{p}$ must only satisfy

$$\sum_{n=0}^{I} \binom{I}{n} (1/J)^n (1-1/J)^{I-n} v(\underline{P}, n)$$

$$\geq (1-\delta) \sum_{n=0}^{I} \binom{I}{n} (1/J)^n (1-1/J)^{I-n} v(P_m(1/J), n)$$

Of course, if $\delta=0$, then $\underline{p} = p_m(1/J)$. However, for larger, more realistic δ, first period prices can be driven well below $p_m(1/J)$, the price which maximizes period one profit given the buyer strategy s*. This is clearly contrary to the Diamond conclusion.

In addition to the example above, equilibria can also be supported in which one firm receives all buyers in each period, but must price low in period one lest he lose all buyers in period two. Clearly, a variety of other distributions of buyers is also consistent with equilibrium behavior. The key point is, <u>regardless of scale economics</u>, the game G'[0] admits a vast number of equilibria, in many of which low first period prices are supported by the credible threat of the withdrawal of future purchases. There is absolutely no sense in which Diamond's conclusion holds for equilibria of the game G'[0].

STABLE SYMMETRIC EQUILIBRIA OF G'[0] UNDER DECREASING RETURNS TO SCALE

We now proceed to refine the equilibrium set of G'[0] by proposing a notion of stability. Then, as in the previous section, we show that very strong predictions can be made about the stable equilibria of G'[0], when buyers use symmetric strategies and v(p,n) exhibits stochastic decreasing returns to scale.

We begin with the following definition.

DEFINITION 6 An equilibrium $\{p^*,s^*,\sigma^*,t^*\}$ of G'[0] is said to be stable if and only if there exists some sequence $\{p(\epsilon),s(\epsilon),\sigma(\epsilon),t(\epsilon)\}$ such that $\{p(\epsilon),s(\epsilon),\sigma(\epsilon)t(\epsilon)\}$ is an equilibrium for $G'[\epsilon]$, $\lim_{\epsilon\to 0}\{p(\epsilon),s(\epsilon)\}=\{p^*,s^*\}$ and $\{\sigma(\epsilon),t(\epsilon)\}$ converge pointwise to functions $\{\sigma^*,t^*\}$.

This is simply the natural extension of the notion of stability to the game G'[0].

We also require a definition of symmetry.

DEFINITION 7 A symmetric equilibrium for the game G'[0] is a set of equilibrium strategies $\{p^*,s^*,\sigma^*,t^*\}$ such that $t^*_{ij}(p',s')=t^*_{kj}(p',s')\equiv t^*_j(p',s')$ for every buyer pair {i,k}, for every seller j, and for every history (p',s').

This is a very weak concept of symmetry, in that we do not require buyers to adopt symmetric first period strategies or sellers to adopt symmetric strategies in either period. As the above example illustrates, even a stronger notion of symmetry is insufficient to eliminate the possibility of low early prices supported by punishment threats. Symmetry is clearly consistent with punishment in the game G'[0].

The combination of stability, symmetry, and stochastic decreasing returns to scale is sufficient to eliminate the credibility of all future threats, however, as the following proposition shows.

PROPOSITION 4 Suppose $v(p,n)$ exhibits stochastic decreasing returns to scale. If $\{p*,s*,\sigma*,t*\}$ is a stable, symmetric equilibrium of $G'[0]$, then for every j, p_j^* maximizes

$$\sum_{K\epsilon P(I)} Q_j^K[s*]\ v\ (p_j,|K|)$$

PROOF Fix (p',s'). The notions of stability and symmetry required for the second period of $G'[0]$ are then formally equivalent to those established above for $G[0]$. But, by Proposition 3, second period profit is then uniquely defined. Since (p',s') is payoff irrelevant for the period two game, second period profit is

$$\sum_{n=0}^{I} \binom{I}{n} (1/J)^n\ (1-1/J)^{I-n}\ v\ (P_m(1/J),n)$$

for all firms and all (p',s'). The optimal p_j^* is therefore selected without regard to the future. QED

