Second Opinions and Price Competition: Inefficiency in the Market for Expert Advice

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Abstract

We analyze a market where the consumer must rely on experts to identify the correct type of service. Medical services, repair services and various types of consulting and advisory services belong to this broad category. Our focus is on situations where the diagnosis of the consumer's needs is costly and the expert's effort is unobservable. We develop a model where experts offer competing contracts and the consumer may gather multiple opinions. We explore the incentives that a competitive sampling of prices and opinions provides for experts to exert effort and find that there is a tension between price competition and the quality of the advise provided in equilibrium. The equilibrium fails to realize the second best welfare optimum. On the other hand, limiting price competition via price control increases total welfare.

1 Introduction

This paper analyzes the provision of a 'credence' service. A credence service has the property that the consumer (the *principal*) must rely on *experts* to identify the correct type of service. Medical services, repair services and various types of consulting and advisory services belong to this broad category.

The provision of credence services is beset by a number of information problems. Here, we are concerned with a situation where the expert's diagnostic effort is unobservable and the final success of the service is not contractible (say, because it is not easily or objectively measurable). We focus on the role of a specific mechanism—the gathering of multiple opinions—in mitigating the information problem and disciplining the expert's behavior.

Consider an individual whose car needs repair. He decides to visit a mechanic to ask for a diagnosis of the problem. The mechanic has a skilled and an unskilled employee. At a high cost the mechanic may ask the skilled employee to look at the car. In this case, the problem is correctly diagnosed. Alternatively, at a low cost, the mechanic may ask the unskilled employee to perform the diagnosis. In this case, the problem is not diagnosed correctly. The owner of the car cannot determine the skill level of the employee who inspected the car. To check whether the recommended repair is indeed appropriate, the owner of the car can visit another mechanic. If the second mechanic recommends the same repair, then it is more likely that the recommendation is the correct one.

There are of course other potential information problems, such as the unobservability of the expert's actions in the provision of the service. Moreover, there are other forces, such as reputation[1], that work to mitigate these problems. We disregard these issues and corrective forces in the interest of isolating the particular information problem and particular corrective force outlined above. We do not underestimate the importance of these missing elements. However, some of them have been discussed in the literature[2] and the manner in which reputation might work is relatively well understood from different contexts.

We model the basic scenario as follows. A principal is in need of a service but is uncertain as to which of a continuum of possible types of service matches his need. There is one correct service which gives the principal a payoff of $V > 0$; any other service yields a payoff of zero. The set of possible services is modeled as a continuum to assure that an unguided guess will not yield the right choice with positive probability. There are experts who can identify the right choice by incurring a cost c. The principal can consult experts, but does not observe whether or not the expert incurred the cost.

Experts are sampled sequentially from a large population of experts. A sampled expert offers the principal a contract. Upon observing this contract, the principal decides whether to consult the expert or to continue sampling. Consulting an expert is costly for the principal. This cost represents the time it takes to visit a doctor, take the car to a mechanic, or wait for a contractor. Once the principal agrees to be diagnosed, the expert decides whether to invest effort and then provides a

[1]Reputation effects provide an important inducement by substituting for the difficulty to objectively measure the success of of the service.

[2]See, e.g., Pitchik and Schotter [1987, 1989], Wolinsky [1993] and Emons [1997] for analyses of the case of unobservable expert's actions.

recommendation. After learning the recommendation the principal either buys the service or continues his search.

The first scenario analyzes a situation where the provision of the correct service requires the correct diagnosis. Thus, the principal cannot simply learn the information and then instruct some other expert to conduct the service. This assumption is more compelling for services that require specialized knowledge, such as medical services. In this scenario, we assume that the contract between the principal and the expert takes the following simple form. The contract stipulates two prices, a diagnosis fee and a price for the service. The diagnosis fee is paid by the principal up front. In exchange, the expert makes a recommendation. The principal then has the option to buy the recommended service at the price stipulated in the contract. In this environment, experts may have an incentive to provide the correct recommendation because it results in a higher probability of making a sale.

We do not allow contracts to depend on the success of the treatment, that is, the principal's payoff. This assumption is plausible if the success of the treatment is difficult to verify, for example, if only the principal can observe his payoff. Of course, if contracts could depend on the principal's payoff, then the incentive problem could easily be solved.

The second scenario assumes that a correct recommendation reveals all the relevant information and therefore, the principal need not buy the service from the diagnosing expert. This scenario corresponds to a situation in which only recommendation are traded. If the contracts are as in the first scenario, experts can never have an incentive to provide high effort because the probability of making a sale does not depend on the quality of the recommendation. We therefore assume that the service adopted by the principal is observable and can be contracted on. A contract may therefore reward the expert for providing a recommendation that is adopted by the principal and thereby provide a link between the expert's effort and payoff.

The first scenario corresponds to the familiar processes of gathering recommendations and bids that go can often be observed in credence markets. Repair services, medical and consulting services fall in this category. It is more difficult to find close real world analogues to the second scenario. Nevertheless we discuss this scenario as a robustness check to our findings for the first scenario. The difficulty of tying payments to the ultimate service selection of the principal is a possible reason why this scenario is not encountered frequently in practice.

For both scenarios we conduct equilibrium and welfare analyses. The first best outcome in both cases is such that the correct service is performed without wasteful search and duplication of the diagnoses. Obviously, the first best outcome cannot be sustained in equilibrium. The second best outcome is the welfare maximizing outcome among those that can be sustained in an environment in which the planner controls prices (but the principal and experts still freely control the other aspects of their behavior). We show that also the second best outcome cannot be sustained in equilibrium. This is perhaps the main qualitative insight that emerges from this analysis. The source of the additional inefficiency here is an informational externality: the incentives faced by an expert and hence the effort that she exerts, depend on the other experts' effort level. The second best contract maximizes the overall welfare when it is offered by all experts, but it does not maximize the joint surplus of a given expert and the principal when all other experts offer it. We conclude that competition may be in conflict with good expert incentives. In particular, we describe a welfare improving intervention that imposes a floor on the price of the service.

The related theoretical literature on the provision of credence goods or services is not very large. The formal models we are aware of analyze the incentives experts may have to misrepresent minor problems as major ones in order to profit at the expense of the principal.[3] Our conclusion that a minimum price may enhance efficiency is reminiscent of Telser's (1960) argument in favor of a minimum retail price. Telser points out that a retailer has little incentive to provide the consumer with information if other retailers provide this service. Hence retailers may free ride on one another's services. By contrast, experts only have an incentive to provide a high quality recommendation if other experts confirm the recommendation, that is, also provide a high quality recommendation. Hence, the source of the inefficiency in our setting is quite different from the retail setting.

Our paper is also related to the literature on product quality provision under asymmetric information, particularly to the moral hazard strand of this literature. In this literature (see, e.g., Wolinsky [1983]) two better informed sellers face less

[3] In addition to the papers cited above, Darby and Karni [1973] provide a model of misrepresentation.

informed buyers and the analysis explores how search or reputation interact with the competition to determine prices and quality levels. One of the features that separate our paper from this literature is the fact that in our case multiple opinions may share common information, whereas in the traditional analyses of product quality provision sellers posses independent information. This gives rise to a different form of competition and different sets of relevant contracts.

2 The Model

The principal is in need of a service but is uncertain as to which service meets his need. The range of possible services is $[0, 1]$. The principal benefits from the service only if it matches his need $\alpha \in [0, 1]$, hence his utility is

$$\begin{cases} V \text{ if } a = \alpha \\ 0 \text{ if } a \neq \alpha \end{cases}$$

where $V > 0$. The principal does not know his own type α and has a uniform prior on $[0, 1]$. The set of possible types is modeled as a continuum to assure that an unguided guess will not yield the right choice with positive probability.

