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THE COORDINATION PROBLEM AND EQUILIBRIUM THEORIES OF RECESSIONS*

by

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and

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Abstract

Our aim in this paper is simply stated. It is to examine a model for the determination of the level of output in which there is potential for coordination difficulties and a resulting theory of "low output" equilibrium.

Although we build on the recent literature on coordination problems, we present a model that does not share what we think are unsatisfactory features. More precisely, existing models have four common features which seem to be important determinants of the properties of the equilibrium. The first two--the separation of workers and customers and the simultaneity of the firms' decisions--are, we think, realistic and important abstractions. The other two seem to be more ad hoc. These are, first, the presence of monopoly power and, second, the particular institutional arrangements through which agents are assumed to act.

We study a model that is suitable for analyzing the effects of relaxing the last two assumptions. We show that the conditions that guarantee interior Walrasian equilibria rule out equilibria with extremely low levels of activity (zero activity), which is a distinguishing feature of existing models. We also analyze the equilibrium in a special case, and we study its properties when the economy becomes large--that is, when individual agents do not have market power. In the special cases we consider, the equilibrium outcomes converge to the Walrasian allocation as the economy becomes large.

1. Introduction

Our aim in this paper is simply stated. This is to examine a model of the determination of the level of output in which there is the potential for coordination difficulties and a resulting theory of "low output" (or Keynesian) equilibrium.

In particular we are interested in conducting a critical reevaluation of the recent literature in this vein (e.g., Roberts [9] and [10] and Heller [6]) This literature is primarily directed at providing a formal equilibrium theory of the Keynesian version of unemployment (or underemployment) equilibrium based on problems of output coordination when there are important feedbacks between income and the level of output. In essence, this is a situation in which no firm individually has an incentive to unilaterally increase its employment and output even though all firm owners (as well as workers) could be made better off if they could all agree to simultaneously increase output. Moreover, there are workers that at the market wage would be willing to work but find themselves unemployed, that is, they are rationed in the labor market.

These models have four common features which seem to be the important determinants of the equilibrium features. The first two we think are realistic and important abstractions. The first of these is the separation of workers and customers. That is, from the point of view of an individual firm, increasing its own output probably has very little impact (through the generation of worker or owner income) on its own demand. Second, output decisions by firms are made simultaneously. There is no "exogenous" mechanism (e.g., the Walrasian price) on which the various economic agents can coordinate their decisions.

The other two important features of these models are, we think, more ad hoc. These are, first, the presence of monopoly power and, second, the particular institutional arrangements (i.e., the description of possible actions, the assumptions concerning the timing of these actions and the particular beliefs about the consequences of the various action profiles that agents are assumed to have) through which the agents are assumed to act.

In this context, our principal goal in this paper is to discover which of these sets of features gives rise to the substantive economic conclusions of this work.

To accomplish this goal, we will analyze a variant on the model pioneered by Roberts in [9] and [10]. The difference between our model and his has a simple interpretation. The assumptions he makes concerning the timing of decisions can be interpreted as an economy in which goods are produced on a "made to order" basis. In ours, goods are manufactured for inventory. Restated, the difference between the models is that in Roberts' a precommitment on the part of buyers is required before any production can take place. In ours, this commitment is not necessary. (Of course, in either case firms are both rational and perfectly accurate in their forecasts of sales.) Although this change may seem minor at first glance, it is, in fact, quite fundamental. Many of the problems arising in the Roberts' model can be traced directly to the fact this precommitment gives rise to difficulty in coordination of customers orders.

The reason that this is such a crucial issue in this context is that in Roberts' model, like in so many of the strategic market games models (e.g., Shapley and Shubik [12], Hart [3] and Mas-Colell [7]), the point of zero activity seems to play an extremely important role in the results.

Thus, our first goal is to construct a model which maintains the basic structural features of the Roberts model but which no longer has zero equilibria (except in very special cases). This done, we hope to test the robustness of the substantive economic conclusions of this line of work to both the changes in strategic form and the introduction of increased competition.

In the case where the number of firms and workers is small, we show that there is no rationing in equilibrium. As could be expected, firms find it profitable to change prices to capture the potential surplus associated with rationing. This happens notwithstanding the fact that reaction functions are upward sloping. There is, therefore, potential for multiple equilibria which may prevent price changes from generating higher profits. For the structure we analyze, we show that the equilibrium is unique given prices and wages.

We also study the behavior of the model as the number of players increases. For the versions we consider, equilibrium outcomes converge to the Walrasian allocation.

The remainder of the paper is organized as follows. In section 2 notation is discussed and the model is introduced and compared to Roberts' [9]. Section 3 is concerned with the possibility of "zero equilibria." A detailed analysis of an important class of special cases is conducted in Section 4. In Section 5 we explore the behavior of the model when the number of firms and workers becomes large. Finally, concluding remarks are offered in Section 6.

2. The Model and Notation

Since our model so closely parallels that of Roberts, we will follow his notation as closely as is possible.

Economic Fundamentals

Goods: There are five goods. Quantities will be denoted by:

m: money, our numeraire.

r: productive input, the first type of "labor."

s: productive input, the second type of "labor."

x: consumption good.

y: consumption good.

Agents: There are four agents in the economy, two are worker/consumers' and two are "capitalists." They will be indexed as:

A: Capitalist 1, cares only about money--$V_A(m,r,s,x,y)=U_A(m)$--and is endowed only with money--$e_A=(\bar{m}_A,0,0,0,0)$.

B: Capitalist 2, cares only about money and is endowed only with money.

J: Worker 1, is endowed with labor of type r and money--$e_J=(\bar{m}_J,\bar{r}_J,0,0,0)$--and cares only about money, r and y--$V_J(m,r,s,x,y)=U_J(r,y,m)$.

K: Worker 2, is endowed with labor of type s and money and cares only about money, s and x.

Technology and Firms: There are two firms. The first is owned wholly by A, can use only r as an input and can produce only x. This production takes place according to the production function--$x=f_A(r)=r$. The second firm is owned by B, uses s as an input and produces y. Again, the production function is the CRS technology given above.

Thus, the fundamentals of our model agree completely with those used in Roberts. The most interesting feature of the model is the structural complementarities implicit in the description of the fundamentals. That is, worker J does not consume any of his own output, only that of worker K and vice versa. This separation is a stylized but reasonable abstraction concerning modern economies. Note that the "contribution" of the two capitalists is solely in their knowledge of productive technique.

Note that from the point of view of Walrasian analysis, the two consumption goods are redundant. That is, if price taking behavior was assumed, the model would be fully equivalent to one with three goods--r,s, and m--and two agents--J and K. (The fact that A and B play no role in Walrasian Equilibrium depends on the CRS assumption.)

We are interested in modelling the behavior in this economy as the outcome of a game. There are, of course, benefits and costs associated with doing this. First, we will have to specify a strategic form. This is always somewhat arbitrary, but we hope that our choice is not too unreasonable. We will comment on this question in more detail in section 6. The benefits from the approach lie largely in the explicitness of the description of institutional detail. Thus, firm owners set prices not an "auctioneer," rationing "rules" are precisely spelled out, etc.

The Game: The game we will analyze features four stages. Our analysis is entirely in terms of behavioral strategies. Within each stage, all actions to be described below are taken simultaneously.

Verbally, the model can be described as:

Stage I: Firms set prices.

Stage II: Workers make "suggestions" about how much labor they would like to sell given wages and prices.

Stage III: Firms hire workers, production takes place, workers are paid, output goes into inventory.

Stage IV: Workers go shopping.

Formally, we have:

Stage I: Firms set prices for both inputs and outputs:

A sets p_x and w_r.

B sets p_y and w_s.

Stage II: Workers set limits on their labor supplies:

J chooses r_J subject to $r_J \le \bar{r}_J$.

K chooses s_K subject to $s_K \le \bar{s}_K$.

Stage III: Firms set quantities:

A: $r_A \le r_J$ and $x_A = r_A = f_A(r_A)$.

B: $s_B \le s_K$ and $y_B = s_B = f_B(s_B)$.

Stage IV: Workers choose consumption levels:

J: y_J subject to $p_y y_J \le \bar{m}_J + w_r r_A$ and $y_J \le y_B$.

K: x_K subject to $p_x x_K \le \bar{m}_K + w_s s_K$ and $x_K \le x_A$.

<u>Payoffs</u>: Given a full array of choices by all agents at all stages production takes place and goods are distributed as:

A: m: $\bar{m}_A - w_r r_A + p_x x_K$
r: 0
s: 0
x: $x_A - x_K$
y: 0

B: m: $\bar{m}_B - w_s s_B + p_y y_J$
r: 0
s: 0
x: 0
y: $y_B - y_J$

J: m: $\bar{m}_J + w_r r_A - p_y y_J$
r: $\bar{r}_J - r_A$
s: 0
x: 0
y: y_J

K: m: $\bar{m}_K + w_s s_B - p_x x_K$
r: 0
s: $\bar{s}_K - s_B$
x: x_K
y: $0.$

Of course, payoffs in terms of utilities are described according to this distribution of goods.

Embodied in this description of the distribution of resources are some implicit assumptions. Intuitively, the model is one of production for inventories. That is, firms must commit themselves to labor demand/output supply decisions before realizing sales. (Note that in our model these sales are forecast perfectly however since there is no uncertainty.) This follows since it is r_A and s_B, the firms' choices of labor inputs, that enter payoffs. That is, worker J is paid for r_A units of labor, not the amount of x actually sold (i.e., x_K).

Further, note that given the formal description of the game we need either that capitalist's utility functions be defined for negative money holdings or that their initial endowments of money be so large that no matter what happens they will end up with positive money balances. This follows because of our assumption that firm owners must finance inventories completely. Of course, this will never occur in equilibrium as long as owners utility functions are strictly monotone, an assumption that we will make.

Equilibrium: We will consider subgame perfect equilibria.

The difference between this model and that analyzed by Roberts is slight, but, as it turns out, important. In the Roberts model, Stage IV is collapsed into Stage II. That is, workers simultaneously limit their labor supplies and their consumption demands. Interpreting the second stage statement of consumption limits as the placing of orders gives us a natural interpretation of the differences between the two models.

Further, it is easy to see why "no activity" is an equilibrium in the Roberts model. If one of the workers, say J, decides neither to work nor buy, the other, K, knows that no matter what he does he will sell no labor and can buy no output. Therefore, it is optimal for K to supply no labor and demand no output as well.

It is interesting to see where this argument breaks down in our model. If J decides to sell no labor, K may still offer a positive amount if he expects J to buy output at Stage IV. Thus, this change in the timing of decisions is quite important. Of course, if $\bar{m}_J=0$ and J does not offer a positive amount of labor at stage II his demand will necessarily be zero.

It follows that if $\bar{m}_J$ and $\bar{m}_K$ are both zero, no activity could still be an equilibrium of the model. Because of this, we will restrict our attention in what follows to situations in which both $\bar{m}_J$ and $\bar{m}_K$ are positive.

As mentioned above, "no activity" is an equilibrium in Roberts' model, but is not the only one. In an approximate sense, any allocation that is an equilibrium outcome for the continuation game (once prices and wages have been chosen) that is considered at least as good as "no activity" by each agent can be supported as an equilibrium. This is the content of Proposition 1.

In the Roberts version of the game the third stage equilibrium is given by a mapping from prices, wages, labor supplies and output demands to outputs which we denote σ_3. In the second stage, the equilibrium is described by a mapping from prices and wages to labor supplies and output demands, which we label σ_2. Finally, σ_1, the first stage equilibrium, is a vector in $\mathbb{R}^4$ that consists of prices and wages. That is, for each σ_1, (σ_2,σ_3) constitute an equilibrium for the continuation game, i.e., $(\sigma_2,\sigma_3)(p_x,p_y,w_r,w_s)$ is an equilibrium allocation.

We are now ready to prove the following (which is basically Proposition 3 of Roberts [9]):

Proposition 1: Consider the model that collapses stage IV into stage II:

1. Let p_x, p_y, w_r, w_s be any 4-tuple that satisfies $p_x \geq w_r$ and $p_y \geq w_s$. Then (p_x, p_y, w_r, w_s), $(\sigma_2, \sigma_3)(p_x, p_y, w_r, w_s)$ is an equilibrium outcome.

