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THE IMPORTANCE OF THE AGENDA IN BARGAINING*

by

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Abstract

In this paper we discuss a multi-issue bargaining game in which the players set up an agenda and negotiate on the issues sequentially according to the agenda. We demonstrate that the agenda really matters and discuss the relationship between the agenda and the final outcome of the bargaining game. Assuming that players have different evaulations regarding the importance of the issues under negotiation, we show what kind of agenda each player prefers and how it relates to his subjective evaluation of the issues. By demonstrating the strategic use of the agenda the paper explains the phenomenon of "bargaining on the agenda."

THE IMPORTANCE OF THE AGENDA IN BARGAINING

1. Introduction

In many bargaining situations the parties involved negotiate on more than one issue. In labor negotition, for example, workers and management negotiate on wages, retirement programs, as well as on other issues related to the workers' working conditions. In the peace talks between Israel and Egypt the two countries discussed the end of the state of war, the withdrawal from occupied territories, trade and tourist relationship, an arrangement regarding the oil fields in the Sinai desert as well as other issues. In such multi-issue negotiations the players have the possibility to discuss all the issues simultaneously or, alternatively, they can set up an agenda and negotiate on the issues sequentially according to this agenda. In some cases, the simultaneous negotiation is not feasible and an agenda is the only way in which the negotiation can be done.

The main objective of this paper is to discuss the relationship between the agenda and the outcome of the bargaining game. We show that the agenda really matters, so if players change the order of issues in the agenda, it will affect the outcome of the bargaining game.

In a multi-issue negotiation we can sometimes face a situation in which players have different evaluations regarding the importance of the issues under negotiation. Some issues are more important to one player while the other player regards the other issues as the important ones. We investigate such a situation and discuss the kind of agenda each player prefers and how it relates to the players' evaluations regarding the importance of the different issues under negotiation. By demonstrating the strategic use of the agenda the paper can explain why there is a bargaining on the agenda. A phenomenon which is frequently observed but not yet explained.

Using the axiomatic approach to bargaining, Kalai (1977) discussed the cooperative outcome of the bargaining when the negotiation is done by stages. The players consider first only a subset of the feasible alternatives, reach an agreement on this subset, and then consider the remaining alternatives. This structure can be formally described as an agenda. Restricting the set of solutions to ones which are invariant under decomposition of the bargaining process into stages, Kalai proved that only the proportional solution satisfies this restriction and that this solution involves interpersonal comparison of utility.

The framework that we adopt in this paper is Rubinstein's (1982) strategic approach. Our point of deviation from Rubinstein's paper is by assuming that instead of one cake the players have to agree on the partition of several cakes and that the bargaining on these cakes is done sequentially.

We will differentiate in this paper between two situations: (i) the cakes are of different sizes but the players are identical in their evaluations regarding the sizes of these cakes. (ii) Players have different evaluation regarding the size of the cakes.

Since the bargaining on the cakes is done sequentially, an important part of such a model is the assumption regarding the time the players are allowed to eat these cakes. There are three possibilities regarding this assumptions:

(a) After the players agree on a partition of a particular cake each one of them is allowed to eat his share.

(b) The players can eat their shares from all the cakes only when the negotiations on <u>all</u> the cakes are over.

(c) The third possibility is a situation in which a cake is actually a flow of services. Once a partition of a certain cake is decided, players start to enjoy the benefits from their share of the cake. However if the players fail to reach an agreement on the partition of all the cakes and the negotiation terminates all the partitions that were agreed upon are cancelled and each player has to give up the pieces of cakes he started to enjoy.

In the first case mentioned above, if the utility from a partition of one cake does not affect the utility generated from the partition of another cake then the bargaining on each cake can be discussed separately. Whatever is agreed upon in the bargaining on one cake does not affect the bargaining game on the other cakes. In the second case, when players are allowed to eat their shares only at the end, a partition of one cake affects both the threat point and the impatience rate in the bargaining game on the second cake. It affects the threat point since if the bargaining on the second cake is broken, the players do not get to eat their shares from the first cake. It affects the impatience rate since if there is a delay in the agreement on the second cake, it implies a delay in eating the first cake. In the third case, when the cakes represent flows of services, the partition of one cake affects only the threat point in the bargaining game on the second cake. It does not affect the impatience rate since the players start immediately to enjoy the serevices provided by their shares of the cake.

It is clear however that when the players bargain on the partition of one cake they must take into account the implications of each partition on the continuation of the bargaining game.

In this paper we discuss the more interesting case in which players are allowed to eat their shares only when the bargaining on all the cakes is finished. In section 2 we discuss the case in which the cakes have different sizes but players have identical evaluations regarding the sizes of the cakes. In subsection 2.2 we discuss the bargaining game when players negotiate first the partition of the small cake and only when this negotiation is finished they start the negotiation on the partitions of the large cake. In subsection (2.3) we reverse the order of the cakes letting the players to bargain on the large cake first. In subsection (2.4) we compare the equilibria resulting from these two agendas.

