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The Optimal Design of a Market

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Abstract

We study the optimal design of the rules of trade in a two-period market given that agents arrive at different times and may only trade with agents present contemporaneously. First period agents face a fixed cost of trading across periods, and their decisions of whether or not to trade in the second period result in externalities relative to the agents arriving in the second period. Given the non-convexities associated with the fixed cost, competitive trading rules can result in inefficiencies in such a market and, in fact, anonymity must be sacrificed to achieve efficiency. Efficient trading rules have a market maker (i.e., an agent who is given some market power and the right to trade across periods) who faces some competition within period trading, but not across periods. The efficient choice of who should be market maker can be made by auctioning rights to this position. If there is uncertainty across periods, then efficient mechanisms may involve multiple market makers, and the optimal number of market makers depends on the cost of trading, level of risk aversion, and presence of asymmetric information.

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1 Introduction

Many organized markets, such as the major world security exchanges, limit the number of agents who may execute trades. We take a normative perspective on the design of a market and study the structure of the rules of trade that result in efficient allocations in a simple two-period model. We find that the efficient rules of trade exhibit specific roles for market makers.

We examine a market where agents arrive at different points in time, and introduce a constraint that is normally assumed away in a competitive analysis: agents can only trade with other agents who are present in the market contemporaneously. This constraint, coupled with a fixed cost for a first period agent to be present in the second period market, introduces externalities and potential inefficiencies.[1] First period agents' decisions have an impact on the prospects of trade for agents who arrive later.

We begin by showing that if the second period market is competitive relative to the agents who show up in the second period, then the externality may result in Pareto inefficient allocations. When deciding whether to incur a cost and come to the second period market, first period agents consider only their own gains from trade and not the gains from trade of the future agents.[2] If one moves away from a competitive market, and for instance, gives one of the first period agents some limited market power, then the inefficiency can be eliminated. This sets the stage for the analysis of the optimal design of a market. We examine trading mechanisms which efficiently fill the role of the market maker, provide the market maker with correct incentives, and implement efficient allocations.

Although our analysis fits into a large literature on market microstructure, it differs from the theoretical work in this literature in important ways. The previous study of market microstructure has been predominantly of a positive type. A number of authors have explored the outcome of some market game to be played between traders and market makers for a given set of trading rules. Such rules are modeled to resemble

[1] The fixed cost introduces non-convexities relative to the usual Arrow-Debreu world, which can lead to non-existence of a competitive equilibrium of the overall economy. The fixed cost can be viewed as a non-convex production technology for transforming first period goods into second period goods.

[2] Jackson and Palfrey (1996) examine a decentralized market where a similar externality exists, as agents are repeatedly matched to bargain until they decide to consummate a trade. An important contrast between these works is that Jackson and Palfrey show that in the search setting the externality results in inefficiency regardless of the design of trading rules, while here in a more centralized setting the problem can be overcome by careful market design.

the trading practices followed by the major security exchanges.[3] Our approach studies the market microstructure from a more normative perspective.[4] We are interested in characterizing the allocations in an intertemporal asset market (where there are costs to being present in the market) that maximize societal welfare and structures of trading rules that achieve those allocations. We bring ideas and techniques from the mechanism design and implementation literatures to bear on this market design problem. In addressing these normative issues, our results tie back to the positive literature, since market makers turn out to play central roles in the optimal mechanisms we identify.

Our starting point is a setting with no uncertainty. We examine a simple two period world in which there are new agents born in each period with heterogenous preferences and endowments, so that there are potential gains from trade within and across periods. Agents born in the first period choose either to consume their goods or to wait to trade with agents born in the second period. When they decide to wait and trade in the second period, they pay a fixed cost for participating to the market. The problem of optimally designing trading rules is to manage the incentives of agents to guarantee efficiency. Given the lack of previous study of the efficiency of market making, this simple starting point allows us to outline a basic need for market making across periods.[5] We study how trading rules have to be structured to choose a market maker and give that agent the incentives to correctly choose when and how much to trade across periods. The first half of the paper is devoted to this complete information analysis.

Once the foundation for market making is built, we introduce uncertainty across periods and analyze how this can change the structure of the efficient allocations and optimal trading rules. We discuss several reasons for having multiple market makers as part of a Pareto efficient trading structure in the face of uncertainty across periods.

The results we obtain are summarized as follows:

- Competitive (or even simply anonymous) trading rules relative to the second

[3]A few representative references are Bernhardt and Hughson (1996), Glosten and Milgrom (1985), and Kyle (1989).

[4]There are papers that have explicitly compared various trading institutions. A few examples are: Demsetz (1968), Madhavan (1992), Gehrig (1993), Gehrig and Jackson (1994), Pagano and Röell (1996) and Bernhardt and Hughson (1996). These papers however, compare specific institutions, rather than setting efficiency as a goal and deriving the trading rules which realize efficiency, as we do here.

[5]The model can also be interpreted as an analysis of trade across different market locations.

period may result in Pareto inefficient allocations, even in large economies.

- The gain in efficiency of a market making mechanism over a competitive mechanism can be significant, and may increase as the economy becomes larger.

- In designing efficient trading rules one needs to identify a market maker, and the role of market maker can be efficiently filled through an auction. The market maker's monopoly power must be limited so that she can expropriate the gains from trade across periods but cannot expropriate gains from trade within periods. This can be done by providing other agents with the ability to opt for refusing the services of the market maker trading in a static competitive market.

- Any mechanism that limits the market maker to only offering nonlinear pricing schedules (or more generally any anonymous schedule) can result in Pareto inefficiency.

- If there is uncertainty across (or within) periods, Pareto efficiency may involve multiple market makers, the number of which depends on the discount factor, the level of risk aversion, and the presence of asymmetric information within periods.

The rest of the paper is organized as follows. Section 2 introduces the basic model, and section 3 provides examples that illustrate the problems arising with competitive markets and the role played by market making in overcoming such problems. The analysis is then generalized in the following sections. Section 4 shows that markets that are competitive within each period (or, even simply ones that satisfy a weak anonymity condition) produce inefficient outcomes for some economies, and section 5 shows that by properly assigning market making rights, and putting some checks on the monopoly power of the market maker, efficiency can be attained. In section 6 we examine how uncertainty modifies the analysis. Concluding remarks are in section 7, and proofs are collected in the appendix.

2 The Two Period Model

Timing

There are two periods, $t \in \{1, 2\}$. N_t agents are born in period t, and the notation N_t denotes both the set of agents born at time t and the cardinality of that set. An agent born in the first period can trade in the cardinality of that set. We use $M \subset N_1$

to denote the set of first period agents who choose to be present in the second period market (at a cost described below). Agents in N_1 may trade with other agents belonging to N_1 in period 1. Agents in $N_2 \cup M$ may trade with other agents belonging to $N_2 \cup M$ in period 2. No other trade may occur.

As will become clear, the model is easily extended to overlapping generations without affecting the results.

Cost of trading

There is a cost of being present in the market. This may be the opportunity cost of time, the resources needed to physically visit a market or any other cost associated with participation in the market. The cost is represented by having an agent $i \in N_1$ pay a fixed cost $c_i > 0$ if $i \in M$. Thus, first period agents who choose to be present in the second period market pay a cost c_i, independent of any trades they make.[6]

Preferences

There are two goods which are labeled x and m, where x is the good to be traded and m is a numeráire. Each agent i's preferences for consumption of $(x, m) \in I\!R_+ \times I\!R$ are represented by a quasilinear utility function [7]:

$$V_i(x) + m - c_i I_M(i),$$

where V_i is strictly increasing, strictly concave, continuously differentiable at $x > 0$, satisfies $V_i(0) = 0$, and $I_M(i)$ is the indicator function of M ($I_M(i) = 1$ for $i \in M$ and $I_M(i) = 0$ for $i \notin M$). We assume that for any $p \in I\!R_{++}$ there exists $x > 0$ such that $V_i'(x) = p$. (This ensures that there will not exist prices for which demand is infinite, and also implies that $\lim_{x \to 0} V_i'(x) = +\infty$.) Preferences satisfy the von Neumann-Morgenstern axioms.

Endowments

Agent i is endowed with e_i units of good x; where $e_i > 0$ for some i. We treat endowments of the m good as 0 for all agents, as these endowments are irrelevant in

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[6] We could add a fixed cost for any agent to be present in any period. This would complicate the analysis without changing the fundamental externality and efficiency issues we are examining. This may be a worthwhile issue for future research, however, as it could raise questions concerning endogenous choice of when to trade.

[7] At times, agents may be offered trades which set their consumption of x to be less than 0. We assume that an agent prefers any (x, m) for which $x \geq 0$ to any (x', m') for which $x' < 0$. This could be modeled by extending V_i to have as range the extended reals and setting $V_i(x') = -\infty$ for $x' < 0$.

the quasi-linear model. We permit unlimited short sales of either good in the first period by agents who wait to trade in the second period.