There is an infinite population of identical experts, indexed by $k \in [0, 1]$. Experts serve a dual role: they recommend a service to the principal and, if chosen by the principal, perform the recommended service. The expert's recommendation can be of high or low quality. A high quality recommendation is always correct, that is, equal to α. A low quality recommendation is independent of the principal's type and chosen at random from a uniform distribution on $[0, 1]$. To make a high quality recommendation, the expert has to incur a cost $c > 0$. The low quality recommendation can be made at no cost. We assume that the cost of performing any of the potential services is zero and independent of the type of the principal.

The basic incentive problem studied in this paper stems from the fact that the principal does not observe the quality of the experts recommendation. Notice that we do not allow the expert to make an incorrect recommendation if she learns α. This is done to simplify the model, in particular, to avoid uninteresting multiplicities in the communication between expert and principal.

There is an infinite number of discrete periods. Within each period events unfold in the following order:

1. An expert is chosen at random and offers a contract (d, p). If accepted, a contract requires the principal to pay d to the expert. In return, the expert recommends a service and the principal has the option to buy the recommended service at the price p at any future date.

2. The principal decides on one of the following actions: (i) accept the contract; (ii) sample a new expert; (iii) buy the service from an expert whose contract the principal previously accepted; (iv) quit the process without purchase. The decision to buy the service and the decision to quit terminate this process.

3. If the principal accepts the contract, he pays the fee d and incurs a cost $s > 0$.

4. Next, the expert chooses the level of diagnostic effort $e \in \{0, 1\}$ where $e = 1$ denotes the high effort level required for a high quality recommendation.

5. Finally, the principal learns the recommendation $r \in [0, 1]$.

The model incorporates two features that strengthen price competition. First, the principal observes the expert's contract (d, p) at no cost. The search cost s and the fee d are only paid if the principal accepts the contract offer. Second, prior to the principal's decision to purchase the service, he observes the contract offered by a new expert. The first feature eliminates the familiar paradox that even small search costs endow the sellers with monopoly power (Diamond (1971)) and thus allows us to focus on a *competitive* environment. The second feature ensures that the sequential manner in which the prices are being observed does not dampen the competition relative to a situation in which the principal would observe a few price offers simultaneously.

We assume that the service can only be purchased from an expert who recommends it. This assumption ensures that the principal cannot first learn the appropriate service and then instruct an arbitrary expert (who did not provide a high quality recommendation) to perform the service. This is justified in a setting where the recommendation does not uniquely identify the service to be performed. In the mechanic example, the recommendation may not contain all the necessary instructions for an unskilled worker to implement the repair. This can be formalized by modeling the principal's need as a point in a two−dimensional space. An informed expert identifies both dimensions but communicates only the first dimension to the principal. Section 5 considers a model where this assumption is relaxed.

Suppose the principal is of type α, received recommendations from n experts whose fees were $d_1, ..., d_n$, and purchases from an expert who recommends a. Then, the principal's utility is

$$\begin{cases} V - p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha = a \\ -p - \sum_{i=1}^{n} d_i - ns & \text{if } \alpha \neq a \end{cases}$$

If the principal quits after n recommendations without purchase, then his utility is $-\sum_{i=1}^{n} d_i - ns$. The principal seeks to maximize his expected utility.

An expert who operates under the contract (d, p) and exerts effort $e \in \{0, 1\}$, receives the following payoff

$$\begin{cases} d - e \cdot c + p & \text{if the principal purchases the service from this expert \textit{in some period.}} \\ d - e \cdot c & \text{if the principal does not purchase the service from this expert \textit{in any period.}} \end{cases}.$$

Experts seek to maximize the expected profit.

The relevant past *history* of the principal records the sequence of experts whose contract he accepted[4], their initial offers and recommendations.

Every period, after a new expert was sampled, the principal chooses one of the available options. After n recommendations and after observing a new contract offer, the principal must choose from $n+3$ options: quit, buy from any one of the n experts who previously made a recommendation, accept the new contract offer, or continue sampling. Formally, the principal's strategy σ is a sequence of functions $\sigma = (\sigma_n)_{n=0}^{\infty}$, where the function σ_n takes as input a history of length n (that records the encounters with the n previously sampled experts) and a newly sampled offer of the form (d, p), and prescribes a probability distribution (an element of the $n+3$ dimensional simplex) over the $n + 3$ available options.

Experts do not observe the history of the principal. A *strategy* for expert k consists of a contract offer $(d_k, p_k) \in R_+^2$ and an effort choice should the principal accept the contract offer. This effort choice depends on the contract offered and is denoted by

[4] If the principal samples an expert but decides not to get a recommendation from this expert, the contract offered is not recorded as part of the relevant history. This is done for notational simplicity and has no consequences for the subsequent analysis. This is so, because we will focus on symmetric equilibria below.

$\xi_k : R_+^2 \rightarrow [0,1]$, where $\xi_k(d_k, p_k)$ is the probability of a high diagnostic effort. In this paper, we analyze symmetric equilibria which are defined below. By (d, p, ξ) we denote a symmetric strategy profile.

Experts have beliefs (probability distribution) over the set of possible histories of the principal *conditional* on being sampled by the principal. Let $\beta(H)$ denote an expert's belief (the probability conditional on being sampled) that the principal's history belongs to the set H. In addition, for each contract (d', p'), let $\beta(\cdot|d', p')$ denote the beliefs about the principal's history conditional on the principal *accepting* the expert's contract. We refer to β and $\beta(\cdot|\cdot)$ as the expert's conditional beliefs.

The principal's and the experts' strategies, σ and (d, p, ξ), determine a stopping rule defined over histories. If the associated stopping time has a finite expectation, then this stopping rule together with the random sampling of new experts determine a well defined distribution over histories conditional on a particular expert being sampled. We shall say that the conditional belief β is *compatible* with the strategies σ and (d, p, ξ) if, for any set of histories H, $\beta(H)$ coincides with the probability of H conditional on a particular expert being sampled, given these strategies. Similarly, the belief $\beta(\cdot|\cdot)$ is *compatible* with σ and (d, p, ξ), if for all H and all contracts (d', p') that are accepted by the principal with positive probability, $\beta(H|d', p')$ coincides with the probability of H conditional on a particular expert being sampled and on the principal accepting this contract. For contracts (d', p') that are not accepted by the principal with positive probability, $\beta(H|d', p')$ is unrestricted. Finally, note that if the expectation of the stopping time induced by σ and (d, p, ξ) is not finite, then $\beta(H)$ may not be well defined. However, since search is costly, this situation will not occur when σ is a best response.

A symmetric perfect Bayesian equilibrium consists of a strategy σ for the principal, a strategy (d, p, ξ) for experts and conditional beliefs β and $\beta(\cdot|\cdot)$ such that: (i) σ maximizes the principal's utility after any possible history, given the experts' strategy profile; (ii) (d, p, ξ) maximizes the expert's profit given the conditional beliefs β and $\beta(\cdot|\cdot)$. (iii) β and $\beta(\cdot|\cdot)$ are *compatible* with the above strategies. In the following we refer to symmetric equlibria simply as equilibria.

3 Equilibrium

Let (d, p) describe the contract offered by experts in an equilibrium. Let $x = \xi(d, p)$ denote the probability of high diagnostic effort along the equilibrium path. We first characterize the principal's best response to (d, p, x). The symmetry of the profile implies that only the experts' recommendations vary along a search history. The principal's best response is an optimal stopping rule applied to sequences of recommendations.