In section 3 we discuss the bargaining game when players have conflicting evaluations regarding the sizes of the cakes. We show that in such multi-issue bargaining game, the players' expected outcome depend on the agenda of the bargaining. Each player prefers an agenda such that the first cake to bargain on is the one which is the least important to him but the most important to his opponent.

2. Identical Preferences: The Order of Cakes

Let $A = \{a_1,\ldots,a_n\}$ be a collection of cakes such that a_i is the size of the i'th cake.

Definition 1: An agenda is a sequence of subsets $\{A_1,\ldots,A_m\}$ such that $A_i \subset A$, $m < n$, $\bigcup_{j=1}^{m} A_j = A$, and for every i,j $A_i \cap A_j = \emptyset$.

Given an agenda $\{A_1,\ldots,A_m\}$ the bargaining is conducted as follows: the players start to negotiate simultaneously on all the cakes included in A_1. Once they reach an agreement on the partitions of all $a_j \in A_1$ they start to negotiate on the partition of the cakes included in A_2. The bargaining terminates when the players reach an agreement on the paritition of all the cakes.

If $A_i = \emptyset$ for every $i > 1$, we will say that the bargaining game has a degenerated agenda so that players negotiate on all the issues simultaneously.

In order to simplify our analysis we assume that the players bargain on the partition of only two cakes. One cake is of the size one the other cake is of the size $a > 1$. The two players have identical evaluations regarding the size of these cakes. The rules of the bargaining game is such that the bargaining is done sequentially. We assume that the cakes are eaten only after the negotiation over the second cake is finished.

Let y be a partition of the large cake such that the first player receives y while the second player gets $a - y$, and let x be a partition of the cake of size one such that the first player gets x and the second player gets $1 - x$.

<u>Assumption 1</u>: Given a partition (y,x) of the two cakes the utilities of the players are

$$u_1(y,x) = (y + x)\delta^t$$

$$u_2(y,x) = (a - y + 1 - x)\delta^t$$

where δ is a constant and identical rate of time preference and t is the period in which the two players agree on the partition of the two cakes. We assume that a is sufficiently large so that $a > 1/\delta$.

Let S_i, $i = 1,2$, be the set of all possible partitions of the i'th cake. Let F be the set of all sequence of functions $f = \{f^t\}_{t=1}^{\infty}$ where $f^1 \in S_1$, for t odd $f^t\colon S_1^{t-1} \to S_1$ and for t even $f^t\colon S_1^t \to \{Y,N\}$ (S_i^t is the set of all sequences of length t of elements in S_i). Let G be the set of all sequences of functions $g = \{g^t\}_{t=1}^{\infty}$ such that for t odd $g^t\colon S_1^t \to \{Y,N\}$ and for t even $g^t\colon S_1^{t-1} \to S_1$.

Let Q be the set of all sequences of functions $q = \{q^t\}_{t=1}^{\infty}$ where $q^1\colon S_1 \to S_2$ for t odd $q^t\colon S_1 \times S_2^{t-1} \to S_2$ and for t even $q^t\colon S_1 \times S_2^t \to \{Y,N\}$. Similarly let M be the set of all sequences of functions $m = \{m^t\}_{t=1}^{\infty}$ such that for t odd m^t: $S_1 \times S_2^t \rightarrow \{Y,N\}$ and for t even m^t: $S_1 \times S_2^{t-1} \rightarrow S_2$.

Definition 2: (F,Q) is the set of all possible strategies for the player who makes the first offer in the bargaining on both cakes. (F,M) is the set of all possible strategies for the player that starts first in the bargaining over the first cake and moves second in the bargaining over the second cake. (G,M) is the set of all possible strategies for the player who moves second in the bargaining over the two cakes and (G,Q) is the set of all possible strategies for the player who moves second in the first bargaining and moves first in the second bargaining.

We will now consider two cases: In the first one the two players bargain on the small cake first (the cake of size one) and when this bargaining is over they start the bargaining on the large cake. In the second case we will reverse the order of the cakes letting the players bargain on the large cake first.

2.2 Bargaining On the Small Cake First

Proposition 1: If the two players have identical evaluations regarding the size of the cakes and they bargain on the small cake first then the following strategies constitute a subgame perfect equilibrium:

(a) For any two partitions $z, \ell \in [0,1]$ the strategy f such that for t odd $f^t = z$ and for t even $f^t = Y$ and the strategy g such that for t even $g^t = \ell$ and for t odd $g^t = Y$ are the equilibrium strategies for the bargaining on the first cake.

(b) In the bargaining on the second cake the equilibrium strategies are as follows: given a partition y of the first cake and let

$$\alpha^* = \frac{1 + a}{1 + \delta} - y$$

$$\beta^* = \frac{\delta(1 + a)}{1 + \delta} - y.$$

The first player offers α^* and accepts any offer that leaves him better off than β^* and the second player offers β^* and accepts any offer that leaves him better off than α^*.