Economies

Given N_1 and N_2, an *economy* is a list $\{(V_i, e_i, c_i)_{i\in N_1}, (V_i, e_i)_{i\in N_2}\}$ which specifies preferences and an endowment for each agent and a cost of trading across periods for first period agents. The set of all economies (satisfying our assumptions) is denoted E.

Allocations

Given any positive integer N, let

$$T^N = \left\{ t = (t_x, t_m) \in R^{2N} \mid \sum_{i=1}^{N} t_{xi} = 0 \text{ and } \sum_{i=1}^{N} t_{mi} = 0 \right\}.$$

An *allocation* is a specification of the agents in N_1 who wait to consume until $t = 2$ as well as trades for agents present in each period. Thus it is a specification M, t^1, t^2 such that $M \subset N_1$, $t^1 \in T^{N_1}$ and $t^2 \in T^{N_2+M}$ (where M denotes a set and its cardinality). The set of allocations is denoted A, and the notations (t^1_{xi}, t^1_{mi}) and (t^2_{xi}, t^2_{mi}) denote the trades that i gets under t^1 and t^2, respectively.

Pareto Efficiency

In this quasilinear setting, (constrained) Pareto efficiency implies a unique allocation of the x good. The allocation of the m good, however, is not tied down by Pareto efficiency and any allocation of the m good that respects the trading constraints can be part of a (constrained) Pareto efficient allocation. Thus, in characterizing Pareto efficient allocations we need only characterize the distribution of the x good and the set of agents who trade across periods, M.

Given an economy, we refer to a (constrained) Pareto efficient allocation as an *efficient allocation*. An allocation is efficient if there is no other allocation in which every agent has at least as high a utility and some agent has a higher utility. In this setting the efficient allocation is characterized as the solution to:

$$\max_{(t^1,t^2,M)\in A} \quad \sum_{i\in N_1} V_i(e_i + t^1_{xi} + t^2_{xi}) + \sum_{i\in N_2} V_i(e_i + t^2_{xi}) - \sum_{i\in M} c_i.$$

where $t^2_{xi} = 0$ if $i \notin M \cup N_2$.

It is clear that an efficient allocation has at most 1 agent in the set M, since having several agents just results in additional fixed costs paid without increasing the trade that can occur.[8] (We re-examine this in the face of uncertainty in Section 6.)

Lemma 1 *If an allocation is efficient, then M has at most one element, and the allocation satisfies:*

$V_i'(\epsilon_i + t_{xi}^1) = V_j'(\epsilon_j + t_{xj}^1)$ *for all* $i, j \in N_1 \setminus M$,

$V_i'(\epsilon_i + t_{xi}^2) = V_j'(\epsilon_j + t_{xj}^2)$ *for all* $i, j \in N_2$, *and*

$V_i'(\epsilon_i + t_{xi}^1) = V_k'(\epsilon_k + t_{xk}^1 + t_{xk}^2) = V_j'(\epsilon_j + t_{xj}^2)$ *if* $k \in M$, $i \in N_1 \setminus M$ *and* $j \in N_2$,

and if $i \in M$ *then* $c_i \le c_j$ *for all* $j \in N_1$.

Trading Rules and Mechanisms

We model trading rules as a *mechanism*, which is a specification of:

- a finite length extensive game form for period 1 with N_1 as the set of players and with terminal nodes that list net trades for each agent and agents who wait until period 2 to consume; i.e., a terminal node lists (t^1, M) where $t^1 \in T^{N_1}$ and $M \subset N_1$; and
- for each terminal node of the first period game form, a specification of a finite length extensive game form for period 2 with $N_2 \cup M$ as the set of players and with terminal nodes that list trades t^2, where $t^2 \in T^{N_2+M}$.

Given an economy, a mechanism induces an extensive form game. To be precise, fixing an economy replace period 1 terminal nodes with the period two extensive game form. Payoffs to players at a terminal node of this overall game as a function of the (t^1, t^2, M) associated with the play path are then:

$V_i(\epsilon_i + t_{xi}^1) + t_{mi}^1$ for $i \in N_1/M$,

$V_i(\epsilon_i + t_{xi}^1 + t_{xi}^2) + t_{mi}^1 + t_{mi}^2 - c_i$ for $i \in M$, and

$V_i(\epsilon_i + t_{xi}^2) + t_{mi}^2$ for $i \in N_2$.

For the following definitions fix N_1 and N_2.

An *equilibrium* relative to a given economy is a specification of behavioral strategies which form a subgame perfect equilibrium of the induced extensive form game.[9]

[8] This depends on an agent waiting across periods being able to shortsell in the first period. (It is possible that $\epsilon_i + t_{xi}^1 < 0$ for $i \in M$.) If there are limits on shortselling, then efficiency may require more agents trading across periods.

[9] This definition allows agents to randomize as part of an equilibrium. For the mechanisms in our proofs, there exist only pure strategy equilibria.

An *allocation rule* is a specification of an allocation as a function of the economy (i.e., a function from E to A).

A mechanism *implements* an allocation rule if for each economy in E there is a unique equilibrium outcome relative to the economy of the mechanism which is the specified allocation for the economy.

3 Motivating Examples

We now illustrate the externality and show that it is present even in large economies.

Example 1. Agent $i = 1$ is born in period 1 with $e_1 = 1$ unit of the x good. Agent $i = 2$ is born in the second period with $e_2 = 0$ units of the x good. The agents have the same preferences represented by $V_1 = V_2 = V$, where $V(0) = 0$. Agent 1 can either consume her endowment in the first period or wait until the second period, trade with agent 2 and then consume.

Consider the structure of the trading rules at time 2. Suppose first that, after a quick reading of a micro theory text, the market designer decides that trading rules should be designed so that trade results in competitive allocations.[10] If agent 1 decides to wait and trade in the second period, then at time 2 the agents will trade one-half unit of good x at the competitive price of $p = V'(\frac{1}{2})$ (where the price is in units of the m good for the x good). Thus, by waiting and trading in period 2, agent 1 expects a utility of

$$V\left(\frac{1}{2}\right) + \frac{1}{2}p - c_1$$

If, instead, agent 1 does not wait and consumes in period 1 then she obtains a utility of $V(1)$. Therefore, in the face of a competitive set of trading rules in the second period, agent 1 will wait only if:

$$V\left(\frac{1}{2}\right) + \frac{1}{2}V'\left(\frac{1}{2}\right) - c_1 \geq V(1).$$

Depending on c_1, this can result in autarchy (agent 1 consuming in the first period). It is important to note that this market organization can result in a Pareto inferior outcome. Given the transferability of the m good, Pareto efficiency requires agent 1 to

[10] See the discussion following Proposition 1 for a description of the trading rules that would implement the competitive allocations between 2 agents.

wait and trade if, and only if,

$$2V\left(\frac{1}{2}\right) - c_1 \geq V(1).$$

Thus, if c_1 is in the range

$$2V\left(\frac{1}{2}\right) - V(1) > c_1 > V\left(\frac{1}{2}\right) + \frac{1}{2}V'\left(\frac{1}{2}\right) - V(1),$$

then a competitive market at time $t = 2$ results in an inefficiency. Notice that this interval is not empty since the strict concavity of V implies that $V(\frac{1}{2}) > \frac{1}{2}V'(\frac{1}{2})$. Furthermore, strict concavity implies that $V(1) < V\left(\frac{1}{2}\right) + \frac{1}{2}V'\left(\frac{1}{2}\right)$, so that the interval is a subset of R_+.

How could the inefficiency be avoided? A simple way is to designate agent 1 a 'market maker' at period 2, by giving her monopoly power. In that case, she appropriates the full surplus in the second period and internalizes the externality. This results in an efficient outcome.

The presence of a 'fixed cost of trading' introduces a non-convexity which accounts for the inefficiency of competitive markets. The main problem is that an agent $i \in N_1$ who decides to trade in period 2 generates a positive externality for period 2 agents, who see their trading possibilities enhanced. The social return for market participation in period 2 of $i \in N_1$ is therefore greater than the private return to agent i considering the fixed cost of trading c_i. However, agent i only consider private returns when she decides whether or not market participation is worth paying the fixed cost c_i. This results in a level of market participation in period 2 which is less than optimal.

To see where the non-existence of an overall competitive equilibrium occurs, consider the choice of p. It must be that $p = V'(1/2)$ in a competitive equilibrium. However, for some c_1 (as in Example 1) this results in agent 1 not trading across periods, and so p does not clear the second period market. The competitive price in the second period would have to be $p = \infty$ to clear the second period market if agent 1 does not trade across periods; however, at $p = \infty$, agent 1 would like to trade across periods.

The inefficiency could be avoided if agent i was able to contract the second period agents to compensate her for the fixed cost before the fixed cost is incurred (a 'Coase theorem' type of logic).[11] Such a contract is impossible, since agents in N_2 are not

[11]The contracting literature on the 'hold up' problem has considered similar problems based on

present when the cost is incurred. Thus, c_i is a sunk cost at the first time agent i can interact with second period agents. This leaves agent $i \in M$ with no bargaining power to recover c_i.

