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MEASURES FOR THE PSYCHOLOGICAL RISK OF AN ACCIDENT

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Abstract

Formal models of risk and danger behaviour are stated. The models yield numerical measures for these two concepts, and are especially relevant to accident situations since they deal with an individual trying to avoid an event of low probability. Even though the measures turn out to be almost identical, the rationales behind the two models show that risk and danger are conceptually distinct. The formulae given here are reasonable ways to combine probability and utility to a single measure of risk.

Constructing a risk measure from separately assessed probability and utility scales is preferable to asking subjects for risk directly because of the inherent vagueness of words like "risk" and "danger."

1. Introduction

The concepts of risk and danger have appeared frequently in the literature of safety psychology. Risk and danger are regarded as causing accidents in two ways: first, accidents occur when an individual chooses a course of action involving too much risk, and second, performance skills can deteriorate under the stress of danger, so that the probability of an accident increases. Whichever mechanism is at work, an understanding of risk and danger seems to be important to designing safer systems.

In most of the literature, "risk" and "danger" have been used vaguely or ambiguously. The first aim of this paper is to give these words precise meanings in the context of safety research.

The second aim is to show that there are two distinct concepts that have generally not been separated in the safety literature. The measures for these two concepts are very similar, but the rationales behind these measures are quite different, and involve two different patterns of human behaviours. It is proposed here that one be called "risk" and the other "danger".

Two simplified theories of human behaviour will be described and "risk" and "danger" will be defined within these theories, which will lead naturally to numerical measures for each term. The theories do not relate the concepts to other variables such as alcohol consumption or decrement of skill, but focus entirely on the patterns of risk and danger behaviours themselves in order to show how they differ and how these concepts can be measured.

The first theory is a model of risk and is an extension of the microeconomic work of Arrow (1971) and Pratt (1964).[1] It regards human decision-making as partly rational and partly irrational, the irrational part being identified in this context with excessive risk-seeking or excessive risk-avoidance.

This leads to a method of measuring the objective aspects of the threatening situation that determine the individual's degree of irrationality, and this objective measure will be labeled risk. The theory also leads to a measure of the individual's disposition to respond to risk, or risk propensity. In Section 3 this theory is developed and is shown to lead to a definition of risk as the sum of squares of the accident disutilities weighted by their probabilities.

The second theory, related to the work of Pollatsek and Tversky (1970), describes an individual's reaction to danger, typically reactions that are not part of decision-making. It assumes that the individual produces some response to threatening situations but the exact form of this response is left unspecified. It may be for example a verbal estimate of fear, or a physiological change, or a decrement in performance. In line with a common psychological procedure, the exact numerical value of this response is ignored and only the comparative feature is used, i.e., is the response to one situation greater than, equal to, or less than the response to another? If the comparisons among a large number of threatening situations show a certain pattern, it is possible to assign numbers to the situations representing their dangers, such that the numbers determine the comparative responses. In Section 4 it is shown that the simplest measure for danger is the sum of accident utilities weighted by their probabilities. Comparison with past research and discussion are given in Section 5, and practical methods of assessing risk are discussed in Section 6.

2. Theories of Risk versus Theories of Danger

As used here *danger* is a general term referring to the amount of threat in a situation. *Risk* is a special type of danger, appropriate to use when a decision is being made whether or not to enter the situation. The decision-maker measures the amount of risk and decides what to do. Sometimes the risk measurement is not conscious, but the researcher observes the decision-maker's pattern of choices and assigns a degree of risk for situations *post facto*. The justification for doing this is that the individual is acting *as if* these risks were being assigned.

Formulae for the measurement of risk go back two hundred years or more. Tetens (1786) seems to have been the first to put one on record. Investigating the subject of annuities, he suggested calculating one-half the mean deviation of the distribution of money. Generally the formulae have involved a combination of the probability and the severity of loss. Some more recent proposals are mentioned in Section 5.

Having stated a formula one must stipulate how the risk-measure combines with other features of the courses of action to determine the best choice. Generally there have been four categories:

1) *Maximal* risk theories in which the individual tries to maximize the risk of the action (the other salient parameters being equal).

