Co-ordination, Spillovers and Cheap Talk*

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Abstract

We analyze the role of cheap-talk in two player games with one-sided incomplete information. We identify conditions under which (1) players can fully communicate and coordinate on efficient Nash equilibria of the underlying complete information game; and (2) players cannot communicate so cheap-talk does not alter the equilibrium set of the Bayesian game. We present examples that illustrate several issues that arise when there is two-sided incomplete information and also analyze the role of cheap-talk in the electronic mail game of Rubinstein [1989].

1. Introduction

When are cheap talk statements credible? Following Farrell [1993], it is often argued that a cheap talk statement about your planned behavior is credible if it is *self-committing*: it you expected your cheap talk statement to be believed, you would have an incentive to carry out your plan. Under this view, players should always be able to coordinate on a Pareto-dominant Nash equilibrium of a complete information game. Aumann [1990] suggested a more stringent *self-signalling* requirement for credibility: your cheap talk statement about your planned behavior is only credible if you would only want it to be believed if in fact it was true.

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The contrast between the views is especially striking if one considers games with both strategic complementarities and positive spillovers: each player's best response is increasing in other players' actions; and each player would prefer that other players choose higher actions. Such games arise naturally in many economic settings, including bank runs and various oligopoly problems. Thus a depositor in a bank will always want other depositors to leave their money in the bank. This being the case, why should he be believed when he claims he is going to leave his money in the bank? There may be an equilibrium where all depositors leave their money in the bank, but what is it about his cheap talk statement that makes it more credible that the speaker will play according to this desirable equilibrium? Similarly, a duopolist always wants his competitor to choose lower output. Why should he be believed when he claims he will produce low output (even if it is part of a Nash equilibrium)?

We briefly review the complete information debate about cheap talk and coordination in Section 2. However, our purpose in this paper is to derive lessons about cheap talk and coordination with *incomplete information*. In this context, we can formalize the idea that self-signalling is necessary for cheap talk statements to be credible and that positive spillovers prevent communication. We will focus on two player games with one-sided incomplete information (though we will offer comments and examples to demonstrate the new issues that arise in games of two-sided incomplete information). Crawford and Sobel [1982] also looked at cheap talk games of one-sided incomplete information but allowed only one player, the receiver, to take an action after the player with information, the sender, sends a message in the cheap talk stage. We allow the sender and receiver to play a *game* and therefore for *both* to take actions after the cheap talk stage. This, therefore, generates our key new issue: if and when is it possible for the sender to credibly communicate his type-dependent action in the game and to coordinate on efficient outcomes?

Formally, we consider two questions. When is there full communication, in the sense that the informed player truthfully reveals his type and the two players then play a Nash equilibrium of the underlying complete information game? And when is there no communication, so that the equilibria of the cheap talk game are outcome equivalent to equilibria where cheap talk is not allowed?

Consider first our full communication result. A complete information game is *self-signalling* if a player does not have an incentive to deceive his opponent about which action he intends to take. In particular, conditional on a player being forced to choose action a, he would prefer that his opponent choose the best response to a to his choosing the best response to any other action a'. An action a for the informed player is a *Stackelberg action* if he could not do better by committing to another action a' and allowing his opponent to choose a best response to a'. We show that in a two player, private values, one-sided incomplete information game with self-signalling satisfied and a Stackelberg, Nash equilibrium existing for every type profile, there exists an equilibrium with communication where players truthfully announce their types and play according to the Stackelberg action, Nash equilibrium for the announced type profile (Proposition 3.2). The self-signalling condition rules out the difficulty identified by Aumann [1990]. The Stackelberg action requirement rules out play of a dominated equilibrium for the informed player. Otherwise, he would have the incentive to lie about his type to get his opponent to coordinate on his preferred equilibrium. Therefore, our full communication result shows that these are the only two conflicts between players that impede the successful transmission of information in a private values environment.

Our sufficient condition for no communication is much stronger. We show that when the uninformed player only has two actions and all types of the informed player have the same preferences over these actions, no communication can occur in equilibrium (Proposition 3.5). Thus a failure of self-signalling implies that communication is impossible. However, we show by example that in many action games, even when the sender's preferences over mixed strategies are type-independent, communication is possible (though it is very fragile). We also present a series of examples that demonstrate that communication when there is two-sided incomplete information presents new issues involving transmission of correlated information and two-way communication.

The above results are described in Section 3. In Section 4, we describe a correlated type example not covered by our general results (cheap talk in the electronic mail game). We begin with a review of the complete information cheap-talk literature.

2. Cheap Talk and Coordination with Complete Information: A Review

We provide a very brief review in this section of some key ideas in the literature on cheap talk with complete information. The reader is referred to Farrell and Rabin [1996] for a more complete discussion.