To further explore conditions under which having a market maker offers improvements over having competitive markets within each period, let us examine a slightly generalized version of the Example 1. The following example illustrates the inefficiency of competitive markets when the size of the economy is large.

Example 2

N_1 agents are born at time 1 with $e_i = 1$ and N_2 agents are born at time 2 with $e_i = 0$. Utility functions are identical $V_i = V$, and $V(0) = 0$. Let $c^* = \min_{i \in N_1} \{c_i\}$.

Consider a situation where a single agent with $c_i = c^*$ waits between periods, as this is the only agent who trade across periods in an efficient allocation. Also, this is the relevant benchmark to determine if no agents waiting across periods is an equilibrium outcome.

Given one agent waiting across periods, let x^* be the efficient consumption per agent in both periods (so $x^* = N_1/(N_1 + N_2)$). For it to be efficient for the agent to trade across periods it must be that:

$$(N_1 + N_2)V(x^*) - c^* \geq N_1 V(1). \tag{1}$$

An efficient allocation could be achieved with a market making mechanism (Proposition 3, below) and this is the inequality the market maker evaluates in making the decision of whether or not to trade across periods.

Given that (1) depends on x^*, we provide a sufficient condition for (1) relating V, N_2, and c^*:

Lemma 2 *If* $V'(\frac{1}{N_2+1}) \geq V(1) + c^*$, *then it is efficient to have an agent trade across periods.*

The intuition is that agents in period 2 have large marginal utility from obtaining small amounts of the x good, and for any fixed cost c^*, if there are enough second period agents, then it is efficient for a first period agent to trade across periods.

a variety of contracting imperfections. Here the opportunity to contract is only present after the relevant cost has been incurred (by the very nature of the cost) and so the contracts that agents can sign ex post will not solve the problem. Our focus is instead on how to design rules for trade (i.e., the market) to efficiently align preceding incentives.

Now let us consider the incentive that an agent has to wait across periods if that agent expects the allocation to be efficient, and expects the allocations to be as if they were obtained by trading at the competitive price in each period. That is, the allocation is as if all trade occurred at $V'(x^*)$. The agent would wish to wait only if:

$$(V(x^*) + N_2 x^* V'(x^*)) - ((N_2+1)x^* - 1)V'(x^*) - c^* \geq V(1).$$

This simplifies to

$$V(x^*) + (1 - x^*)V'(x^*) - c^* \geq V(1). \tag{2}$$

Notice that the left hand side of inequality (2) is precisely what the agent who waits earns only from trading her own endowment! That is, she is not getting any reward for any of the other trade that she is carrying across periods, since allocations are competitive at each time. This can result in an inefficient decision, since she is not accounting for the substantial gains from trade that are realized by other agents.

To obtain a better understanding of when a first period agent's choice to wait is efficient, we consider the cases $\frac{N_1}{N_2} \to \infty$, $\frac{N_1}{N_2} \to 0$ and $\frac{N_1}{N_2} = 1$.

First, consider the case where N_2 becomes large relative to N_1. In this case x^* tends to 0 (simply from budget balance) and (2) tends to $V'(0) - c^* \geq V(1)$, which will hold since $\lim_{x \to 0} V'(x) = +\infty$. Thus, as N_2 becomes large relative to N_1, a first period agent would (efficiently) choose to wait.

Next, consider the case where N_1 becomes large relative to N_2. In this case x^* tends to 1 and (2) tends to $V(1) - c^* \geq V(1)$, which cannot be satisfied. Thus, in cases where N_1 is large relative to N_2, a first period agent anticipating competitive allocations would choose not to wait, which is an inefficient choice for a sufficiently large N_2 by Lemma 2.

Finally, consider a large economy where $N_1 = N_2$. In this case $x^* = \frac{1}{2}$ and from (1) intertemporal trade is efficient if:

$$N_1 \left(2V\left(\frac{1}{2}\right) - V(1) \right) \geq c^*. \tag{3}$$

Notice that the strict concavity of V implies $2V\left(\frac{1}{2}\right) - V(1) > 0$, so that inequality (3) is always satisfied for large enough N_1. On the other hand, inequality (2) implies that a first period agent will not wait if:

$$c^* > V\left(\frac{1}{2}\right) + \frac{1}{2}V'\left(\frac{1}{2}\right) - V(1).$$

Whenever this inequality is satisfied, the competitive mechanism will be unable to implement the efficient allocation for large enough N_1.

To understand the inefficiency, notice that a mechanism which has competitive equilibria relative to the agents present in each period gives any agent precisely the gains from trade provided by his or her own endowment, but not any surplus from additional trading that they provide. With a large economy, the intertemporal gains from trade become substantial and intertemporal trade is efficient, and yet a first period agent only compares his or her own gains from trade with the cost they must incur to trade across periods. Thus, the cost can be very small relative to the total gains from trade and still impede efficient trading.

The efficiency or inefficiency of a mechanism that is competitive within each period in this example depends on the direction of the trade imbalance across periods. Even with large numbers of agents in each period, such a mechanism may be very inefficient. In fact, the only case in this example where the competitive mechanism always leads to efficiency is the case where the size of the second period market is much larger than the first, so that any single first period agent's own endowment becomes so valuable that they have an incentive to trade across periods without even worrying about the gains from trade available from other first period agents.

4 The Inefficiency of Competitive or Anonymous Trading

The inefficiency illustrated in Examples 1 and 2 occurs more generally. That is, for a large class of trading rules and any N_1 and N_2, there exist economies such that the competitive mechanism yields inefficient equilibria.

Before proceeding further, we first show that the competitive allocations in the second period are implementable. Otherwise, the discussion of the inefficiency of competitive allocations is only a hypothetical. The following proposition will also be useful later, as our 'efficient mechanism' will use such a 'competitive mechanism' as a component.

4.1 A Competitive Mechanism

Given a static economy of size N, the *competitive* allocation is the unique $t \in T^N$ such that $V_i'(\epsilon_i + t_{xi}) = V_j'(\epsilon_j + t_{xj})$ for all i and j in N, and $t_{mi} = -p\, t_{xi}$ where $p = V_1'(\epsilon_1 + t_{x1})$.

A static mechanism[12] *implements* the competitive allocation rule if for each economy there is a unique equilibrium outcome to the mechanism which is the competitive allocation for the economy.

Proposition 1 *Consider a static setting with $N \geq 3$. There exists a static mechanism that implements the competitive allocation rule.*

The ideas underlying the mechanism are simple. Agent 1 announces a price for trade. If this is the competitive price, (as in equilibrium) the other agents will then select their competitive trades at this price. If this is not the competitive price, then there exists a Pareto improving trade between some agent $i > 1$ and agent 1, relative to the optimal trade at that price for i.[13] Agent i can suggest this price and trade instead of a trade at 1's price. If some agent i suggests an improving trade, then agent 1 is fined (through an endogenous payment to an agent $j \neq i$) regardless of any further choices. Agent 1 can then either accept or reject this suggested improving trade. If it is rejected then i is also fined, which keeps agent i honest in suggesting improvements.

The proposition is stated only for $N \geq 3$. If $N = 2$, then balance may be difficult to satisfy out of equilibrium, as there is no third party to whom fines can be paid.[14] For $N = 2$, the utility functions V_i need to be bounded, so that a fine could be imposed which is exogenous. In the mechanism we employ for $N \geq 3$, the fines are set endogenously and so utility functions can be unbounded and the mechanism is still balanced in and out of equilibrium.

Outside of its relation to the rest of this paper, this proposition contributes to the study of the implementation of Walrasian allocations (e.g., Hurwicz (1979), Schmeidler (1980), and Hurwicz, Maskin, and Postlewaite (1984)). The standard theorems of subgame perfect implementation (for instance Moore and Repullo (1988)) do not apply here since both endowments and preferences vary across economies. Hong (1995)

[12]A static mechanism is any finite length extensive game form with N as the set of players and outcomes $t \in T^N$ as terminal nodes. Given an economy this static mechanism induces an extensive form game (evaluating utilities of outcomes at terminal nodes), and an equilibrium is a subgame perfect equilibrium of the extensive form game.

[13]Otherwise, $V_1' = V_i'$ for all i at the suggested trades which characterizes the competitive trades.

[14]The mechanism can be constructed to be feasible, i.e., such that the constraint $\sum_{i=1}^{N} t_m^i \leq 0$ is always satisfied. The mechanism for the case $N \geq 3$ satisfies the stronger constraint $\sum_{i=1}^{N} t_m^i = 0$ for all possible strategy profiles. It is an open question whether this is possible for $N = 2$, although a reasonable conjecture seems that it is impossible.

provides a mechanism for Nash implementation of competitive allocations when both endowments and preferences vary. Our use of subgame perfect implementation provides a simpler mechanism than the mechanisms in the Nash implementation literature. Our mechanism has features not found in the Nash mechanisms: it is balanced in and out of equilibrium, it does not involve any integer or modulo games[15], and it delivers the correct outcome even when mixed strategy equilibria are taken into account.[16] Of course, our analysis is limited to a quasilinear setting, but the ideas behind this mechanism are extendable to more general environments.