2) *Minimal* risk theories in which the individual tries to minimize the risk. (The theory offered here can be either a minimal or maximal risk theory depending on the behaviour exhibited by the decision-maker.)

3) *Optimal* risk theories in which the individual chooses some intermediate optimal level of risk (for example the portfolio theory of Coombs and Huang, 1970).

4) Tolerable risk theories in which the individual is indifferent as long as risk is at an acceptable level, but begins to avoid risk once it exceeds that level. (Wilde, 1975; O'Neill, 1978)

A theory of risk is a specification of both of the above: how the individual measures the risk and how this measure is used in making a decision.

A theory of danger concerns situations in which the individual is responding to some threatening aspect of the environment. Some measure combining the probability and severity of the loss facing the person determines the response, but the individual is not necessarily using the measure for the purpose of making a decision. Thus danger is a more general concept than risk.

A number of examples of each type can be found in the safety literature. The research of Cohen, Dearneley and Hansel (1956) would be classified as a study of risk. London bus drivers were asked to state their probabilities of successfully driving through a gap slightly larger than the width of their bus and the estimates were compared with their success rates and experience. This research would be relevant to a theory of risk since its ultimate aim is to determine the factors influencing people's willingness to choose threatening courses of action, and thus their decision-making.

A further example of research on risk is given by Hurst (1976) who suggests that variability in risk-taking is a more significant cause of accidents than is the average level of risk-taking. Hurst's theory involves risk since he is investigating the individual's choice behaviour.

Economic models of workers' decisions to leave a job, as a function of health hazards (Viscusi, 1979) can be termed theories of risk, since they generally portray the workers as weighing the benefits of staying or leaving.

On the other hand, an example of danger research is given by Stikar, Hoskovic, and Biehl (1971). A fifth wheel was added to a car so that the experimenter controlling this wheel from the passenger's seat, could suddenly induce a skid. The pulse rates of experienced and inexperienced drivers during the skid were compared. This is clearly a theory of danger since it investigates a subject's response to a present situation.

Some other physiological responses that have been studied as functions of danger are breadth of attention (Gershon, Weltman, and Egstrom, 1966) and galvanic skin response (Taylor, 1964).

Many of the authors' terminologies differ from the usage advocated here, but the content and structure of their theories allows us to classify them either as risk or danger theories.

The two theories that follow give an example of each type.

3. A Theory of Risk

Risk can be regarded as a function of both the probability and the severity of an accident. Suppose an accident will occur with probability p and will have (negative) utility u if it occurs. (Both these quantities are to be interpreted as objective, in the sense that p is a realistic estimate of the likelihood of the accident, and u is the harm that would really befall the person in the accident. The individual's beliefs about p and u may be different from their true values.)

An individual facing such an accident must decide the maximum effort he is willing to expend to avoid the possibility of the accident. This effort will be a function of his beliefs about p and u. Each of these beliefs is assumed to be a specific function of p and u, at least for some class of accidents broad enough to be worth studying, and thus the effort can be denoted e(p,u). It may be realized in terms of money, time, attention to the task, or some other valued commodity.

If the individual were accurate in his judgment of p and u and were a rational decision-maker, his maximum effort would be $e(p,u) = -pu$. However, he is not rational -- there is distortion in his judgment. First, his subjective probability of the accident may differ from p by being too optimistic or pessimistic. (For research on the causes and nature of probability bias, the reader may consult the review by Slovic, Fischoff, and Lichtenstein (1977).)

A second deviation from rationality might be a subjective utility differing from u. The individual may imagine the accident to be better or worse than it really would be. Third, to decide on the value of maximum effort, he may combine probabilities and utilities in some way other than multiplying, contrary to the principles of decision analysis. In summary, e(p,u) is some function of an as yet undetermined form.