Lemma 1 shows that the principal's best response is of a simple form. If participation is worthwhile, then the principal either buys after the first recommendation or he searches for a matching pair of recommendations and buys from one of the two experts making that recommendation. The intuition for this result is as follows. Two matching recommendations reveal the correct diagnosis. Therefore, search must stop after two matching recommendations have been obtained. Suppose the principal has received n conflicting recommendations. Intuitively, the larger is n the more unreliable any particular recommendation becomes. After all, at most one of the recommendations can be correct. The principal must therefore either stop after just one recommendation or search for two matching recommendations.

Lemma 1 *Given the profile (d, p, x), the principal's best response is one of the following three strategies: (i) quit; (ii) accept one contract and purchase the recommended service; (iii) accept contracts until two recommendations match. Then, purchase from one of the two experts who provided the matching recommendations.*

The proof of the Lemma and of all subsequent results are in the appendix.

Lemma 1 implies that we may characterize the principal's best response to (d, p, x) by the participation decision and by the probability he buys after the first recommendation, denoted by f. We assume that the principal always participates if he has a weak incentive to do so. This assumption is made for convenience only.

The value of strategy (ii) is

$$xV - p - (s + d) \tag{1}$$

The value of strategy (iii) is

$$V - p - 2\frac{s + d}{x} \tag{2}$$

The principal participates if the value of (1) or (2) is nonnegative. The principal stops after the first recommendation ($f = 1$) if (1) is greater than (2) and he searches for a matching recommendation ($f = 0$) if (2) is greater than (1).

Since a randomly sampled expert makes the correct recommendation with probability x, the expected duration of the search to the first correct recommendation is $1/x$ and for two matching recommendations it is $2/x$. Hence, the expected search and diagnosis costs that the principal incurs when searching for two matching recommendations is $2(s + d)/x$.

Let B denote the sampled expert's belief that she is the first expert sampled by the principal. That is, $B = \beta(\emptyset)$, where $\emptyset$ is the empty history. Note that for the equilibrium offer (d, p), $\beta(\emptyset) = \beta(\emptyset|d, p)$.

Lemma 2

$$B = \frac{x}{fx + 2(1-f)} \tag{3}$$

Proof. Let h denote the history prior to the sampling of expert k and let $\ell(h)$ denote its length. The probability of $\ell(h) = n$ conditional on k being sampled is computed by looking at the sampling process on the set of experts excluding k and decomposing it into the disjoint events that search over this set would end after exactly m observations, $m = 1, 2, ...$ Let T denote the random stopping time of the search over the set of experts excluding k. Obviously, T depends on x and f. We may now express the probability of $\ell(h) = n$, conditional on k being sampled as

$$\begin{aligned} \Pr\{\ell(h) &= n \mid k \text{ is sampled}\} \\ &= \sum_{m=n+1}^{\infty} \Pr\{k \text{ is the } n\text{th expert sampled} \mid T = m, k \text{ is sampled}\} \Pr\{T = m \mid k \text{ is sampled}\} \end{aligned}$$

Notice that $\Pr\{k$ is the nth expert sampled$|\ T = m, k$ is sampled$\} = 1/m$ and $\Pr\{T = m \mid k \text{ is sampled}\} = m \Pr\{T = m\}/E(T)$. Therefore,

$$B = \Pr\{\ell(h) = 0 \mid k \text{ is sampled}\} = \frac{\sum_{m=1}^{\infty} \Pr\{T = m\}}{E(T)} = \frac{1}{E(T)}$$

The desired expression is obtained by noting that $E(T) = f + (1-f)\frac{2}{x}$. ■

Next, we analyze the experts effort decision. If an expert incurs the cost c, she provides the principal with the correct diagnosis. The expected profit in this case is

$$d + p \cdot f \cdot B + (1 - f \cdot B)\frac{p}{2} - c \tag{4}$$

where fB denotes the probability that the principal has never sampled before and stops after the first recommendation and $(1 - f \cdot B)$ denotes the probability that the principal searches for a matching recommendation. In the latter case, the expert makes a sale with probability $1/2$.

On the other hand, if the expert does not incur the cost c, she will make an incorrect recommendation. The expected profit in this case is

$$d + p \cdot f \cdot B \tag{5}$$

since she only sells the service to a principal who buys the service after the first recommendation. Thus, the expert's optimal effort decision for the contract (d, p) is $e = 0$ or 1 depending on whether (5) is greater or smaller than (4).

As an intermediate step in the analysis, consider a situation where prices are fixed and the expert can only decide which effort to take. A *fixed price equilibrium* is a profile (d, p, x, f) such that (d, p) are exogenously fixed, the principal's search strategy, f, is optimal given (d, p, x) and the experts effort decision $x \in [0, 1]$ is optimal given (d, p, x, f). There always are degenerate fixed price equilibria in which experts do not invest in the diagnosis (i.e., $x = 0$) and the principal quits immediately. In a degenerate fixed price equilibrium, on and off the path, an expert expects other experts not to invest in the diagnosis and hence she has no incentive to do so. We say that (d, p, x, f) is *non-degenerate* if $x > 0$.

If (d, p, x, f) is a non-degenerate fixed price equilibrium, then the optimality of the expert's effort decision requires

$$fBp \leq fBp + (1 - fB)\frac{p}{2} - c \tag{6}$$

If the principal always buys after the first recommendation ($f = 1$), then the probability that the principal searches for a matching recommendation is zero ($B = 1$ and $1 - fB = 0$) and (6) cannot hold. Therefore, $f < 1$ is a necessary condition for a non-degenerate fixed price equilibrium. Therefore, it must be the case that the principal weakly prefers to search for two matching recommendations, i.e.,

$$xV - p - (s + d) \leq V - p - 2\frac{s + d}{x} \tag{7}$$

Inequality (7) implies that $x < 1$ and therefore (6) must hold with equality in any non-degenerate fixed price equilibrium.

$$fBp = fBp + (1 - fB)\frac{p}{2} - c \tag{8}$$

Finally, the principal's participation requires

$$V - p - 2\frac{s+d}{x} \geq 0 \tag{9}$$

Therefore, on the path of a non-degenerate fixed price equilibrium the system (7)–(9) must hold.

Proposition 1 characterizes non-degenerate fixed-price equilibria. Proposition 1 uses the following magnitudes. Let $\overline{s} \equiv V/(2\sqrt{2}+3)$ and note that (7) ,when it holds with equality, is a quadratic equation in x. This equation has a solution when $d + s \leq \bar{s}$. In this case, we denote by $\underline{x}(d)$ the smaller root and by $\overline{x}(d)$ the larger root.

Proposition 1 *(i) For $s \leq \overline{s}$ and $c \leq V/2 - s/\overline{x}(d)$ there exist non-degenerate fixed price equilibria.*

(ii) The profile (d, p, x, f) is a non-degenerate fixed price equilibrium iff $d \leq \overline{s} - s$, $p \leq V - 2(s+d)/x$, $f = (p-2c)/(p-2c+xc)$ and either $p > 2c$, and $x \in \{\underline{x}(d), \overline{x}(d)\}$ or $p = 2c$ and $x \in [\underline{x}(d), \overline{x}(d)]$.

Non-degenerate fixed price equilibria are of two types. In the first type, $p > 2c$ and $f \in (0,1)$. In this case, the principal is indifferent between searching and stopping after the first recommendation, and x can take only one of two values. Observe that $p > 2c$ implies that (7) must hold with equality. Otherwise, the principal strictly prefers to search for a matching recommendation (resulting in $f = 0$). But then the expert strictly prefers high effort and (8) is violated. Therefore, if $p > 2c$ then x must be either $\underline{x}(d) \in (0,1)$ or $\overline{x}(d) \in (0,1)$. Substituting from (3), we can then solve (8) to yield $f = (p-2c)/(p-2c+xc)$.