4.2 An Inefficiency Result

Although competitive allocations are implementable, we know from Examples 1 and 2 that it is not always desirable to employ a mechanism that implements competitive allocations. Actually, the root of the problem in Examples 1 and 2 is not the competitive nature of the allocation in the second period, but merely the fact that it is anonymous. This prohibits an agent who waits between periods from realizing the full value of the welfare gain due to the waiting, thereby resulting in an inefficient allocation in some cases. This is true more generally, as captured in the following proposition.

A mechanism has *anonymous equilibrium outcomes* in the second period if given any second period extensive game form (corresponding to a first period terminal node with some $M \cup N_2$ as the set of players) the set of subgame perfect equilibrium outcomes of this second period extensive game form depends only on the preferences and endowments of the agents, not on their labels. That is, for any permutation of agents' endowment and preference pairs, the set of equilibrium allocations correspondingly permute.

[15]See Jackson (1992) for an argument against the use of integer and modulo games. Our mechanism also satisfies a condition of Maniquet (1996), which states that from any combination of strategies there must exist a finite sequence of changes to alternative strategies for which each change is weakly improving for the agent taking it, with the last element of the sequence being an equilibrium of the mechanism.

[16]In the context of Nash implementation, several papers have explored the issues of what types of strategy spaces are necessary to implement correspondences such as the Walrasian one. Examples include Dutta, Sen, and Vohra (1995), Saijo, Tatamitani, and Yamato (1995), and Sjöström (1995). Although we do not pursue those same issues here, the mechanism that we propose uses sequential announcements and revisions of prices, quantities, and acceptance or rejection of trades. This distinguishes it from the canonical mechanisms in the literature which involve announcements of preferences (or states) and other information.

A mechanism is *individually rational* if for every economy and equilibrium outcome, no agent prefers her endowment to her equilibrium allocation.

Proposition 2 *Fix any N_1 and N_2. For any individually rational mechanism which has anonymous equilibrium outcomes in the second period, there exist economies for which some equilibria of the mechanism result in inefficient allocations.*

Since competitive allocations are anonymous and unique (in this setting), the following is also true.

Corollary 1 *Fix any N_1 and N_2. For any individually rational mechanism which results in competitive equilibria relative to the agents present in the second period, there exist economies such that all equilibria of the mechanism result in inefficient allocations.*

Actually, for this result the order of the quantifiers can be switched so that the same economy results in inefficient allocations for any mechanism that is competitive in the second period.

In the next section we show that efficient allocations are implementable if we sacrifice anonymity. In particular, the appointment of a market maker is a crucial ingredient of a market structure that delivers efficient outcomes.

5 The Efficiency of Market Making

It follows from Lemma 1 that a mechanism that implements an efficient allocation rule will have at most one agent trade across periods. Therefore, a mechanism implementing an efficient allocation rule must be able to identify situations where it is efficient to have an agent trade across periods, identify the correct agent in those cases, and give the agent the proper incentives to trade across periods.

Proposition 3 *There exists an individually rational mechanism which implements a Pareto efficient allocation rule. For the case where $N_1 \geq 3$ and $N_2 \geq 3$, the implemented allocation rule is also individually rational relative to the competitive allocations within each period.*

The formal proof appears in the appendix, but we outline the structure of the mechanism here for the case where $N_1 \geq 3$ and $N_2 \geq 3$.

- Part 1: Market making rights are auctioned among first period agents via a sequential auction, and a market maker is identified.

- Part 2: The market maker from Part 1 offers a list of trades to the other first period agents, and then in sequence the other first period agents either approve or veto the offer of the market maker.

- Part 3: If approved by all first period agents other than the market maker, the trades are executed and then the market maker decides either to consume or to wait. If vetoed by any agent, the competitive mechanism (Proposition 1) is operated among the first period agents and the market maker must consume in the first period.

- Part 4: If the market maker consumed in the first period, then the competitive mechanism is operated among the second period agents. If the market maker waited, then the market maker offers a list of trades to the second period agents and pays them each $\frac{1}{N_2}$ of the bid from the auction.

- Part 5: In sequence, the second period agents either approve or veto the offer of the market maker. If approved by all second period agent, then the trades are executed. If vetoed by any agent, the competitive mechanism is operated among the second period agents (and the market maker is excluded).

The idea behind having agents bid for the right to be market maker is that agents will be bidding based on the surplus utility they can generate from trading across periods, and the most efficient agent will be able to bid the most. The fine points of setting up the bidding are delicate since one has to set up the auction so that all of its equilibria have the efficient agent winning with probability 1. One cannot use a standard auction to achieve this. For instance, if there is only one efficient agent, then it can be the case that in any equilibrium of a sealed bid first price auction an inefficient agent may win with some probability. A second price auction has multiple equilibria - some of which do not result in the efficient agent winning. Details of how we design the auction are presented in the appendix.

A mechanism that implements efficient allocations also has to handle another subtlety. There must exist checks on the surplus that the market making agent can extract by waiting until the second period. If the market maker could simply make unchallenged take it or leave it offers in the second period, then the market maker would be able to extract not only the gains from trade *across* periods, but also the gains from trade *within* the second period. This would induce the market making agent to wait in situations where it would not be socially efficient to do so. The mechanism limits the market maker's power by allowing the second agents an option of ignoring the market maker and trading on their own. Thus, the market maker can only extract the gains from trade from the benefit of having trade across periods. Since agents can veto the market maker's offers and operate a competitive mechanism within any given period, each agent is guaranteed a utility level at least as high as the one corresponding to the competitive allocation within the period.

Let us remark on the role of subgame perfection in Proposition 3. Given the two period timing, it is important to impose sequential rationality to obtain efficient outcomes. Otherwise, if first period agents anticipated that second period agents will choose not to trade whenever a first period agent waits to trade across periods, then first period agents would not wait to trade across periods regardless of the potential gains from trade. Such incredible beliefs are possible if Nash equilibrium is considered, but not with subgame perfect equilibrium.

Subgame perfect equilibrium plays an even deeper role in our results. In some cases to ensure that the efficient choice of trading across periods is made by first period agents, such agents need to extract at least a certain portion of the resulting gains from trade in the second period. These gains from trade may differ across economies that have similar efficient allocations of the x good. If Nash equilibrium is the solution concept used in the second period, sufficient gains from trade are not always extractable. A sequential structure plays an important role in making sure that agents trading across periods are sufficiently rewarded for the cost that they have sunk, and thus subgame perfection is important:

Proposition 4 *Consider a mechanism for which the second period game forms have no proper subgames and depend only on the (t^1, M) realized from the first period game form. Such a mechanism cannot implement an individually rational and efficient allocation rule (or correspondence).*

If the second period game forms have no proper subgames, then subgame perfection can have no bite beyond Nash equilibrium in the second period. The additional condition that the second period game forms depend only on the (t^1, M) realized from the first period game form, implies that agents trading across periods cannot 'announce' (irrevocably) their second period moves in the first period. Since there are proper subgames when considered across periods, such an inter-period dependence would allow subgame perfection to have implications (beyond Nash equilibrium) relative to second period trading. The idea of the proposition is that multiple equilibria cannot be ruled out when Nash equilibrium is used as the solution concept, and some of the Nash equilibria do not result in sufficient gains from trade to a potential market maker to induce the market maker to make an efficient choice. When subgame perfection is applied, a mechanism can be constructed (following Proposition 3) for which all equilibria offer sufficient gains from trade to a potential market maker to induce the market maker to make an efficient choice.

We can also say something about the way in which the second period mechanism must be structured. One has to be careful in how one limits the market maker's power. In order to obtain an efficient allocation, the market maker has to have the correct incentives to wait across periods and this can require that the market maker price discriminate among agents. If such perfect price discrimination is eliminated, then the market maker cannot extract the full gains from trade across periods and inefficiency may result.

Proposition 5 *If $N_2 \geq 2$ and the market maker from period 1 is constrained to announce a price schedule (not necessarily linear) in the second period and each agent present in the second period then can choose a trade and price in the offered set, then there exists an economy for which all equilibrium outcomes are inefficient.*

Any set of anonymous offers that a market maker could make in our setting is equivalent to offering some non-linear price schedule. Inefficiency from such a schedule results from incentive compatibility constraints associated with agents choices from the schedule.

Thus, there are two important breaks from symmetry in the efficient mechanism. First, as we knew from Proposition 2, the market maker must be treated asymmetrically (non-anonymously) from other agents. Second, Proposition 5 points out that the market maker must be able to make different offers to agents who have different characteristics.