A series of assumptions will now be made about the properties of e(p,u). First of all, distortions in the judgment of p and u are caused by the threat of the accident. Thus, when there is very little threat, optimistic biases and defense mechanisms are reduced, and e(p,u) tends to assume the standard form $e(p,u) = -pu$:

$$\frac{e(p,u)}{-pu} \rightarrow 1 \quad \text{as } u \rightarrow 0 \tag{1}$$

Also, it is assumed that if an event is certain not to occur, the individual recognizes this and will expend no effort to avoid the accident. That is, although his judgment may sometimes be distorted, he does not go so far as to invent non-existent dangers:

$$e(0,u) = 0 \tag{2}$$

A standard technique in calculus is the approximation of a general function by a polynomial. It is assumed that e(p,u) changes in a "non-abrupt" way (a rigorous statement of this condition is given in most books of advanced calculus), and thus can be approximated by a Taylor series about the values p = u = 0. In this case a series of the third degree will be chosen:

$$e(p,u) \cong e_0 + \left.\frac{\partial e}{\partial p}\right|_0 p + \left.\frac{\partial e}{\partial u}\right|_0 u + \left.\frac{\partial^2 e}{\partial p^2}\right|_0 \frac{p^2}{2} + \left.\frac{\partial^2 e}{\partial p \partial u}\right|_0 \frac{pu}{2} + \left.\frac{\partial^2 e}{\partial u^2}\right|_0 \frac{u^2}{2}$$

$$+ \left.\frac{\partial^3 e}{\partial p^3}\right|_0 \frac{p^3}{6} + \left.\frac{\partial^3 e}{\partial p^2 \partial u}\right|_0 \frac{p^2 u}{6} + \left.\frac{\partial^3 e}{\partial p \partial u^2}\right|_0 \frac{pu^2}{6} + \left.\frac{\partial^3 e}{\partial u^3}\right|_0 \frac{u^3}{6}. \quad (3)$$

The zero subscripts on the right-hand side mean that the expressions are to be evaluated at p = u = 0, and thus are fixed numbers, not functions of p and u.

It follows from (1) that e(p,0) = 0. Therefore, since e is a constant function of p at u = 0,

$$e_0 = \left.\frac{\partial e}{\partial p}\right|_0 = \left.\frac{\partial^2 e}{\partial p^2}\right|_0 = \left.\frac{\partial^3 e}{\partial p^3}\right|_0 = 0. \quad (4)$$

According to (2), e(0,u) = 0. Therefore,

$$e_0 = \left.\frac{\partial e}{\partial u}\right|_0 = \left.\frac{\partial^2 e}{\partial u^2}\right|_0 = \left.\frac{\partial^3 e}{\partial u^3}\right|_0 = 0. \quad (5)$$

Substituting (4) and (5) in (3) gives

$$e(p,u) \cong \left.\frac{\partial^2 e}{\partial p \partial u}\right|_0 \frac{pu}{2} + \left.\frac{\partial^3 e}{\partial p^2 \partial u}\right|_0 \frac{p^2 u}{6} + \left.\frac{\partial^3 e}{\partial p \partial u^2}\right|_0 \frac{pu^2}{6}. \quad (6)$$

Substituting (6) in (1) it follows that for all p

$$- \left.\frac{\partial^2 e}{\partial p \partial u}\right|_0 \frac{1}{2} - \left.\frac{\partial^3 e}{\partial p^2 \partial u}\right|_0 \frac{p}{6} - \left.\frac{\partial^3 e}{\partial p \partial u^2}\right|_0 \frac{u}{6} \rightarrow 1 \quad \text{as} \quad u \rightarrow 0. \quad (7)$$

Since (7) is true at u = 0 for all p, the coefficient of p in (7) must be zero and the constant term, the leftmost term in (7), must be 1:

$$\left.\frac{\partial^3 e}{\partial p^2 \partial u}\right|_0 = 0 \qquad \text{and} \qquad \left.\frac{\partial^2 e}{\partial p \partial u}\right|_0 \frac{1}{2} = -1. \tag{8}$$

Substituting (8) into (6)

$$e(p,u) \cong -pu - Kpu^2, \quad \text{where} \quad K = \left.\frac{\partial^3 e}{\partial p \partial u^2}\right|_0 \frac{1}{6}. \tag{9}$$

The meaning of (9) can be stated verbally. The effort an individual is willing to expend can be approximated by the sum of two terms. The first term -pu is the amount of effort a rational person would expend. The quantity -pu is positive since u is negative. The second term, $-Kpu^2$, reflects the individual's deviation from rationality. If it is negative, the individual is prone to risky behaviour and sometimes takes chances not justified by the objective facts. If it is positive the individual is excessively cautious.