2. In particular, any 4-tuple (r, s, m_j, m_k) satisfying

$$U_{J,1}(\bar{r}_J - r, s, m_J) = w_r U_{J,3}(\bar{r}_J - r, s, m_J)$$
$$U_{J,2}(\bar{r}_J - r, s, m_J) = p_y U_{J,3}(\bar{r}_J - r, s, m_J)$$
$$m_J = \bar{m}_J + w_r r - p_y s$$
$$U_{K,1}(\bar{s}_K - s, r, m_K) = w_s U_{K,3}(\bar{s}_K - s, r, m_K)$$
$$U_{K,2}(\bar{s}_K - s, r, m_K) = p_x U_{K,3}(\bar{s}_K - s, r, m_K)$$
$$m_K = \bar{m}_J + w_s s - p_x r$$

with $p_x \geq w_r$ and $p_y \geq w_s$ is an equilibrium outcome.

3. Every Walrasian equilibrium of the economy is an equilibrium of the game.

Proof:

1. To prove this we simply construct the equilibrium strategies. As a matter of notation we use 0 to denote the "no activity" strategy. Fix any 4-tuple $(\tilde{p}_x, \tilde{p}_y, \tilde{w}_r, \tilde{w}_s)$ and let (σ_2, σ_3) be any continuation equilibrium. The strategy for the continuation game is

$$(\tilde{\sigma}_2, \tilde{\sigma}_3) = \begin{cases} (\sigma_2, \sigma_3) & \text{if} \quad (p_x, p_y, w_r, w_s) = (\tilde{p}_x, \tilde{p}_y, \tilde{w}_r, \tilde{w}_s) \\ (0, 0) & \text{otherwise} \end{cases}$$

Then it is immediate to check that $(\tilde{\sigma}_1 = (\tilde{p}_x, \tilde{p}_y, \tilde{w}_r, \tilde{w}_s), \tilde{\sigma}_2, \tilde{\sigma}_3)$ is an equilibrium. Because the equilibrium outcome for (σ_2,σ_3) gives at least as much utility as the no activity equilibrium and profits are nonnegative.

2. To prove this let $\sigma_1 = (p_x,p_y,w_r,w_s)$ be as in the proposition. Assume that K chooses $s_K = s$ and $x_K = r$. Then in the third stage firm A will choose to hire $r_A = \min(r_J,r)$ and firm B will be sell no more than $y_B = \min(s,y_J)$ where (r_J,y_J) are J's announcements in the second stage. Given K's choices J solves:

$$\max_{(r_J,y_J,m_J)} \quad U_J(\bar{r}_J - \min(r_J,r),\ \min(s,y_J),\ m_J)$$

$$\text{s.t. } m_J = \bar{m}_J + w_r \min(r_J,r) - p_y \min(s,y_J)$$

But the conditions of the proposition show that $r_J = r$ and $y_J = s$ is a solution to a less restrictive problem (one that ignores the possibility of rationing). Therefore, it must be a solution to the above problem. This establishes that (r,s) is the best response to (r,s) and hence that it is an equilibrium outcome.

3. This is a special case of 1 and 2 above. []

We now study the original version of the model and show that there is no "no activity" equilibrium. Additionally, we will assume that in the borderline case in which prices equal wages firms will choose to produce as much as possible given the demand for the product and the supply of input. This amounts to a continuous extension of the optimal action when prices approach wages from above. Although we think this is a reasonable assumption, we want to point out that different selections from the equilibrium correspondence may give rise to different outcomes. See the comments in section 6.

3. The Impossibility of Equilibrium Autarky

In this section we show that under relatively mild assumptions about preferences and, if the competitive equilibrium is not the endowment, there is no zero equilibrium. That is, if an equilibrium exists, at least one of the goods is produced. Our proof relies on the possibility that one firm deviates and that at the endowment point (or zero equilibrium) one consumer finds it profitable to give up some money to buy the good even if he cannot sell any labor while--at the same time--the other will want to work just to increase his money holdings even if he believes that he will later be unable to purchase the consumption good. Of course, we assume that initial money holdings are positive.

In such a situation it is intuitive that a firm could choose prices and wages to induce the consumer to buy the good--even if he is unemployed--and the worker to supply a positive amount of labor--even if he knows he will be later rationed in the goods market.

What is somewhat surprising is that the conditions that guarantee the existence of such prices are relatively weak. In particular, they require smooth preferences (even on the boundary of the consumption set) and that the competitive equilibrium is not the endowment point.

We first describe some properties of the strategies at each stage. This description will be useful later in the proof of nonexistence of a zero equilibrium. Then we prove the main result of this section.

(a) <u>The choice of consumption by the workers in the last stage.</u>

In this stage prices are given and the firms have already produced the goods. The demand for goods is constrained by the available supply. We first solve the worker's unconstrained problem. This is, for worker J,

$$\max_{y} U_J(\bar{r}_J - r_A, y, \bar{m}_J + w_r r_A - p_y y) \tag{1}$$

Denote the maximizer by $\Phi_J(\bar{m}_J + w_r r_A, r_A, p_y)$. Since firm B hired s_B units of labor the actual amount consumed of good y is

$$y_J = \min\{s_B, \Phi_J(\cdot)\}. \tag{2}$$

Using a similar argument, the demand for x and its equilibrium value is given by

$$x_K = \min\{r_A, \Phi_K(\cdot)\}. \tag{3}$$

Formally, the fourth stage strategy is a mapping σ_4: $\mathbb{R}^6_+ \to \mathbb{R}^2_+$ given by

$$\sigma_4(p_x, p_y, w_s, r_A, s_B) = [\min\{s_B, \Phi_J(\bar{m}_J + w_r r_A, r_A, p_y)\},$$
$$\min\{r_A, \Phi_K(\bar{m}_K + w_s s_B, s_B, p_x)\}].$$

(b) <u>Firms' choices of production and labor demand in the third stage</u>.

There is no loss of generality in assuming that prices are greater than or equal to wages, since firms would never choose to set prices lower than wages.

Firm A will choose to produce r_A units (and consequently to hire r_A units of labor) where r_A is given by

$$r_A = \min\{r_L, x_K\}.$$

This is equivalent to:

$$(4)\qquad r_A = \min\{r_L, \phi_K(\bullet)\},$$

where r_L is the maximum amount of labor that worker J is willing to sell. This announcement was made in the previous stage. Simultaneously, firm B chooses s_B that is given by

$$(5)\qquad s_B = \min\{s_L, \phi_J(\bullet)\}$$

The third stage strategies are then summarized by a function σ_3: $\mathbb{R}_+^6 \rightarrow \mathbb{R}_+^2$ given by

$$\sigma_3(p_x, p_y, w_r, w_s, r_L, s_L) = (\sigma_3^A, \sigma_3^B) = (r_A, s_B)$$

which is a vector (r_A, S_B) that is a fixed point of

$$r_A = \min\{r_L, \phi_K(\bar{m}_K + w_s s_B, s_B, p_x)\}$$

(6)

$$s_B = \min\{s_L, \Phi_J(\bar{m}_J + w_r r_A, r_A, p_y)\}$$

(Note that these reaction functions are upper hemicontinuous and convex-valued. Hence existence of an equilibrium (in pure strategies) is guaranteed.)

c. Workers' choice of labor supply in the second stage

In stage 2 workers choose their labor supplies r_L and s_L. Notice that--ignoring prices and wages for now--the third stage strategies that affect workers' welfare and incomes depend on both r_L and s_L. Given an arbitrary value of s_L worker J chooses r_L to be

$$(7) \qquad r_L = \underset{r}{\text{argmax}}\; U_J(\bar{r}_J - \sigma_3^A(r,s_L),$$
$$\sigma_3^B(r,s_L), \bar{m}_J + w_r\sigma_3^A(r,s_L) - p_y\sigma_3^B(r,s_L))$$

Similarly, for any r_L, worker K chooses s_L according to

$$(8) \qquad s_L = \underset{s}{\text{argmax}}\; U_K(\bar{s}_K - \sigma_3^B(r_L,s),$$
$$\sigma_3^A(r_L,s), \bar{m}_K + w_s\sigma_3^B(r_L,s) - p_x\sigma_3^A(r_L,s))$$

The second stage strategy σ_2 is a mapping from $\mathbb{R}_+^4 \rightarrow \mathbb{R}_+^2$ given by the set of fixed points of the previous two equations

$$\sigma_2(p_x,p_y,w_r,w_s) = (\sigma_2^J,\sigma_2^K) = (r_L,s_L)$$

d. Firms' choice of prices and wages

In the first stage firm A chooses p_x, w_r to maximize profits given by

$$\bar{m}_A + (P_x - w_r)\sigma_3^A(p_x,p_y,w_r,w_s,\sigma_2(p_x,p_y,w_r,w_s))$$

Firm B faces an equivalent problem. This first period strategy, σ_1, is simply a vector in $\mathbb{R}^4_+$.

We are now ready to prove that there is no "zero" equilibrium.

Proposition 2: Assume that U_J and U_K are C^1, strictly monotone, and strictly quasi-concave.

(a) If the endowment is a Nash equilibrium, the unique competitive equilibrium allocation is also the endowment.

(b) Conversely, assume that the endowment is a competitive equilibrium allocation. Then, if a Nash equilibrium exists, the equilibrium outcome is equal to the endowment.

Proof:

(a) We will argue by contradiction, That is, we will show that if the endowment is not a competitive equilibrium, it is not a Nash equilibrium of the game. Note that given our assumptions (in particular strict monotonicity and strict quasi concavity of utility functions) if the endowment is a competitive equilibrium, it is unique.

We begin by showing that if the endowment is not a competitive equilibrium, then

$$\text{either} \quad \frac{U^0_{J,2}}{U^0_{J,3}} > \frac{U^0_{K,1}}{U^0_{K,3}}$$

$$\text{or} \quad \frac{U^0_{K,2}}{U^0_{K,3}} > \frac{U^0_{J,1}}{U^0_{J,3}}$$

Where $U^0_{H,i}$ is the partial derivative of U_H with respect to the i-th argument evaluated at the endowment.

Suppose on the contrary that neither of these two inequalities holds. We will then show that the endowment is a competitive equilibrium. This, of course, is a contradiction.

Consider the prices defined by

$$w^0_s \equiv \frac{U^0_{K,1}}{U^0_{K,3}} \geq \frac{U^0_{J,2}}{U^0_{J,3}} \equiv p^0_y,$$

and

$$w^0_r \equiv \frac{U^0_{J,1}}{U^0_{J,3}} \geq \frac{U^0_{K,2}}{U^0_{K,3}} \equiv p^0_x.$$

We claim that these prices support the endowment as a competitive equilibrium. Since wages are greater than or equal to prices, profit maximization requires no activity by either firm. To show that workers' optimal decision is to stay at the endowment, consider the problem they face. For worker J, this is:

$$\max_{(r,y,m)} U_J(\bar{r}_J - r, y, m)$$

subject to

$$p_y^0 y + m \le \bar{m}_J + w_r^0 r.$$

(Note that we have normalized the price of money to 1.) Given our choice of p_y^0 and w_r^0 it is immediate that $y = r = 0$, $m = \bar{m}_j$ is the unique solution to the problem. This completes the argument and gives us the desired contradiction.

Thus, as claimed, at least one of the two inequalities above must hold. Assume, without loss of generality, that the first inequality holds. We will next find prices that firm B can announce in the first period and that result in profits and are such that y_J is positive.

Formally, let $\tilde{\sigma} = (\tilde{\sigma}_1, \tilde{\sigma}_2, \tilde{\sigma}_3, \tilde{\sigma}_4)$ be a Nash equilibrium; and assume that $\tilde{x}_K = \tilde{y}_J = 0.$[1] Let $\tilde{\sigma}_1 = (\tilde{p}_x, \tilde{p}_y, \tilde{w}_r, \tilde{w}_s)$.

Define $\hat{p}_y$ and $\hat{w}_s$ as any two numbers satisfying:

$$\frac{U^0_{J,2}}{U^0_{J,3}} > \hat{p}_y > \hat{w}_s > \frac{U^0_{K,1}}{U^0_{K,3}}.$$

Consider the subgame that results when firm B deviates from $\tilde{\sigma}$ by announcing $\hat{\sigma}_1 = (\tilde{p}_x, \hat{p}_y, \tilde{w}_r, \hat{w}_s)$. We claim $\hat{y}_J = \hat{\sigma}^J_4(\bullet) > 0$. Suppose to the contrary that $\hat{y}_J = 0$. From (2) and (5) it follows that $\hat{s}_B = 0$. We next show that this implies $\hat{\Phi}_K = \tilde{\Phi}_K$. To see this use the appropriate version of (1) to get

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[1] In this proof a "~" over a symbol denotes the equilibrium outcome in the $\tilde{\sigma}$ equilibrium, and a "^" in the corresponding value in the subgame is defined by the deviation by firm B.

$$\hat{\Phi}_K = \underset{x}{\text{argmax}}\ U_K(\bar{s}_K - \hat{s}_B, x, \bar{m}_K + \hat{w}_s\hat{s}_B - \tilde{p}_x x) =$$
$$\text{argmax}\ U_K(\bar{s}_K, x, \bar{m}_K - \tilde{p}_x x) = \tilde{\Phi}_K$$

The last equality follows because we assume $\tilde{y}_J = \tilde{x}_K = 0$, and this implies $\tilde{s}_B = 0$.