6 Uncertainty and Multiple Market Makers

The efficiency of having a single market maker across periods was due to the market maker's ability to correctly forecast how much of the good should be carried across periods. In the presence of uncertainty, efficiency may require several market makers.

Uncertainty Across Periods

Consider economies where in the first period there is uncertainty about second period endowments. A period 1 agent is able to see second period endowments once that agent arrives at the second period market, i.e., if she pays the fixed cost of participating in the second period market. Alternatively, she can trade and consume in period 1. In such situations, the allocation of any first period agent who does not wait to trade in the second period must be fixed before the uncertainty is resolved. This introduces a benefit to having several agents wait to trade in the second period, as it permits additional reallocation of goods. This is illustrated in the following example.

Example 3.

Let $N_1 = 2$ and $N_2 = 1$. Both agents in the first period have $e_1 = e_2 = 1$ and $c_1 = c_2 = c$. The second period agent (agent 3) has a probability of $1/2$ that $e_3 = 0$ and a probability of $1/2$ that $e_3 = 1$. Agents 1 and 2 can trade in the first period, and decide on whether to wait before knowing agent 3's endowment.

Consider the efficient allocation rule in this example. If both agents 1 and 2 consume, then total expected utility is $\frac{5}{2}V(1)$. If both agents 1 and 2 wait, then total expected utility is $3(\frac{1}{2}V(1) + \frac{1}{2}V(\frac{2}{3})) - 2c$. If one agent waits and takes x from the other agent then total expected utility is

$$V(1-x) + 2\left(\frac{1}{2}V\left(\frac{1+x}{2}\right) + \frac{1}{2}V\left(\frac{2+x}{2}\right)\right) - c.$$

Let us show that for low enough c, the unique efficient allocation is to have two agents wait across periods. It is better to have both agents wait than neither whenever

$$\frac{3}{2}V(1) + \frac{3}{2}V\left(\frac{2}{3}\right) - 2c > \frac{5}{2}V(1),$$

which can be written as:

$$\frac{3}{2}V\left(\frac{2}{3}\right) - V(1) > 2c.$$

The left hand side is strictly positive by strict concavity of V, so that the inequality holds for low enough c. It is better to have both agents wait than just one when

$$\frac{3}{2}V(1) + \frac{3}{2}V\left(\frac{2}{3}\right) > V(1-x) + V\left(\frac{1+x}{2}\right) + V\left(\frac{2+x}{2}\right) + c$$

It is clear that the right hand side is less than the left hand side at $c = 0$, since in that case the left hand side is the optimal allocation and the right hand side is the optimal allocation subject to the additional constraint that the consumption of one agent is the same in both states. Furthermore, the optimal level of x does not depend on c. Thus, when c is raised from zero to ϵ, the inequality still holds.

The efficient allocation in this example has no agent waiting for high values of c, one agent waiting for middle values of c, and two agents waiting for low values of c. This pattern is true more generally, as we now explore.

We begin by characterizing the number of first period agents who should trade across periods (referred to as 'market makers') for each c in the special case where $V_i = V$ and $c_i = c$ for each i. In this case all agents are *ex ante* identical, which allows us to concentrate on the optimal number of market makers, disregarding the issue of who should be market maker.

Proposition 6 *Consider an economy where all agents have $V_i = V$, $c_i = c$, and where the total second period endowment of the x good is a random variable with finite expectation and positive variance. For all but a finite number of values of c, efficient allocations have a unique number of agents trading across periods.*[17] *This number of agents is non-increasing in c, taking on all values between N_1 and 0 as c increases.*

The proof (in the appendix) shows that total welfare is a concave function of the number of market makers, M. It also shows that total welfare is continuous and decreasing in c. These facts, coupled with observations tying down the optimal M near the extremes of $c = 0$ and $c \to \infty$ establish the proposition. The intuition behind the proposition is that there is a benefit of having larger M since one can smooth second period consumption among a larger group of agents. The trade-off comes from the cost of market participation. The intuition behind the concavity of total welfare in M is that the marginal benefits from smoothing across agents in the second period are decreasing with the number of agents already present.

For the more general case, where agents have heterogeneous preferences and cost, the monotonicity of Proposition 6 may fail. Nevertheless, we can still identify the efficient market makers and implement an efficient and individually rational allocation rule via a mechanism that is a variation on the mechanism used to prove Proposition 3.

Proposition 7 *Let the total second period endowment of the x good be a random variable with finite expectation and positive variance. There exists a mechanism that*

[17]There are N_1 critical levels of c where is indifference between having M or $M+1$ agents wait.

implements[18] *an efficient and individually rational allocation rule. If* $N_1 \geq 3$ *and* $N_2 \geq 3$, *then the implemented allocation rule is also individually rational relative to the competitive allocations within each period.*

The idea behind the mechanism described in the proof of Proposition 7 is to have a single agent who has market making rights, say agent 1, who can then offer allocations to other agents. The agent's offers to other first period agents can specify that the other agent wait across periods. That is, the market maker not only offers trades in the first period, but also offers other agents payments for trading across periods. As with the mechanism behind Proposition 3, agents other than the market maker can refuse the market maker's offer in which case the static mechanism is run.

7 Concluding Remarks

With uncertainty across periods, we have seen that there is a reason why efficiency may require several market makers: Having additional first period agents postpone consumption to the second period allows for more consumption smoothing across agents after the uncertainty is resolved.

There are (at least) two other factors that may contribute to having additional market makers.

First, if agents are risk averse across periods, then they may choose to hold an inadequate inventory across periods from the social perspective. With risk aversion, a market maker cares not only about the expected gains from trade in the second period, but also about their distribution across states. Given an individual rationality constraint for second period agents, a market maker will have rewards from trading across periods that vary across states. For a risk averse agent, this variance decreases an individual's incentives to carry extra goods across periods with the hope of trading them. Additional market makers can share such risks.

Second, if there is asymmetric information between first and second period agents, then incentive compatibility constraints may reduce the market maker's incentives to trade across periods. A market maker cannot expect as high a return from serving as an intermediary across periods, as she will not be able to extract as high a price in

[18]Implementation, efficiency, and individual rationality, are defined subject to the constraints imposed by the uncertainty (first period consumption choices cannot depend on the state realized in the second period), but otherwise hold in each state.

all states as in the symmetric information. The constraints introduced by asymmetric information will tend to reduce the benefit of having a fixed number of market makers trade across periods.

The overall impact on the efficient number of market makers of risk aversion or asymmetric information; however, is ambiguous. By reducing the benefit of having a given number of market makers wait across periods, it may be that the efficient number of market makers either increases or decreases relative to the risk neutral, symmetric information case. The issues of risk aversion and asymmetric information suggest areas for further research.

As a last remark, our analysis has been in the context of a simple two-good, quasi-linear model. This has made the analysis as transparent as possible (and has kept the mechanisms relatively simple), without playing a critical role in the results. It is clear that the negative results concerning the inefficiency of anonymous or competitive markets is not limited to this setting, as the counter-examples behind these results are easily generalized. Also, the structure behind the competitive and efficient mechanisms (Propositions 1 and 3) are based on straightforward intuitions and are generalizable - with the main complications coming in the bidding stages (that endogenize the fines or the role of the market maker), but with little in terms of other changes.

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Appendix

Proof of Lemma 1: The equalities of the derivatives follows from the first order necessary conditions for the program characterizing efficient allocations. We show that M has no more than one member by contradiction. Suppose that M has more that one element. Pick any $i^* \in M$ such that $c_{i^*} = \min_{i \in M} \{c_i\}$, and define a new allocation by:

- $\bar{M} = \{i^*\}$
- $\bar{t}^1_{xi} = t^1_{xi}$ if $i \in N_1 \setminus M$
- $\bar{t}^1_{xi} = t^1_{xi} + t^2_{xi}$ if $i \in M \setminus i^*$
- $\bar{t}^1_{xi^*} = t^1_{xi^*} - \sum_{j \in M \setminus i^*} t^2_{xj}$
- $\bar{t}^2_{xi^*} = t^2_{xi^*} + \sum_{j \in M \setminus i^*} t^2_{xj}$
- $\bar{t}^2_{xi} = t^2_{xi}$ if $i \in N_2$.