Each of the factors of the second term of (9) can be given an interpretation. The factor K depends on the individual's effort function e, but not on the accident's values of p or u. Thus it is a property of the individual's personality independent of the accident he happens to be facing at the time. Since pu^2 is always positive, then whether the individual is overly risky, perfectly rational or overly cautious depends on whether K is positive, zero, or negative. Thus K will be called the individual's <u>risk propensity</u>.

The factor pu^2 behaves in just the opposite way. It is dependent only on the accident and not on the individual, and is thus the objective stimulus to which the individual is reacting when he displays risky or cautious behaviour. So a sensible proposal is that the risk of the accident be defined:

$$r(p,u) = pu^2. \tag{10}$$

Risk is here a property of situations rather than a property of the people in the situations. Note that pu^2 increases as p increases and as u becomes more strongly negative as would be expected of a risk measure.

The individual may be regarded as facing a loss of zero with probability $1 - p$ and loss u with probability p. The variance of his prospects is $p(1 - p)u^2$, so that if p is very small the risk measure $r(p,u)$ is approximately equal to the variance.

Whenever there is expected loss (i.e., whenever $pu < 0$) there is risk, but the above theory implies that risk is not to be measured in the same way as expected loss. Risk-seeking or avoidance is here irrational by definition, and there may be two accidents with equal expected loss to which an individual shows different degrees of deviation from rationality, so that the two accidents will have different degrees of risk. But accidents with equal risk as measured by (10) induce equal deviations from rationality by an individual as measured by the extra effort given to avoiding them. This follows from formula (9) and is the motivation for defining risk according to this particular measure.

Formula (10) refers to a situation in which there is only one possible accident. To generalize the theory, suppose there are n threats, denoted $t_1, t_2, \ldots, t_n$. (A threat is a specific accident along with its probability and utility.) Threat t_i has probability p_i of occurring, and its specific accident has utility u_i. A threat situation is a set of threats faced simultaneously by an individual. A typical threat situation will be denoted $A = \{t_1, t_2, \ldots, t_n\}$. All t_i are assumed to be mutually exclusive, since each describes an accident completely, and it is assumed that $u_i \leq 0$ for all i.

The assumption that all accidents in a threat situation are mutually exclusive would be satisfied most naturally if the accidents' descriptions were extremely detailed so that the occurrence of one would rule out the others. We can then follow a line of reasoning similar to the derivation of formula (9) to produce

$$r(A) = \sum_{i=1}^{n} p_i u_i^2 , \qquad (11)$$

a general formula for risk, of which (10) is a special case.

4. A Theory of Danger

The second measure also deals with an individual in a situation where he faces a set of possible accidents. He compares two threat situations on their relative danger, in some unspecified way indicating which is the greater or whether the two are equal. This might be done by some verbal statement, by a physiological or behavioral response, and a measure of danger arises from the pattern of these comparisons among a large group of threat situations.

A series of axioms will be stated that restricts the possible comparison patterns. If an investigator finds that an individual's behaviour is consistent with the axioms it will be possible to assign a number to each situation that represents its relative danger, in the sense that situations with higher numbers are judged to be more dangerous. (This general approach to measurement is discussed in an introductory way by Coombs, Dawes and Tversky (1971) and in detail by Krantz, et al. (1971).)

Threats will again be denoted $t_1, t_2, \ldots t_n$. Threat t_i has probability of accident p_i and utility of accident u_i. It is assumed that threats of all probabilities and losses as well as various threats with the same probability and loss are available for comparison by the individual. Even though a threat may not currently exist the individual is assumed to be willing to answer the investigator's hypothetical questions.

A <u>threat situation</u> is a set of mutually exclusive threats. An individual who is simultaneously facing t_1, t_5, t_6, for example, is said to be in the threat situation $A = \{t_1, t_5, t_6\}$. The <u>combination</u> of two threat situations is $A \cup B$, read "A union B". If A is $\{t_1, t_5, t_6\}$ and B is $\{t_5, t_7\}$, then $A \cup B$ is $\{t_1, t_5, t_6, t_7\}$.