This proposition is easily understood. When $v(p,n)$ exhibits stochastic decreasing returns to scale, there is but one profit level possible in any stable symmetric equilibrium of the static game. Replacing the last period of the dynamic game with the static game, then, yields the conclusion that firms maximize game expected profit by maximizing first period expected profit. Thus, in both the first and the second periods, firms choose price to maximize current period profit. In the second period, the common price is $p_m(1/J)$ and buyer strategies are $t_j^*(p',s')=1/J$, for all j and histories (p',s'). To know the exact first period price, we would need to place restrictions on first period buyer strategies, such as symmetry. Nevertheless, we do know that each firm's first period price is selected so as to maximize first period profit. Given buyers' first period strategies, firms are thus selecting the corresponding monopoly price. Diamond's conclusion of monopoly pricing is given a dynamic equilibrium foundation.

4. THE DYNAMIC MODEL WITH PRIVATE HISTORY

In the previous section, we assumed that first period selections were commonly known upon the beginning of the second period. This assumption was defended as an approximation of a communicatively efficient market and as a polar case designed to "stack the odds" against monopoly pricing by making punishment easy. We now consider the opposite case of a communicatively inefficient market, in which buyers and sellers observe only their respective private histories. Thus, buyer i begins period two knowing only the price at the store which he visited in period one, and firm j likewise begins the second period knowing only his first period price and the identity of the buyers to whom he sold. In this setting, it is more difficult to punish a deviant firm j, since the buyers who initially visited a firm other than j cannot be influenced by j's deviation. In particular, these buyers may choose j in period two, unaware that j deviated in period one. The observation that punishment is more difficult in a private history setting is consistent with this section's fundamental result. Specifically, after defining a notion of scale diseconomies, we show that a strong form of symmetry alone is often sufficient to rule out punishment and to give monopoly pricing. The full force of stability is not required.

The formal game proceeds much as before, except that histories are now private and nature is no longer given a move. To minimize notation, we will continue to use the symbols developed above. Now, the main conceptual difficulty with the private history model is the fact that we must explicitly deal with the beliefs held by each player at each information set. That is, we must look at sequential equilibria, as defined by Kreps and Wilson (1982). In the second period, a buyer i's belief will consist of a probability distribution over the space $\mathbb{R}_+ \times P(I-\{i\})$ for every seller in the game. Each seller will similarly have beliefs about the prices and buyers at every seller's store. It is, however, tedious and really unnecessary to pursue this analysis formally. We instead take a shortcut that exploits the natural belief structure that is imposed during the course of play in the game. Specifically, we assume that every player expects each of the players whose first period play he did not observe to have abided by the initial equilibrium agreement. This seems a natural requirement to place on beliefs, though it is not required by the rules of sequential equilibrium.[10]

We now define strategies for the game G'[0]. A seller j's first period strategy consists of a point, p_j, in $\mathbb{R}_+$. At the end of the first period, the

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[10]If a buyer, for example, sees a play that does not correspond with the equilibrium agreement in the first period, he is, by the rules of sequential equilibrium, allowed to revise his beliefs in almost any way. The problem is what he will believe about the future actions of all firms whose prices he has not observed. These beliefs could be essentially anything, and by choosing beliefs appropriately virtually any type of second period behavior could be a sensible response to an off-equilibrium play in the first period. These possibilities seem to implicitly rest on a view of correlated deviations, which we feel is inappropriate for our purposes.

seller observes his price as well as the identity of the buyers who visited his firm. A seller j's second period strategy is then a mapping σ_j from $\mathbb{R}_+ \times P(I) \to \mathbb{R}_+$. Buyer i selects, in the first period, a probability distribution $s_i \epsilon S^J$. At the end of the first period, the seller observes his price as well as the identity of the buyers who visited his firm. A seller j's second period strategy is then a mapping σ_j from $\mathbb{R}_+ \times P(I) \to \mathbb{R}_+$. Buyer i selects, in the first period, a probability distribution $s_i \epsilon S^J$. At the end of the first period, i observes the price charged by one firm. Hence, a buyer i's second period strategy is a mapping t_i: $J x \mathbb{R}_+ \to S^J$. We can now represent seller and buyer strategies, respectively, with $\{p,\sigma\} \equiv \{(p_1,\sigma_1),\ldots, (p_J,\sigma_J)\}$ and $\{s,t\} \equiv \{s_1,t_1), \ldots,(s_I,t_I)\}$.