The optimal choice of r_A is given by

$$\hat{r}^A = \min\{\hat{r}_L, \tilde{\Phi}_K\}$$

But $\hat{r}_L$ is given by (7).

$$\hat{r}_L = \underset{r}{\text{argmax}}\ U_J(\bar{r}_J - \min(r,\tilde{\Phi}_K), \hat{y}_J, \bar{m}_J + \tilde{w}_r \min(r,\tilde{\Phi}_K))$$
$$= \underset{r}{\text{argmax}}\ U_J(\bar{r}_J - \min(r,\tilde{\Phi}_K), 0, \bar{m}_J + \tilde{w}_r \min(r,\tilde{\Phi}_K)) = \tilde{r}_L$$

But if $\tilde{x}_K = 0$ it follows that $\tilde{r}_A = 0$. This shows that $\hat{r}_A = \tilde{r}_A$ and hence $\hat{r}_A = 0$.

Consider now the optimal choice of worker J in the fourth stage given that $\hat{r}_A = 0$. Equation (1) shows that $\hat{\Phi}_J$ is given by

$$\hat{\Phi}_J = \underset{y}{\text{argmax}}\ U_J(\bar{r}_J, y, \bar{m}_J - \hat{p}_y y)$$

The first part of Condition (10), which we assumed holds, guarantees $\hat{\Phi}_J > 0$.

Next we will show that $\hat{s}_B > 0$. By (5) it suffices to show that $\hat{s}_L > 0$. But $\hat{s}_L$ is given by (8)

$$\hat{s}_L = \underset{s}{\text{argmax}}\ U_K(\bar{s}_K - \min(s,\hat{\phi}_J),0,\bar{m}_K + \hat{w}_s \min(s,\hat{\phi}_J))$$

which is strictly positive by (10).

Therefore, if firm B deviates from $(\tilde{p}_y,\tilde{w}_s)$ by announcing $(\hat{p}_y,\hat{w}_y)$ it obtains positive profits. Since profits are zero in the $\tilde{\sigma}$ equilibrium, this is a profitable deviation. Consequently, we cannot have a zero equilibrium.

(b) We show that if the endowment is not the unique Nash equilibrium outcome (assuming that one exists), then the endowment is not a competitive equilibrium allocation. Let the Nash equilibrium allocation be $[(\tilde{r},\tilde{s},\tilde{m}_J), (\tilde{s},\tilde{r},\tilde{m}_K)]$. Because the endowment is always feasible for each player (there is a strategy that generates the endowment as an equilibrium outcome independently of what the other players do) we must have

$$U_J(\bar{r}_J - \tilde{r},\tilde{s},\tilde{m}_J) \geq U_J(\bar{r}_J,0,\bar{m}_J)$$
$$U_K(\bar{s}_K - \tilde{s},\tilde{r},\tilde{m}_K) \geq U_K(\bar{s}_K,0,\bar{m}_K).$$

By strict convexity of preferences the allocation $[(\bar{r}_J - \lambda\tilde{r},\lambda\tilde{s},\lambda\tilde{m}_J + (1 - \lambda)\bar{m}_J), (\bar{s}_K - \tilde{s},\lambda\tilde{r},\lambda\tilde{m}_K + (1 - \lambda)\bar{m}_K)]$ is Pareto superior to the endowment. Therefore, the endowment is not Pareto optimal. Because the assumptions of the first welfare theorem are satisfied by this economy we conclude that the endowment cannot be a competitive equilibrium. []

4. Rationing in a Special Case

In this section we will explore a class of examples of the model discussed in sections 2 and 3. We have several goals in mind.

The first purpose is solely to develop some stronger intuition concerning the inner workings of the model. In addition to this, we hope to begin to answer some of the questions listed in the introduction: When will there be rationing in equilibrium? How is the model affected by increased competition? And so on.

Before proceeding, it is worthwhile to note that it is immediate that for most price-wage combinations there will be some rationing. For general configurations, it will not hold that the output demand of worker J will equal the input supply of worker K. The question then becomes: Will firms have incentives to adjust prices and/or wages as the preferred method of rationing?

The class of economies that we will restrict attention to is that where the utility functions of the worker/consumers are given by:

$$U_S(m,r,s,x,y) = u_1(\bar{r}_J - r) + u_2(y) + u_3(m),$$

and

$$U_K(m,r,s,x,y) = v_1(\bar{s}_K - s) + v_2(x) + v_3(m)$$

where the u_i, v_i are strictly increasing, strictly concave, C^2 and $v_i'(0) = u_i'(0) = \infty$ for $i = 1,2,3.$[2] Note that this form is similar to that analyzed in the examples of Roberts [9] in that separability is assumed. (In fact, Robert's example is almost a special case of this form except that u_2 is assumed to be linear.)

For this specification of utility functions, we will be able to

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[2]The assumption that marginal utilities are infinite at zero is convenient, but not crucial to the results that follow. Details are available from the authors upon request.

completely characterize a unique equilibrium of the continuation of the game for any choices of prices and wages of the firms. (This is the content of Lemma 2, below.) This allows us to reduce the game to one that is one-shot in prices and wages with rationing, as determined by the equilibrium of the continuation and is of some independent interest.

Note further that the specification of owners' utility functions is irrelevant in this environment without uncertainty as long as utility is strictly increasing in money (for both positive and negative values).

In accordance with the discussion of section 3, we will assume that $u_2'(0) > v_1'(\bar{s}_K)$ and $v_2'(0) > u_1'(\bar{r}_J)$ so that both goods will be produced in a Nash (and competitive) equilibrium.

Our first aim is to characterize worker behavior in the final stage of the game. Consider the problem facing worker J. This is to choose his level of consumption of y and m optimally given:

(1) The budget constraint $m + p_y y \leq \bar{m}_j + rw_r$

(2) The fact that he has already worked r_A hours (i.e., $r = r_A$).

(3) The fact that firm B has only s_B units of output for sale.

Formally,

$$\max_{m,y} [u_1(\bar{r}_J - r_A) + u_2(y) + u_3(m)]$$
s.t. (1), (2) and (3) above.

Note that one effect of our assumption of additively separable preferences is to guarantee that the only effect of the labor supply choice by the worker (made at stage 2) is through the budget constraint, (1). Clearly, the solution to this problem $(y^*(r_A,s_B),m^*(r_A,s_B))$ gives $y^*(r_A) = \min(s_B,y(r_A))$ where $y(r_A)$ solves the budget problem ignoring constraint (3). Thus, (y^*,m^*) is "quasi-Walrasian." Separability implies that y(r) is given by the income expansion path of the utility function $u_2 + u_3$ when income is given by $\bar{m}_J + w_r r_A$.

It follows for the special utility functions being considered that $y(r_A)$ is defined by:

$$u_2'(y(r_A)) = p_y u_3'(\bar{m}_J + w_r r_A - p_y y(r_A)).$$

Of course,

$$m(r_A) = \bar{m}_J + r_A w_r - p_y y(r_A).$$

Similarly, $x(s_B)$ is defined by $v_2'(x(s_B)) = v_3'(\bar{m}_K + w_s s_B - p_x x(s_B))$. Note further that if $p_y < u_2'(0)$ and $s_B > 0$, $\Phi_J(p_y,w_r,r_A)$ is positive at $r_A = 0$ and increases as r_A increases. These considerations give us the following diagram which will be useful in analyzing stage III play.

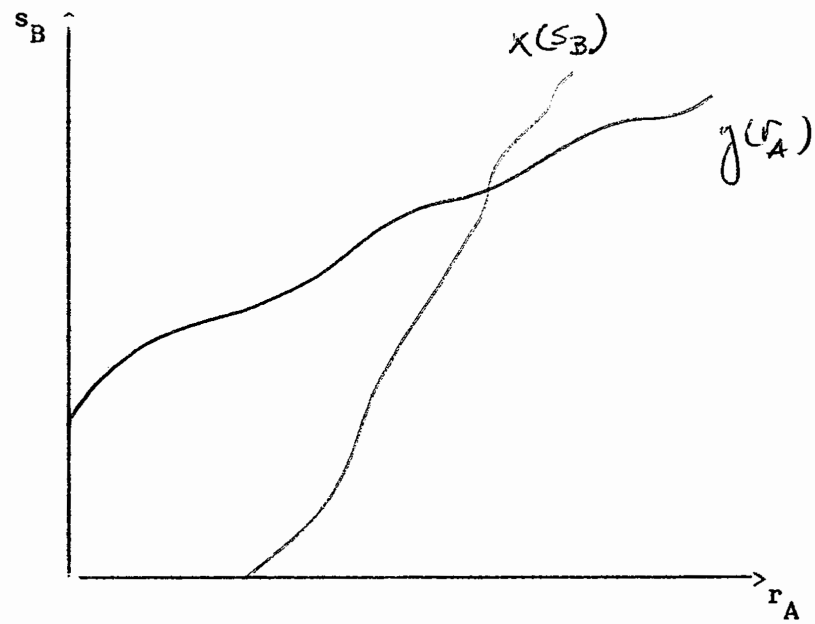


Figure 1

Note also that y(0) = 0 if and only if y* is identically zero (similarly for x).

The analysis of stage III of the game is now relatively straightforward. Note that because of our assumption of constant returns to scale, it is always in the firm's interest to produce as much as possible. (This follows since $p_y \geq w_r$ and we have assumed continuity at $p_y = w_r$.)

It follows that the firms' stage III reaction functions are singly simple in form. That is, given that firm B has chosen to hire s_B units of labor, the income of firm A's customer is fixed as is his final stage demand. It follows that firm A will sell as much as possible subject to the constraints that this quantity be both no larger than $x(s_B)$ and r_J. That is,

$$r_A(s_B) = \min[r_J,\ x(s_B)]$$

and

$$s_B(r_A) = \min[s_K,\ y(r_A)].$$

Differentiating the first-order conditions defining the functions x and y gives

$$y' = \frac{w_r}{p_y} \frac{u_3'' p_y^2}{u_2'' + u_3'' p_x^2} \quad \text{and}$$

$$x' = \frac{w_s}{p_x} \frac{v_3'' p_x^2}{v_2'' + v_3'' p_x^2}$$

It follows (since u_2'', u_3'', v_2'' and v_3'' are all negative) that both y and x are upward sloping. Moreover,

$$x'y' = \frac{w_r w_s}{p_x p_y} \frac{u_3'' p_y^2}{u_2'' + u_3'' p_y^2} \frac{v_3'' p_x^2}{v_2'' + v_3'' p_x^2}.$$

It is immediate that each of these terms is less than one in absolute value. Hence, $y' > 1/x' = dx^{-1}/dr$. This implies that x can cross y only from below. Thus, it follows that the equilibrium at the third stage is unique. (It is immediate that such an equilibrium always exists.) Note that in the case that p=w we have used the assumption that firms produce the maximum possible level of output.

Denote this equilibrium by $(r^*(r_J,s_K),\ s^*(r_J,s_K))$ and note that

$$r^* = \min[r_J, x(s_K), r_1]$$

and

$$s^* = \min[s_K, y(r_J), s_1]$$

where the point (r_1, s_1) is determined as the (unique) point of intersection between $x(s)$ and $y(r)$ (possibly infinite). Note that (r_1, s_1) is determined by prices and wages, but not r_J and s_K. This is a special feature of the separable case.

We summarize these facts in the following lemma for future reference.

Lemma 1: Given our assumptions on preferences, the stage III game has the following features:

(1) The reaction functions of the firms are given by

$$r_A(s_B) = \min[r_J, x(s_B)]$$
$$s_B(r_A) = \min[s_K, y(r_A)].$$

(2) Given any actions by the agents in the first two stages of the game, there is a unique pure strategy equilibrium of the continuation with the resulting allocations to the workers given by:

Worker J: $\bar{r}_J - r^*$

r^*

$\bar{m}_J + w_r r^* - p_y s^*$

Worker K: $\bar{s}_K - s^*$

r^*

$\bar{m}_K + w_s s^* - p_x r^*$

where

$$r^* = \min[r_J, x(s_K), r_1]$$

$$s^* = \min[s_K, y(r_J), s_1]$$

and r_1, s_1 are defined by the (possibly infinite) intersection of x(s) and y(r).