2.1. Leading Example

Throughout the paper, we will be interested in the following example. Two players/firms must decide whether to invest (I) or not invest (N). The two players might be firms in different industries but in the same local area so one firm's investment decision affects the other through demand externalities. Or they could be two firms that do business with each other and are deciding whether to upgrade to a new technology, say, new software. Not investing is the choice of staying with the existing technology, while investing involves adopting the new technology, i.e., purchasing new software. There is a cost c of investing and the firms receive a return on their investment only if *both* invest: we assume they each receive a gross return of 100 in this case. Thus there are strategic complementarities (each firm has more incentive to invest if the other firm invests). In addition, there is a "spillover" x that one firm receives if the *other* firm invests, independent of whether the first firm invests. Thus payoffs are given by the following matrix:

		Opponent's Action	
		I	N
Own Action	I	$100 - c + x$	$-c$
	N	x	0

FIGURE 1

When the two firms are in different industries in the same locality, firm 1's investment might increase the firm 2's profit by increasing demand in the local economy, so there will be a positive spillover ($x > 0$). If the two firms are doing business together, there may be a cost to firm 2 if one firm 1's upgrades its technology, so there will be a negative spillover ($x < 0$).

If it is common knowledge that the cost of investing is "low" ($c = 90$) for both players, payoffs are given by the following bi-matrix:

		Player 2	
		I	N
Player 1	I	$10 + x, 10 + x$	$-90, x$
	N	$x, -90$	$0, 0$

FIGURE 2

This game has two strict Nash equilibria: both invest and both not invest. If $x > -10$, the both invest equilibrium is an efficient outcome in the game. Conventional wisdom holds that if one player (say, player 1) is able to communicate about his intentions in this game, the efficient equilibrium will be played. Farrell [1993] argued that the promise "I will invest" by player 1 will be credible to player 2 because if player 1 expected his statement to be believed, he would have an incentive to carry out his promise. Farrell and Rabin [1996] label this notion of credibility *self-commitment*. The promise "I will invest" will always be credible in the above game, and since it leads to player 1's most preferred payoff, he will always choose to make it.

Aumann [1990] has argued that this criterion of credibility is insufficient, at least in some cases. Consider the above game in the case where $x = 1$, so the payoff matrix becomes

Player 2

		I	N
Player 1	I	$11, 11$	$-90, 1$
	N	$1, -90$	$0, 0$

FIGURE 3

In this case, player 1 would like player 2 to invest independent of the action that player 1 plans to carry out. This being the case, a promise to invest by player 1 conveys no information about player 1's actual intent. In particular, suppose that player 1 thought it was likely that player 2 would ignore any cheap talk statement and would play safe by not investing but that there was a positive probability that player 2 would believe his cheap talk statement. Then he would have an incentive to announce "I will invest" and not do so.

Aumann's critique suggests a stronger credibility requirement (again, the terminology is taken from Farrell and Rabin [1996]): A statement is *self-signalling* if the speaker would want it to be believed only if it is true. The statement "I will invest" is self-signalling for player 1 in the game of figure 2 only if $x \leq 0$. If $x > 0$, the statement "I will invest" is not self-signalling because player 1 would want that statement to be believed even if he were planning to not invest.

The existing literature on cheap talk refinements in complete information games has followed Farrell's lead in focussing on variations of the self-commitment notion of credibility while ignoring the self-signalling issue. Both evolutionary models of equilibrium selection with cheap talk (Kim and Sobel [1995]) and experimental work (Charness [1998]) seem to confirm Farrell's view that self-commitment and not self-signalling is the key credibility requirement under complete information.

2.2. Formalizing the Credibility Properties

There are two players, 1 and 2. Each player i has a finite action set A_i. A complete information game is described by a pair of payoff functions (g_1, g_2), with each $g_i : A \to \mathbb{R}$ (where $A = A_1 \times A_2$). We will focus on generic complete information games, assuming in particular that $g_i(a) \neq g_i(a')$ for all $a, a' \in A$ with $a \neq a'$. With this restriction, pure strategy best response functions, $b_i : A_j \to A_i$, are well-defined:

$$b_i(a_j) \equiv \underset{a_i \in A_i}{\arg\max} \; g_i(a_i, a_j)$$

We want to consider the credibility of statements by one player, say, player 1.

Definition 2.1. *Action a_1 is self-committing if $g_1(a_1, b_2(a_1)) \geq g_1(a_1', b_2(a_1))$ for all $a_1' \in A_1$.*

Action a_1 is self-committing if it is the optimal action for player 1 if he expects his opponent to choose a best response to action a_1. Action a_1 is self-committing exactly if $(a_1, b_2(a_1))$ is a pure strategy Nash equilibrium of g. In the game of figure 2, the action invest is always self-committing.

We will later be interested in which action a player would wish to commit to (if he were able to commit).

Definition 2.2. *Action a_1 is the Stackelberg action if $g_1(a_1, b_2(a_1)) \geq g_1(a_1', b_2(a_1'))$ for all $a_1' \in A_1$.*

In the game of figure 2, the action invest is the Stackelberg action for player 1 if $x > -10$. If a_1 is self-committing and a_1 is the Stackelberg action, then $(a_1, b(a_1))$ is the pure strategy Nash equilibrium g most preferred by player 1.

Formalizing the idea of self-signalling (a statement is self-signalling if the speaker would want it to be believed only if it is true) is tricky as it depends on how other statements would be evaluated. The following definition is a very strong property that requires that all statements about the action to be played would be self-signalling.