In the second type of equilibrium, $p = 2c$ and $f = 0$. In this case, the principal searches until he gets matching recommendations, and x may take on any value in the interval $[\underline{x}(d), \overline{x}(d)]$. When $p = 2c$ and $f = 0$, experts are indifferent between high and low effort. In this case, every effort probability x that satisfies (7) and (9) is compatible with a fixed price equilibrium.

We next turn to the characterization of (the unconstrained) equilibrium. An equilibrium consists of a strategy for experts (d, p, ξ), a strategy for the principal σ, and conditional beliefs for experts. Lemma 1 implies that we can characterize the strategy of the principal on the equilibrium path by the probability of stopping search after one recommendation, f. We say that (d, p, x, f) is an equilibrium outcome if there are equilibrium strategies (d, p, ξ) and σ and conditional beliefs such that on the equilibrium path the experts choose $(d, p, x) = (d, p, \xi(d, p))$ and the principal's strategy is characterized by f.

Proposition 2 characterizes non-degenerate equilibrium outcomes. In addition to the requirements for fixed price equilibria, price deviations by experts must be unprofitable. Consequently, non-degenerate equilibria exist for a smaller region of the parameter space.

Proposition 2 *There is $\widetilde{s} \in (0, \overline{s})$ such that (d, p, x, f) is a non-degenerate equilibrium outcome iff $s \leq \widetilde{s}, c \leq V/2 - s/\underline{x}(0)$, and*

$$(d, p, x, f) = \left(0, \rho, \underline{x}(0), \frac{\rho - 2c}{\rho - 2c + \underline{x}(0)c}\right)$$

with $\rho \in [2c, V - 2s/\underline{x}(0)]$.

Notice that the fixed price equilibria with $x \in (\underline{x}(s), \overline{x}(s)]$ cannot be sustained as full equilibria. For example, a fixed price equilibrium with $p = 2c$ and $x \in (\underline{x}(s), \overline{x}(s))$ would permit a profitable deviation to a slightly lower price (d', p'). In the equilibrium of the continuation game following this deviation, the expert's optimal effort $\xi(d', p')$ will depend on her belief about the history of the principal. The belief that would encourage the least effort is that the principal has previously received no recommendation. In this case, $y = \xi(d', p')$ must satisfy

$$yV - p' = V - [(1 - y)\frac{2}{x} + y\frac{1}{x}]s - yp' - (1 - y)p \tag{10}$$

Notice that (10) is the counterpart of (7) for a search starting with the deviating expert, and it must hold with equality at an equilibrium in the continuation game. But since (7) holds with strict inequality for $x \in (\underline{x}(s), \overline{x}(s))$, it follows that for p' just below p, we have $y > x$. This means that the deviation will be attractive to the principal. For prices just below p the deviation would be profitable for the principal since it yields a payoff of $p' - c > 0$. Since this deviation is attractive even with the most detrimental belief, there is no equilibrium that supports this fixed price equilibrium.

In a non-degenerate equilibrium, (7) must hold as an equality. In the continuation game following a deviation to a lower price (d', p'), the deviating expert choose high effort with probability $\xi(d', p') < x = \xi(d, p)$ and the argument in the proof shows that the combination of the lower price and lower effort will be unattractive for the principal.

4 Welfare

Welfare will be measured by the sum of expected payoffs (the principal's expected payoff and experts' profits). Since prices here are just transfers, the endogenous variables on which welfare depends are the experts' probability of high diagnostic effort x and the principal's probability of stopping search after one recommendation, f. Let U denote the total expected payoff, then

$$U(x, f) = f\,(xV - s - xc) + (1 - f)\left(V - 2\frac{s}{x} - 2c\right) \tag{11}$$

The term xc and $2c$ capture the expected diagnosis costs associated with stopping after the first sampling and searching for two matching recommendations respectively.

The "first best" outcome is such that the correct diagnosis is obtained at the minimal cost of search and diagnosis. That is, $x = 1$ and $f = 1$. The first best is clearly not sustainable by an equilibrium or even just a fixed price equilibrium since as we noted earlier $x > 0$ requires $f < 1$. A "second best" outcome maximizes welfare from among all those that can be sustained by prices that respect the informational constraints. Thus, a second best outcome, (x^{SB}, f^{SB}), maximizes U over the set of all (x, f) such that for some d and p, (d, p, x, f) is a fixed price equilibrium.

Proposition 3 *The second best outcome is*

$$(x^{SB}, f^{SB}) = \left(\overline{x}(0), \frac{\overline{x}(0)V - s - 2c}{\overline{x}(0)V - s - 2c + \overline{x}(0)c}\right)$$

The second best outcome is sustained by a fixed price equilibrium with $d = 0$ *and* $p = \overline{x}(0)V - s$.

The second best outcome consists of the highest probability of high diagnostic effort and the lowest probability of continued search that can be sustained in a fixed price equilibrium. This result is intuitive since high x means that recommendations are more informative and high f means that fewer resources are wasted on search. To sustain the second best (x, f) the fee d has to be the minimal (i.e., $d = 0$) while the price p has to be at the maximal level compatible with the individual rationality constraint.

Proposition 3 has the following two implications for the welfare attained by the equilibria characterized in Proposition 2. The first implication is immediate.

Corollary 1 *None of the equilibria achieves the second best.*

The second implication is an immediate extension of an argument in the proof of Proposition 3.

Corollary 2 *The surplus maximizing equilibrium is the one with the highest possible price, i.e.,* $p = \underline{x}(0)V - s$.

The reason why competition does not lead to second best contracts in our setting is as follows. The attractiveness of a contract to the principal depends on the contracts and the resulting effort levels of other experts. After all, the principal must rely on the recommendations of other experts to verify the quality of a recommendation. Consider a principal who is offered a price discount Δ and expects the high diagnostic effort with probability y by the deviating expert. Assume the diagnostic effort of other experts is x and $d = 0$. Suppose the principal accepts the contract of the deviating expert, but decides to verify that the deviator's recommendation is correct. If the deviator has made a correct recommendation, the verification cost is s/x. Thus, for a given Δ and y a higher x makes this strategy more attractive for the principal. It turns out that in the subgame after a deviation by the expert, the principal must be indifferent between purchasing immediately and pursuing the "verification strategy" above. Thus, as x goes up, the probability of high diagnostic effort by the deviating expert, y, must also increase to keep the principal indifferent. Thus, a price discount of a fixed magnitude is more attractive the higher is x. Now observe that if a small price discount increases the principal's payoff, it is also profitable for the deviating expert. The reason is that at the reduced price the expert can ensure a sale by providing a high quality recommendation. Hence, the expert's profits are at least $p' - c$ which exceeds $pfB + (1 - fB)p/2$ when p' is close to p. Hence, we conclude that a high probability of diagnostic effort makes the equilibrium more susceptible to deviations by experts to a lower price. This prevents second-best contracts from being sustainable in equilibrium.

The inefficiency result is rather robust. As we mention in the later discussion section, this result survives alternative specifications of the price determination mechanism and the diagnosis technology. The inefficiency of equilibria might be somewhat alleviated under weakened competition. For example, a softer price competition due to an additional dimension of differentiation among experts might allow equilibria with a higher effort level. However, there is no reason to suppose that such a modification will necessarily eliminate the inefficiency.

Of course, the model is too stylized to provide firm grounds for regulatory intervention. However, the analysis suggest that restrictions on price competition might be beneficial when the forces highlighted in the model seem important.

5 The Sale of Recommendations

To this point we have assumed that the correct service can only be provided by an expert who made the correct recommendation. However, there are situations where it is straightforward to perform the service once the correct diagnosis is known. For example, if an automobile part has been found to be defective, any mechanic and perhaps the car owner himself can replace it. In this section, we consider such an environment. In particular, we assume the recommended service can be performed by the principal.