That is, the equilibrium is given by either:

(a) the intersection of y(r) and x(s): (r_1, s_1);

(b) the intersection of y(r) and r_J: $(r_J, y(r_J))$;

(c) the intersection of s_K and x(s): $(s_K, x(s_K))$; or

(d) the intersection of r_J and s_K: (r_J, s_K).

It follows that if $r_J \geq r_1$ and $s_K \geq s_1$, there is no rationing in the goods market and the equilibrium is given by (r_1, s_1). If $r_J < r_1$ and $s_K \geq s_1$, it is the scarcity of J's labor that determines the equilibrium (i.e., $(r_J, y(r_J))$) in both markets. The case where $r_J \geq r_1$ but $s_K < s_1$ is similar. Finally, if $r_J < r_1$, and $s_K < s_1$ at least one, but possibly both workers are rationed in the goods market. Which of these occurs depends on the exact choices of r_J and s_K.

Note that the conclusion of this lemma is that the equilibrium is unique even though the reaction functions of the firms are upward sloping (a property emphasized in Cooper and John [1], and Heller [5]).

It is worthwhile to examine a slightly different explanation for why this equilibrium is unique. This will highlight the role of separability in the argument.

Fix worker K's income at some level I and let $I_K(I)$ denote the amount of income that worker J ends up with after K's purchases are made when he has income I. Define $I_J(\cdot)$ similarly. In the notation above, $I_K(I) = \bar{m}_J + w_r x((I - \bar{m}_K)/w_s))$.

Now, consider a revised version of the stage III in which firms are viewed as choosing their own workers' incomes rather than output levels. (This is perfectly equivalent to the first formulation.) It is easy to see that an equilibrium for this game is a pair $(\bar{I}_J^*, I_K^*)$ with $I_K(I_K^*) = I_J^*$ and $I_J(\bar{I}_J^*) = I_K^*$. It is easy to see that this equilibrium will be unique as long as $I_J' < 1$ and $I_K' < 1$. This will hold if $w_s x' \leq p_x x' \leq 1$ and $w_s y' \leq p_y y' < 1$. That is, equilibrium is unique as long as marginal expenditures on consumption do not exhaust marginal income. In the separable case this is automatically satisfied and hence the result goes through.

Note that this condition is similar to, although not identical with, the condition that money be a normal good. The reason these conditions are not coincident in general (they are in the separable case) is that as we are "forcing" r (respectively, s) up, we are not moving out a true income expansion in the classical sense. Nonetheless, this argument should give the reader some idea as to the exact role of the separability assumption in the argument presented above. (Note that we have ignored the problem of the constraints imposed by the second stage action in this discussion.)

We turn now to the repercussions of this discussion for the game at Stage II.

At this stage, workers are faced with making labor supply decisions with full understanding of the repercussions in the goods market. At this point they have (indirect) utility functions given by

$$U(r_J, s_K) = u_1(\bar{r}_J - r^*(r_J, s_K)) + u_2(s^*(r_J, s_K)) \\ + u_3(\bar{m}_J + w_r r^*(r_J, s_k) - p_y s^*(r_J, s_k))$$

and $V(r_J, s_K) = v_1(\bar{s}_K - s^*) + v_2(r^*) + v_3(\bar{m}_K + w_s s^* - p_x r^*)$, where we have suppressed the dependence of r^* and s^* on prices.

Let (r^0, y^0, m_J^0) and (s^0, x^0, m_K^0) denote the agents' unconstrained (Walrasian) demand choices at the given prices.

Our next goal is to characterize the reaction functions of the workers at Stage II. Note that there is automatically an element of indifference on the part of worker J for some choices of worker K. For example, it follows from Lemma 1 that worker J will never be able to sell more than r_1 units of labor no matter how much he offers. For example, if J would like to sell r_1 units and K would like to sell s_1 units, any combination of (r,s) with $r \geq r_1$ and $s \geq s_1$ is an equilibrium. This source of multiplicity is trivial however since all of these choices give rise to the same real outcome (r_1, s_1).

Moreover, there are situations in which worker K's choice of s_K will completely determine the equilibrium of the stage III game. This occurs if, given s_K, J would like to sell more than $x(s_K)$ units of labor. In this situation, his labor sales will always be limited by $x(s_K)$. Because of this, any choice of r_J at least as large as $x(s_K)$ is an optimal response by J to the announcement by K of s_K. Again, no matter what choice J makes, the resulting stage III equilibrium will be given by $(s_K, x(s_K))$. Thus, J's indifference has no effect on any of the players of the game. Again, there is a potential source of multiplicity but, again, it has no real effect.

For this reason, in characterizing the reaction functions of the workers at stage II, we will limit attention to choices where $r_J \le x(s_K)$ and $s_K \le y(r_J)$.

Fix s_K and consider the choice problem faced by worker J. Consider two cases.

<u>Case 1</u>: $s_K \le y(0)$.

In this case, the resulting equilibrium continuation as a function of r_J is given by:

$$r^*(r_J, s_K) = r_J$$
$$s^*(r_J, s_K) = s_K \text{ for } r_J \le x(s_K)$$

and

$$r^* = x(s_K)$$
$$s^* = s_K \text{ for } x(s_K) \le r_J.$$

In this case, note that J will consume s_K units of y no matter how much labor he offers. Because of this he must solve the problem:

$$\max_r \; u_1(\bar{r}_J - r) + u_2(s_K) + u_3(\bar{m}_J + w_r r - p_y s_K)$$
$$\text{s.t. } (1)\; r \le x(s_K).$$

Let $r(s_K)$ denote the solution to this problem ignoring the constraint (1) and note as before that this solution traces out the income expansion path of the utility function $u_1 + u_3$ in r,m space. (Note, however, that higher values of s correspond to lower levels of wealth.)

The first order condition for this problem gives the following implicit definition of r(s):

$$u_1'(\bar{r}_J - r(s)) = w_r u_3'(\bar{m}_J + w_r r(s) - p_y s)$$

which we will use later on.

As before, it is immediate that the solution to the problem given above is given by:

$$\hat{r}(s_K) = \min\ [r_1, r(s_K), x(s_K)]$$

Case II: $s_1 \geq s_K > y(0)$

In this case, the equilibrium of the continuation contains three regions:

For $r_J \leq y^{-1}(s_K)$

$r^* = r_J$

$s^* = y(r_J)$

For $y^{-1}(s_K) \leq r_J \leq x(s_K)$

$r^* = r_J$

$s^* = s_K$.

Finally, for $r_J \geq x(s_K)$

$r^* = x(s_K)$

$s^* = s_K$.

Thus, worker J must solve the problem:

$$\max\ u_1(\bar{r}_J - r^*) + u_2(s^*) + u_3(\bar{m}_J + w_r r^* - p_y s^*)$$

subject to the constraint that r*,s* is given by the above. It is straightforward to show that the solution this problem is for J to set

$$\hat{r}(s_K) = \min\ [r_1, r^0, r(s_K), x(s_K)]$$

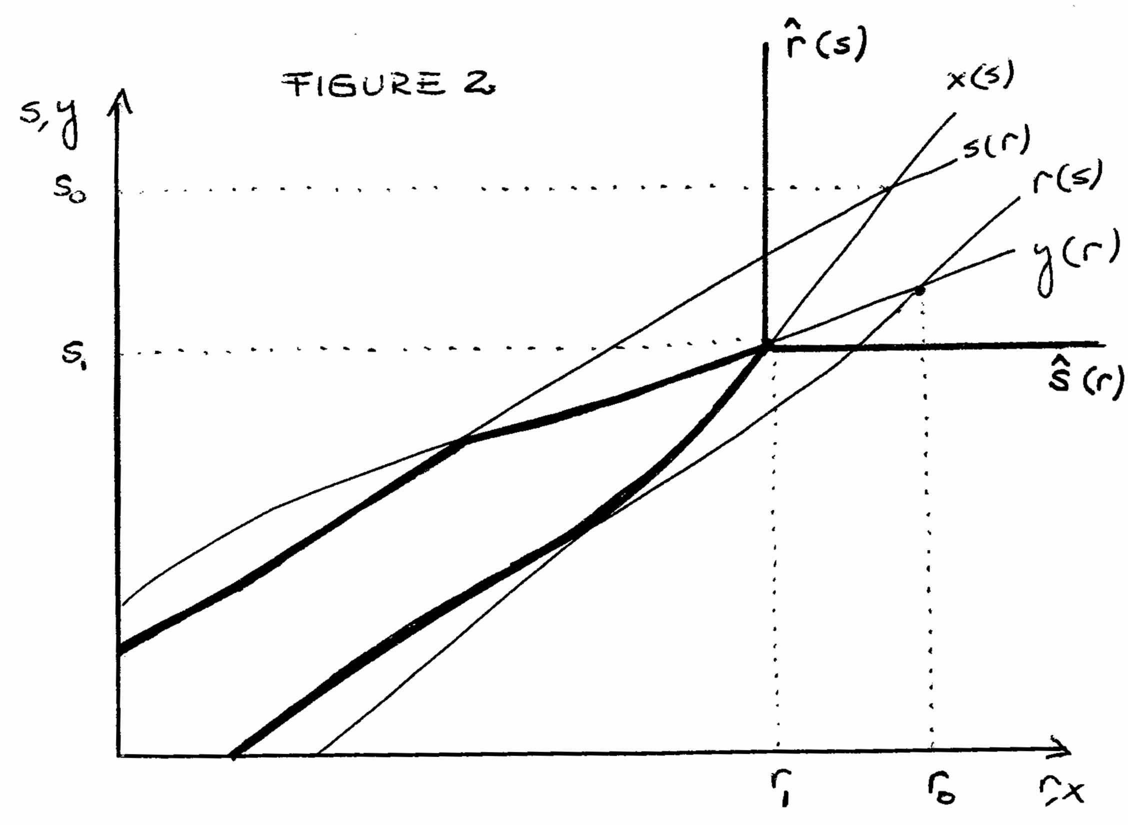
where r^0, as before, represents the unconstrained Walrasian labor supply of the worker. It is immediate that this more general form includes Case 1 as a special case.

Similarly, the reaction function of worker K is given by

$$\hat{s}(r_J) = \min\ [s_1, s^0, s(r_J), y(r_J)].$$

Figure 2 displays the reaction functions.

It is straightforward to check that $\hat{r}$ and $\hat{s}$ are both weakly increasing and, as above,



$d\hat{s}/dr < d\hat{r}^{-1}/dr.$

As before, this implies the existence of a unique equilibrium of the stage II game for any price/wage combination. (See Figure 2.) Summarizing, we have:

Lemma 2: For any choice of prices and wages at the first stage there is a unique pure strategy equilibrium continuation of the game. Moreover, the equilibrium level of r is given by the minimum level of r from the intersection of: $x^{-1}(r)$ and $y(r)$; $r^{-1}(r)$ and $s(r)$; $r^{-1}(r)$ and $y(r)$; and $r^{-1}(r)$ and $s(r)$. A similar characterization holds for the equilibrium level of s.

We now turn to the question of whether or not we will ever observe rationing in equilibria of our game. One's first intuition is that this should not occur. That is, if a consumer is being rationed in the goods market, why not just raise the price? It would seem that this would give the firm the same sales with higher revenue and hence would represent an unambiguous improvement for the firm. A similar argument should hold for the labor market.

The reason the argument is necessarily more complex than this is that due to changes in the equilibrium structure downstream, sales will in fact adjust. For example, lowering the wage of a worker reduces his income (other things held equal). This in turn may reduce his demand for the other good. This reduces the (equilibrium) income of the firm's customer, thus lowering sales. One must then compare the benefits from the reduction in wages to the costs from reduced sales to find if this deviation is profitable.

To begin note that rationing occurs if and only if one of the workers is "off" the appropriate "income-expansion path." Let $(\hat{r}^*(p_y,p_x,w_T,w_s)$, $\hat{s}^*(p_y,p_x,w_r,w_s))$ (or more briefly $(\hat{r}^*,\hat{s}^*)$) denote the equilibrium outcome of the continuation given prices and wages. Given this notation, it is easy to see what corresponds to rationing. For example, worker J is rationed in the goods market when $\hat{s}^* < y(\hat{r}^*)$; he is rationed in the labor market when $\hat{r}^* < r(\hat{s}^*)$. Note that if he is rationed in neither, the equilibrium of the continuation lies at the intersection of r and y which occurs at (r^0,y^0,m^0).