We next define payoffs for $G'[0]$. To begin, consider the second period of the game. Firm j has a payoff of form

$$\sum_{n=0}^{I} a_j^n \, v\,(r,n)$$

where r is j's second period price, and a_j^n is the probability that n buyers visit j in period two. It is this latter number that we characterize now.

The firm begins the period with an observation $\{K,p_j'\}$, where primes again denote the possibility of disequilibrium behavior. The firm knows that each buyer in K observed p_j', and will respond by choosing j with probability $t_{ij}[j,p_j']$ in the second period. The firm believes that each buyer in I-K observed some equilibrium play p_k. But j is not exactly certain as to the observation made by these buyers, since j does not know exactly which sellers they have visited. From knowledge of equilibrium buyer strategies, j can compute a probability distribution. Conditional on not visiting j, the probability that i visits firm k is

$$[s_{ik}/\sum_{\ell\neq j} s_{i\ell}].$$

Hence, the probability that i visits j in period two is

$$t^{o}_{ij}[s_i,p]=\sum_{k\neq j}[s_{ik}/\sum_{\ell\neq j} s_{i\ell}]t_{ij}[k,p_k]$$

We now have

$$a^{n}_{j}[\{K,p'_j\},s,p,t]\equiv\sum_{B:|B|=n}\prod_{k\epsilon K\cap B} t_{kj}(j,p'_j)\prod_{k\epsilon(I-K)\cap B} t^{o}_{kj}[s_k,p] \times$$

$$\prod_{k\epsilon K\cap(I-B)}[1-t_{kj}(j,p'_j)]\prod_{k\epsilon\{I-K\}\cap\{I-B\}}[1-t^{o}_{kj}[s_k,p]]$$

and so the firm's second period payoff is

$$V^{2}_{j}[r,\{K,P'_j\},p,s,t]\equiv\sum_{n=0}^{I} a^{n}_{j}[\{K,p'_j\},s,p,t]\ v(r,n)$$

Note that the second period payoff depends on the firms', and all other buyers', first period strategies, even if the firm does not play its equilibrium strategy in the first period. A seller j's second period strategy is then a <u>best reply</u> if and only if

$$V^{2}_{j}\ [\sigma_j[\{K,p'_j\}],\{K,p'_j\},p,s,t]\geq V^{2}_{j}\ [r,\{K,p'_j\},p,s,t]$$

for all $r\geq 0$ and for each history $\{K,p'_j\}\ \epsilon P(I)\times \mathbb{R}_{+}$.

Now consider buyer i's second period outcome. Buyer i enters period two with the observation $\{j,p_j'\}$, but he is not sure what strategy any seller, including j, will play in the final period, since seller histories are not observed by buyers. However, i does know that seller j has observed a K containing i, while each of the other sellers has observed a K not containing i. Thus, the probability that j has been visited by each of the buyers in the set $K \epsilon P(I-\{i\})$ is given by

$$\tilde{Q}_j^K[s] \equiv \prod_{k \epsilon K} s_{kj} \prod_{k \epsilon \{I-K-i\}} [1-s_{kj}]$$

This gives the second period payoff to i from visiting firm l to be

$$U_{i\ell}^2[\{j,p_j'\},s,\sigma] \equiv \sum_{K \epsilon P(I-\{i\})} \tilde{Q}_\ell^K[s]\ u[\sigma_\ell[KU\{i\},p_j']] \text{ if } \ell=j$$

$$\sum_{K \epsilon P(I-\{i\})} \tilde{Q}_\ell^K[s]\ u[\sigma_\ell[K,p_\ell]] \text{ if } \ell \neq j$$

Notice that i believes firm ℓ to have charged p_ℓ in period one when i initially visits some other firm j, no matter what j charges in period one. Buyer i's second period strategy t_i is then a <u>best reply</u> if and only if

$$\sum_{\ell \epsilon J} t_{i\ell} U_{i\ell}^2[\{j,p_j'\},s,\sigma] \geq U_{ik}^2[\{j,p_j'\},s,\sigma]$$

for each $k \epsilon J$, and for each history $\{j,p_j'\}$.