This allocation is feasible and is clearly improving: each agent is consuming the same quantity as in the original allocation, but agents belonging to $M \setminus i^*$ are strictly better off because they don't pay the cost of trading in period 2. ▮

Proof of Lemma 2: Note that having each agent consume x^* results in at least as high a utility as setting $x^1 = 1$ and allocating 1 unit of x among the agents present in period 2:

$$(N_1 + N_2)V(x^*) \geq (N_1 - 1)V(1) + (N_2 + 1)V\left(\frac{1}{N_2 + 1}\right).$$

By the strict concavity of V, $V\left(\frac{1}{N_2+1}\right) > \frac{1}{N_2+1}V'\left(\frac{1}{N_2+1}\right)$, so that

$$(N_1 + N_2)V(x^*) > (N_1 - 1)V(1) + V'\left(\frac{1}{N_2 + 1}\right).$$

Therefore, if:

$$(N_1 - 1)V(1) + V'\left(\frac{1}{N_2 + 1}\right) > N_1 V(1) + c^* \tag{4}$$

then (1) is satisfied. (4) simplifies to $V'\left(\frac{1}{N_2+1}\right) \geq V(1) + c^*$. ▮

Proof of Proposition 1: Let $F \geq \sum_i V_i(x_i^{comp}) - V_i(e_i)$ where x^{comp} is the competitive allocation of the x good, and choose any $\epsilon > 0$. First, we show that for such an F the following mechanism fully implements the competitive allocation. After establishing this, we augment the mechanism so that in the unique equilibrium outcome, F is revealed in a first stage and then the mechanism is played.

Let $j(i) = i+1$ if $i < N$ and $j(N) = N-1$.

Stage 1. Agent 1 announces $p_1 \in I\!R_{++}$.

Stage $i > 1$. Agent i either announces $t_i \in I\!R$ or a pair $p_i \neq p_1, \hat{t}_i \neq 0$.

- If each $i > 1$ announces a t_i, then each i trades t_i units of the x good for $-p_1 t_i$ units of the m good. 1 balances these trades and the mechanism ends.
- Otherwise, consider the lowest indexed i who announced $p_i \neq p_1, \hat{t}_i \neq 0$, and proceed to $\Gamma_i(p_i, \hat{t}_i)$.

$\Gamma_i(p_i, \hat{t}_i)$:

Substage 1. Agent 1 says 'Yes' or 'No'.

Substage $k > 1$. Agent k says $\bar{t}_k$.

- Each $k > 1$, $k \neq i$, trades $\bar{t}_k$ for $-p_1 \bar{t}_k$.
- If agent 1 said 'Yes', then i trades $\hat{t}_i$ for $-p_i \hat{t}_i$, and 1 pays $2F$ to $j(i)$.
- If agent 1 said 'No', then i trades $\bar{t}_i$ for $-p_1 \bar{t}_i$, 1 pays $2F$ to $j(i)$, and i pays $\epsilon > 0$ to $j(i)$.

Lemma 3 *Fix an economy. If $p_1 = p^{comp}$ in stage 1, then whenever Γ_i is reached, i is strictly worse off in any subgame perfect equilibrium continuation than if they instead announced $t_i = t_i^{comp}$ (and $\bar{t}_i = t_i^{comp}$, if some Γ_j were reached).*

Proof: If the equilibrium continuation in Γ_i has agent 1 say 'No,' then agent i will pay $\epsilon > 0$ and has a unique most preferred trade of $\bar{t}_i = t_i^{comp}$. This leaves i strictly worse off than trading t_i^{comp} and not paying ϵ. If the equilibrium continuation in Γ_i has agent 1 say 'Yes,' then since all agents (including i) will say $\bar{t}_k = t_k^{comp}$ in the equilibrium continuation after 'No' - agent i must be offering agent 1 a trade $p_i, \hat{t}_i$ that provides 1 with a utility of at least agent 1's competitive utility to have induced 1 to say 'Yes'. (Note that all agents $k \neq i$ will say $\bar{t}_k = t_k^{comp}$ in the equilibrium continuation after 'Yes'.) However, since p_i cannot be equal to p^{comp}, then this must make i worse off than the competitive allocation. ∎

Lemma 4 *Fix an economy. If in stage 1* $p_1 = p^{comp}$, *then there exists a subgame perfect equilibrium continuation and on the equilibrium path of the continuation agents say* t_i^{comp}.

Proof: This follows from Lemma 3. To be careful in verifying that there is a subgame perfect equilibrium continuation, one has to check that there is an equilibrium continuation in each proper subgame, including those off the equilibrium path. This follows from the uniqueness of the optimal trade at any price, and the discrete choice of 'Yes' or 'No'. ∎

Lemma 5 *Fix an economy. If* $p_1 \neq p^{comp}$, *then in any subgame perfect equilibrium continuation (and at least one exists) agent 1 is worse off than at the competitive allocation.*

Proof: Given that agents other than 1 can always ask for 0 trades, the trades (not including fine revenues to any $j(i)$) in any equilibrium continuation must provide each agent with at least the utility of their endowment. This means that 1's continuation utility is at most $\sum_i V_i(x_i^{comp}) - V_i(e_i)$ less any fines that are paid, since the competitive equilibrium maximizes the total gains from trade. Given that i pays $2F$ in fines if any Γ_i is reached, in that case agent 1 is worse off than at the competitive allocation. We show that every equilibrium continuation must have some Γ_i reached. Suppose the contrary and let us show that there is some i who strictly gains by triggering Γ_i. For each $k \neq i$ let $\bar{t}_k$ be the utility maximizing trade given the price p_1. There exists i and $p_i, \hat{t}_i$ which strictly improves both i and 1 compared to the optimal trade for i at p_1, anticipating announcements of $\bar{t}_k$ by $k \neq i$. This follows since p_1 is not the competitive price, and so there exists i such that $V_i'(e_i + t_i) \neq V_1'(e_1 + t_1)$, where t_i is i's optimal trade at p_1 and t_1 is agent 1's trade to balance all other agents' optimal trades at p_1. Any such i can strictly benefit by deviating and announcing such $p_i, \hat{t}_i$ (triggering Γ_i), given the anticipated announcements of t_k by $k \neq i$.

To complete the proof of the lemma, we need to verify that there exists an equilibrium continuation given that $p_1 \neq p^{comp}$. Given the existence of an optimal trade for any agent at any given price, and the discrete choice of 'Yes' or 'No', we need only check that for each agent there is an optimal $p_i, \hat{t}_i$, should they choose to announce one. Let agent 1 say 'Yes' whenever indifferent given the anticipated announcements of $\bar{t}_k$ by each k. Thus, i is faced with choosing a trade to maximize i's utility, subject to the constraint of offering 1 a utility of at least that from balancing all of the trades (including i's) $\bar{t}_k$ at p_1 and to the additional constraint of announcing $p_i \neq p_1$. If we ignore this last constraint it is clear that, given the convexity of preferences, this has a solution. If the solution has $p_i \neq p_1$, then we are fine. If the solution has $p_i = p_1$, then there is an equilibrium continuation where i does not trigger Γ_i, but instead simply announces an optimal t_i in response to p_1. This is the best that i could have hoped for under any continuation where Γ_i would be triggered, given that the optimal $p_i = p_1$. ▮

Next, let us examine the augmented mechanism that in equilibrium reveals the fine $F \geq \sum_i V_i(x_i^{comp}) - V_i(\epsilon_i)$. Consider the following game to be played before Stage 1 listed above:

Bidding (Stage 0):

Substage 1. Agent 1 announces any $F \geq 0$.

Substage 2. Agent 2 either announces some $\bar{F} > F$ and the game proceeds to substage 3, or 'ok' and the game proceeds to Stage 1 (of the previously described mechanism).

Substage 3. Agent 1 announces either 'yes' or 'no'.

- If 1 announces 'yes', then agent 1 makes take it or leave it offers to the other agents who sequentially respond yes or no. All yes trades are completed and agent 1 pays $\bar{F}$ to agent 2 and ϵ to agent 3. The game ends.

- If 1 announces 'no', then there is no trade and agent 2 pays $\bar{F}$ to agent 3 and agent 1 pays ϵ to agent 3. The game ends.

It is easily checked that in any equilibrium of this augmented game, in the first stage agent 1 announces $F \geq \sum_i V(x_i^{comp}) - V(\epsilon_i)$, and agent 2 says 'ok' in substage 2. If agent 1 announces $F < \sum_i V(x_i^{comp}) - V(\epsilon_i)$, then every equilibrium continuation has agent 2 announcing $\overline{F} = \sum_i V(x_i^{comp}) - V(\epsilon_i)$ in substage 2, and agent 1 saying 'yes' in substage 3. ▮

Proof of Proposition 2. Consider economies where the first $N_1 - 1$ first period agents have endowment $\epsilon_i = 1$, the first $N_2 - 1$ second period agents have $\epsilon_i = 1$, and $V_i = V$ for all $i \in N_1 \cup N_2$. Also, assume that all agents have identical costs of trading in second period, c. The remaining two agents have endowments 0 and 2. For convenience, refer to the agent with $\epsilon = 2$ as agent 2, and the agent with $\epsilon = 0$ as agent 0. Consider two economies

Economy A – agent 2 is born in the first period and 0 in the second.

Economy B – agent 0 is born in the first period and 2 in the second.