Definition 2.3. *The game g is self-signalling (for player 1) if $g_1(a_1, b_2(a_1)) \geq g_1(a_1, a_2)$ for all $a_1 \in A_1$ and $a_2 \in A_2$.*

This property requires that if a player is going to choose an action a_1, then he would like his opponent to choose a best response to that action. The game of figure 2 is self-signalling only if $x \leq 0$: if $x > 0$, then if player 1 planned to not invest, he would prefer that this opponent invest (which is not a best response). Observe that if a_1 is the Stackelberg action and the game g is self-signalling, then a_1 is self-committing and $(a_1, b_2(a_1))$ is the most preferred action profile of player 1, since for all $a_1' \in A_1$ and $a_2 \in A_2$

$$\begin{aligned} g_1(a_1, b_2(a_1)) &\geq g_1(a_1', b_2(a_1')), \text{ since } a_1 \text{ is Stackelberg} \\ &\geq g_1(a_1', a_2), \text{ since } g \text{ is self-signalling} \end{aligned}$$

Some of the significance of these properties can be seen by considering games where there is a natural order on the actions of both players. Following the terminology of Cooper and John [1998], we use the following definitions.

Definition 2.4. *The game g has strategic complementarities for player i if b_i is increasing.*

Definition 2.5. *The game g has positive spillovers for player i if $g_i(a_i, a_j)$ is strictly increasing in a_j.*

Under the natural ordering, the game of figure 2 always has strategic complementarities (for both players), but it has positive spillovers only if $x > 0$ (i.e., exactly when the game is not self-signalling). If a game g has strategic complementarities and positive spillovers for both players, then there is a "largest equilibrium" which is Pareto-preferred by both players. Milgrom and Roberts [1996] show that this largest and Pareto-preferred equilibrium is [1] a semistrong Nash equilibrium, i.e., robust to any coalitional deviation that is itself robust against any individual deviation; and [2] is a coalition-proof correlated equilibrium for any coalition communication structure. This suggests that cheap talk might be especially effective in achieving the efficient equilibrium in this setting (and Milgrom and Roberts cite many economic applications where strategic complementarities and positive spillovers are both satisfied).

Yet it is immediate that a game with positive spillovers fails the self-signalling condition. In a game with positive spillovers, a player always has an incentive to get his opponent to choose a high action, independent of what he plans to do. Thus if the Aumann [1990] critique has any relevance, it surely applies to games with positive spillovers. In the next section, we will see that with incomplete information, some form of self-signalling will be required for effective cheap talk communication and positive spillovers will tend to preclude communication.

3. Cheap Talk and Coordination with Incomplete Information

In the literature on cheap talk in complete information games discussed in the previous section, authors propose refinements of Nash equilibrium based on intuitive credibility requirements. There is no formal way of evaluating the correctness of the intuition behind the solution concepts. In studying coordination with incomplete information, we can restrict attention to the standard solution concept of perfect Bayesian equilibrium (with no special refinements), and then see which features of the coordination game allow cheap talk about actions to be effective in equilibrium. We will see that the need for self-signalling and the incentive problems created by positive spillovers emerge naturally from the equilibrium analysis.

3.1. Leading Example

Now let there be some uncertainty about the cost of investing. The cost of investing is either low ($c = 90$) or high ($c = 110$), giving the following payoff matrices:

		Opponent's Action	
		I	N
Own Action	I	$10 + x$	-90
	N	x	0

Low Cost

		Opponent's Action	
		I	N
Own Action	I	$-10 + x$	-110
	N	x	0

High Cost

FIGURE 4

Thus a player with high costs has a dominant strategy to not invest. But if it was common knowledge that both firms had low costs, we would have the game of Figure 2 and there would be an equilibrium where both invested.

We assume that there is incomplete information about costs. While player 2 is known to be low cost, player 1 is low cost with probability $\frac{4}{5}$ and high cost with probability $\frac{1}{5}$. Notice that we can analyze the equilibria of this incomplete information game without knowing the value of x: it is strategically irrelevant. If player 1 is high cost, he has a dominant strategy to not invest. Thus player 2 assigns probability at least $\frac{1}{5}$ to player 1 not investing. Thus the net gain to the (low cost) player 2 (from investing over not investing) is at most $\frac{4}{5}(10)+\frac{1}{5}(-90) = -10 < 0$; so there is no investment in any equilibrium.

This outcome is inefficient: both players would gain if they could co-ordinate on investment when both their costs are low (as long as $x > -10$). We will allow player 1 to make cheap talk statements before the players simultaneously choose actions, and see how this influences the outcome. It turns out that what is crucial is whether there are positive spillovers: the *sign* of x is critical.

3.1.1. Adding Cheap Talk without Positive Spillovers

Let $x = -1$, so that the above payoff matrices become:

Opponent's Action

Own Action

	I	N
I	9	−90
N	−1	0

Low Cost

Opponent's Action

Own Action

	I	N
I	−11	−110
N	−1	0

High Cost

FIGURE 5

The following is an equilibrium: player 1 truthfully announces his type. If he announces that he is low cost, both players invest. If he announces that he is high cost, both players don't invest.

3.1.2. Adding Cheap Talk with Positive Spillovers

Let $x = 1$, so that the above payoff matrices become:

Opponent's Action

Own Action

	I	N
I	11	−90
N	1	0

Low Cost

Opponent's Action

Own Action

	I	N
I	−9	−110
N	1	0

High Cost

FIGURE 6

Truth-telling is longer an equilibrium. The problem is that now the high cost type of player 1 - who has a dominant strategy to not invest - would now strictly prefer that player 2 invests nonetheless. Thus the low cost type of player 1 can no longer credibly convey information. One can verify that every equilibrium of the game with cheap talk has no investment in equilibrium.