It is now straightforward to write the game payoff. First, for the seller, the expected payoff is

$$V_j^1[p_j',p,s,\sigma,t] \equiv \sum_{K\epsilon P(I)} Q_j^K(s)v(p_j',|K|) + \delta\ V_j^2\ [\sigma_j[\{K,p_j'\}],\{K,p_j'\},p,s,t]$$

while for the buyer, the payoff for the game is

$$U_i^1[s_i',p,s,\sigma,t]\equiv \sum_{j=1}^{J} s_{ij}'\ [u(P_j) + \delta \sum_{k=1}^{J} t_{ik}(j,p_j)\ U_{ik}^2[\{j,P_j\},s,\sigma]]$$

The set of strategies $\{p^*,s^*,\sigma^*,t^*\}$ is an <u>equilibrium</u> for the repeated search game G'[0] if and only if each of the second period strategies is a best reply and

$$V_j^1[p_j^*,p^*,s^*,\sigma^*,t^*] \geq V_j^1[p_j',p^*,s^*,\sigma^*,t^*]$$

for all $p_j'\epsilon\mathbb{R}_+$ and for every $j=1,\ldots,J$; and

$$U_i^1[s_i^*,p^*,s^*,\sigma^*,t^*] \geq U_i^1\ [s_i',p^*,s^*,\sigma^*,t^*]$$

for all $s_i'\epsilon S^J$ and for every $i=1,\ldots,I$.

<u>EQUILIBRIA OF G'[0]</u>

The equilibria of G'[0] under private history are potentially quite different from those that arise under common history. For example, it is no longer clear that all active firms set the same second period price, and there is thus a possibility of price dispersion. The equilibria with private history are similar to those of the last section, however, in that the credible threat of second period punishment can drive down first period prices, contrary to Diamond's conclusion.

To illustrate this effect, consider the following example. Suppose there exists $K \subseteq J$ such that I/K is an integer. Let buyers employ pure strategies, with the I buyers dividing evenly over some K firms. Give buyers the following second period rule:

$$t^*_{i\ell}\,[j,P'_j] = \begin{cases} 1, \text{ for } \ell=j \text{ if } p'_j = \underline{p} \\ 1, \text{ for some } \ell\neq j \text{ if } p'_j \neq \underline{p} \end{cases}$$

Thus, buyer i returns to j with probability one if $p'_j=\underline{p}$; however, if $p'_j \neq \underline{p}$, then i goes with certainty to some alternative firm ℓ.

Notice first that this boycott threat is credible: if indeed $p'_j \neq \underline{p}$, then j will expect no second period visitors and so j will be indifferent about setting a high second period price. Second, observe that any $\underline{p}$ satisfying

$$v\,(\underline{p},I/K) \geq (1-\delta)\;v\,(P_m^{I/K},I/K)$$

can be sustained as a first period price of K active firms in an equilibrium of G'[0]. Thus, is $\delta>0$, $\underline{p}$ can be pushed significantly below $p_m(I/K)$, the static monopoly price given buyer strategies. As before, this boycott equilibrium of G'[0] is independent of scale economies.

SYMMETRIC EQUILIBRIA OF G'[0] UNDER DECREASING RETURNS TO SCALE

Reasoning as in the previous sections, one can show that stability would require a boycotted firm to set a second period price of p_m^1, which would of course destroy the equilibrium illustrated above. Our goal in this section is to show that all punishment equilibria are inconsistent with a strong definition of symmetry, even when the full force of stability is not applied.

Now, in the above example, buyers adopt asymmetric strategies. Furthermore, unless I is an integral multiple of J, these strategies treat sellers asymmetrically in the first period. We are thus led to explore the possibility of punishment when buyers use symmetric strategies and these strategies initially treat firms symmetrically.

DEFINITION 8 A <u>strongly symmetric equilibrium</u> for the game G'[0] is a set of equilibrium strategies $\{p^*, s^*, \sigma^*, t^*\}$ such that

i.) $s_{ij}^* = s_{k\ell}^*$ for all buyers i,k and sellers j,ℓ

ii.) $t_{i\ell}^*[j,p_j'] = t_{k\ell}^*[j,p_j'] \equiv t_\ell^*[j,p_j']$ for all buyers i,k, sellers j,ℓ and histories $\{j,P_j'\}$.