We first consider the problem of rationing in the goods market.

For simplicity, assume that worker J is rationed in the goods market. This situation is depicted in Figure 3 (the reader should ignore the dotted lines at this point). This implies that at the equilibrium min $[s,y] = \hat{s}^*$ Thus, the equilibrium occurs at either the intersection between the loci s and x (Figure 3a) or s and r (Figure 3b). Consider the effect of increasing p_y in this situation. It is straightforward to verify the effect of this is to shift y down and shift r out (since the worker is now poorer). It follows that for small changes in p_y the equilibrium of the continuation still has min $[s,y] = \hat{s}^*$. In Figure 3 we display the "new " reaction functions as dotted lines, and we label the "new" functions as r', s', etc. Since r has increased, x^{-1} has not changed and s is monotone, it follows that $\hat{s}^*$ does not decrease for small changes in p_y. Thus, profits of B unambiguously rise after the deviation contradicting our assumption of equilibrium. Thus, there can be no rationing in the goods market in equilibrium.

We turn now to the labor market. The issues are more subtle here as a

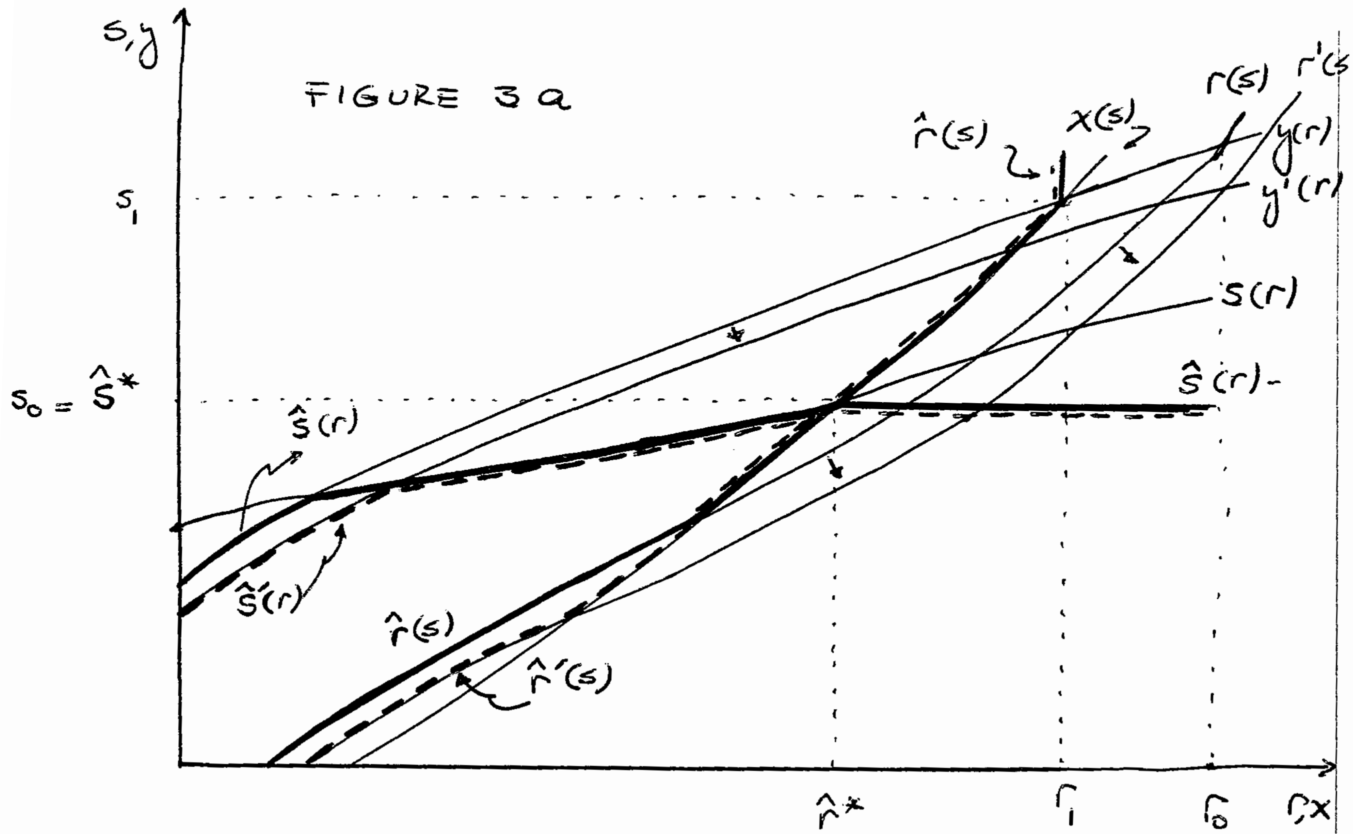


FIGURE 3a
s,y
s1
s0 = ŝ*
ŝ(r)
ŝ'(r)
r̂(s)
r̂'(s)
x(s)
r(s)
r'(s)
y(r)
y'(r)
s(r)
ŝ(r)
r̂*
r1
r0
r,x

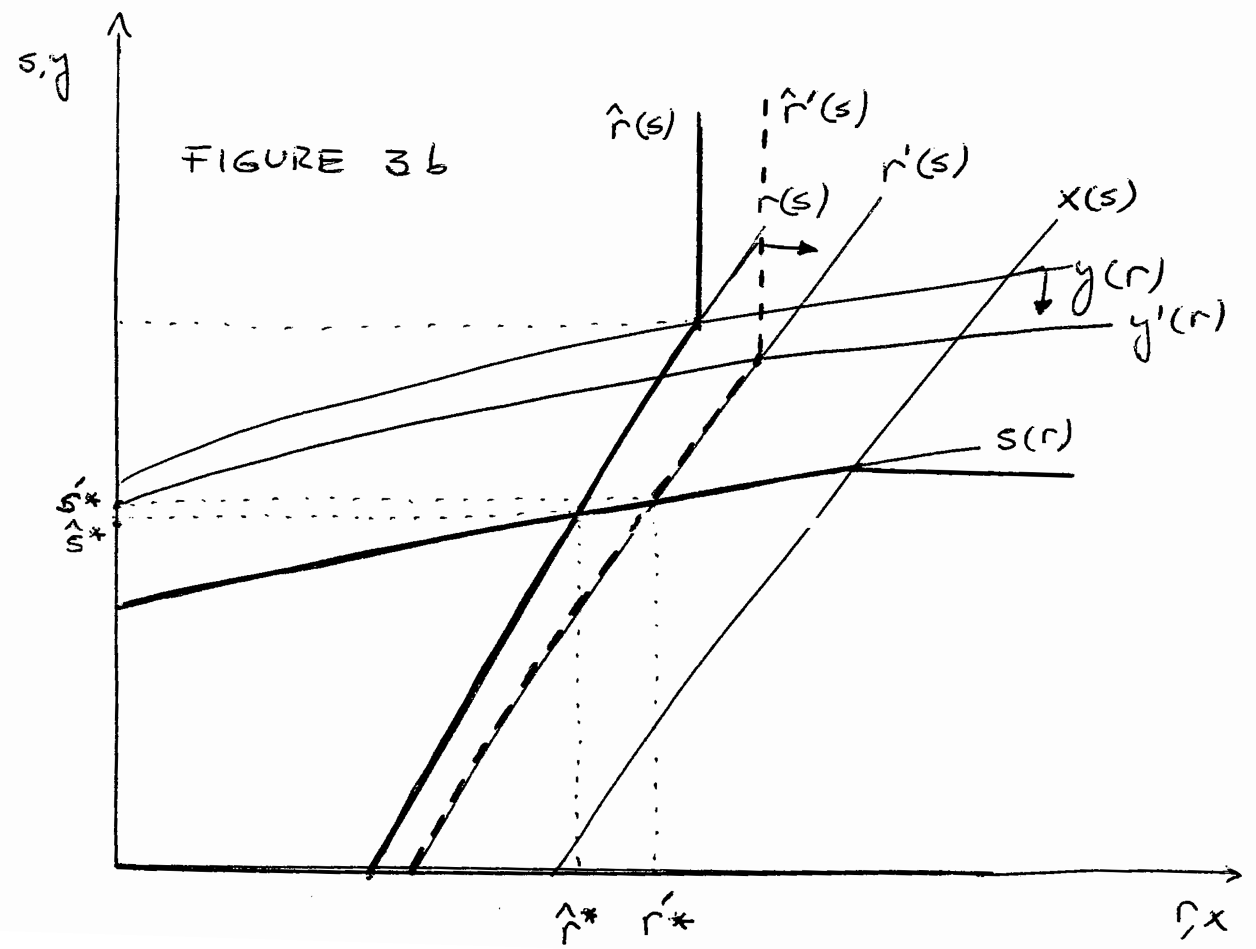


FIGURE 3b
s,y
s'*
ŝ*
r̂(s)
r̂'(s)
r(s)
r'(s)
x(s)
y(r)
y'(r)
s(r)
r̂*
r'*
r,x

reduction in wages by firm A will necessarily lead to a reduction in its sales. This occurs through the indirect effect of a reduction in income of its customer (the standard Keynesian argument). Nevertheless, as we will see, there are still profitable deviations that the firm can make.

Assume that worker J is rationed in the labor market. As noted above, this corresponds to a situation where $\hat{r}^* = x(\hat{s}^*) < r(\hat{s}^*)$. From the argument above it also follows that $\hat{s}^* = y(\hat{r}^*) \leq s(\hat{r}^*)$ (i.e., neither worker is rationed in the goods market). Assume for the moment that $s(\hat{r}^*) > \hat{s}^*$ so that both workers are being rationed in the labor market. This case is displayed in figure 4 (as before the reader should ignore the dotted lines), where it follows that the equilibrium is given by the intersection of x and y. It follows that for small changes in prices and wages the equilibrium of the stage II game is still given by the intersection of y(r) and x(s). Let $r(p_x, w_r)$ denote the equilibrium level of sales by firm A.

For the candidate prices to be an equilibrium, A must be maximizing profits. Now, $\pi_A = (p_x - w_r)r(p_x, w_r)$.

We consider a policy of simultaneous changes in prices and wages. Specifically, let $p_x - w_r = a$. Then we analyze changes that leave $p_x - w_r$ unchanged. Under this policy the profit function for firm A can be written as:

$$\tilde{\pi}_A = ar(p_x,\ p_x - a).$$

The effect of changing p_x and w_r is given by

$$d\tilde{\pi}_A/dp_x = a(\partial r/\partial p_x + \partial r/\partial w_r)$$

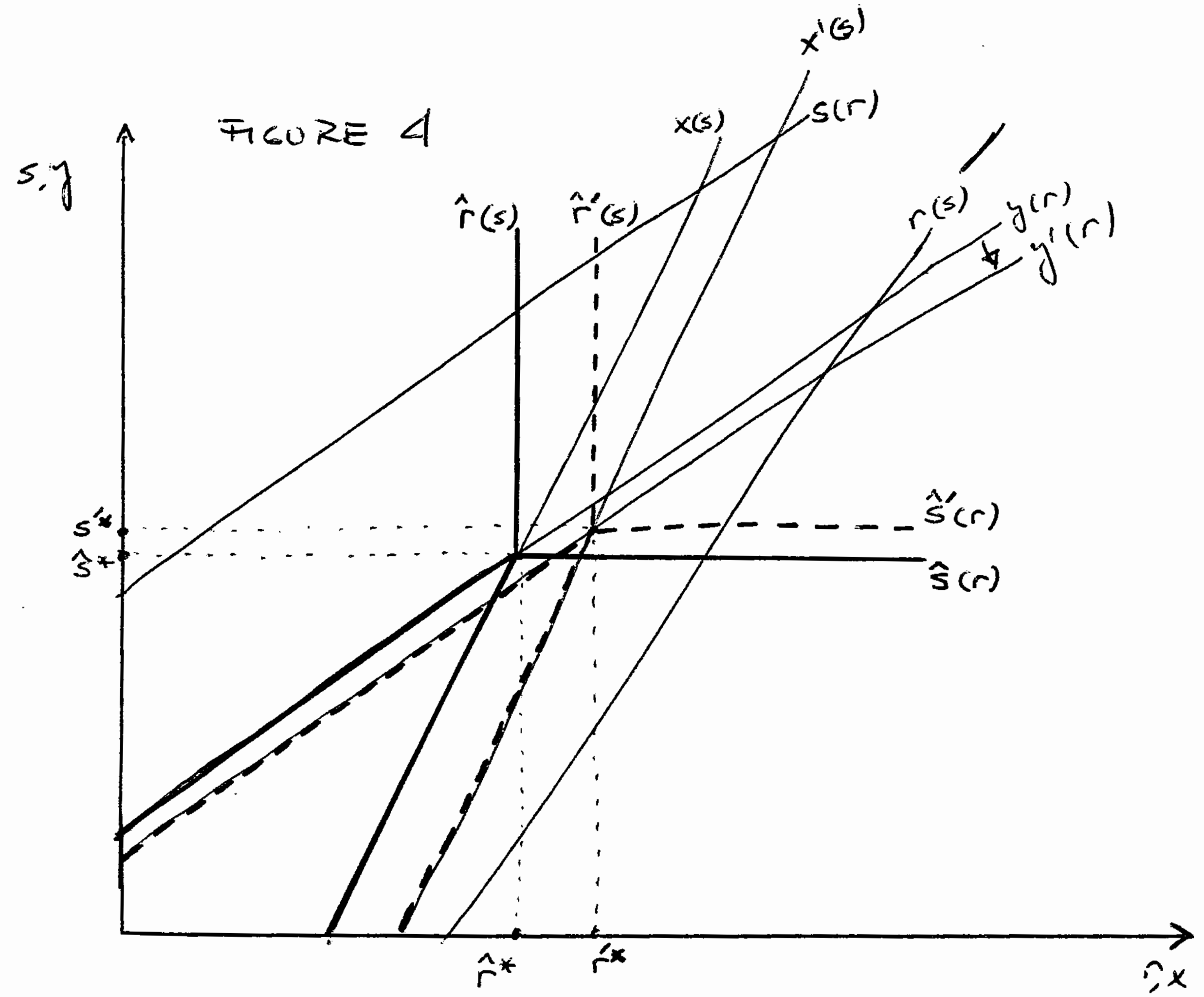
In Appendix I we show that this last term is negative at the intersection of the x and y loci. Therefore, a simultaneous decrease in prices and wages increases profits. Diagrammatically, what happens is that the downward shift in y captures the income effect of lower wages (w_r decreases), while the outward shift in x captures the substitution effect of lower prices (p_x decreases). The equilibrium shifts from $(\hat{s}^*, \hat{r}^*)$ to (s'^*, r'^*) and it is such that substitution effects dominate. Profits of firm A change from $a\hat{r}^*$ to ar'^*. Because $r'^* > \hat{r}^*$ this is a profitable deviation.

The previous argument applies to the case in which both consumers are rationed in the labor market. It is possible for one consumer to be rationed in the labor market, say J, while the other is at his unconstrained utility maximizing bundle.

Formally, we consider a point (r^*, s^*) such that $s^* = s(r^*)$, $r^* = x(s^*)$ (and, consequently, K is at his unconstrained bundle), while $r^* < r(s^*)$ (J is rationed in the labor market). In this case a deviation may result in both workers being rationed in the labor market (the case we analyzed above), or one worker in the labor market and the other in the goods market. To show that this situation cannot be an equilibrium we show that the same deviation we analyzed for the "pure" labor market rationing , namely, lowering p_x and w_r while keeping the difference constant, results in higher profits for firm A. The necessary calculations are in Appendix I.

The underlying intuition is as follows: because worker J is rationed in the labor market, it is possible for firm A to reduce his wage and to increase the demand for labor. This, however, may result in a decrease in J's income, consequently reducing his purchases of y. This decrease in the

FIGURE 4



demand for y reduces K's income and his purchases of the good that firm A sells. To counter this potentially negative effect firm A not only reduces w_r but also the price it charges for the good, p_x . In this way it keeps its profit margin constant and the decrease in p_x more than compensates for A's possible loss of income to result in a higher equilibrium quantity. Profits, of course, go up.

Summarizing, we have:

Proposition 3: In the separable case there is never any rationing in either the goods market or the labor market in equilibrium.

5. Direct Competition Among Firms

In this section we will present an analysis of the behavior of models such as those discussed in the preceding sections when firms are faced with direct competitors. As noted in the introduction, our goal in this endeavor is to try and illuminate the roles played by both the economic fundamentals and the strategic structure in the previous work.

Ideally, we would like to do this by analyzing a multiple firm/multiple worker version of the model discussed in sections 3 and 4. The difficulty with this approach is that since it involves the possibility of multiple prices for some good, a very difficult rationing problem must be confronted (i.e., who gets to buy/sell from/to which firm and at what levels). Since this would take us too far from our original objectives, we will instead analyze two simple models of the strategic interaction of the agents while maintaining the basic economic structure. (See however the comments on the more recent work of Roberts [11] in section 6.)

The first model we will discuss is one in which firms behave as Cournot quantity setters. Thus, each firm chooses input demand and output supply levels. Prices and wages are adjusted until markets clear at these quantities and then payoffs are realized. In a model with free entry and small fixed costs, we will show that equilibria of the game are approximately Walrasian under mild assumptions. This is in the spirit of the work by Novshek and Sonnenschein [8].

The second model considered involves three stages and in this sense is more in keeping with the models analyzed in the previous section. At the first stage, firms set wages simultaneously and independently. At the second, workers, taking wages as given, choose labor supply levels. At the third, firms choose the demand for labor and the supply of output. After this, output prices are adjusted until markets clear and then payoffs are realized. Again, we will show that when the economy is "large" the outcomes are approximately Walrasian. However, in this case, the sufficient conditions we will be able to give are much more severe (basically gross substitutes).

5.1 Cournot Competition

The economic fundamentals of the model we will consider are exactly as described in section 2 except for the following changes:

(i) There are N capitalists of each type which care only about money.

(ii) The technology for each firm is constant returns to scale up to a capacity constraint c. In addition there is an entry cost ε which must be paid by active firms.

For simplicity, we will maintain our assumption of one worker of each type.

Formally, firms of each type must choose output levels $x_1,\dots,x_N$ and $y_1,\dots,y_N$. Let $x = \Sigma x_i$ and $y = \Sigma y_i$. Given these choices by the firms, define $p_y(x,y)$, $p_x(x,y)$, $w_r(x,y)$ and $w_s(x,y)$ so that:

(i) The solution to the problem:

$$\max U_J(r,y,m) \text{ s.t. } w_r r + p_y y + m = w_r \bar{r}_J + \bar{m}_J$$

is given by $\bar{r}_J - r = \Sigma_{i=1}^N x_i$ and $y = \Sigma_{i=1}^N y_i$ and,

(ii) The solution to the problem:

$$\max U_K(s,x,m) \text{ s.t. } w_s s + p_x x + m = w_x \bar{s}_K + \bar{m}_K$$

is given by $\bar{s}_K - s = \Sigma_{i=1}^N y_i$ and $x = \Sigma_{i=1}^N x_i$.

That is, p_y, p_x, w_r and w_s clear markets at the given quantities assuming that the workers are price takers. Equivalently, these prices can be read off of the aggregate inverse demand function at the given quantities.

In what follows, we will assume that these prices are uniquely defined for all combinations of x and y.

Given this, we can describe what payoffs are to the firms. For firm i of type A, profits are given by:

$$\pi_i^A(x_1,\dots,x_N;\ y_1,\dots,y_N) = [p_x(x,y) - w_r(x,y)]x_i - \delta_i(x_i)$$

for $0 \le x_i \le c$ where

$$\delta(x_i) = \begin{cases} \varepsilon & \text{if } x_i > 0 \\ 0 & \text{if } x_i = 0 \end{cases}$$

A similar description holds for firm i of type B.

Consider a sequence of games as described above parameterized by N, c_N and ε_N. We will assume that for all N, $N\varepsilon_N > \bar{m}_J + \bar{m}_K$, that $\lim \varepsilon^N/c^N \to 0$ and that $c^N \to 0$. The first of these assumptions guarantees that there are inactive firms in equilibrium whereas the second implies the technology of the limiting economy is constant returns (as in the preceding sections) since in this case, fixed costs per unit go to zero as $N \to \infty$.

Consider a sequence of pure strategy equilibria of these games given by $x_1^N, \ldots, x_N^N, y_1^N, \ldots, y_N^N$. Let $x^N = \Sigma x_i^N$, $y^N = \Sigma y_i^N$, $p_y^N = p_y(x^N, y^N)$, etc.

<u>Proposition 4</u>: Assume that p_x, p_y, w_r and w_s are continuous functions. Then as $N \to \infty$, every limit point of the outcome of the equilibrium of the game is a Walrasian equilibrium of an economy in which the workers own the firms and these firms have CRS technologies.

<u>Proof</u>: Consider a convergent subsequence of the relevant variables and let the limits be given by x*, y*, etc. Since by definition x^N, y^N are utility maximizing for the consumers given the prices and wages, it is immediate that this holds in the limit as well. Thus, we need only show that the limiting mythical firm is also maximizing taking prices as given. Since the

technologies in the limit are CRS, this amounts to showing that $p_x^* = w_r^*$ and $p_y^* = w_s^*$.

It is immediate that $p_x^* \geq w_r^*$ and $p_y^* \geq w_s^*$.

From our assumption that $N \bullet \varepsilon^N > \bar{m}_J + \bar{m}_K$ for all N, it follows that for each N $x_i^N = 0$ for some i. Without loss of generality, assume that $x_1^N = 0$ for all N. Consider the alternative strategy by firm 1 which sets $x_1^N = c^N$ for all N. Let $\tilde{p}_x^N$, $\tilde{p}_y^N$, $\tilde{w}_r^N$, $\tilde{w}_s^N$ be the prices that result from this change in strategy on the part of firm 1. Since $c^N \to 0$ it follows that $\tilde{p}_x^N \to p_x^*$, etc.

Calculating this firm's profits gives

$$\pi^N = (\tilde{p}_x^N - \tilde{w}_r^N)c^N - \varepsilon^N.$$

If the original array of strategies was in equilibrium we must have that $\pi^N \leq 0$ for all N. This gives

$$\tilde{p}_x^N - \tilde{w}_r^N \leq \varepsilon^N/c^N$$

which, on taking limits, gives $p_x^* \leq w_r^*$ as desired. []

As can be readily seen, this argument differs very little from that given in the other literature on Cournot markets in large economies (e.g., Novshek and Sonnenschein [8], and Mas-Colell [7]). Further, that literature gives some immediate insight into the reasonableness of our assumptions. Of particular interest is that inverse demand is well defined and continuous. This rules out backward bending demand and/or supply curves.

In fact, the model we have analyzed is almost a special case of that analyzed in Novsheck and Sonnenschein [8]. The sole difference arises due to our special structure on preferences which imply a lack of monotonicity. The argument above shows (at least for the question of convergence) that this difference is unimportant.

There are two final observations to make concerning this model. The first is to note that as shown in section 3, autarky is a Nash equilibrium if and only if the endowments are the unique competitive equilibrium of the economy. (Thus, there is no problem with complementarities as emphasized by Hart [3] arising in this structure.)

Second, we note that in the special case of separable preferences (as analyzed in section 4) all of our assumptions concerning p_x, p_y, w_r and w_s are satisfied. In particular, they are uniquely defined and are continuous functions of x and y.

5.2 A Three-Stage Model of Competition

The Cournotian model analyzed above shows one thing. This is that it cannot be the economic fundamentals alone that give rise to Robert's results on rationing and low activity equilibria. That is, the analysis shows that, at the very least, it must be an interaction between the fundamentals and the strategic structure that gives rise to the problems.

To gain a better understanding of this we will analyze a richer model of strategic interaction. In particular, the form of competition we will consider contains three stages and is much closer in spirit to the model of the previous section.

As above, our aim to make all agents in the economy small relative to the aggregate. To do this requires, in addition to many firms, many workers of each type.

Fundamentals

A. There are N identical consumers of each type.

B. There are 2N firms of each type.

C. Each worker can work for only two of the firms and each firm can only hire one worker's labor. Thus, worker i of type J can work for either firm i,L type A or firm i,R type A. Firm i,t of type A can hire only worker i of type J (t = L,R).

D. Technology, preferences and endowments are as given in section 2.

The Game

Stage I: Firms simultaneously and independently set wages $w_{r_{i,t}}, w_{s_{i,t}}$, $i = 1, \ldots N$, $t = L,R$.