Proof: We will start by proving (b). Since $a > 1/\delta$ it is evident that $0 \leqslant \alpha^*, \beta^* \leqslant a$ for every $0 \leqslant y \leqslant 1$ so the strategies are well defined. Now note that α^* and β^* are the unique solution of the characteristic equations

(1) $$y + \beta^* = \delta(\alpha^* + y)$$

(2) $$(1 - y) + (a - \alpha^*) = \delta(1 - y + a - \beta^*).$$

Thus following Rubinstein (1982) the suggested pair of strategies is the only subgame perfect equilibrium of the bargaining game on the second cake.

Following these equilibrium strategies the partition of the second cake is such that the first player's share from the second cake is

$$\begin{cases} \dfrac{1 + a}{1 + \delta} - y & \text{, if the first player moves first} \\ \dfrac{\delta(1 + a)}{1 + \delta} - y, & \text{if the second player moves first} \end{cases}$$

Adding to this, the first player's share from the first cake, i.e., y, indicates that his total share from the two cakes is:

$\dfrac{\delta(1 + a)}{1 + \delta}$, if the first player moves first in the second bargaining game.

$\frac{\delta^2(1 + a)}{1 + \delta}$, if the first player moves second in the second bargaining game.

In a similar way we can calculate the shares of the second player. Thus, the final allocation, of the two cakes together, does not depend on the outcome of the bargaining on the first cake. Q.E.D.

Remarks: (a) Note that although we have multiplicity of equilibria all the equilibrium strategies yield the same payoffs.

(b) The first part of Proposition 1 implies that the players are indifferent about the outcome of the bargaining game on the first cake. The bargaining is actually postponed to the second period.

(c) The final partition is identical to Rubinstein's result with the exception that players receive their share in the second period rather than in the first one.

(d) The result presented in Proposition 1 can be easily generalized to a situation in which we have a sequence of cakes $\{a_1,\ldots,a_m\}$ as long as for every $m < n$, $\sum_{j=1}^{m-1} a_j < a_m$.

The intuition behind Proposition 1 is that the partition of the first cake affects the players' impatience rate in the bargaining on the second cake. The higher the player's share from the first cake is, the more impatient he becomes. He receives a piece of cake that is placed in front of him but he is not allowed to eat it until the bargaining on the second cake is over. In the bargaining on the second cake, each player takes advantage of the impatience of his opponent. What the above proposition indicates is that any additional piece from the first cake is exactly offset by the higher impatience it will imply in the bargaining on the second cake.

2.3 Bargaining On the Large Cake First

Assume now that the first cake is of size $a > 1/\delta > 1$ while the second cake is of size 1. Let $\underline{b} = \text{Min}\{\frac{\delta(1 + a)}{1 + \delta}; \frac{a - \delta}{1 + \delta}\}$ and $\bar{b} = \text{Max}\{\frac{\delta(1 + a)}{1 + \delta}; \frac{a - \delta}{1 + \delta}\}$.

Proposition 2: Let $y \in [0,a]$ be a partition of the first cake and let

$$(3) \quad x^* = \begin{cases} 1 & 0 \leqslant y \leqslant \underline{b} \\ \frac{1 + a}{1 + \delta} - y & \underline{b} < y \leqslant \bar{b} \\ (1 - \delta)[1 + a - y] & \bar{b} < y \leqslant a \end{cases}$$

$$(4) \quad z^* = \begin{cases} \delta - (1 - \delta)y & 0 \leqslant y \leqslant \underline{b} \\ \frac{\delta(1 + a)}{1 + \delta} - y & \underline{b} < y \leqslant \bar{b} \\ 0 & \bar{b} < y \leqslant a \end{cases}$$

The unique subgame perfect equilibrium of the bargaining on the second cake is such that the first (second) player offers $x^*(z^*)$ and accepts any offer that leaves him better off than $z^*(x^*)$.

Proof: We divide the proof into three parts:

(i) $0 \leqslant y \leqslant \underline{b}$: The suggested strategies is the unique subgame perfect equilibrium of the bargaining game on the second cake since the first player is indifferent between getting z^* today and getting x^* tomorrow and x^* is the maximum that the first player can demand such that the second player prefers to get $1 - x^*$ today than to wait another period and get $1 - z^*$, i.e., the suggested strategies satisfy the following conditions:

(5) $$x^* = \text{Max}\{x \mid a - y + 1 - x \geqslant \delta(a - y + 1 - z^*)\}$$

(6) $$y + z^* = \delta(y + x^*).$$

(ii) $\underline{b} < y \leqslant \bar{b}$: The suggested strategies for this region are the unique solution of the characteristic equations (6) and

(7) $$a - y + 1 - x^* = \delta(a - y + 1 - z^*).$$

(iii) $\bar{b} < y \leqslant 1$: The suggested strategies satisfy the following conditions:

(8) $$z^* = \text{Min}\{z \mid y + z \geqslant \delta(y + x^*)\}$$

(9) $$a - y + 1 - x^* = \delta(a - y + 1 - z^*).$$ Q.E.D.