Note that this definition requires $s_{ij}^* = 1/J$ for all buyers i and sellers j. Thus, in a strongly symmetric equilibrium, buyers mix evenly over all sellers. We do not, however, go so far as to require symmetric treatment of sellers in period two. While buyers' second period strategies are symmetric, they are allowed asymmetric distributions over firms following a history $\{j,p'\}$. A boycott, for example, is not ruled out by strong symmetry.

We now seek a notion of decreasing returns to scale. Recall in the common history setting, we defined a notion of scale diseconomics which directly ensured that a firm expecting buyers to arrive with a higher probability charged a higher price. We now propose an analogous definition for the private history case. From firm j's perspective, private history buyers fall into two groups. Those who visited j in period one will return to j with probability $t_j^*[j,p']$, while those who did not begin with j will "cross-over" to j with probability $t_j^o[s^*,p^*]$. (Strong symmetry removes the buyer indices.) The latter group of buyers cannot respond to j's first period price; however, the "return buyers" do observe this price, and the notion of scale diseconomies we propose simply requires j's second period price to rise with the probability $t_j^*[j,p']$ of return patronage.

To be more precise, if j initially received buyers from the set $K \subseteq I$, then j chooses his second period price to maximize

$$\sum_{m=0}^{|I-K|} \binom{|I-K|}{m} [t_j^o(s^*,p^*)]^m \, [1-t_j^o(s^*,p^*)]^{|I-K|-m} \quad X$$

$$\sum_{n=0}^{|K|} \binom{|K|}{m} [t_j^*(j,p_j')]^n \, [1-t_j^*(j,p_j')]^{|K|-n} \, v(p,n)$$

For every $K \subseteq I$, it is thus useful to define

$$P_m^{|K|}(t^o,t^*) \equiv \mathrm{argmax}_p \sum_{m=0}^{|I-K|} \binom{|I-K|}{m} (t^o)^m \, (1-t^o)^{|I-K|-m} \quad X$$

$$\sum_{n=0}^{|K|} \binom{|K|}{n} (t^*)^n \, (1-t^*)^{|K|-n} \, v(p,m+n)$$

Again, apart from simple examples, it is very difficult to formally relate the monotonicity of $p_m^{|K|}(t^o,t^*)$ to the assumption of decreasing returns to scale. Noting that $p_m^{|K|}(t^o, t^*)$ is independent of t^o and t^* when returns to scale are constant, however, it is natural to assume directly that $p_m^{|K|}(t^o,t^*)$ is increasing in t^*. That is, holding fixed the probability of cross-orders, if j expects more repeat patronage, then j charges a higher price. This assumption is of course consistent with decreasing returns to scale in tractable examples. It also implies the assumption of stochastic decreasing returns (put K=I and t^o=0).

DEFINITION 9 v(p,n) exhibits strong stochastic decreasing returns to scale if C(q) exhibits decreasing returns to scale and $p_m^{|K|}(t^o,t^*)$ is monotonically increasing in t^*, for all $K \subseteq I$ with $|K| \neq 0$ and for all (t^o,t^*) such that $t^* \epsilon [0,1]$, $t^o \epsilon [0,1]$ and t^* and t^o are not both zero.

Having discussed symmetry and scale effects, we turn now to the issue of stability. As shown in the previous section, when history is common, strongly symmetric equilibria can be constructed in which early prices are pushed down by the threat of future punishment. These equilibria do not exist, however, if boycotted firms set price equal to p_m^1, as would be required under stability. We find here that a significantly weaker restriction can be placed on the price of inactive firms when history is private.

DEFINITION 10 Let $\{p^*,s^*,\sigma^*,t^*\}$ be an equilibrium of G'[0]. The equilibrium is weakly stable if, for any history that leads a firm j to expect no second period visitors, j's second period price, p_m^o, satisfies

$$u(P_m^o) > u(p_m^1) + \sum_{n=1}^{I-1} \binom{I-1}{n} (1/J)^n \ (1-1/J)^{-n} \ [u(p_m^{n+1})-u(p_m^n)]$$

and

$$u(p_m^o) > u(p_m^1(0,t*)) + \sum_{n=1}^{I} \binom{I-1}{n} (1/J)^n \ (1-1/J)^{-n}[u(p_m^{n+1}(o,t*))-u(p_m^n(o,t*))]$$

It is straightforward that this restriction is weaker than stability, since $p_m^o > p_m^1 = p_m^1$ (0,t*) is allowed. (Under strong decreasing returns to scale, the summed expressions are negative.) Notice also that, if I is very large, then weak stability may actually place no restriction on p_m^o. To see why, observe that p_m^o is allowed to be higher as I gets larger. Thus, if there is a reservation price associated with the buyer demand function q(p), then for sufficiently large I weak stability <u>does not restrict p_m^o at all</u>.