Stage II: Workers simultaneously and independently choose labor supplies $r_{i,t}$ and $s_{i,t}$, $i = 1, \ldots, N$, $t = L,R$.

Stage III: Firms simultaneously choose supply of consumption (and the demand for labor) $\hat{r}_{i,t} \leq r_{i,t}$ and $\hat{s}_{i,t} \leq s_{i,t}$ $i = 1, \ldots N$, $t = L,R$.

Outcomes: Given the supply choices of firms, output is then auctioned off in a Cournot fashion to the workers taking as given their income and prices. Firms' profits are calculated in the obvious way.

More formally, given the demand for labor at stage III, $\hat{r}_{i,t}, \hat{s}_{i,t}$, $i = 1, \ldots, N$, $t = L,R$, and given prices p_x, p_y let $y_i(p_y; \hat{r}_{i,L}, \hat{r}_{i,R})$ solve

$$\max_y U_i(r,y,m) \text{ s.t. } p_y y \le \bar{m}_i + w_{r_{i,L}} \hat{r}_{i,L} + w_{r_{i,R}} \hat{r}_{i,R}$$

$$\text{and } r = \bar{r}_J - \hat{r}_{i,L} - \hat{r}_{i,R}.$$

Define $x_i(p_x;\hat{s}_{i,L},\hat{s}_{i,R})$ similarly.

Let $y(p_y;\hat{r}) = \Sigma_{i=1}^N y_i(p_y;\hat{r}_{i,L},\hat{r}_{i,R})$ and define $p_y(\hat{r};\hat{s})$ implicitly through the equation $y(p_y(\hat{r},\hat{s});r) = \Sigma_{i,t} \hat{s}_{i,t}$.

Define $x(p_x;\hat{s})$ and $p_x(\hat{r};\hat{s})$ similarly. We will assume that p_x and p_y are uniquely defined for all choices of $\hat{r},\hat{s}$. (This is true, for example, in the separable case considered in section 4.)

For simplicity, we will again restrict attention to the case in which workers' preferences are separable. Given these definitions, worker i receives the allocation

$$\bar{r}_i - \hat{r}_{i,L} - \hat{r}_{i,R},\ y_i(p_y(\hat{r};\hat{s});\hat{r}_{i,L},\hat{r}_{i,R}),\ \bar{m}_i + w_{r_{i,L}} \hat{r}_{i,L} + w_{r_{i,R}} \hat{r}_{i,R} - p_y(\hat{r},\hat{s})y_i.$$

The profits of firm i,t are $\hat{r}_{i,t}(p_x(\hat{r},\hat{s}) - w_{r_{i,t}})$.

Similar formulae hold for the sellers of s and producers of x.

Notice that in this version of the model there is no room for rationing in the goods market: prices p_x and p_y are chosen to clear the market. There is, however, potential for the workers to be rationed in the labor market. Specifically, it is possible for a firm to choose $\hat{r}_{i,t} < r_{i,t}$. We next argue that, when N is large, firms will choose not to ration workers if prices exceed wages. Formally, we consider a sequence of games indexed by

N. We assume that prices are uniformly bounded above and that workers' choice of labor supply satisfies $\Sigma r_{i,t} \to \infty$ and $\Sigma s_{i,t} \to \infty$ (so that any individual worker is becoming small). (Note that while this assumption rules out the possibility of equilibria in which output is bounded, it does not imply that output per capita is bounded away from zero. Thus, we have ruled out only the most severe forms of low activity equilibria.) We summarize the no-rationing result as:

Lemma 3: Assume prices are greater than or equal to wages. Given any $\varepsilon > 0$, there exists an N* such that all $N \geq N^*$ $|\hat{r}^N_{i,t} - r^N_{i,t}| < \varepsilon$ and $|\hat{s}^N_{i,t} - s^N_{i,t}| < \varepsilon$ for all (i,t).

Proof: Notice first that if $\Sigma r_{i,t} \to \infty$ we cannot have $\Sigma \hat{r}_{i,t}$ bounded. If this was the case, consider the proportional impact upon profits of changing the quantity produced from $\hat{r}_{i,t}$ to $\alpha_{i,t}(r_{i,t} - \hat{r}_{i,t}) + \hat{r}_{i,t}$. Let the price of the good be p_x before the deviation and $p_{x,i,t}$ after firm (i,t) has deviated. Notice that for any sequence $\alpha_{i,t} \to 0$ we have $p_{x,i,t} \to p_x$. Let $b_{i,t} \equiv p_x - w_{r_{i,t}}$ and $\tilde{b}_{i,t} = p_{x,i,t} - w_{r_{i,t}}$ be the profit per unit. Then, if $b_{i,t} > 0$ we have that for $\alpha_{i,t}$ small enough $\tilde{b}_{i,t} > 0$. Therefore, $\tilde{b}_{i,t}/b_{i,t} \to 1$ and is not negative.

Next compute the percentage change in profits.

$$\frac{\tilde{\pi}_{i,t}}{\pi_{i,y}} = \frac{\hat{r}_{i,t} + \alpha_{i,t}(r_{i,t} - \hat{r}_{i,t})}{\hat{r}_{i,t}} \frac{\tilde{b}_{i,t}}{b_{i,t}} = [\alpha_{i,t} \frac{r_{i,t}}{\hat{r}_{i,t}} + 1 - \alpha_{i,t}] \frac{\tilde{b}_{i,t}}{b_{i,t}}$$

But the assumptions on $\Sigma r_{i,t}$ and $\Sigma \hat{r}_{i,t}$ imply that the ratio, $r_{i,t}/\hat{r}_{i,t} \to \infty$.

Thus for $\alpha_{i,t}$ converging to zero at a "slow enough" rate $\tilde{\pi}_{i,t}/\pi_{i,t} \to \infty$. Consequently, the original policy cannot be an equilibrium

But if $\Sigma\hat{r}_{i,t} \to \infty$ each firm has, for large enough N, negligible impact upon the market price and, therefore, a deviation that corresponds to a production "at capacity" has a first order positive effect on profits given by its impact upon quantity. Notice that the argument is strictly correct when $p_x > w_{r_{i,t}}$. In keeping with the argument in Section 3, we assume that when $p_x = w_{r_{i,t}}$, $r_{i,t} = \hat{r}_{i,t}$. This is just the continuous extension of the case when prices approach wages from above. This completes the argument.

[]

In section 4 we showed that there is no rationing using a very different argument. There it was argued that if a worker is being rationed in the goods market the seller can increase the price and, therefore, increase profits because it will not loose sales. In the case of labor market rationing the policy is to lower the wage. In the present set up we cannot guarantee that a wage decrease results in higher profits because there may be multiple equilibria and any change in the first stage may induce a regime shift. However, there is another argument that implies that firms will not ration workers (this is equivalent to producing at less than capacity with zero cost and price given by the profit margin), namely, that when the firm is small relative to the market it has no effect upon market price and, consequently, it will choose to produce at capacity.

Given this result we will now ignore the possibility that $\hat{r}_{i,t} < r_{i,t}$ and $\hat{s}_{i,t} < s_{i,t}$ and we will use $(s_{i,t}, r_{i,t})$ to denote both the amount of labor supplied by the workers and the amount purchased (and sold in the market) by firms.

An important fact to note is:

Lemma 4: Workers never work for the low wage firm.

Proof: Fix the actions of all workers at stage II other than that of worker i. Without loss of generality, suppose that $w_{r_{i,L}} < w_{r_{i,R}}$. Consider a strategy by i $(r_{i,L}, r_{i,R})$ in which $r_{i,L} > 0$. Consider an alternative strategy with $\tilde{r}_{i,R} = r_{i,R} + (w_{r_{i,L}}/w_{r_{i,R}})r_{i,L}$, $\tilde{r}_{i,L} = 0$. It is clear that this earns the worker the same income but gives him a higher level of consumption of leisure. Since his income is the same, it follows that inverse demand for y has not changed nor has its supply. (This depends on separability.) It follows that p_y is unchanged and hence the workers' final allocation of y and m have not changed. Thus, the worker is better off. (Note that the change in strategy suggested above does affect p_x, but this is irrelevant from the workers' point of view.) []

Given the lemma, we will for the most part ignore the fact that a worker can potentially work for two firms (until the final step in our argument).

To show that the outcomes of the game are approximately Walrasian when N is large we must show two things. These are, first, that when N is large workers are approximately wage and price takers and second, that prices equal wages for all active firms.

As can be seen from the proof of Lemma 4, the proof of the first part comes down to showing the indirect effect (through income) of a worker's labor choice on the price of the good that he buys is small when N is large. This is, due to our formulation, the only effect that could cause problems.

The second problem above will be solved by showing that if prices do not equal wages, one firm can drastically increase its market share by offering a higher wage than its competition.

As above, consider a sequence of games as described with $N \to \infty$ and a sequence of pure strategy equilibria.

As before, assume that prices are uniformly bounded above and that both $\Sigma_i r_i \to \infty$ and $\Sigma_i s_i \to \infty$ (so that any individual worker is becoming small).

At the second stage, any individual worker faces the following problem:

$$\max_{r_i} u_i(\bar{r}_J - r_i) + u_2(y_i(p_y^N(r;s), r_i)) + u_3(\bar{m}_J + w_{r_i}^N r_i - p_y^N(r;s) y_i(p_y^N(r;s); r_i))$$

The first order conditions for this problem are

$$0 = -u_1' + u_2'\left[\frac{\partial y_i^N}{\partial p_r}\frac{\partial p_y^N}{\partial r_i} + \frac{\partial y_i}{\partial r_i}\right] + u_3'\left[w_{r_i}^N - \frac{\partial p_y^N}{\partial r_i} y_i - p_y^N \frac{\partial y_i}{\partial p_y}\frac{\partial p_y^N}{\partial r_i} - p_y^N \frac{\partial y_i}{\partial r_i}\right]$$

Since $\Sigma s_i \to \infty$, it follows that $\partial p_y^N/\partial r_i \to 0$ as $N \to \infty$.

Thus, when N is large, i's optimal choice of r_i is characterized by (approximately) $0 = -u_1' + u_2'(\partial y_i/\partial r_i) + u_3'[w_{r_i} - p_y(\partial y_i/\partial r_i)]$, where p_y is the limiting price of y and w_{r_i} is the limit of the $w_{r_i}^N$.

As can be easily checked, this defines (through the way that y_i is defined) the worker's Walrasian demand when faced with the price p_y and the wage w_{r_i}.

More formally, we can show:

Proposition 5: Given any $\varepsilon > 0$, there is an N^* such that for all $N \geq N^*$, the equilibrium allocation for all workers is within ε of their Walrasian demands at the price they face.

That is, in the limit workers are price takers.

Given this fact, all that is left to show is that in the limit prices equal wages so that firms are also price takers.

The argument for this point is clear--if the wages for one worker are strictly less than prices in the limit, have the low wage firm for this worker raise its wage offer. It will become the sole employer of the worker and, in the limit, the move will have little effect on prices, and the firm's profits will go up.

The problem with this argument is that it assumes that only this worker will change his labor supply decision. Of course, there is no reason to believe that this will be true. Indeed, we would expect that all workers would alter their decisions to some degree. This does not matter per se as long as the effect on prices is small. The sole difficulty then lies in the possibility that there might be two equilibria in the limiting economy. If this holds, a change in wage by one firm could trigger a "regime-shift" so that the effect on prices is very large.

To rule this possibility out, we will adopt a form of the gross substitutes assumption. In essence, this guarantees uniqueness (in the limit) of the second stage and implies that wage changes by any individual firm will necessarily have a small impact on prices.

We will need some notation to carry this out.

Let μ_r and μ_s be probability distributions which we will interpret as wage distributions.

Given a price p_x and a wage w_s let $x(p_x,w_s)$ and $s(p_x,w_s)$ denote a worker of type K's Walrasian demands and supplies. That is:

$$\max_{x,s} v_1(\bar{s}_K - s) + v_2(x) + v_3(\bar{m}_K + w_s s - p_x x)$$

is at $x = x(p_x,w_s)$, $s = s(p_x,w_s)$. Let $\bar{\bar{x}}(p_x,\mu_s) = \int x(p_x,w_s)d\mu_s$ and $\bar{\bar{s}}(p_x,\mu_s) = \int s(p_x,w_s)d\mu_s$. Define $\bar{\bar{y}}(p_y,\mu_r)$ and $\bar{\bar{r}}(p_y,\mu_r)$ similarly.