Thus by increasing x from -1 to 1 (i.e., making the both invest equilibrium more attractive for both players), we have paradoxically destroyed the possibility of efficient investment in equilibrium.[1]

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[1] As V. Bhaskar has pointed out to us, all that actually matters is that x has increased for the

3.2. The Model

There are 2 players, 1 and 2. Each player i has a finite set of possible actions, A_i. We write $A = A_1 \times A_2$. Player 1 is one of a finite set of possible types, T. The prior over the type space is $\pi \in \Delta(T)$. The informed player 1's utility function is $u_1 : A \times T \to \mathbb{R}$; the uninformed player 2's utility function is $u_2 : A \to \mathbb{R}$.

Without cheap talk, a behavioral strategy for the informed player is a function $\widehat{\alpha}_1 : T \to \Delta(A_1)$. A behavioral strategy for the uniformed player is just a mixed strategy $\widehat{\alpha}_2 \in \Delta(A_2)$. A (Bayes Nash) equilibrium is defined in the usual way.

Now add a cheap talk stage. There is a discrete message space for the informed player 1, M. Therefore, player 1's has a *talking strategy,* $\mu : T \to \Delta(M)$; and an *action strategy,* $\alpha_1 : M \times T \to \Delta(A_1)$. The action strategy for player 2 is $\alpha_2 : M \to \Delta(A_2)$. Beliefs for player 2 are $\lambda : M \to \Delta(T)$. We will be interested in *perfect Bayesian equilibria* of the game with cheap talk: $(\mu, \alpha_1, \alpha_2, \lambda)$ is a perfect Bayesian equilibrium [PBE] if each player is playing optimally at all his information sets given the strategy of the other and beliefs are updated using Bayes' rule whenever possible.

Definition 3.1. $(\mu, \alpha_1, \alpha_2, \lambda)$ *is a perfect Bayesian equilibrium if*

[1] $\mu(m|t) > 0 \Rightarrow m \in \underset{m' \in M}{\arg\max} \sum_{a \in A} \left[\alpha_1(a_1|m', t)\, \alpha_2(a_2|m')\right] u_1(a, t)$

[2] $\alpha_1(a_1|m, t) > 0 \Rightarrow a_1 \in \underset{a_1' \in A_1}{\arg\max} \sum_{a_2 \in A_2} \alpha_2(a_2|m)\, u_1(a_1', a_2, t)$

and $\alpha_2(a_2|m) > 0 \Rightarrow a_2 \in \underset{a_2' \in A_2}{\arg\max} \sum_{t \in T} \pi(t) \sum_{a_1 \in A_1} \alpha_1(a_1|m, t)\, u_2(a_1, a_2')$

[3] $\lambda(t|m) = \frac{\pi(t)\mu(m|t)}{\sum_{t' \in T} \pi(t')\mu(m|t')}$

for all $m \in M_\mu \equiv \{m' : \mu(m'|t') > 0 \text{ for some } t' \in T\}$.

3.2.1. Full Communication

If the type of the informed player were common knowledge, then the players would choose a Nash equilibrium of the corresponding complete information game for each realized type. Suppose we fix a Nash equilibrium for each realized type. When is there an equilibrium of the game with incomplete information of player 1's type and cheap talk, where player 1 truthfully announces his type and the full information equilibrium is replicated?

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committed (high cost) type. If x were positive for player 2 and the low cost type of player 1 but negative for the high cost type of player 1, we could still have communication in equilibrium.

Suppose that there was a message corresponding to each possible type of player 1, i.e., $T \subseteq M$. Write μ^* for the fully revealing talking strategy, i.e.,

$$\mu^* \left(m \left| t \right. \right) = \begin{cases} 1, \text{ if } m = t \\ 0, \text{ if } m \neq t \end{cases} .$$

Let $f_i : T \to A_i$ for each player i. We say that $f = (f_1, f_2)$ is played in a full revelation equilibrium of the cheap talk game if the following strategies-beliefs are a perfect Bayesian equilibrium of the cheap talk game:

$$\begin{aligned} \mu &= \mu^* \\ \alpha_1 \left(a_1 \right| m, t) &= \begin{cases} 1, \text{ if } a_1 = f_1 (t) \\ 0, \text{ otherwise} \end{cases} \\ \alpha_2 \left(a_2 \right| m) &= \begin{cases} 1, \text{ if } a_2 = f_2 (m) \\ 0, \text{ otherwise} \end{cases} \\ \lambda \left(t \left| m \right. \right) &= \begin{cases} 1, \text{ if } t = m \\ 0, \text{ otherwise} \end{cases} \end{aligned}$$

Proposition 3.2. *Let $A_2^* = \{a_2 : a_2 = f_2 (t) \text{ for some } t \in T\}$. Then f is played in a full revelation equilibrium if and only if*

$$u_1 (f_1 (t), f_2 (t), t) \geq u_1 (a_1, a_2, t) \tag{3.1}$$

for all $t \in T$ and $a_2 \in A_2^*$; and

$$u_2 (f_1 (t), f_2 (t)) \geq u_2 (f_1 (t), a_2) \tag{3.2}$$

for all $t \in T$ and $a_2 \in A_2$.