Weak stability is analogous to stability in that it generates randomness in the second period. However, the randomness is now <u>endogenous</u>. Specifically, we find that buyers' second period strategies must necessarily be characterized by mixing (crossing-over). It is this endogenous randomness in buyers' strategies that ensures that each firm is active in period two, and thus that punishments cannot be rationalized by arbitrary prices. In fact, when returns to scale are strongly decreasing, we show that every firm chooses it first period price to maximize its first period profit in any weakly stable, strongly symmetric equilibrium of G'[0]. Under sufficiently strong assumptions about symmetry and scale diseconomies, Diamond's result holds in a repeated game with private history, without invoking stability.

<u>**PROPOSITION 5**</u> Suppose v(p,n) exhibits strong stochastic decreasing returns to scale. If $(p^*, s^*, \sigma^*, t^*)$ is a weakly stable, strongly symmetric equilibrium of G'[0], then for every j, p_j^* maximizes

$$\sum_{K \epsilon P(I)} Q_j^K[s^*] \ v\ (p_j, |K|)$$

<u>PROOF</u> Suppose not. Let p^o maximize

$$\sum_{K \epsilon P(I)} Q_j^K \ [s^*] \ v\ (p_j, |K|)$$

and consider what happens to firm j's second period price if he changes his first period price from p_j^* to p_o.

Exploiting strong symmetry, let $t_{ij}^* \ [k, p_k^*] \equiv t_j^* \ [k, p_k^*]$ and $t_j^o[s^*, p^*] \equiv t_j^o \ [s^*, p^*]$. Then $\sigma_j^*[\{K, p^o\}]$ maximizes

$$\sum_{m=0}^{|I-K|} \binom{|I-K|}{m} [t_j^o(s^*, p^*)]^m \ [1 - t_j^o(s^*, p^*)]^{|I-K|-m} \qquad X$$

$$\sum_{n=0}^{|K|} \binom{|K|}{n} [t_j^*(j, p^o)]^n \ [1 - t_j^*(j, p^o)]^{|K|-n} \quad v(p, m+n)$$

That is, $\sigma_j^*[K, p^o\}] = p_m^{|K|}(t_j^o(s^*, p^*), t_j^* \ (j, p^o))$ and similarly $\sigma_j^* \ [\{K, p_j^*\}] = p_m^{|K|}(t_j^o(s^*, p^*), t_j^* \ (j, p_j^*))$. Put $\sigma_j^* \ [\{\emptyset, p^o\}] = \sigma_j^* \ [\{\emptyset, p_j^*\}]$.

Having characterized second period prices, consider now $t_j^o \ (s^*, p^*)$, which we will establish to be positive. To begin, we argue that there must exist some firm pair $\{\ell, k\}$ such that $t_\ell^* \ [k, p_k^*] > 0$. If not, each buyer would stay with probability one with the seller with whom he began. Equilibrium would then require $\sigma_k^*[(K,p_k^*)] = p_m^{|K|}$. But the expected utility of a buyer remaining with k after seeing p_k^* would then be

$$\sum_{n=0}^{I-1} \binom{I-1}{n} (1/J)^n (1-1/J)^{I-1-n} u(P_m^{n+1})$$

while if he selected a different firm ℓ, his expected utility would be

$$\sum_{n=0}^{I-1} \binom{I-1}{n} (1/J)^n (1-1/J)^{I-1-n} u(P_m^n)$$

which is strictly better by weak stability. This same logic establishes that there can be at most one firm for whom return patronage occurs with probability one.