Since the principal need not return to the expert to implement the recommendation, the contracts studied in the previous section cannot provide experts with incentives to exert high effort. Thus, the provision of incentives requires additional contractible information. We continue to assume that the success of the treatment itself is not contractible and hence there are two types of additional information that contracts could depend on. First, if the type of service that the principal ultimately chooses is verifiable to all experts, then the contract may stipulate payments conditional on that choice. Second, the contract between the principal and an expert could condition payments on the recommendations made by other experts.

In this section, we allow contracts to condition payments on the principal's ultimate choice of service but not on other experts' recommendations. We make this restriction because contracts that depend on other experts' recommendations are susceptible to collusion. For example, the principal could collude with a dishonest expert to obtain whatever recommendation that would benefit him in his contractual relations with another expert. In this paper, we therefore restrict attention to *bilateral contracts* that condition payments only on the actions of the principal and the expert who is party to that contract.

A bilateral contract stipulates a fee d for the diagnosis and a price p to be paid if the principal adopts the recommendation of the expert. Of course, this assumes that the service adopted by the principal is verifiable[5].

Other features of the model and the notions of history, strategy, beliefs and equilibrium remain unchanged from the previous scenario. The expert strategy is described by (d, p, ξ), but now p is the above described fee for an adopted diagnosis rather than the price of the treatment. The contract form (d, p) is not the most general contract possible in this environment. Rather than a fixed price p, the contract could stipulate a schedule $p(a, r)$ that prescribes payments as a function of the ultimately chosen service type a and the recommendation r. However, since the only real information contained in the pair (a, r) is whether $a = r$ or $a \neq r$, we adopt the simpler form (d, p).

The analysis follows closely the previous analysis. It is straightforward to extend Lemma 1 to this situation. Here too, the principal's best response is to either quit, stop after the first diagnosis, or search two matching opinions. Conditional on participation, f denotes the probability that the principal stops after the first recommendation.

Let (d, p, x, f) describe a non-degenerate equilibrium outcome. The principal must now pay the price p to the two experts who make matching recommendations. Thus, the principal's expected utility in case he obtains two matching recommendations is

$$V - 2p - 2\frac{s+d}{x} \tag{12}$$

while the utility associated with stopping immediately remains $xV - p - s - d$ as before.

[5]Notice that it need not be the case that an expert can monitor principal's choice. It is enough that the contract will compel the principal to pay p to the expert unless he proves that he made another choice.

The expert's belief that she is the first to be sampled, B, remains the same and is given by (3). An expert who chooses high diagnostic effort is assured to get the price, and hence her payoff is

$$d + p - c \tag{13}$$

while the profit of an expert who chooses low diagnostic effort is $d + fBp$.

The expert provides high diagnostic effort only if $fBp \leq p - c$. This implies that $f < 1$, since $f = 1$ implies $B = 1$. Thus, the principal weakly prefers to search for a matching recommendation, i.e.,

$$xV - p - (s + d) \leq V - 2p - 2\frac{s + d}{x} \tag{14}$$

Therefore, $x < 1$ which implies that the expert is indifferent between the two effort levels and hence

$$fBp = p - c \tag{15}$$

The individual rationality of the principal requires

$$V - 2p - 2\frac{s + d}{x} \geq 0 \tag{16}$$

Thus, a non-degenerate equilibrium outcome satisfies (14)-(16).

It is again useful to start with *fixed price equilibria*. A fixed price equilibrium is a profile (d, p, x, f) that satisfies (14)-(16), where B is given by (3). Let $\overline{s}$ be the smaller s root of $(V + s)^2 - 8Vs = 0$. Observe that, when $s + d \leq \overline{s}$, there is a range $[0, \overline{p}]$ such that for $p \in [0, \overline{p}]$ the equation version of (14) has a solution. Let $\underline{x}(d, p)$ and $\overline{x}(d, p)$ denote the smaller and larger roots of this equation. Let

$$\begin{aligned} p_1(d) &= \max\{p | p \leq V, 0 \leq (V + s + d - p)^2 - 8(s + d)V\} \\ p_2(d) &= \max\{p | p \leq V, 0 \leq V - 2p - 2s/\overline{x}(d, p)\} \end{aligned}$$

and let $\overline{c}(d) = \min[p_1(d), p_2(d)]$. Notice that $\overline{c}(\overline{s} - s) = 0$ and, since both p_1 and p_2 are strictly decreasing, so is $\overline{c}$.

Proposition 4 is analogous to Proposition 1 and characterizes fixed price equilibria for the contractual setting of this section.

Proposition 4 *(i) There exist fixed price equilibria with $x > 0$ iff $s \leq \overline{s}$ and $c \leq \overline{c}(d)$.*

(ii) The non-degenerate profile (d, p, x, f) is a fixed price equilibrium iff $s + d \leq \overline{s}$, $p \leq \overline{c}(d)$, $f = 2(p - c)/[2(p - c) + xc]$ and either $p > c$, and $x \in \{\underline{x}(d, p), \overline{x}(d, p)\}$ or $p = c$ and $x \in [\underline{x}(d, p), \overline{x}(d, p)]$.

Proposition 5 characterizes the parameter range for which non-degenerate equilibria exist. For small search costs, Proposition 5 also characterizes the unique non-degenerate equilibrium outcome.

Proposition 5 *(i)* $(d, p, x, f) = (0, c, \overline{x}(0, c), 0)$ *is a non-degenerate equilibrium outcome iff* $s \leq \overline{s}$ *and* $c \leq \overline{c}(0)$. *(ii) There is* $\hat{s} \in (0, \bar{s})$ *such that for* $s \leq \hat{s}$ *this is the unique non-degenerate equilibrium outcome.*

Remark. In the proof of Proposition 5 we characterize all possible non-degenerate equilibrium outcomes. For some range of (s, c) with $s \leq \bar{s}, c \leq \bar{c}(0)$ the outcomes $(d, p, x, f) = (0, V + s - 3\sqrt{sV}, \sqrt{s/V}, \frac{2(p-c)}{2(p-c)+xc})$ and $(d, p, x, f) = (0, c, \underline{x}(0, c), 0)$ are also equilibrium outcomes. Together with the equilibrium outcome described in Proposition 5 these are the only non-degenerate equilibrium outcomes.

We now examine the welfare properties of the equilibrium. As before, the total surplus depends only on the probability of high diagnostic effort, x, and on the probability that the principal stops searching after one recommendation, f, and it is expressed by

$$U(x, f) = f(xV - s - xc) + (1 - f)\left(V - 2\frac{s}{x} - 2c\right) \tag{17}$$

The "first best" is $x = 1$ and $f = 1$, i.e., the correct diagnosis is obtained at the minimal cost of search and diagnosis. The second best is defined a the pair (x^{SB}, f^{SB}) that maximizes $U(x, f)$ over all (x, f) such that (d, p, x, f) is a fixed price equilibrium for some d and p. Proposition 6 shows that the equilibrium does not achieve the second best for small values of the search cost s.

Proposition 6 *There is* $s' \in (0, \bar{s})$ *such that for* $s < s'$ *the second best outcome is not sustained by an equilibrium.*

Remark. The other possible equilibrium outcomes described in the remark that follows Proposition 5 are not second best. However, for a large s, it may be that the outcome $(d, p, x, f) = (0, c, \overline{x}(s, c), 0)$ is second best.

Again, the second best outcome is not in general attained by an equilibrium. In the scenario of the previous sections the equilibrium failed to sustain the second best effort level x. Here, the second best effort level $\overline{x}(s, c)$ is sustained in equilibrium, however, equilibria involve too much search (too low f) relative to the second best level. In order to reduce search intensity while maintaining a high probability of diagnostic effort it would be necessary to increase the price p. However, because of price competition, this is inconsistent with equilibrium. As in the previous section, a regulatory intervention that reduces competition by introducing a price floor may improve welfare.