Proof. Condition (3.2) simply states that player 2 chooses a best response to player 1's equilibrium action. If (3.1) is also satisfied, then we can construct a full revelation equilibrium as follows. For all $m \in M \backslash T$, let $\lambda \left(\cdot \left| m \right. \right)$ be any belief that puts probability 1 on some element of A_2^*. Now no deviation by player 1 will ever induce player 2 to choose an action not in A_2^*. To show the necessity of (3.1), suppose that (3.1) failed. Then there exist $\widehat{t} \in T$, $\widehat{a}_1 \in A_1$ and $\widehat{a}_2 \in A_2^*$ such that $u_1 \left(f_1 \left(\widehat{t} \right), f_2 \left(\widehat{t} \right), \widehat{t} \right) < u_1 \left(\widehat{a}_1, \widehat{a}_2, \widehat{t} \right)$. Since $\widehat{a}_2 \in A_2^*$, there exists $t' \in T$ such that $f_2 (t') = \widehat{a}_2$. Now type $\widehat{t}$ of player 1 has a payoff improving deviation, by announcing that he is type t' and choosing action $\widehat{a}_1$. ■

The exact characterization of Proposition 3.2 can be related to the properties of the previous section as follows. Recall that we showed in the previous section that if

1. $f_2(t)$ is a best response for player 2 to $f_1(t)$;
2. $f_1(t)$ is a self-committing action for player 1 in the game $u(\cdot, t)$;
3. $f_1(t)$ is the Stackelberg action for player 1 in the game $u(\cdot, t)$; and
4. the game $u(\cdot, t)$ satisfies self-signalling for player 1;

then

$$u_1(f_1(t), f_2(t), t) \geq u_1(a_1, a_2, t)$$

for all $a_1 \in A_1$ and $a_2 \in A_2$. Thus if the above four conditions hold for all t, then f is played in a full revelation equilibrium.

The four conditions are also almost necessary in the following sense. Suppose that f is played in a full revelation equilibrium and all actions of both players are chosen under f (i.e., for all $a_i \in A_i$, there exists t such that $f_i(t) = a_i$). The latter property will be true for any f played in a full revelation equilibrium if, for each action $a_i \in A_i$ there exists $t \in T$ such that a_i is a dominant strategy for player i in the game $u(\cdot, t)$. Then for all $t \in T$, we must have (1), (2) and (3) holding, and $u_1(f_1(t), f_2(t), t) \geq u_1(f_1(t), a_2, t)$ for all $a_2 \in A_2$. This last property requires that each action a_1 is self-signalling when player 1 is called upon to choose it.

It is straightforward to extend the sufficient condition for the existence of a fully revealing equilibrium to two-sided incomplete information (see an earlier version of this paper, Baliga and Morris [1998]). In particular, there is a fully revealing equilibrium if the complete information games are self-signalling for both players and each player is called upon to choose a Stackelberg action and the Stackelberg action profile forms a Nash equilibrium.

We already saw in the positive spillovers example of section 3.1.2 a game where the efficient outcome was not played in a full revelation equilibrium because of a failure of self-signalling condition. We now consider an example where the game is self-signalling, but we nonetheless cannot support an efficient full revelation equilibrium because the Stackelberg condition fails. Examples such as this motivate Banks and Calvert's [1992] analysis of cheap talk in the Battle of the Sexes games.

Let player 2 have the low cost payoffs of figure 5 (for sure). Player 1 either has the high cost payoffs of figure 5 or has the following payoffs (arising with low cost, $c = 90$, and $x = -11$)

Opponent's Action

Own Action

	I	N
I	-1	-90
N	-11	0

Low Cost, Low x

FIGURE 7

In this game, the self-signalling condition is always satisfied. But if both firms are low cost, the both don't invest equilibrium is preferred by player 1 and the both invest equilibrium is preferred by player 2: firm 1 has a preference for the old software while firm 2 prefers the new software. There is no equilibrium where player 1 tells the truth and both players invest when player 1 has low cost and not invest when player 1 has high cost. The low cost type of player 1 has an incentive to lie and pretend he is high cost to persuade player 2 to not invest and not invest himself. The difficulty is that as the informed agent prefers the equilibrium where they both do not invest to the invest equilibrium that is played if he tells the truth. In our terminology, "invest" is not a Stackelberg action for the low cost type of player 1.

3.2.2. No Communication

Without self-signalling, full revelation is not possible. We now examine when the incentive problems are so great that it is impossible to transmit *any* private information in the cheap-talk game. Notice that this question is qualitatively different from the question asked above: there we looked for conditions for the existence of a fully revealing *equilibrium* (though the equilibrium set could and does contain other equilibria) while here we ask that the equilibrium *set* contain only non-communicative equilibria. We will show that positive spillovers (an extreme failure of self-signalling) imply that no communication is possible, but only under restrictive conditions.

Recall that given a talking strategy μ, we write M_μ for the set of messages sent with positive probability by some type, $M_\mu \equiv \{m : \mu(m|\, t) > 0 \text{ for some } t \in T\}$.