The event that A is <u>at least as dangerous</u> as B in the person's judgment is written $A \succ_D B$. If both $A \succ_D B$ and $B \succ_D A$, then A and B are <u>danger-equal</u>, $A \sim_D B$.

Suppose two threat situations A and B are such that the threats can be paired off, one from A and one from B in each pair, so that in each pair the two threats have equal probabilities and utilities. Then A and B are said to be <u>equivalent</u>, $A \equiv B$. Axiom I states that the person's relative danger judgments are based only on the probabilities and utilities of the threats on the list, so that equivalent threats are always judged danger-equal.

<u>Axiom 1.</u> If $A \equiv B$, then $A \sim_D B$.

Note that the converse is not true. A person may judge two situations to be equally dangerous even though the two sets are not similar, accident by accident.

The sign for equivalence must also be distinguished from the equality sign: $A = B$. The latter means that the two sets have identical membership. Thus, three signs with increasingly stronger meanings are "$\sim_D$", "$\equiv$", and "$=$".

The next axiom states that every pair of situations can be compared for relative danger and that comparisons of danger are transitive.

Axiom 2. For all A, B and C

i) either $A \succcurlyeq_D B$ or $B \succcurlyeq_D A$, and

ii) if $A \succcurlyeq_D B$ and $B \succcurlyeq_D C$, then $A \succcurlyeq_D C$.

The next axiom, Axiom 3, states that adding a new threat to each of two situations does not change their relative danger. For example suppose an individual is trying to evaluate the danger of four alternative vacation plans. Plan 1 is a trip to Colorado by car, plan 2 the same trip by plane. Plans 3 and 4 are like 1 and 2 except the individual includes a mountain-climbing expedition. Thus if t_c, t_p and t_m are the threats of a car, plane and mountain-climbing accident, plan 1 is $\{t_c\}$, plan 2 is $\{t_p\}$, plan 3 is $\{t_c, t_m\}$ and plan 4 is $\{t_p, t_m\}$, Axiom 3 requires that if plan 1 provokes a greater danger-response than 2, then 3 must elicit a greater response than 4.

Axiom 3. If A and C have no common members, and B and C have no common members, then $A \succcurlyeq_D B$ if and only if $A \cup C \succcurlyeq_D B \cup C$.

The zero-threat situation is defined as the one containing no threats, $Z = \{\}$. According to Axiom 4 all situations are at least as dangerous as Z.

Axiom 4. For all A, $A \succcurlyeq_D Z$.

It follows from the axioms stated so far that adding threats can never make a situation less dangerous:

Theorem 1. For all A, B, $A \cup B \succcurlyeq_D A$.

Proof: Define the situation C = B-A, that is, C is the threats in B but not in A. By Axiom 4, $C \succ_D Z$. This along with Axiom 3 implies $C \cup A \succ_D Z \cup A$. By the definition of C, $C \cup A = A \cup B$. Also, $Z \cup A = A$, so that $A \cup B \succ_D A$ and Theorem 1 is proved.

The next axiom, known as an Archimedean axiom, states that no situation is "infinitely more dangerous" than another unless the latter is equally dangerous with the zero-threat situation. If one situation is more dangerous than another a large group of situations each one equivalent to the lesser can be combined, and taken together this group exceeds the greater one.

Axiom 5. For any A and B with $A \succ_D B$ and not $B \sim_D Z$, there exist $B_1, B_2, \ldots B_n$ all different and all danger-equivalent such that $B_1 \cup B_2 \cup \ldots \cup B_n \succcurlyeq_D A$.

The following theorem states that if Axioms 1 through 5 are consistent with the individual's pattern of danger judgments, numbers can be assigned to the situations to reflect the pattern of judgment. Also, situations without common members will combine to produce a danger measure that is the sum of the individual dangers. Further, two sets of numbers representing the judgments may be different but must be related to one another in a simple way: the numbers must be proportional.