Note that as long as $\underline{b} \leqslant y \leqslant \bar{b}$ the final shares of the two cakes together are not affected by y and the situation is identical to the one described in Proposition 1. But when $0 \leqslant y < \underline{b}$, the second player gets a big piece from the first cake. Thus, in the bargaining on the second cake he becomes very impatient so that he is willing to get nothing from the second cake than to wait another period and to get something from this cake. A similar situation occurs when $\bar{b} < y \leqslant a$. In this case the first player gets most of the first cake so he is impatient enough to accept the offer of the second player and to get nothing from the second cake than to wait another period.

We are now able to discuss the bargaining on the first cake. Each player in this bargaining game is aware of the way the partition of the first cake determines the equilibrium of the bargaining game on the second cake. We assume that the first mover is selected randomly by a (1/2, 1/2) lottery. Letting $R_i(y)$ be the i'th player's expected share from the two cakes together as a function of the partition of the first cake yields that

$$(10) \qquad R_1(y) = \begin{cases} \dfrac{\delta(1+\delta)(1+y)}{2} & 0 \leqslant y \leqslant \underline{b} \\ \dfrac{\delta(1+a)}{2} & \underline{b} < y \leqslant \bar{b} \\ \frac{1}{2}\delta[(1+a)(1-\delta) + (1+\delta)y] & \bar{b} < y \leqslant a \end{cases}$$

$$(11) \qquad R_2(y) = \begin{cases} \dfrac{\delta[2a + (1-\delta) - (1+\delta)y]}{2} & 0 \leqslant y \leqslant \underline{b} \\ \dfrac{\delta(1+a)}{2} & \underline{b} < y \leqslant \bar{b} \\ \dfrac{\delta[(1+\delta)(1+a) - (1+\delta)y]}{2} & \bar{b} < y \leqslant a \end{cases}$$

<u>Proposition 3:</u> Let

$$\alpha^* = \frac{(1+\delta^2)(1+a)}{(1+\delta)^2}$$

$$\beta^* = \frac{2\delta a - \delta^2 - 1}{(1+\delta)^2}$$

The unique subgame perfect equilibrium of the bargaining on the first cake is such that the first player offers α^* but he is accepting any offer that gives him more than β^* while the second player offers the partition β^* but he is accepting any offer that gives him more than $a - \alpha^*$.

<u>Proof:</u> Note that $\bar{b} < \alpha^* < a$ and $0 < \beta^* < \underline{b}$. Given the payoff function $R_i(y)$

we can find the total expected utility from the two cakes associated with each proposed offer. Straightforward calculation indicates that (α^*,β^*) is the unique solution of the following equations:

$$\frac{1}{2}\delta[(1+\delta)(1+\beta^*)] = \frac{\delta^2}{2}[(1+a)(1-\delta)+(1+\delta)\alpha^*] \tag{12}$$

$$\frac{1}{2}\delta[(1+\delta)(1+a)-(1+\delta)\alpha^*] = \frac{\delta^2}{2}[2a+(1-\delta)-(1+\delta)\beta^*]. \tag{13}$$

Thus the suggested strategies satisfy the characteristic equations and therefore they constitute a subgame perfect equilibrium.

Careful examination of (10) and (11) indicates that if we choose an (α,β) such that $\alpha \notin [\bar{b},a]$ or $\beta \notin [a,\underline{b}]$, then this (α,β) will not be the solution of

$$\alpha = \mathrm{Max}\{z \mid R_2(z) \geqslant \delta R_2(\beta)\}$$

$$\beta = \mathrm{Min}\{x \mid R_1(x) \geqslant \delta R_1(\alpha)\}$$

Thus the strategies defined in Proposition 3 is the unique subgame perfect equilibrium in the bargaining on the first cake. Q.E.D.

2.4 Comparison of the Two Agendas

We discuss above two possible agendas. In the first one the players start by bargaining on the small cake and in the second case they start by bargaining on the large cake. A possible way to describe the above bargaining game is by presenting it as a lottery. Each player engages in a two step lottery. In the first step it is determined who is the first mover in the bargaining on the first cake and in the second lottery the first mover in the bargaining on the second cake is selected. Once the first movers are selected the equilibrium strategies and the equilibrium payoffs can be easily calculated using Propositions (1)-(3). In the following two figures we describe the first player's equilibrium payoffs in the four possible events. $A_1(B_1)$ is his payoff when he is the first mover in the bargaining on both cakes. $A_2(B_2)$ is his payoff when he is the first mover in the bargaining on the first cake and the second mover in the bargaining on the second cake. $A_3(B_3)$ and $A_4(B_4)$ are defined similarly.