Definition 3.3. *There is no communication in a PBE* $(\mu, \alpha_1, \alpha_2, \lambda)$ *if* $\alpha_2(\cdot, m) = \alpha_2(\cdot, m')$ *for all* $m \in M_\mu$.

Observe that every no communication equilibrium is outcome equivalent to an equilibrium without cheap talk.

Definition 3.4. *There are binary action positive spillovers if* $A_2 = \{0, 1\}$ *and* $u_1(a_1, 1, t) > u_1(a_1, 0, t)$ *for all* $a_1 \in A_1$ *and* $t \in T$.

Proposition 3.5. *If there are binary action positive spillovers, then there is no communication in any equilibrium of the cheap talk game.*

Proof. Let $(\mu, \alpha_1, \alpha_2, \lambda)$ be an equilibrium of the cheap talk game. Suppose m and m' were both elements of M_μ and

$$\alpha_2(1|m') > \alpha_2(1|m). \tag{3.3}$$

Let t be a type who sends message m with positive probability ($\mu(m|t) > 0$) and let a_1 be an action played with positive probability by type t if he sends message m ($\alpha_1(a_1|m, t) > 0$). His equilibrium payoff is

$$\sum_{a_2 \in A_2} \alpha_2(a_2|m) u_1(a_1, a_2, t)$$

Type t's expected payoff to following the pure strategy "send message m' and choose action a_1" is then

$$\sum_{a_2 \in A_2} \alpha_2(a_2|m') u_1(a_1, a_2, t)$$

and, by the positive spillovers property and (3.3)

$$\sum_{a_2 \in A_2} \alpha_2(a_2|m') u_1(a_1, a_2, t) > \sum_{a_2 \in A_2} \alpha_2(a_2|m) u_1(a_1, a_2, t).$$

This contradicts our assumption that $(\mu, \alpha_1, \alpha_2, \lambda)$ is an equilibrium. ■

It turns out that it is very hard to weaken the conditions under which this negative result holds, as we will demonstrate in a series of four examples. Crawford and Sobel [1982] and a number of applied papers have shown that even when there is a conflict of interest between the sender and receiver in a sender-receiver game, it is still possible to construct *partially revealing* equilibria. Our examples demonstrate similar effects when the cheap talk concerns an endogenous action and not an exogenous type.

First, consider the restriction to binary actions. Our first example illustrates why with many actions for the uninformed player, it is not enough to have common preferences over the opponent's pure actions.

Example 1: Let $T_1 = \{t_1, t_1'\}$, with each type equally likely. Let $A_1 = \{U, D\}$ and $A_2 = \{L, C, R\}$. Payoffs are given by:

Player 1's Action

Player 2's Action

	L	C	R
U	$1,1$	$2,0$	$6,1$
D	$0,0$	$1,1$	$5,0$

Type t_1

Player 1's Action

Player 2's Action

	L	C	R
U	$0,1$	$4,0$	$5,1$
D	$1,0$	$5,1$	$6,0$

Type t_1'

FIGURE 8

There is an equilibrium where player 1 announces m if his type is t_1 and m' if his type is t_1'; he then chooses action U if type t_1 and D if type t_1'. Player 2 randomizes 50/50 between actions L and R if the message is m, and chooses C if the message is m'.

In example 1, player 1 has constant preferences over his opponent's *pure* actions (independent of his action and type): he always ranks his opponent's action choice $R \succ C \succ L$. But the example relies on the fact that player 1 has varying preferences over his opponent's *mixed* strategies. Whatever action he takes, type t_1' strictly prefers pure action C to the 50/50 combination of actions L and R.

The following example shows that even if he has constant preferences over his opponent's mixed actions, information can be communicated.

Example 2: Let $T_1 = \{t_1, t_1'\}$, with each type equally likely. Let $A_1 = \{U, D\}$ and $A_2 = \{L, C, R\}$. Let payoffs be given by:

Player 1's Action

Player 2's Action

	L	C	R
U	$1,1$	$2,0$	$3,1$
D	$0,0$	$1,1$	$2,0$

Type t_1

Player 1's Action

Player 2's Action

	L	C	R
U	$0,1$	$1,0$	$2,1$
D	$1,0$	$2,1$	$3,0$

Type t_1'

FIGURE 9

As in example 1, there is an equilibrium where player 1 announces m if his type is t_1 and m' if his type is t_1'; he then chooses action U if type t_1 and D if type t_1'. Player 2 randomizes 50/50 between actions L and R if the message is m, and chooses C if the message is m'. This is an equilibrium, and valuable information is conveyed from player 1 to player 2. However, note that player 1 is indifferent between which message to send. In this sense, this equilibrium is not very satisfactory: If we focus on equilibria where types with the same preferences over equilibrium messages send the same message, there is no communication in equilibrium (see Baliga and Morris [1998] for a formal version of this result).

The no communication result is also maintained with two-sided uncertainty as long as the types of the two players are independent and as long as only one player is allowed to talk. This is because equilibrium announcements have (equilibrium) interpretations that they reveal information about the actions that the sender will take. But with correlated types, messages may have more complex interpretations, i.e., depending on the type of the receiver, they convey different information about the speaker's intended actions. As example 3 shows, this allows information to be conveyed, even in equilibria where the speaker's preferences over the receiver's mixed actions are the same for all types and where there is no indifference over the messages sent in equilibrium.