Theorem 2. If the relation $\succ_D$ satisfies Axiom 1 through 5, there exists a real-valued function d such that

(i) $A \succcurlyeq_D B$ if and only if $d(A) \geq d(B)$,

(ii) if A and B have no common members $d(A \cup B) = d(A) + d(B)$,

(iii) $d(Z) = 0$ and $d(A) \geq 0$,

(iv) if d and d′ satisfy Axioms 1 through 5, then there is a constant $k > 0$ such that for all A, $d'(A) = k\,d(A)$.

Theorem 2 is a consequence of the basic theorem of extensive measurement, a version of which is given by Krantz et al. (1971, Chapter 3, Theorem 1). The present axioms are not identical to theirs, but each is at least as strong as their corresponding axiom so that Theorem 2 follows.

The function d is not completely determined by Theorem 2. Part (iv) suggests that there is an infinity of admissible functions. A situation A* must be selected to have unit danger, either arbitrarily or on some practical grounds. By Theorem 2, (iv), only one function d will have $d(A^*) = 1$ so the function will then be completely specified. The function d will be called the danger measure for $\succ_D$. Evaluated for a specific situation, d(A) will be called the danger of A.

The additivity property of d expressed by Theorem 2, (ii), is very convenient in that the danger of a complex situation can be calculated by adding the dangers of its simple components. Some consequences of this will be described now.

The value of $d(\{t\})$ is the danger of a situation comprising a single threat. Whenever such a single-threat situation is involved, d can be written d(p,u), a function of the probability and utility of t. This follows from Axiom 1.

Suppose two different threats t_i and t_j have probabilities p_i

and p_j respectively, and a common degree of loss u. Then by Theorem 2, (ii),

$$d(\{t_i, t_j\}) = d(\{t_i\}) + d(\{t_j\})$$
$$= d(p_i, u) + d(p_j, u)$$

Since t_i and t_j are mutually exclusive, the probability of a loss u in $\{t_i, t_j\}$ is $p_i + p_j$ so that

$$d(\{t_i, t_j\}) = d(p_i + p_j, u)$$

and therefore

$$d(p_i + p_j, u) = d(p_i, u) + d(p_j, u) \tag{12}$$

The function d must satisfy (12) for all values of p_i, p_j and u. This is a form of Cauchy's functional equation. A classical theorem (Aczél, 1966, Ch. 2, Theorem 1) implies that d is proportional to p. That is, there is some function w, a function only of u, such that d can be written,

$$d(p,u) = p \cdot w(u).$$

The function w will be called the <u>utility weighting function</u> for $\succcurlyeq_D$ since it determines the weight given to various losses in the measurement of danger.

These arguments along with Theorem 2 lead to Theorem 3.

<u>Theorem 3</u>. Suppose $\succcurlyeq_D$ satisfies Axiom 1 through 5 and that d is its danger measure. Then there is a function w such that

$$d(A) = \sum_{i=1}^{n} p_i \cdot w(u_i) \tag{13}$$

where A is a situation with n threats with respective probabilities $p_1, \ldots p_n$ and utilities $u_1 \ldots u_n$.

To specify the danger measure d, the form of the function w must be decided. One would expect it to have these two properties:

$$w(0) = 0 \qquad \text{and} \qquad \frac{d\,w(u)}{d\,u} < 0 \qquad (14)$$

That is, events without loss have no danger, and danger increases with amount to lose.

The only sure way of determining the exact form of w is to conduct a controlled experiment to find the individual's pattern of responses to danger. A researcher might need a danger measure but decide it is not worthwhile to run an experiment solely for the purpose of finding w. In this case a function might be chosen a priori on grounds of simplicity. Such a choice would have no empirical support but at least its meaning would be clear, since it would be known to satisfy the above axioms, which describe certain general patterns of human response to danger.

The simplest possible choice would be w(u) = u, leading to the following proposal:

$$d(A) = \sum_{i=1}^{n} p_i \; u_i \qquad (15)$$

Danger is the expected loss associated with the possible accidents, the sum of utilities weighted by the corresponding probability.

5. Comparison with Other Theories

Certain formulae that have appeared in the literature can be interpreted as choices of the weighting function w and are therefore consistent with axioms for a danger measure of section 4.

Typically researchers have used the words "risk" "hazard" and "danger" each in their own way although we would prefer "risk" in some cases and "danger" in others.