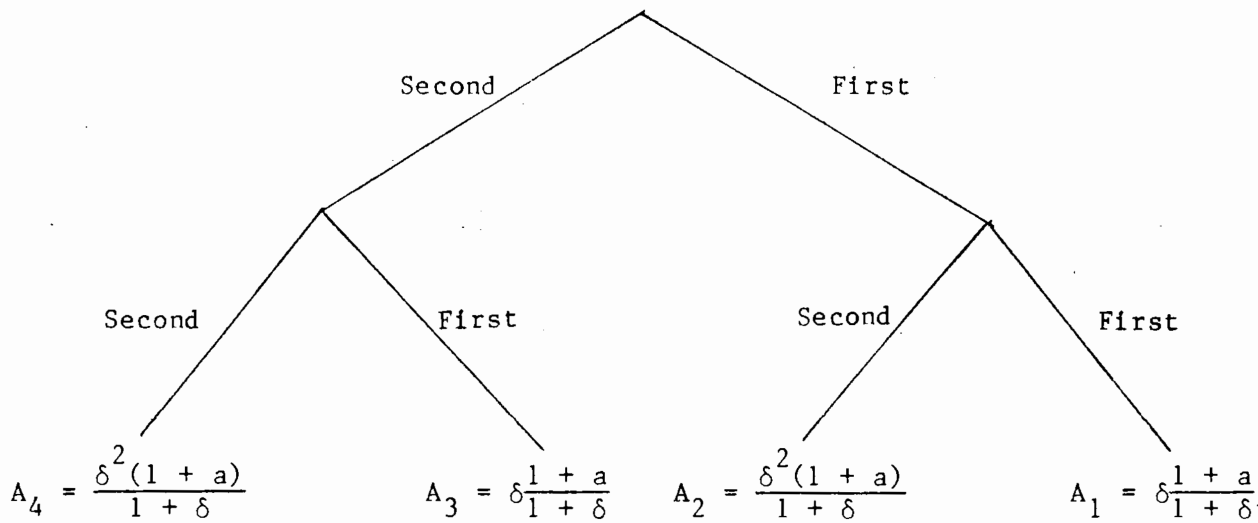

Figure 1: The first player's payoffs from the two cakes when the bargaining on the small cake is done first.

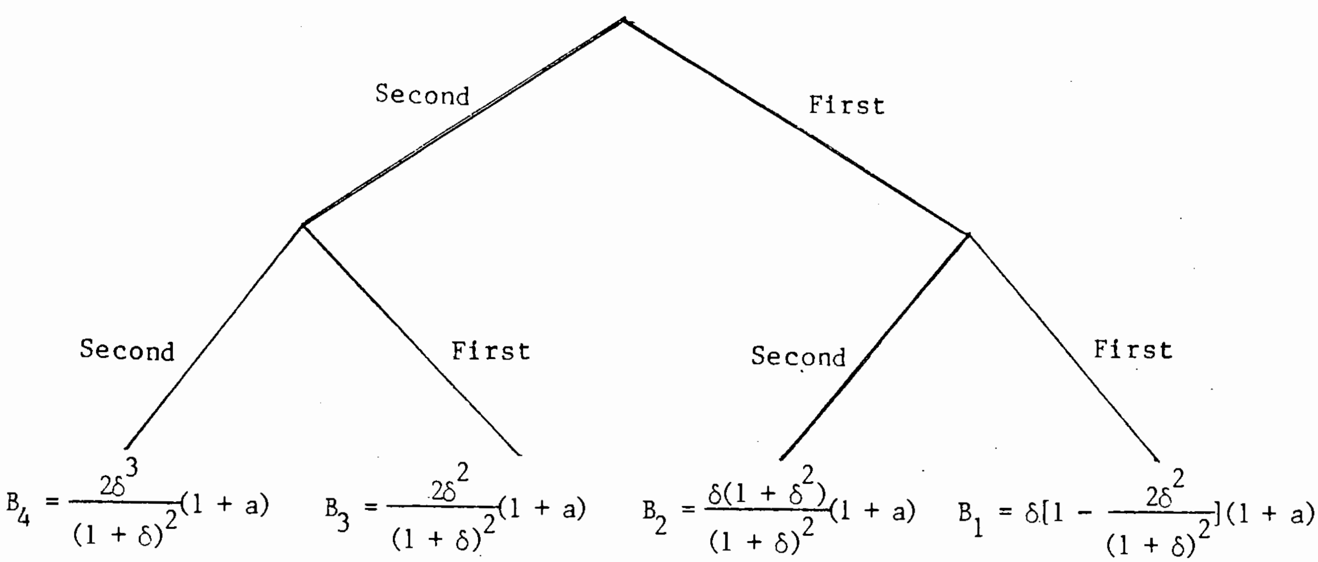

Figure 2: The first player's payoffs from the two cakes when the bargaining on the large cake is done first.

Comparing the two figures implies the following:

Corollary 1: (i) When the players bargain on the small cake first the important lottery is the one that determines who is the first mover in the bargaining of the second cake. Since $A_1 = A_3$ and $A_2 = A_4$ the first lottery does not affect the final payoffs. When they bargain on the large cake first it is the first lottery which is the more important one. Simple calculation implies that $B_1 > B_2 > B_3 > B_4$, thus if the first player is the second to move in the first bargaining game he will be worse off regardless of what the outcome in the second lottery is.