6 Discussion

In this section we revisit some of the modeling assumptions and consider some alternative variations. One of the objectives is to discuss further the robustness of the main insight that, when the uncertainty concerning a credence service is dealt with via the gathering of multiple opinions, the outcome of competition is typically inferior to the second best.

Alternative price determination. Due to the information asymmetry, the equilibrium outcomes may be sensitive to what might seem insignificant changes in the price determination mechanism. In an earlier version, we analyzed a mechanism in which an expert could offer an additional price discount after the principal's decision to be diagnosed by her. This environment leads to even more extreme inefficiency in the first scenario since only degenerate equilibria can be sustained. In the second scenario the results are unchanged.

The 0-1 nature of the diagnosis. The model assumes that by exerting effort the expert learns the correct diagnosis with certainty, whereas by not exerting effort the expert does not learn anything. We adopt this assumption because it leads to a very simple form of the optimal search. Notice however that the zero value of the information obtained without effort is merely a normalization. One may think instead that the zero level is already valuable for the principal and that the effort discussed here is just the extra effort required for further refinement of the diagnosis.

The assumption that effort yields a perfect diagnosis allows us to derive the simple form of the optimal stopping rule. In a more elaborate model the value of the service would depend monotonically on its distance from the true problem and the diagnosis would be imperfect. Clearly, in such a model the stopping rule would be more complicated. However, the general form will remain similar: the search stops once a sufficient number of close recommendations is obtained. More importantly, the informational externality at the core of the described inefficiency in this market would still be present.

The diagnosis fee. Recall that the diagnosis fee d is restricted to be nonnegative. A previous version (Pesendorfer and Wolinsky (1998)) allowed d to be negative (i.e., experts pay the principal to be examined by them). The results obtained there were similar as long as d is restricted to be greater or equal to $-s$ (i.e., so long as the principal does not profit just from being diagnosed). The rationale for placing a lower bound on d (be it 0 as we do here or $-s$ as in the previous version) is that in a richer framework contracts that make a sufficiently large up front payment are vulnerable to extreme abuse. If one imagines that there exists a fringe of principals who are not interested in the service but would be quick to take advantage of an expert that offers up front payment, then such offer cannot be profitable. Without a lower bound on d at or above $-s$, non-trivial equilibria with $x > 0$ will not exist.

Alternative interpretation of the model. The model and the analysis were developed in the context of a single principal who samples sequentially from a population of experts. It is possible to embed this basic scenario in a market setting with a population of principals. In such a model, we envision the market as operating over time without beginning or end. At any period, the principals who obtained the service depart from the market and there is a flow of new principals into the market. Thus, at any time the population consists of principals who have experienced different search histories. In a steady state, the distribution of histories over the principal population remains constant over time (although the principals themselves change). A steady state equilibrium of this model would correspond to the equilibrium of the model analyzed above, where the beliefs coincide with the equilibrium steady state distribution of the principal population.

Finally, observe that we could allow the principal to observe a number of contracts simultaneously, similar to an auction setting. This would not alter our analysis or results, nor would it affect the model with a population of principals described in the previous paragraph.

7 Appendix

Proof of Lemma 1: The existence of an optimal stopping rule is standard in this problem. We first show that stopping after two or more non-matching recommendations (with or without purchase) cannot be optimal. Consider a sample of n experts giving different recommendations. Let $\varphi(n)$ be the probability that a randomly drawn expert out of these n has the correct diagnosis.

$$\varphi(n) = \frac{(1-x)^{n-1}x}{(1-x)^n + n(1-x)^{n-1}x} = \frac{x}{1+(n-1)x}.$$

Note that $\varphi(n) < x$ due to the negative inference from having $n-1$ non-matching observations. Let $\pi(n)$ be the probability that the $(n+1)$st recommendation will match one of the first n.

$$\pi(n) = \frac{x^2 n}{1+(n-1)x}.$$

Let W^n be the expected continuation value of the optimal search after receiving n different recommendations. Optimality requires that

$$W^n = \max\Big\{0, \varphi(n)V - p, -(s+d) + (1-\pi(n))W^{n+1} + \pi(n)(V-p)\Big\}$$

We now show that $W^{n+1} = \max\{0, \varphi(n+1)V - p\}$ implies $W^n = \max\{0, \varphi(n)V - p\} > -(s+d) + (1-\pi(n))W^{n+1} + \pi(n)(V-p)$. That is, if after $n+1$ different recommendations the best continuation is to stop (either by quitting or purchasing), then it is strictly better to stop after n and hence after one observation. This shows that the optimal search never prescribes stopping (quitting or making a purchase) after two or more different recommendations. If , for $n \geq 1$, $W^{n+1} = \varphi(n+1)V - p$ then

$$\begin{aligned} W^n &= \max\{0, \varphi(n)V - p, -(s+d) + (1-\pi(n))(\varphi(n+1)V - p) + \pi(n)(V-p)\} \\ &= \max\{0, \varphi(n)V - p, -(s+d) + Vx - p\} \end{aligned}$$

Observe that $W^k \geq -(s+d)+Vx-p$ for all k. Therefore, since $\varphi(n+1)V - p = W^{n+1}$ and since $\varphi(n)$ is decreasing in n, it follows that

$$\varphi(n)V - p > \max\{0, -(s+d) + Vx - p\}$$

Therefore, $W^n = \varphi(n)V - p$. If , for $n \geq 0$, $W^{n+1} = 0$, then $\pi(n+1)(V-p) \leq s+d$. But since $\pi(n)$ is increasing in n, $\pi(n)(V-p) < s+d$ and hence

$$W^n = \max\{0, \varphi(n)V - p\}$$

Thus, for $n \geq 1$, $W^{n+1} = \max\{0, \varphi(n+1)V - p\}$ implies $W^n = \max\{0, \varphi(n)V - p\}$ and, for $n \geq 1$, $W^{n+1} = 0$ implies $W^n = 0$.

Since $s+d > 0$, searching beyond two matching recommendations is never optimal. Therefore, the optimal search policy may prescribe only stopping (with purchase) after two matching observations or after the first observation, or stopping (with quit) before any observations were obtained.

It is straightforward to verify that any one of these three alternative policies is optimal for some choice of V, p, s, d and x. ■

Proof of Proposition 1: (i) Let $(d, p, x, f) = (0, 2c, \overline{x}(s), 0)$. It is straightforward to show that (d, p, x, f) satisfies the equilibrium conditions (7), (8) and (9).

(ii) If a profile (d, p, x, f) with $x > 0$ is a fixed price equilibrium, then it satisfies (7), (8) and (9). (7) implies $d \leq \overline{s} - s$. (8) yields

$$f = (p - 2c) / (p - 2c + xc)$$

and hence $f < 1$ and $p \geq 2c$. From (9), $p \leq p(d, x)$. Now, $p > 2c$ implies $f > 0$, which in turn implies that (7) holds with equality and hence $x \in \{\underline{x}(d), \overline{x}(d)\}$. If $p = 2c$ then $f = 0$ and (7) may hold with inequality, implying that x has to be in $[\underline{x}(d), \overline{x}(d)]$. Conversely, suppose that a profile (d, p, x, f) with $x > 0$ is of one of the forms described in (ii). Therefore, $x \in [\underline{x}(d), \overline{x}(d)]$, $f = (p - 2c)/[p - 2c + xc]$ and $p \leq p(d, x)$. Now, $x \in [\underline{x}(d), \overline{x}(d)]$ implies that (7) holds, $f = (p - 2c)/[p - 2c + xc]$ implies that (6) holds, and $p \leq p(d, x)$ implies that (9) holds. So (d, p, x, f) is a fixed price equilibrium. ■

Proof of Proposition 2: Let (d, p, x, f) be a fixed price equilibrium of the form described in the proposition. To show that it is an equilibrium, we have to complete the description of the strategies and beliefs in the continuation game after the principal encounters a deviating offer (d', p'). The principal's strategy is such that (d', p') is not accepted. Otherwise the principal's strategy is unchanged. In particular, if the principal is searching for a matching recommendation, he continues sampling the next expert. Otherwise, he buys from a previously sampled expert.