Cohen et al. (1956), Rockwell (1962) and others have defined risk to be dependent only on the probability of loss. In the present theory this would be equivalent to setting $w(u) = 1$. This would be an appropriate definition for comparing accidents sharing a common average degree of loss and differing only on probability of loss. For example, one could compare different pedestrian crosswalks by the rate of death, and label the latter the risk of the crosswalk, but to compare accidents of different types severity should be included, i.e., w should increase with u.

In a study of the psychology of decision-making, Pruitt (1962) defined risk as the expected value calculated over all elements worse than the status quo. This is the function $w(u) = u$, the measure proposed above. The same choice of weighting function is the basis of the Frequency-Severity Index used by the Consumer Product Safety Commission to measure the potential for physical injury of commercial products (Kelman, 1974). More generally, it appears in the analysis of cost-benefit planning (Starr, Rudman and Whipple, 1976).

Risk has been frequently defined as variance (Tobin, 1965). As shown in Section 3, if the probability of accidents is small, this is equivalent to our risk measure and to our danger measure setting $w(u) = u^2$.

Pollatsek and Tversky (1970) offer a theory in which risk is expressible as a linear function of the mean and variance of the gamble. If the probability of an accident is small, it can be shown that this is equivalent to our danger measure setting $w(u) = u + ku^2$. An interesting aspect of their theory is that it leads to the possibility of gambles with negative risk -- when added to other gambles facing the decision-maker, they reduce the total risk.

Luce (1980) has described several alternative sets of assumptions that yield definitions of risk. One group of his axioms is closely related to the danger measure d presented in this paper. Translating his terminology freely into that used here we can say that he takes Theorem 3 as a basic assumption (his Assumption 3). Luce also requires that there be some $k > 0$, such that if threat situation A_c is derived from A by multiplying all utilities by a constant $c > 0$, then $d(A_c) = S(c)d(A)$ where S is some increasing function with $S(1) = 1$, (this is his Assumption M). That is to say, if loss was previously measured in dollars but now is measured in cents, so that $c = 100$, then the risks as measured in the new units are some multiples of the old risks the proportionality depending only on c. Zero-risk situations remain zero-risk under this assumption. He shows that this leads to the formula

$$d(A) = k \sum p_i u_i^{\theta}$$

for some $k, \theta > 0$. This is consistent with our family of danger measures of the second section and leads to the one proposed there by choosing $\theta = 1$.

6. Comparing Risk and Danger

Risk and danger are functions of the same variables, the probability and losses of a set of accidents. The algebraic forms of the risk and danger measures are very similar. Indeed they are identical except that the utilities of the accidents are squared in the former and not in the latter. Even this difference is somewhat arbitrary, since an alternative choice for a weighting function for a danger measure would be $w(u) = u^2$. This would still satisfy the axioms for a danger measure and the risk and danger measures would become identical.

However, the rationales behind the two show that they are conceptually distinct. In the case of risk the individual is called upon to make a judgment as to how much effort should be spent to avoid a threat. This judgment is compared with rational behaviour -- an unbiased assessment of the threat, leading to a decision by the precepts of decision analysis.

In the case of danger, a decision is generally not involved. The individual responds in some way to various threats and the strengths of the responses are compared. For the risk measure the individual must produce a quantitative ratio scale measure of effort, but in the danger measure, only the ordinal scale, greater-or-less aspects of the response are relevant.

The theory of risk allows the definition of a risk propensity, a measure of the individual's personality. This does not arise from the theory of danger since in the latter there is no norm, no concept of what rational, proper behaviour would be.

Since risk and danger measures can be made to look so much alike, it is not surprising that these ideas should be confused, but there is a fundamental conceptual difference as shown by the distinctness of the two formal models.

7. Experimental Assessment of Risk and Danger

Research on risk can be divided into two types: fundamental research on the internal structure of risk unrelated to any particular context, and applications of the concept of risk as a dependent or independent variable to discover its relation to individual or environmental variables. This section deals with the latter case, and discusses the applied researcher's choice of a method of assessing risk or danger.