(ii) Comparing the payoff distributions for the two possible agendas yields that since $B_1 > A_1$ and $B_4 < A_4$ the highest payoffs that the second agenda might yield is higher than the best that the first agenda might yield. However, at the same time if a player is the second mover in the bargaining on the two cakes, he will be better off with the agenda in which the smaller cake is discussed first.

(iii) Notice that $\frac{1}{4}\sum_{j=1}^{4} A_j = \frac{1}{4}\sum_{j=1}^{4} B_j = \frac{\delta(1 + a)}{2}$. Thus the agenda in this case does not affect the players' expected share. Since the player that makes the first offer is randomly selected, the two players are completely symmetric and the expected share of each player is exactly one-half of the total amount being divided.

3. Opposite Evaluations and the Importance of the Agenda

In many bargaining situations players do not assign the same importance to all the issues under negotiation. Players might also have conflicting evaluations regarding the importance of different issues. Some issues might be important to one group of players while other players might consider the other issues as the real important issues in the negotiation. In such situations, the bargaining game becomes more interesting since players have the option to give up on the issues which are less important to them and, in return, to get a favorable settlement on the issues which are important to them.

In this section we consider a two players two cakes bargaining game. We assume that each player evaluate the size of these cakes differently. The first player regards the first cake as the important one while the second player regards the second cake as the important one. We assume that the different evaluations are common knowledge.

Let (y,x) be a partition of the two cakes such that the first player receives y percentage of the first cake and x percentage of the second cake while the second player receives (1 - y) percentage from the first cake and (1 - x) percentage from the second one. Given (y,x) we assume that the utility functions of the two players are

$$u_1(y,x) = (ay + x)\delta^t$$

$$u_2(y,x) = (1 - y + b(1 - x))\delta^t$$

where $a,b > 1$, δ is a constant and identical time preference factor and t is the period in which the bargaining process is finished.

For simplicity we assume that a = b. Our main concern in this paper is the importance of the agenda and the way it affects the players' expected payoffs. In discussing these issues the above is not a restrictive assumption. Like in the previous section we assume that a is sufficiently large so that $a > 1/\delta$.

In Figure 3 we described the bargaining set which is the set of all possible utilities derived from all possible partitions of the two cakes. The Nash bargaining solution (Nash, 1950), is the point (a,a) such that the first player gets the first cake, while the second player gets the second cake.

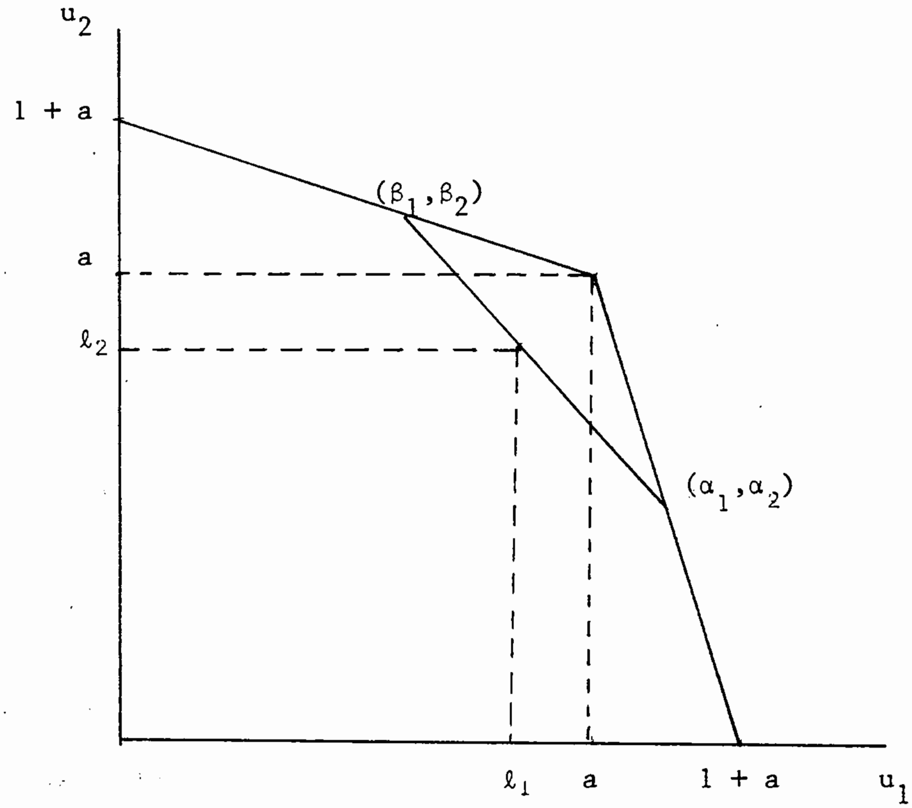

Figure 3

Now let us consider the strategic approach and let assume that the two players bargain over the two cakes simultaneously. An offer in this case is (y,x) which represents a partition of the two cakes.