If the principal accepted the offer (d', p') and if this is the first offer accepted by the principal, he plans to stop the search and purchase from this expert with probability f'. If the principal accepts (d', p'), the expert chooses $e = 1$ with probability $\xi(d', p') = y$. The deviating expert believes with probability 1 that this is the principal's first sample.

If $y = 0$, it is optimal for the principal not to accept (d', p'). Therefore, we assume $y > 0$ in the following. In that case, the counterpart of (6) must hold,

$$\begin{cases} f'p' \le p' - c & \text{if } \; p' < p \\ f'p' \le p'/2 - c & \text{if } \; p' = p \\ f'p' \le -c & \text{if } \; p' > p \end{cases} \tag{18}$$

This implies $p' \in [c, p]$. If $p' = c$ then (18) requires $f' = 0$. But $p' = c$ and $f' = 0$ together mean zero profit for the expert. Hence, the deviation would be unprofitable even if it attracts the principal. We may therefore assume that $p' > c$. Observe that (18) implies $f' < 1$ and therefore the counterpart of (7) must hold

$$yV - p' \le V - \left((1-y)\frac{2}{x} + y\frac{1}{x}\right)(s+d) - yp' - (1-y)p \tag{19}$$

Observe that (19) requires $y < 1$. A strict inequality in (19) implies $f' = 0$. But $p' > c$ and $f' = 0$ together imply that (18) holds strictly and hence $y = 1$. Therefore, $f' \in (0,1)$ and (19) holds with equality.

Claim: *Let $W^n(d', p', y)$ be the principal's expected continuation value if the principal has n distinct recommendations, samples the deviating expert and continues optimally thereafter.*

$$W^n(d', p', y) = V - (s+d)\left(\frac{2-y}{x} - \frac{n}{1+(n-1)x}\right) - (s+d') - (yp' + (1-y)p) \tag{20}$$

Proof of Claim: Recall that y and p' are such that,

$$\begin{aligned} W^0(d', p', y) &= Vy - (s+d') - p' \\ &= V - (s+d') - \left(\frac{2-y}{x}\right)(s+d) - (yp' + (1-y)p) \end{aligned} \tag{21}$$

where $Vy - (s+d') - p'$ is the expected utility of being diagnosed by the deviating expert and purchasing from her immediately. Let $\psi(n)$ denote the probability that one of the n past recommendations is correct.

$$\psi(n) = \frac{n(1-x)^{n-1}x}{(1-x)^n + n(1-x)^{n-1}x} = \frac{nx}{1+(n-1)x}$$

If a principal with n distinct recommendations plans to search for two matching recommendations then the expected search duration is

$$\begin{aligned} &\psi(n)\left(y + (1-y)(1+1/x)\right) + (1-\psi(n))\left(y(1+1/x) + (1-y)(1+2/x)\right) \\ = \; & 1 + \frac{2-y-\psi(n)}{x} \end{aligned}$$

The expected price that the principal pays under this plan is $yp' + (1-y)p$, since with probability y the deviating expert provides the correct recommendation and $p' \le p$. The principal's expected utility if she samples the deviating expert and continues search until a matching recommendation is obtained, is therefore

$$V - (s+d') - \left(\frac{2 - y - \psi(n)}{x}\right)(s+d) - (yp' + (1-y)p) \tag{22}$$

Since $\psi(n) > 0$ for $n \ge 1$, (21) and (22) imply that

$$W^n(d', p', y) = V - (s+d') - \left(\frac{2 - y - \psi(n)}{x}\right)(s+d) - (yp' + (1-y)p) \tag{23}$$

Substitution for $\psi(n)$ gives the required expression. ■

After a history of n distinct observations, the principal strictly prefers to sample the deviating expert, if $W^n(d', p', y) > W^n(d, p, x)$. From (20)

$$W^n(d', p', y) - W^n(d, p, x) = \frac{y - x}{x}(s+d) + y(p - p') + d - d'$$

Recall that (19) holds with equality. Solve it for y and substitute the result above to get

$$\begin{aligned} \Delta(d', p'; d, p, x) &\equiv W^n(d', p', y) - W^n(d, p, x) \\ &= \left(\frac{s+d}{x} + p - p'\right)\frac{V - 2\frac{s+d}{x} + p' - p}{V - \frac{s+d}{x} + p' - p} - s - d' \end{aligned}$$

Claim: *(i) If $d = 0$, $x = \underline{x}(s)$ and $x \le \sqrt{s/V}$, then $\Delta(d', p'; d, p, x) \le 0$, for all d' and $p' \le p$.*

(ii) If any of these conditions fails, there are p', d' arbitrarily close to p, d with $p' \le p$ such that $\Delta(d', p'; d, p, x) > 0$.

Proof of Claim (ii): If $x \in (\underline{x}(s+d), \overline{x}(s+d))$, then (7) holds with strict inequality. Hence,

$$x < \frac{V - 2(s+d)/x}{V - (s+d)/x}$$

and, for p' close to p,

$$x < \frac{V - 2(s+d)/x + p' - p}{V - (s+d)/x + p' - p} = y.$$

It follows that, for p' close to p and $d' = d$, $\Delta(d', p'; d, p, x) > 0$.

If $x \in \{\underline{x}(d), \overline{x}(d)\}$ then $\Delta(d, p; d, p, x) = 0$. Therefore, if $d > 0$, then for $p' = p$ and $d' < d$, $\Delta(d', p'; d, p, x) > 0$. Assume, therefore, $d = 0$. Since

$$\frac{\partial}{\partial p'}\Delta(d', p'; 0, p, x)\bigg|_{p'=p} = -1 + V\frac{s}{x}/\left(V - \frac{s}{x}\right)^2,$$

it follows that $\frac{\partial}{\partial p'}\Delta(d', p'; 0, p, x)\big|_{p'=p} < 0$ iff $x > \sqrt{s/V}$. Hence, if $x > \sqrt{s/V}$ there are $p' \leq p$ arbitrarily close to p such that $\Delta(d, p'; 0, p, x) > 0$. Since x satisfies (7) with equality, we have

$$x \in \{\underline{x}(0), \overline{x}(0)\} = \left\{\frac{V + s \pm \sqrt{(V+s)^2 - 8sV}}{2V}\right\} \tag{24}$$

It is easy to verify that only $\underline{x}(0)$ satisfies $x \leq \sqrt{s/V}$. This completes the proof of part (ii).

To prove (i) observe that $\Delta(d', p'; d, p, x)$ is a concave function of p'. Since $d = 0$ and $x \leq \sqrt{s/V}$, $\frac{\partial}{\partial p'}\Delta(d', p'; d, p, x)\big|_{p'=p} \geq 0$. Hence, $\Delta(d', p'; d, p, x)$ is maximized at $p' = p$ and $d' = 0$,over all $d' \geq 0$ and $p' \leq p$. Since $x = \underline{x}(s)$, $\Delta(d, p; d, p, x) = 0$ and hence $\Delta(d', p'; d, p, x) \leq 0$, for all d' and $p' \leq p$. ■

Part (i) of Claim 2 implies that any fixed price equilibrium with $d = 0$, $x = \underline{x}(s)$ and $x \leq \sqrt{s/V}$ is a full equilibrium since it would be optimal for the principal to ignore any deviation. Part (ii) implies that there are no other equilibria with these beliefs. But since these beliefs make the deviation less attractive than any other beliefs, the implication is that there are no other equilibria.