Let

$$S_0 = (N_1 + N_2)V(1) - N_1 V\left(\frac{N_1 - 1}{N_1}\right) - N_2 V\left(\frac{N_2 + 1}{N_2}\right).$$

and

$$S_2 = (N_1 + N_2)V(1) - N_1 V\left(\frac{N_1 + 1}{N_1}\right) - N_2 V\left(\frac{N_2 - 1}{N_2}\right).$$

Suppose that all equilibria of the given mechanism relative to both of these economies (for each specification of c and V) are efficient.

Consider economy A and any c where it is efficient to have trade across periods. Given trade across periods, the efficient x amount is 1 for all agents. Consider any equilibrium in the second period and the amount of m transferred from 0 to 2. Let U_2 be any equilibrium utility for agent 2 if no trade occurs across periods. In order to have an efficient choice by agent 2 for any c, it must be that m is precisely

$$m = S_2 - V(1) + U_2.$$

Using similar reasoning for economy B and agent 0

$$-m = S_0 - V(1) + U_0.$$

Combining these equations:

$$2V(1) = S_0 + S_2 + U_0 + U_2.$$

Thus, given individual rationality

$$2V(1) \geq S_0 + S_2 + V(0) + V(2).$$

or

$$2V(1) - (V(0) + V(2)) \geq S_0 + S_2.$$

The left hand side is the surplus that would be generated from trade across periods if agents 0 and 2 were the only agents. The right hand side has the total surplus from trade across periods from both economies we have considered. If V is approximately $V(x) = \min(1, x)$ (adjusted to be strictly concave), then approximately $2V(1) - (V(0) + V(2)) = 1$, $S_0 = 1$, and $S_2 = 1$. This implies that the inequality cannot be satisfied, which is a contradiction. ▮

Proof of Proposition 3. The following mechanism results in a Pareto efficient and individually rational allocation for every complete information economy.

Bidding Stage.

- Substage 1. Agent 1 submits a bid $b_1 \in I\!R$. Go to substage 2.
- Substage i, with $N_1 > i > 1$. Agent i either submits a bid $b_i \in I\!R$ or announces a pair $\left(j, \bar{b}\right)$ with $j < i$ and $\bar{b} > b_j$. If i announces a bid, then go to substage $i+1$. If i announces $(j, \bar{b})$, then go to game $G(\bar{b}, j, i)$.
- Substage N_1. Agent N_1 either submits a bid $b_{N_1} \in I\!R$ or announces a pair $\left(j, \bar{b}\right)$ with $j < N_1$ and $\bar{b} > b_j$. If i announces a bid, then define i^* as the agents who announced the highest bid, breaking ties in favor of the highest index, and go to game $\Gamma^1_{MM}(i^*)$. Otherwise, go to game $G(\bar{b}, j, N_1)$.

Game $G(\bar{b}, j, i)$. Agent j is offered a choice between obtaining market making rights and paying $\bar{b}$ and going to game Γ_A (and then no agent can wait across periods). In the first case, each agent pays $\bar{b}$ to agent i and the game goes to $\Gamma^1_{MM}(j)$.

Game $\Gamma^1_{MM}(i^*)$. Without loss of generality, reorder the agents $j \neq i^*$ from 1 to $N_1 - 1$.

- Substage 0. Agent i^* suggests $t^1 \in T^{N_1}$.
- Substage i. Agent i chooses from {yes, no}. If 'yes' go to substage $i+1$ or, if $i = N_1 - 1$, stop. If 'no' go to game Γ_A.

If all agents $i \neq i^*$ said 'yes', then they consume their after-trade allocation $(e_i + t^1_{xi}, t^1_{mi})$, and agent i^* can either consume her after-trade allocation or carry it to period 2. In that case, run Game $\Gamma^2_{MM}(i^*)$ in period 2. Otherwise, run static mechanism Γ_B in period 2.

Game $\Gamma^2_{MM}(i^*)$. Let $\{1, \ldots, N_2\}$ be an enumeration of agents in N_2.

- Substage 0. Agent i^* suggests $t^2 \in T^{N_2+1}$.
- Substage i. Agent i chooses from {yes, no}. If 'yes' go to substage $i+1$ or, if $i = N_2$, implement the trade t^2 suggested at substage 0. If 'no' go to game Γ_B.

Game Γ_A:

If $N_1 = 1$, then it is autarchy.

If $N_1 = 2$, then agent 1 makes a take it or leave it offer of a trade to agent 2, and then agent 2 can accept or reject.

If $N_1 \geq 3$, then run the competitive mechanism described in Proposition 1.

Game Γ_B

This is the same as Γ_A, replacing N_2 for N_1, and excluding the market maker if she is present.

Let us show that this mechanism implements an efficient allocation.

It is clear that once an agent i^* has obtained market making rights the only outcome in the two periods is the one that extracts from the other agents as much surplus as possible under the constraint that each agent obtains at least as much as in the static equilibrium of the corresponding Γ^A or Γ^B, and that this results in an efficient allocation subject to i^* being the only agent who can trade across periods. The only problem is to show that the right agent obtains market making rights at the bidding stage.

Let S_i be the maximal surplus that agent i can extract if she gets MM rights. That is, S_i is the difference between the total welfare of the efficient allocation subject to only having i able to trade across periods, and the allocation found by running Γ^A in the first period and Γ^B in the second period, with no trade across periods. At the bidding stage every agent $i < N_1$ bids exactly S_i. If this were not so, then agent $i+1$ could find $\bar{b}$ such that at game $G(\bar{b}, i, i+1)$ agent i chooses market making rights and $i+1$ obtains a larger payoff.

This implies that if the agent with the highest S_i has index less than N this agent obtains market making rights. If S_N is the highest, then agent N will announce $\max_{i \neq N} S_i$ and will obtain market making rights.[19] ∎

Proof of Proposition 4. We offer the proof for the case where $N_1 = N_2 = 1$. Agent 1 is the first period agent, and agent 2 is the second period agent. This easily extends to more agents.

Suppose that there exists a mechanism satisfying the conditions of the Proposition which does implement an individually rational and efficient allocation correspondence. Consider economies where: $e_1 = 1$ and $e_2 = 0$, c_1 is either 1 or 2, agent 1 has preferences V_1, and agent 2 has either preferences V_2 or $\overline{V}_2$.

[19] We are using the tie-breaking rule assigning the right to the highest index in case of identical bids.

Let $V_1'(\frac{1}{2}) = V_2'(\frac{1}{2}) = \overline{V}_2'(\frac{1}{2})$, so that $\frac{1}{2}$ is the x allocation for each agent whenever trade across periods is efficient. Also, let

$$V_2\left(\frac{1}{2}\right) - V_2(x) > \overline{V}_2\left(\frac{1}{2}\right) - \overline{V}_2(x) \quad \text{for any} \quad x \neq \frac{1}{2}. \tag{5}$$

Finally, let

$$V_1\left(\frac{1}{2}\right) + V_2\left(\frac{1}{2}\right) > V_1(1) + 2 \tag{6}$$

and

$$V_1(1) + 2 > V_1\left(\frac{1}{2}\right) + \overline{V}_2\left(\frac{1}{2}\right) > V_1(1) + 1. \tag{7}$$

The inequalities imply that intertemporal trade is always efficient when agent 2's preferences are described by V_2. When preferences are described by $\overline{V}_2$ then intertemporal trade is efficient if $c_1 = 1$ and inefficient if $c_1 = 2$.

Consider the economy with $c_1 = 1$ and $\overline{V}_2$, and an equilibrium which results in an individually rational and efficient allocation. The allocation for agent 2 must be of the form $(\frac{1}{2}, -\overline{m})$, where $0 < \overline{m} \leq \overline{V}(\frac{1}{2})$.

Next, consider the economy with $c_1 = 2$ and V_2. Here, since the equilibrium allocation is efficient, agent 1 must wait to trade across periods. Consider the second period game form given that agent 1 has waited. There cannot exist an equilibrium with an allocation for agent 2 of $(\frac{1}{2}, -\overline{m})$, since otherwise agent 1 would not have chosen to wait since $V_1(1) + 2 > V_1\left(\frac{1}{2}\right) + \overline{m}$. Thus, given that there are no proper subgames, agent 2 must have an improving deviation (since only agent 2's preferences have changed relative to the other economy) which results in some allocation (x, m) for agent 2, where

$$V_2(x) + m > V_2\left(\frac{1}{2}\right) - \overline{m},$$

but, since this is an equilibrium allocation for the economy with $\overline{V}_2$ and $c_1 = 1$ (and the second period mechanism is the same by the condition of the independence of the second period mechanism of information other than t^1, M):

$$\overline{V}_2(x) + m \leq \overline{V}_2\left(\frac{1}{2}\right) - \overline{m}.$$

The last two inequalities combined imply that

$$V_2(x) - V_2(\frac{1}{2}) > \overline{V}_2(x) - \overline{V}_2(\frac{1}{2}),$$

which contradicts the assumptions on V_2 and $\overline{V}_2$. (For an example of functions V_2 and $\overline{V}_2$ satisfying (5) (6), (7), and $V_2'(\frac{1}{2}) = \overline{V}_2'(\frac{1}{2})$ pick $V_2(x) = 2x^{\frac{1}{2}} + x$ and $\overline{V}_2(x) = x^{\frac{1}{2}} + (\frac{1}{\sqrt{2}} + 1)x$.) ∎

Proof of Proposition 5. There is one period 1 agent endowed with 5 units of good x. There are two period 2 agents, endowed with 1 unit of x, and 0 units, respectively. Each has $V_i = V$. We name traders by their initial endowment, so that 5 is the agent born in the first period and 0 and 1 are the agents in the second.