Lemma 1: Let

$$(14) \qquad (y_1^*, x_1^*) = \left(1, \frac{a(1 - \delta)}{a + \delta}\right)$$

$$(15) \qquad (y_2^*, x_2^*) = \left(\frac{\delta(1 + a)}{a + \delta}, 0\right)$$

If the players bargain over the two cakes simultaneously, the unique subgame perfect equilibrium is for the first player to offer the partition (y_1^*,x_1^*) and to accept any offer that leaves him better off than (y_2^*,x_2^*) and for the second player to offer the partition (y_2^*,x_2^*) and to accept any offer that leaves him better off than (y_1^*,x_1^*).

Proof: Straightforward from applying Rubinstein's (1982) technique.

From the above lemma we conclude that if the first player moves first the equilibrium partition will be (y_1^*,x_1^*). The resultant utility of the first player is $a + x_1^*$ and the utility of the second player is $a(1 - x_1^*)$. This combination of utilities is presented in Figure 3 by the point (α_1,α_2). Similarly, if the second player moves first the resultant utilities are (β_1,β_2).

Proposition 4: When the players bargain simultaneously on the two cakes their expected utility is less than what they get in the Nash bargaining solution.

Proof: Calculating $(1/2)(\alpha_i + \beta_i)$ implies that

$$Eu_i = \frac{a(1 + a)(1 + \delta)}{2(a + \delta)} < a, \ i = 1,2 \qquad \text{Q.E.D.}$$

The players' expected utility is depicted in Figure 3 as (ℓ_1^*,ℓ_2^*). Note however that although (ℓ_1^*,ℓ_2^*) is not on the Pareto frontier the utilities that the two players will realize, i.e., (α_1,α_2) or (β_1,β_2), are on the Pareto frontier.

Now let us assume that the players bargain on the two cakes sequentially according to a given agenda. Furthermore, let us assume that the agenda is such that they first bargain on the first cake, i.e., the one which is more important to the first player, when this bargaining is over they bargain on the partition of the second cake.

Proposition 5: Let

$$(16) \quad q^* = \frac{2a^2\delta - 1 + \delta - a + 2a\delta - \delta^2 a}{2a^2 - 1 + \delta}$$

$$(17) \quad x^*(y) = \begin{cases} \dfrac{1 + a - (a^2\delta + 1)y}{a(1 + \delta)} & y < \dfrac{\delta(1 + a)}{a^2 + \delta} \\[2ex] \dfrac{(1 - \delta)(1 - y + a)}{a} & y \geqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

$$(18) \quad z^*(y) = \begin{cases} \dfrac{\delta(1 + a) - (a^2 + \delta)y}{a(1 + \delta)} & y < \dfrac{\delta(1 + a)}{a^2 + \delta} \\[2ex] 0 & y \geqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

The unique subgame perfect equilibrium of the above two cakes bargaining game is such that in the bargaining on the first cake the first player offers that he will get the whole cake but he accepts any offer that gives him more than q^* percentage of this cake. Giving a partition y of the first cake the first player offers the partition $x^*(y)$ of the second cake but he accepts any offer that gives him more than $z^*(y)$. The equilibrium strategy of the second player is such that he offers the partition q^* of the first cake but he accepts any offer. Given an allocation y of the first cake the second player offers the partition $z^*(y)$ of the second cake but accepts any offer that leaves him better off than $x^*(y)$.

Proof: Let discuss first the bargaining on the second cake. Let y be a partition of the first cake. For $y < \frac{\delta(1 + a)}{a^2 + \delta}$, the suggested strategies satisfy the characteristic equations

$$(19) \qquad z^* + ay = \delta(ay + x^*)$$

$$(20) \qquad a(1 - x^*) + (1 - y) = \delta[(1 - y) + a(1 - z^*)].$$

For $1 \geqslant y \geqslant \frac{\delta(1 + a)}{a^2 + \delta}$,

$$(21) \qquad z^*(y) = \mathrm{Min}\{z \mid z + ay \geqslant \delta(ay + x^*)\}$$

$$(22) \qquad x^*(y) = \mathrm{Max}\{x \mid a(1 - x) + (1 - y) \geqslant \delta[(1 - y) + a(1 - z^*)]\}.$$

Thus for every given $y \in [0,1]$ the suggested strategies is the unique subgame perfect equilibrium of the bargaining on the second cake.

Using the above equilibrium strategies we can calculate the partition of the second cake when the first player makes the first offer and when the second player makes the first offer. Assuming that the first mover is selected randomly by a (1/2, 1/2) lottery and let $P_i(y)$ be the i'th player's utility from the second cake as a function of the partition of the first cake yields

$$(23) \quad P_1(y) = \begin{cases} \dfrac{(1 + a) - (1 + a^2)y}{2a} & y \leqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \\[2ex] \dfrac{(1 - \delta)(1 + a) - (1 - \delta)y}{2a} & y > \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

$$(24)\quad P_2(y) = \begin{cases} \frac{1}{2}[(a - 1) + (a^2 + 1)y] & y \leqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \\ \frac{1}{2}[a(1 + \delta) - (1 - \delta) + (1 - \delta)y] & y > \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