Typically there are three approaches:

1) use a physiological measure, such as heartrate or galvanic skin response,
2) ask the subject directly for an estimate or comparison of risk,
3) ask the subject for estimates or comparisons of the probabilities and utilities of the threats and combined these into a derived measure for risk by some formula.

Our own view and our motivation for developing the present theory is that the third method is preferable.

The first method, physiological measurement, has proved difficult since there must be independent evidence that the physiological response really reflects risk and not some other variable. Heartrate for example is influenced by physical exertion. It could be arranged that non-decision making variables be held constant, but it seems plausible that some of the anxiety responses measure not risk but criticality of the performance -- how much difference poor versus good performance will make.

The second method, direct questioning, has been used by several authors. Gautien and Wilde (1971) had subjects drive through freeway, suburban and downtown traffic and periodically give "risk ratings" from 0 to 10, 0 defined as no danger at all, 10 as imminent unavoidable collision. Smith (1977) asked workers to compare safety violations in their factory and choose one of a pair as "more hazardous." He scaled the responses and compared the results with actual accident rates.

A third example of direct questioning is the research of Coombs and Bowen (1971) who gave subjects pairs of monetary gambles and asked for a judgment of which is "riskier." They used the responses as evidence against the risk formula of Pollatsek and Tversky (1970), and suggested that risk involves more than the mean and variance of the gamble, that skewness affects the subjects' judgments as well.

The difficulty with direct ratings is that they assume that "risk" or whichever term is used, has a clear meaning understood by the subjects. For example, in his research on industrial safety violations Smith asked subjects for "more hazardous" comparisons and assumed they interpreted this as asking for unconditional probability of the accident, i.e., the accident rate. However it seems plausible that they may have interpreted the terms differently, e.g., as the conditional probability of an accident given the worker engages in the safety violation, or as the frequency of the accident weighted by its severity, etc.

Similarly the "risk ratings" elicited by Coombs and Bowen and by Wilde are difficult to interpret. The subjects may use a consistent risk measure in their decision-making but perhaps did not label it as "risk." We would like some methodology that depends on the subjects' decision-making behaviour, not on their semantic habits.

It is not surprising that people disagree on the meanings of these terms since safety researchers themselves have used them in different ways.

Some use "hazard" as a measure of pure probability while others use "risk" for this. Some dictionaries and synonym guides make distinctions, but ironically different sources give opposing definitions. Safety researchers may define "risk," etc., for themselves but they cannot expect their subject population to be aware of these meanings.

The difficulty is avoided if one uses the third method, basing risk on probability and utility, since relatively unambiguous questions can be devised to estimate these variables.

Probability and utility can be assessed on an ordinal scale, for example, in the case of auto accidents, "Which type of accident is more frequent, A or B?" To get a stronger scale as needed by our measures more refined methods can be used. "You read about an accident in which two people are injured. Would it be worse if one suffered injury A and the other injury B, or if both suffered C?" (Hernandez, Miller and Wolf, 1979).

Another method of utility assessment is its construction as a function of the set of variables known to be relevant to the loss. For example, the disutility of an accident is represented as a linear function of the permanent impairment treatment period, threat to the victim's life, and the physical energy dissipation, the coefficients being determined by regression methods (Huang, 1975).

A general discussion of empirical probability assessment is given by Spetzler and Stael von Holstein, (1975) and methods of utility assessment are reviewed by Kneppreth et al. (1974).

Having generated appropriate scales of probability and utility, risk can be specified by choosing one of the measures r or d. The particular measure chosen will be influenced by basic research findings in the study of risk, by other research in the particular area of application, by simplicity and by considerations of which measure shows the strongest relationship to other variables of interest.

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1. The present approach differs from their work in the following ways. They assume the existence of an externally measurable commodity, typically money. Risk behaviour occurs when the individual's utility gains for the money are not proportional to the face value. In the case of accident psychology, many of the costs of an accident do not have an external measure distinct from their internal utility to the individual, so a more general approach is used which does not assume the existence of an external measure.

Also, the work of Arrow and Pratt does not distinguish subjective and objective probabilities, in effect assuming they are the same and that the individual is judging probability correctly. The present theory allows for the distortion of the probability of an accident. In fact the individual's bias in judging probability is one component of his risky behaviour.

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