Let

$$V(5) = 3V(2) - 2V(\frac{1}{2}) - \epsilon - c$$

where $\epsilon > 0$, so that it is efficient to trade across periods.

The efficient trades in the second period are to have agent 0 buy 2 units of the x good, and agent 1 buy 1 unit. Let p^2 be the price quoted for 2 units of the x good, and p^1 be the price quoted for 1 unit of the x good.

For agent 0 to choose to buy 2 units of x rather than 1, it must be that

$$V(2) - p^2 \geq V(1) - p^1.$$

This implies that

$$p^2 \leq p^1 + V(2) - V(1).$$

Individual rationality for agent 1 implies that

$$p^1 \leq V(2) - V(1).$$

Thus,

$$p^2 \leq 2(V(2) - V(1)).$$

For agent 5 to make the efficient choice to trade across periods, it must be that

$$V(2) + p^2 + p^1 - c \geq V(5).$$

Substituting from the inequalities for p^1 and p^2, it must be that

$$4V(2) - 3V(1) - c \geq V(5).$$

Thus,

$$4V(2) - 3V(1) - c \geq 3V(2) - 2V(\frac{1}{2}) - \epsilon - c$$

This implies that

$$V(2) - V(1) \geq 2\left(V(1) - V(\frac{1}{2})\right) - \epsilon.$$

For any strictly concave V, $V(2) - V(1) < 2\left(V(1) - V(\frac{1}{2})\right)$. Thus, we can find V and a small enough ϵ such that agent 5 does not make the efficient choice. ∎

Proof of Proposition 6. Let $W(M,c)$ denote the total expected utility when an efficient allocation is chosen, subject to the constraint that $M \leq N_1$ agents wait across periods. That is,

$$W(M,c) = (N_1 - M)V(x^M) + (N_2 + M)E\left[V\left(\frac{e^1 - (N_1 - M)x^M + \tilde{e}^2}{N_2 + M}\right)\right] - cM$$

where x^M is chosen to maximize the above expression, $e^1 = \sum_{i \in N_1} e_i$, and $\tilde{e}^2 = \sum_{i \in N_2} e_i$ (and $\tilde{e}^2$ is a random variable with finite expectation and positive variance).

Under our assumptions on V, $W(M,c)$ is continuous in c by the theorem of the maximum. We show that the following are also true: $W(M+1,c) - W(M,c)$ is decreasing in M and decreasing in c. These facts, coupled with the observations that for large enough c we have $W(0,c) > W(M,c)$ for any $M > 0$; and $W(N_1, 0) > W(M, 0)$ for any $M < N_1$, establish the proposition.[20]

We first show that $W(M+1,c) - W(M,c)$ is decreasing in M. We show that for any $M \leq N_1 - 2$

$$W(M+2,c) - W(M+1,c) < W(M+1,c) - W(M,c)$$

This can be rewritten as

$$W(M+1,c) > \frac{1}{2}W(M+2,c) + \frac{1}{2}W(M,c) \tag{8}$$

Let

$$\overline{x} = \frac{N_1 - M}{2(N_1 - M - 1)}x^M + \frac{N_1 - M - 2}{2(N_1 - M - 1)}x^{M+2}.$$

We show that

$$(N_1 - M - 1)V(\overline{x}) + (N_2 + M + 1)E\left[V\left(\frac{e^1 - (N_1 - M - 1)\overline{x} + \tilde{e}^2}{N_2 + M + 1}\right)\right] - (M+1)c > \tag{9}$$

$$\frac{1}{2}W(M+2,c) + \frac{1}{2}W(M,c).$$

which implies (8) since the left hand side evaluated at $\overline{x}$ is no more than when evaluated at x^{M+1} by the definition of x^{M+1}. First, by strict concavity and the definition of $\overline{x}$

$$(N_1 - M - 1)V(\overline{x}) > (N_1 - M - 1)\left[\frac{N_1 - M}{2(N_1 - M - 1)}V(x^M) + \frac{N_1 - M - 2}{2(N_1 - M - 1)}V(x^{M+2})\right].$$

[20] When c is larger than the maximal total surplus of the economy it is clear that having any agent wait results in a loss of welfare. When $c = 0$ there is no loss in waiting, and having agents wait allows for additional smoothing of consumption across states in the second period.

which simplifies to

$$(N_1 - M - 1)V(\overline{x}) > \frac{1}{2}(N_1 - M)V(x^M) + \frac{1}{2}(N_1 - M - 2)V(x^{M+2}). \tag{10}$$

For any M, let y^M denote the random variable

$$y^M = \frac{\epsilon^1 - (N_1 - M)x^M + \tilde{\epsilon}^2}{N_2 + M}.$$

Define $\overline{y}$ by

$$\overline{y} = \frac{\epsilon^1 - (N_1 - M - 1)\overline{x} + \tilde{\epsilon}^2}{N_2 + M + 1}.$$

Note that by the definition of $\overline{x}$, $\overline{y}$, and y^M

$$\overline{y} = \frac{N_2 + M}{2(N_2 + M + 1)}y^M + \frac{N_2 + M + 2}{2(N_2 + M + 1)}y^{M+2}.$$

Then, by the strict concavity of V we have that for each realization of $\tilde{\epsilon}^2$ we have:

$$V\left(\overline{y}\left(\epsilon^2\right)\right) > \frac{N_2 + M}{2(N_2 + M + 1)}V\left(y^M\left(\epsilon^2\right)\right) + \frac{N_2 + M + 2}{2(N_2 + M + 1)}V\left(y^{M+2}\left(\epsilon^2\right)\right).$$

Taking expectation and multiplying by $(N_2 + M + 1)$ on both sides we obtain:

$$(N_2 + M + 1)E[V(\overline{y})] > \frac{1}{2}(N_2 + M)E[V(y^M)] + \frac{1}{2}(N_2 + M + 1)E[V(y^{M+2})]. \tag{11}$$

Combining (10) and (11), we obtain (9).

Finally, we show that $W(M+1,c) - W(M,c)$ is decreasing in c. Notice that

$$\frac{\partial W(M,c)}{\partial c} = -M \tag{12}$$

and so

$$\frac{\partial}{\partial c}\left[W(M+1,c) - W(M,c)\right] = -1,$$

proving the claim. ∎

Proof of Proposition 7: We briefly describe the implementing mechanism. It is a simple variation of the mechanism in the proof of Proposition 3. The proof of implementation is then a simple variation on the proof in Proposition 3.

Designate agent $1 \in N_1$ to be market maker. Run $\Gamma^1_{M,M}(1)$ as described in the proof of Proposition 3, with the following changes. In stage 1, agent 1 announces $t^1 \in T^{N_1}$ and $M \subset N_1$. For agents $i \neq 1$, $i \in M$, saying 'yes' in stage i commits the agent to the trade prescribed by t^1 and waiting to trade again in the second period, and if $1 \in M$ then 1 is committed to trading in the second period.[21] In the event that $M \neq \emptyset$ agents wait across periods, run mechanism $\Gamma^2_{MM}(i^*)$ as described in the proof of Proposition 3, where i^* is the lowest indexed agent in M, trades t^2 announced in $\Gamma^2_{MM}(i^*)$ are in $T^{N_2 \cup M}$, and where substages are added for agents $i \neq i^*$, $i \in M$, to say 'yes' or 'no'. If any agent $i \in N_2 \cup M \setminus \{i^*\}$ says 'no' in the second period, then the mechanism Γ_B is run only among N_2 and the other agents consume their current holdings.

The only difference from the proof of proposition 3 is that agents $i \in M$ correctly anticipate their expected utilities under the equilibrium continuation, and make decisions in period 1 based on whether these are at least what they would get from Γ_A. (The agents in M know that in the second period agent i^* will extract any surplus above their current holdings $(\epsilon_i + t^1_{xi}, t^1_{mi})$. Thus, in effect, agent 1 pays them 'up front' for waiting until the second period.) The market maker again will appropriate the full surplus over the running of mechanisms Γ_A and Γ_B. Thus, she has the incentives to choose the correct M. The rest of the proof is then identical to that in proposition 3, except that we do not need to worry about the selection of a market maker. ∎

[21] This choice is binding relative to agents present in period 1 - this has not, however, placed any constraints on agents born in period 2. Agents will only trade t^1 if all agents say yes in all substages i of $\Gamma^1_{MM}(1)$. M would then wait across periods.