Example 3: Let $T_1 = \{H, L, H', L'\}$ and $T_2 = \{t_2, t_2'\}$, with the prior be given by the following table:

Player 2's Type

Player 1's Type

	t_2	t_2'
H	$\frac{\alpha}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha^2}{2(1+\alpha+\alpha^2+\alpha^3)}$
L	$\frac{1}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha^3}{2(1+\alpha+\alpha^2+\alpha^3)}$
H'	$\frac{\alpha^2}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{\alpha}{2(1+\alpha+\alpha^2+\alpha^3)}$
L'	$\frac{\alpha^3}{2(1+\alpha+\alpha^2+\alpha^3)}$	$\frac{1}{2(1+\alpha+\alpha^2+\alpha^3)}$

FIGURE 10

where $\alpha < \frac{1}{9} < \frac{\alpha+\alpha^2}{1+\alpha^3}$. Both types of player 2 and types L and L' of player 1 have the low cost payoffs of the positive spillover example of section 3.1.2 (see Figure 4). Types H and H' of player 1 have the high cost payoffs.

In the absence of cheap talk, the unique equilibrium has no investment. With cheap talk, there is an equilibrium where types H and L announce m; types H' and L' announce m'. Types L and L' of player 1 invest, types H and H' do not invest (independently of messages). Type t_2 invests only if message m is sent. Type t_2' invests only if message m' is sent.

One might imagine that with independent types and two-sided information, information cannot be conveyed in equilibrium if there are binary action positive spillovers even if *both* players can talk. We show this is not the case.[2] When both players send messages, actions can be made contingent on the message *profile*. Therefore, in our leading example, the final probability of investment faced by player 1 depends also on the message sent by player 2. Therefore, some type of player 1 might be willing to send a message that has a lower ex ante probability of causing player 2 to invest but implies a high ex post probability of investment for certain message profiles.

Example 4: We return to the investment game of Figure 1. Restrict attention to the case where $x = 1$ and the cost of investing takes one of three values: low [L] ($c = 10$), medium [M] ($c = 90$) or high [H] ($c = 110$). Thus payoffs are:

Opponent's Action

Own Action

	I	N
I	91	-10
N	1	0

Low Cost [L]

Opponent's Action

Own Action

	I	N
I	11	-90
N	1	0

Medium Cost [M]

Opponent's Action

Own Action

	I	N
I	-9	-110
N	1	0

High Cost [H]

FIGURE 11

Each player is low cost with probability $\frac{1}{2}$, medium cost with probability $\frac{1}{3}$ and high cost with probability $\frac{1}{6}$ (types are independent).

The following is a symmetric pure strategy equilibrium of the cheap talk game. The low cost and high cost types send message m. The medium cost types send message m'. The low cost types invest if *either* both players have sent message m; *or* both players have sent message m'. The medium cost types invest only if

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[2] We thus show that Proposition 4.11 in Baliga and Morris [1998] was false as stated.

both players have sent message m'. The high cost types never invest. Formally, each player follows the following strategy:

$$\begin{aligned}\widetilde{m}_i\left(t_i\right) &= \begin{cases} m, \text{ if } t_i = L \text{ or } H \\ m', \text{ if } t_i = M \end{cases} \\ \widetilde{a}_i\left(t_i, m_i, m_j\right) &= \begin{cases} I, \text{ if } \left(t_i, m_i, m_j\right) = \left(L, m, m\right),\ \left(L, m', m'\right) \text{ or } \left(M, m', m'\right) \\ N, \text{ otherwise} \end{cases}\end{aligned}$$

To check optimality of the action choices, first notice that the high cost type never invests, the medium cost type invests if the probability of his opponent investing is at least $\frac{9}{10}$; and the low cost type invests if the probability of his opponent investing is at least $\frac{1}{10}$. Now observe that if both players have sent message m, each expects his opponent to invest with probability $\frac{3}{4}$; if both have sent message m', each expects his opponent to invest with probability 1; if they have sent different messages, each expects the other to invest with probability 0.

Now go back to the ex ante stage. Sending message m implies a $\frac{2}{3}$ chance of a having probability $\frac{3}{4}$ that the opponent will invest (and a $\frac{1}{3}$ chance that he will not invest for sure). Sending message m' implies a $\frac{1}{3}$ chance of a having probability 1 that the opponent will invest (and a $\frac{2}{3}$ chance that he will not invest for sure). Now all types would like the opponent to invest but they have different preferences over those options.

The high cost type is not going to invest anyway, so he just wants the highest ex ante probability of investment. Sending message m gives a $\frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$ probability of investment; sending message m' gives a $\frac{1}{3} \times 1 = \frac{1}{3}$ probability. So the high cost type sends message m.

If the medium cost type sends message m, he will not invest for sure and his interim expected utility (conditional on sending message m) will be $\frac{1}{2}(1) + \frac{1}{2}(0) = \frac{1}{2}$. If he sends message m', he will invest if his opponent sends message m' and not otherwise. So his interim expected utility will be $\frac{2}{3}(0) + \frac{1}{3}(11) = \frac{11}{3}$. So he sends message m'.