It follows (due to our assumptions on preferences) that $\bar{\bar{x}}$, $\bar{\bar{y}}$, $\bar{\bar{r}}$ and $\bar{\bar{s}}$ are differentiable functions of prices. Standard arguments that show how gross substitutes implies uniqueness are sufficient to prove the following:

Lemma 5: Assume that $\partial\bar{\bar{r}}/\partial p_y \le 0$ and $\partial\bar{\bar{s}}/\partial p_x \le 0$ (gross substitutes). Then there is a unique p_x,p_y pair such that

$$\bar{\bar{x}}(p_x,\mu_s) = \bar{\bar{r}}(p_y,\mu_r) \text{ and } \bar{\bar{y}}(p_y,\mu_r) = \bar{\bar{s}}(p_x,\mu_s).$$

Proposition 6: Given any $\varepsilon > 0$, there exists an N^* such that for all $N \ge N^*$, $|p_y^N - w_{s_i}^N| < \varepsilon$ and $|p_x^N - w_{r_i}^N| < \varepsilon$ for all i.

Proof: Assume the contrary. It follows that (after taking subsequences if necessary) there is a sequence of firms i_N with $w_{i_N,t}^N < p_y^N - \varepsilon$ for all N. Let μ_r^N and μ_s^N be the empirical wage distributions in the two industries for the N-th game and assume that $\mu_r^N \to \mu_r^*$, $\mu_s^N \to \mu_s^*$. It follows from the proof of Proposition 5 that $p_y^N \to p_y(\mu_r^*,\mu_s^*)$ and $p_x^N \to p_x(\mu_r^*,\mu_s^*)$.

Assume that firm i_N,L is receiving less than one-half of worker i_N's supply of labor for all N (e.g., $w^N_{i_N,L} \le w^N_{i_N,R}$), and consider the alternative strategy of setting wage at $\tilde{w}^N(\delta) = w^N_{i_N,R} + \delta$, $\delta>0$. It follows from Lemma 4 that firm i_N,L now captures all of the workers' labor supply. It follows from Proposition 4 that when N is sufficiently large, this labor supply is arbitrarily close to the workers' Walrasian supply given the new wage and the new equilibrium output price, $\tilde{p}^N_x$. Let $\tilde{\mu}^N_s$ be the new empirical wage distribution. It is immediate the $\tilde{\mu}^N_s \to \mu^*_s$. It follows that $\tilde{p}^N_x \to p_x(\mu^*_r,\mu^*_s)$.

Thus, the equilibrium profits for the firm are given by $\tilde{\pi}^N(\delta) = (\tilde{p}^N_x - \tilde{w}^N(\delta))\ \tilde{r}_{i_N}(\delta)$. As $N \to 0$ this converges to $\tilde{\pi}(\delta) = (p_x(\mu^*_r,\mu^*_s) - \tilde{w}(\delta))\tilde{r}(\delta)$ where $\tilde{w}(\delta) = \lim \tilde{w}^N(\delta)$ and $\tilde{r}(\delta)$ is the worker's Walrasian labor supply when he is faced with prices $\tilde{w}(\delta)$, $p_x(\mu^*_r,\mu^*_s)$. At the old strategy, the firm's limiting profits are no more than

$$\pi = (p_x(\mu^*_r,\mu^*_s) - \tilde{w}(0))(1/2)\tilde{r}(0).$$

It is straightforward to show that for δ sufficiently small the suggested change in strategy gives higher profits when δ is sufficiently small giving the desired contradiction.

It is interesting to note that the only place in the proof that the gross substitutes condition is used is to guarantee that equilibrium is essentially unique in the second stage when N is large. Formally, this is used in the proof to argue that the limiting price of x is not affected by the proposed change in strategy.

This leads us to suspect that an alternative result is probably available in which the size of movements of aggregates in response to wage changes by any one firm are small. This is much more in the spirit of Proposition 4.

6. Concluding Comments

We close the paper with a few notes concerning extensions of and limitations to the work presented thus far.

1. In the Introduction, we motivated the difference in timing between the model analyzed in sections 2-4 and that of Roberts ([9], [10], and [11]) as being interpretable as the difference between economies where goods are produced for inventories and ones in which they are produced on a "made-to-order" basis. Although this is an interesting distinction, the reader should be wary of attaching too much importance to it. (For example, that this distinction is at the root of the difference in results.)

Intuitively, the most important consideration for the existence of autarky in equilibrium is probably the simultaneity of action by parties on both sides of the market. Thus, if one believes the other will not participate, he should not, etc. While this holds in Robert's version of the "made-to-order" economy it is not necessarily a feature of this type of model.

Indeed, consider a game as above in which at Stage I prices and wages are set, at stage II labor supply offers are made, at stage III orders for consumption goods are placed and at stage IV actual production decisions are made (as in Robert's stage III). Intuitively, one would expect that the argument presented in section 3 would cover this model as well (we have not done this). That is, workers cannot rationally expect that their firms' customers will put in zero orders at reasonable prices. Thus, autarky will not in general be an equilibrium, equilibrium of the latter stages will be unique in "nice" cases, etc. One would expect this argument to work in a model in which orders for goods are placed at stage II and labor supply decisions are made at stage III as well. Summarizing, it is likely that the crucial factor is whether or not there is simultaneity in the critical decisions, not the exact timing if this feature is present.

2. It is natural to question the role that money (i.e., an outside good) plays in our results. At first, it would seem that the existence of such a good is crucial to our results. That is, in section 3 we argue that if autarky is not a Walrasian equilibrium and prices are set reasonably, worker J will always buy output at stage IV even if he does not work (i.e., his stage II choice is zero). This result, of course, depends on our assumptions that his initial money holdings are positive. Without this, it follows that the worker will necessarily buy no output if they have not worked.

This argument leads one to suspect the existence of this outside good is both crucial to the success of the arguments we have presented and in large part responsible for the differences between our results and that of previous work. There are two reasons why this line of argument is incorrect.

First, note that the model analyzed by Roberts does have money in it and autarky is still an equilibrium even if both workers have positive initial endowments of money.

Second, it follows that autarky is an equilibrium of the game analyzed in sections 2-4 if initial money holdings are zero. (If either worker chooses zero at stage II, it follows that the unique outcome of the continuation game is autarky. Thus, zero is always an optimal response to zero giving the results.) However, one suspects that other small changes in the game would get rid of this problem. If, for example, the workers set their labor supplies sequentially it seems likely that the argument presented in section III will hold even in the absence of money. That is, although it is true that if the first worker to move says zero the second will rationally respond with zero, it will not in general be in the first worker's interest to do this.

Again, the strict simultaneity in action choices seems to be the deciding factor, not the existence of the outside good per se. (Although this does obviate this problem somewhat.)

3. We should point out that our results are not incompatible with the point emphasized in Heller [6]. This is that these games may well have multiple equilibria which are Pareto ranked. This is possible in our formulation as well.

For example, in the special case analyzed in section 3, although we have shown that given prices and wages, the equilibrium of the continuation is unique (in outcomes), it is still possible that the resulting one stage price/wage game among firms may have multiple equilibria. Further, these may be Pareto ranked. This multiplicity arises for the standard game-theoretic reasons.

4. We should note that in recent work Roberts [11] has analyzed an example of a multiple firm, multiple worker version of the model discussed

in sections 2-4 (i.e. the same timing). He shows that is is possible to get unemployment equilibria in this context even if prices are at their Walrasian levels. These equilibria are heavily dependent on firm's willingness (due to the assumption of constant returns and the fact that prices are at their Walrasian levels) to change output to any level. Thus, if a firm upon seeing an unemployed worker and a rationed consumer deviates from the proposed equilibrium strategy, the firms employing potential customers no longer hire those individuals. It is a property of the utility functions in the example that workers will not consume if they are not hired and face Walrasian prices. Thus, the deviating firm faces a situation in which it will not sell any output. If follows that it is indifferent to changing strategies and equilibrium is assured.

5. The "coordination games" that we study share one property: there are no typical "coordination problems." In the case where the number of participants is small, we show that there is no rationing in equilibrium. This happens notwithstanding the fact that for "most" choices of prices and wages the model requires that there must be rationing. When we look at variants of the model and analyze their properties when the number of players becomes large, we find that equilibrium outcomes are Walrasian.

Therefore, we must conclude that--at least for the structures that we study--economic environments in which participants make simultaneous decisions without some central mechanism, and in which individual players are not self sufficient--and consequently there is a need for coordination--do not necessarily result in a Keynesian version of unemployment (rationing) or in low output equilibria.

Two natural questions to ask are: Are these economies reasonable? Are the models we study useful? We think that the answer is yes to both questions. It is interesting to point out that the models we analyze are minor variants of the models found in the literature that, in many cases, reach exactly opposite conclusions. To evaluate the usefulness of our version we must analyze the source of the disparity of results. It seems clear that it is the particular choice of institutions that gives rise to the differences. Of course, the fact that changing the timing of actions in a game theoretic model can have significant effects is well known. But we think that the examples of this phenomenon that we have presented are particularly striking. What seem like only small changes in the economic structure can give rise to altogether different conclusions about the qualitative properties of equilibrium. This suggests that, at the very least, great care should be taken in the application of these models.

The versions of the model we study seem to question the notion that simultaneity of decisions and environments with potential coordination problems will, in general, imply that, in equilibrium, we will observe involuntary unemployment and low output. They do not challenge the notion that imperfect competition can result is less than optimal levels of output. Thus, although it may well be that models with imperfect competition can and should play an important part in future theories of potential inefficiencies arising in business cycles, our results seem to indicate that more work is needed to understand these environments before a theory that can challenge more standard equilibrium theories can be found.

Appendix I

Our aim here is to give the derivations necessary to show that (from section 4) there is no rationing in the labor market.

The cases of rationing in the labor market are described by the intersection of the x and y loci. In the second case this also corresponds to the intersection of the s loci. The following equations describe those intersections.

(I.1) $u_2'(s^*) = p_y u_3'(\bar{m}_J + w_r r^* - p_y s^*)$ (y locus)

(I.2) $v_2'(r^*) = p_x v_3'(\bar{m}_K + w_s s^* - p_x r^*)$ (x locus)

(I.3) $v_1'(\bar{s}_K - s^*) = w_s v_3'(\bar{m}_k + w_s s^* - p_x r^*)$ (s locus)

In the "pure" labor market rationing case the relevant equations are (I.1) and (I.2). A lengthy but straightforward analysis shows that

$$\frac{\partial r}{\partial p_x} = \frac{(u_2'' + p_y^2 u_3'')(v_3' - p_x r v_3'')}{\Delta_1}, \text{ and}$$

$$\frac{\partial r}{\partial w_r} = \frac{p_x w_s p_y r u_3'' v_3''}{\Delta_1}, \text{ where}$$

$\Delta_1 = (u_2'' + p_y^2 u_3'')(v_2'' + p_x^2 v_3'') + p_x p_y w_r w_s u_3'' v_3''$ is strictly positive.

Therefore,

$$\frac{\partial r}{\partial p_x} + \frac{\partial r}{\partial w_r} = \frac{u_2'' + p_y^2 u_3'')v_3' - u_2'' v_3'' p_x r^* - r^* u_3'' v_3'' p_x p_y (p_y - w_s)}{\Delta_1}.$$

The numerator is negative and, consequently, the whole expression is also negative.

The second case of labor market rationing is described by the solution to all three equations (I.1)-(I.3). To show that the same policy increases profits we need to show that it does so no matter what describes the "new" equilibrium for the continuation game. Specifically, after firm A lowers p_x and w_r keeping $p_x - w_r = a$, the new equilibrium may be given by the intersection of (I.1) and (I.2) or (I.2) and (I.3). In the first case the previous argument shows that the policy results in higher profits. We next consider the second possibility. To do this we compute $\partial r/\partial p_x$ and $\partial r/\partial w_r$ from (I.2) and (I.3). A standard calculation shows that they are given by

$$\frac{\partial r}{\partial p_x} = \frac{-[(v_3' - p_x r^* v_3'')v_1'' + w_s^2 v_3'' v_3']}{\Delta_2},$$

$$\partial r/\partial w_r = 0,$$

where $\Delta_2 = -[v_2''(v_1'' + w_s^2 v_3'') + p_x^2 v_3'' v_1''] < 0$.

Thus, $\partial r/\partial p_x + \partial r/\partial w_r < 0$ independently of where the new equilibrium occurs. This completes the argument.

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