Now, assume $t_j^*[j,p_j^*] > 0$. We will show $t_j^o(s^*,p^*) > 0$. For suppose not. Then if j receives |K| visitors in period one, he must charge $p_m^{|K|}(0,t_j^*(j,p_j^*))$ in period two. Let $i \epsilon K$ have history $\{j,p_j^*\}$ and $i' \epsilon$ I-K have history $\{k,p_k^*\}$. From above, we know that k can be picked such that a firm ℓ exists with $t_\ell^*[k,p_k^*] > 0$. Thus, i' chooses ℓ, but not j, with positive probability. Since neither i nor i' begin with ℓ, it must be that i and i' get the same expected utility from visiting ℓ. As i' does not choose j, his expected utility from ℓ must be at least as large as

$$\sum_{n=0}^{I-1} \binom{I-1}{n} (1/J)^n (1-1/J)^{I-1-n} u(p_m^n(0,t_j^*(j,p_j^*))$$

$$> \sum_{n=0}^{I-1} \binom{I-1}{n} (1/J)^n (1-1/J)^{I-1-n} u(p_m^{n+1}(0,t_j^*(j,p_j^*))$$

under weak stability, where the last sum is the expected utility to i of staying with j. But then i should choose ℓ, not j. Hence, t_j^o $(s^*,p^*) > 0$.

Next, suppose $t_j^*[j,p_j^*]=0$. Then a direct contradiction to $p_j^* \neq p^o$ is had if t_j^o $(s^*,p^*)=0$. Thus, t_j^o $(s^*,p^*) > 0$.

We now use strong stochastic decreasing returns to scale to arrive at a contradiction. Suppose t_j^* $[j,p_j^*] > t_j^*$ $[j,p^o]$. Since t_j^o $(s^*,p^*) > 0$, j is active in period two (even if t_j^* $[j,p^o] = 0$), and so σ_j^* $[\{K,p_j^*\}] > \sigma_j^*$ $[\{K,p^o]$, contradicting the proposed strategies. Suppose then that t_j^* $[j,p^o] >$ t_j^* $[j,p_j^*]$. It must be that σ_j^* $[\{K,p^o\}] > \sigma_j^*$ $[\{K,p_j^*\}]$, again a contradiction. Finally, if t_j^* $[j,p_j^*] = t_j^*$ $[j, p^o]$, then $p_j^* \neq p^o$ is immediately contradictory. QED

5. CONCLUSION

The essential message of this paper is that whether or not monopoly power is diminished in repeated trading situations depends rather critically on the existence of returns to scale in production. If costs are declining, repeated trade situations can lead to outcomes where monopoly power is diminished, whereas when production costs are rising, monopoly power tends to be unaffected by repeat trades. The paper also develops a framework within which the strategic role of buyers with search costs is examined. We hope this effort will inspire more work on the strategic role of searching buyers.

The model suggests a number of important extensions. The two period nature of our game does not appear especially important. Even in this setting, a multitude of equilibria arise, and so a selection technique such as stability has purpose. Moreover, our results for the common history case extend directly to any finite horizon. We do not, however, have results for the infinite horizon game.

The model can also be extended to allow for more general costs. What is important is that the monopoly price associated with one buyer be lower than the price selected along the equilibrium path. Thus, our theory extends immediately to the possibility of U-shaped costs, for example, if buyers are individually large enough so that the monopoly quantity associated with one buyer is on the rising portion of the marginal cost curve. More generally, the possibility of U-shape costs invites an endogenous analysis of the number of firms, with the number being selected so as to keep the equilibrium price above the one buyer monopoly price.

We have not characterized asymmetric equilibria. These equilibria are somewhat unattractive, in that they implicitly require buyers to resolve the large coordination difficulty of assigning strategies to buyers.[11] Nevertheless, asymmetric equilibria are intriguing, and are potentially quite different from symmetric equilibria. We plan to analyze this possibility in future work.

Finally, an important extension would have buyers engaging in repeated search experiments <u>within</u> a period of purchase. We believe this extension can be made, as in Diamond's original model, if buyers have common search costs. However, if, for example, some buyers have no search costs, then a distribution of prices would presumable arise. One wonders if zero search cost buyers might then exert a negative externality by playing the same role as randomness and making punishment noncredible.

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[11] See Dixit-Shapiro (1985) for an analogous defense of symmetric equilibria in the context of an entry game.

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