If the low cost type sends message m, he will invest if his opponent sends message m. So his interim expected utility is $\frac{2}{3}\left(\frac{3}{4}(91) + \frac{1}{4}(-10)\right) + \frac{1}{3}(0) = \frac{263}{4} = 43\frac{5}{6}$. If he sends message m', he will invest if his opponent sends message m'. So his interim expected utility is $\frac{2}{3}(0) + \frac{1}{3}(91) = \frac{91}{3} = 30\frac{1}{3}$. So he will send message m.

4. Cheap Talk in the Electronic Mail Game

An incomplete information game literature has demonstrated how ex ante small probability events may have a major impact on equilibrium payoffs via higher order beliefs (see, e.g., Kajii and Morris [1997]). In particular, there may be arbitrarily high ex ante probability that payoffs are given by a certain complete information game, but nonetheless a strict and Pareto-dominant Nash equilibrium of that complete information game is never played in any equilibrium of the incomplete information game. Such conclusions require that there be two-sided incomplete information and types be highly correlated. Is this conclusion robust to allowing cheap talk?

Our "no communication" result was for the one-sided incomplete information case. In this section, we show that nonetheless in a version of the electronic mail game of Rubinstein [1989], a failure of the self-signalling condition leads to no communication. The example can also be demonstrates the possibility of full communication with two-sided incomplete information and correlated types, as long as the underlying complete information game are always self-signalling (and Stackelberg conditions are satisfied).

Again, we have two players deciding whether to invest (I) or not invest (N) with payoffs as in figure 4. Each player's type space is the set of non-negative integers, $T_1 = T_2 = \{0, 1, 2, ...\}$, with the following probability distribution over types:

Player 2's Type

		0	1	2	·	n
	0	ε	0	0	·	0
Player 1's Type	1	$\varepsilon(1-\varepsilon)$	$\varepsilon(1-\varepsilon)^2$	0	·	0
	2	0	$\varepsilon(1-\varepsilon)^3$	$\varepsilon(1-\varepsilon)^4$	·	0
	·	·	·	·	·	·
	n	0	0	0	·	$\varepsilon(1-\varepsilon)^{2n}$

FIGURE 12

Type 0 of player 1 is high cost (and thus has a dominant strategy to not invest). All other types of player 1, and all types of player 2, are low cost.[3]

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[3] The following story from Rubinstein [1989] may motivate the information structure. With probability $\varepsilon > 0$, player 1 is high cost and player 2 is low cost. With probability $1-\varepsilon$, both firms are low cost. Firms know only their own costs. If firm 1 is high cost, he sends no message.

A well known argument (see Rubinstein [1989]) shows that there is a unique equilibrium in this setting (without cheap talk): always play N. Type 0 of player 1 does not invest as it is a dominant strategy to not invest. Type 0 of player 2 attaches probability $\frac{1}{2-\varepsilon} > \frac{1}{2}$ to player 1 not investing, so she must not invest. The argument iterates.

Now suppose that cheap talk is allowed. That is, before choosing their actions, the players simultaneously send messages in some arbitrary message space. If $x < 0$, then the underlying complete information game is always self-signalling; since the relevant Stackelberg conditions are always satisfied, there exists an equilibrium where each player truthfully announces his type and then invests as long as player 1 is not of type 0.

But suppose that $x > 0$. In this case, a truth-telling equilibrium does not exist: type 0 of player 1 would have an incentive to claim to be some other type (in order to induce investment by player 2). In fact, *every* equilibrium has no investment by any type. To see why, fix an equilibrium and let i^* be the lowest type of player 1 who ever invests with positive probability (after any message). Since type 0 has a dominant strategy to not invest, we must have $i^* \geq 1$. Now all types of player 2 less than $i^* - 1$ must attach zero probability to player 1 investing, and therefore must never invest in the equilibrium.

Suppose then that type $i^* - 1$ is the lowest type of player 2 who ever invests in the equilibrium. Let M_1^* be the set of messages that lead type $i^* - 1$ of player 2 to invest with positive probability. Type $i^* - 1$ of player 1 sends a message in that set with probability 1 (since he knows that type $i^* - 2$ is not investing, he chooses his message to maximize the probability that type $i^* - 1$ invests). Now recall that ex ante, type $i^* - 1$ of player 2 assigned probability $\frac{1}{2-\varepsilon}$ to player 1 being of type $i^* - 1$. Conditional only on observing a message in M_1^*, that probability must weakly go up (type $i^* - 1$ always sends a message in M_1^*, even though type i^* may not). Thus for at least one message in M_1^*, player 2 must assign probability at least $\frac{1}{2-\varepsilon}$ to player 1 being type $i^* - 1$, and therefore not investing. But then it is a best response for player 2 to not invest, a contradiction.

Thus we get a contradiction if $i^* - 1$ is the lowest type of player 2 who ever invests in the equilibrium. If i^* is the lowest type of player 2 who ever invests in the equilibrium, we can similarly construct a contradiction, reversing the roles of players 1 and 2 in the argument. If the lowest type of player 2 who ever invested

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If he is low cost, he sends a message to player 2, lost with probability ε. If received, 2 sends a confirmation, and so on. Now the type of each player corresponds to the number of messages sent.

in equilibrium were larger than i^*, then type i^* of player 1 would never invest in equilibrium, again a contradiction.

Unfortunately, this argument depends crucially on the special structure of types in this example and it is not clear how to generalize it.

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