We can now turn to the analysis of the bargaining on the first cake. Our first step is to find the total expected payoffs, from the two cakes together as a function of the partition of the first cake. We denote these expected payoffs as $R_i(y)$. Once these payoffs are defined we can use the standard strategic approach to find the equilibrium. From the definition of $P_i(y)$ it is evident that

$$(25)\quad R_1(y) = \delta[P_1(y) + ay] = \begin{cases} \dfrac{\delta(1 + a)}{2a} + \dfrac{\delta(a^2 - 1)}{2a}\, y & y \leqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \\ \dfrac{\delta(1 - \delta)(1 + a)}{2a} + \dfrac{\delta(2a^2 - 1 + \delta)}{2a}\, y & y > \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

$$(26)\quad R_2(y) = \delta[P_2(y) + 1 - y] = \begin{cases} \frac{1}{2}\delta[a + 1 + (a^2 - 1)y] & y \leqslant \dfrac{\delta(1 + a)}{a^2 + \delta} \\ \frac{1}{2}\delta[(1 + \delta)(1 + a) - (1 + \delta)y] & y > \dfrac{\delta(1 + a)}{a^2 + \delta} \end{cases}$$

Notice that for $0 \leqslant y \leqslant \dfrac{\delta(1 + a)}{a^2 + \delta}$ the payoffs of both players are increasing functions of y. Thus, in discussing the bargaining game on the first cake, giving an offer in this range cannot be a part of an equilibrium behavior. The player that gives such an offer can benefit from offering an higher y. His utility will be higher and the other player will be more likely to accept the new offer since he will also benefit from the new offer. Thus in the bargaining on the first cake both players offer partitions in $[\frac{\delta(1+a)}{a^2+\delta}, 1]$.

Now observe that the suggested strategies is the unique subgame perfect equilibrium since they are the unique solution of

$$(27)\quad \mathrm{Min}\{q | \frac{(1-\delta)(1+a)}{2a} + \frac{2a^2-1+\delta}{2a}q \geqslant \delta[\frac{(1-\delta)(1+a)}{2a} + \frac{2a^2-1+\delta}{2a}]\} = q^*$$

$$(28)\quad \mathrm{Max}\{y | \frac{1}{2}[a(1+\delta) - (1-\delta) + (1-\delta)y] \geqslant \frac{\delta}{2}[a(1+\delta) - (1-\delta) + (1-\delta)q^*]\} = 1$$

Q.E.D.

Note that when the first player moves first in the bargaining on the first cake and the second player moves first in the bargaining on the second cake the equilibrium partition is identical to the Nash solution, the first player gets all the first cake while the second player gets all the second cake.

The above proposition leads us to the main result of this section:

Proposition 6: In a multi-issue bargaining game, the players' expected utilities depend on the agenda of the bargaining. In the two cakes bargaining game discussed here each player prefers an agenda such that the first cake to bargain on is the one which is the least important to him but the most important to his opponent.

Proof: We will show that for the agenda discussed in this section, the expected utility of the second player is higher than the expected utility of the first player.

Given the equilibrium strategies specified in Proposition 5 and assuming that the player who makes the first offer is selected randomly, the expected utility of the second player is

$$(29) \qquad Eu_2 = \frac{\delta a(1+\delta)}{4}\left[\frac{4a^2 - 1 + 2a - 2a\delta + \delta^2}{2a^2 - 1 + \delta}\right]$$

while the expected utility of the first player is

$$(30) \qquad Eu_1 = \frac{1}{4}\,\delta(2a + 1 - \delta)(1 + \delta)$$

Comparing (29) with (30) yields that $Eu_2 > Eu_1$. Q.E.D.

Notice that when the players have identical evaluations regarding the size of the cakes, the random selection of the first mover implies that players are symmetric and thus they have identical expected utility. But when players have conflicting evaluations, the random selection of the first mover does not imply symmetry. The source of such asymmetry is the assumed agenda which determines the sequence of cakes over which the players bargain. If to the above structure we will add the assumption that the agenda is selected randomly, it will imply that players are symmetric and thus obtain identical expected utility from the bargaining game.

The driving force behind the results of this paper is that players are impatient so that $\delta < 1$. When this impatience disappers and $\delta \to 1$, the agenda ceases to be important. For $\delta = 1$ the players' expected payoffs in the bargaining game with an agenda as well as their payoffs in the simultaneous bargaining game are equal to (a,a) which is the Nash bargaining equilibrium.

Concluding Remarks

By establishing the importance of the agenda in a multi-issue bargaining game and by proving that the players' expected utility depend on the agenda of the bargaining, this paper provides a theoretical explanation of the phenomenon of "bargaining on the agenda."

The focus in this paper is on the importance of the agenda in a bargaining game with complete information. If there is, however, some incomplete information regarding the types of players involved, the agenda can play an additional role in the game. Players can set up the agenda strategically so that in bargaining on one cake they gain some information regarding the other players' type. This information, of course, can be useful when the players continue the game and bargain on the partition of the other cakes. This issue, however, is beyond the scope of this paper, but it only emphasizes the various strategic roles the agenda can play in bargaining games.

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