Finally, the range of s for which such equilibria exist is such that $\underline{x}(s) \leq \sqrt{s/V}$. Now, using (24) to express $\underline{x}(s)$ and rearranging, $\underline{x}(s) \leq \sqrt{s/V}$ is equivalent to $V - 3\sqrt{sV} + s \geq 0$. Let $\widetilde{s}$ and $\widetilde{\widetilde{s}}$ denote the two solutions of this inequality when it holds with equality. The inequality is satisfied for $s \leq \widetilde{s}$ and $s \geq \widetilde{\widetilde{s}}$. Observe that $\widetilde{s} > 0$ and $\widetilde{\widetilde{s}} > V$. Thus, the relevant range of s for which there exists an equilibrium with $x > 0$ is $s \leq \widetilde{s}$. ■

Proof of Proposition 3: Observe that $U(x, f)$ is strictly increasing in x. We show that if (d, p, x, f) is a fixed price equilibrium, then there is p' such that $(0, p', \overline{x}(s), f)$ is also a fixed price equilibrium. To see this, recall from Proposition 1 that $x \leq \overline{x}(s)$ and that $f = (p - 2c)/[p - 2c + xc]$. Define $p' = p + (p - 2c)(\overline{x}(s) - x)/x$ and observe that $(p' - 2c)/[p' - 2c + \overline{x}(s)c] = (p - 2c)/[p - 2c + xc] = f$. Now, if $f = 0$, then

$p' = p = 2c$ and $V - 2s/\overline{x}(s) - p' \geq V - 2s/x - p \geq 0$, where the last inequality follows from (9). If $f > 0$, then, by Proposition 1, $x \in \{\underline{x}(d), \overline{x}(d)\}$ and hence $V - 2(s+d)/x - p = xV - s - d - p \geq 0$, where the last inequality follows from (9). Therefore, $\overline{x}(s)V - s - p' = [\overline{x}(s) - x][xV - p + 2c]/x + xV - p - s \geq 0$ and $(0, p', \overline{x}(s), f)$ satisfies (9). The choice of p' and $\overline{x}(s)$ guarantee that $(0, p', \overline{x}(s), f)$ satisfies (8) and (7). Hence $(0, p', \overline{x}(s), f)$ is a fixed price equilibrium.

We conclude that a second best equilibrium $(d^{SB}, p^{SB}, x^{SB}, f^{SB})$ satisfies $x^{SB} = \overline{x}(s)$ and $d^{SB} = 0$. Since (from the definition of $\overline{x}(s)$) we have $\overline{x}(s)V - s = V - 2s/\overline{x}(s)$, it follows that $\overline{x}(s)V - s - \overline{x}(s)c > V - 2\frac{s}{\overline{x}(s)} - 2c$ and hence that $U(\overline{x}(s), f)$ is strictly increasing in f. Since f is an increasing function of p, the maximal f given $\overline{x}(s)$ is achieved at the maximal price consistent with (9). Therefore, $p^{SB} = V - 2s/\overline{x}(s) = \overline{x}(s)V - s$ and $f^{SB} = \frac{\overline{x}(s)V - s - 2c}{\overline{x}(s)V - s - 2c + \overline{x}(s)c}$). ■

Proof of Proposition 4: (ii) If a profile (d, p, x, f) with $x > 0$ is a fixed price equilibrium, then it satisfies (**??**)-(16). (14) implies $d \leq \overline{s} - s$, $x \in [\underline{x}(d), \overline{x}(d)]$ and hence $x < 1$. Now $x < 1$ implies that (15) holds with equality, which in turn yields $f = 2(p - c)/[2(p - c) + xc]$ and hence $f < 1$ and $p \geq c$. (16) together with (15) imply $p \leq \overline{c}(d)$. Now, $p > c$ implies $f > 0$, which in turn implies that (14) holds with equality and hence $x \in \{\underline{x}(d + s), \overline{x}(d + s)\}$. If $p = c$ then $f = 0$ and (14) may hold with inequality, implying that x has to be in $[\underline{x}(d + s), \overline{x}(d + s)]$. Conversely, suppose that a profile (d, p, x, f) with $x > 0$ is of one of the forms described in (ii). Therefore, $x \in [\underline{x}(d+s), \overline{x}(d+s)]$, $f = 2(p-c)/[2(p-c)+xc]$ and $p \leq \overline{c}(s+d)$. Now, $x \in [\underline{x}(d + s), \overline{x}(d + s)]$ implies that (14) holds, $f = 2(p - c)/[2(p - c) + xc]$ implies that (15) holds, and $p \leq \overline{c}(s + d)$ implies that (16) holds. So (d, p, x, f) is a fixed price equilibrium. Finally, part (i) follows from (ii) by noting that, for any $s \leq \overline{s}$ and $c \leq \overline{c}(0)$, the profile $(d, p, x, f) = (0, c, \overline{x}(s, c), 0)$ is an equilibrium for s and c in that range. ■

Proof of Proposition 5: Let (d, p, x, f) be a fixed price equilibrium of the form described in the proposition. To show that it is an equilibrium, we have to complete the description of the strategies and beliefs in the continuation game after the principal encounters a deviating offer (d', p'). The principal's strategy is such that (d', p') is not accepted. Otherwise the principal's strategy is unchanged. In particular, if the principal is searching for a matching recommendation, he continues sampling the next expert. Otherwise, he buys from a previously sampled expert.

If the principal accepted the offer (d', p') and if this is the first offer accepted by the principal, he plans to stop the search and purchase from this expert with probability f'. If the principal accepts (d', p'), the expert chooses $e = 1$ with probability $\xi(d', p') = y$. The deviating expert believes with probability 1 that this is the principal's first sample.

If $y = 0$, it is clearly optimal for the principal to not accept the offer. Assume therefore $y > 0$. As explained in the proof of Proposition 2, $y < 1$. Suppose that the principal, who has already gathered n different recommendations, chooses to be diagnosed by the deviant expert and plans to continue that search until he has two matching recommendations. Essentially the same derivation as in the proof of Proposition 2 yields that the principal's expected utility from this plan is

$$V - (s + d') - \left(\frac{2-y}{x} - \frac{n}{1 + (n-1)x}\right)(s + d) - (yp' + (1-y)p) - p$$

Recall that y and p' are such that, for $n = 0$, the expected utility of this plan is exactly equal to the expected utility of the alternative of accepting the deviant expert's recommendation. Hence

$$yV - p' = V - \left(\frac{2-y}{x}\right)(s + d) - yp' - (1-y)p - p$$

and

$$y = \frac{V + p' - 2(s+d)/x - 2p}{V + p' - (s+d)/x - p} \tag{25}$$

Therefore, for any $n > 0$, this expected utility is strictly higher than the expected utility of the alternative. Thus, this is the value of the optimal continuation. It follows that, after a history of n distinct observations, the principal would rather be examined by the deviant expert (and continue thereafter optimally) than ignore her and continue according to the strategy, if

$$\Delta(d', p'; d, p, x) \equiv \frac{y - x}{x}(s + d) + y(p - p') + d - d' > 0$$

Using (25) to substitute for y above to get

$$\Delta(d', p'; d, p, x) \equiv \left(\frac{s+d}{x} + p - p'\right)\frac{V - 2\frac{s+d}{x} + p' - 2p}{V - \frac{s+d}{x} + p' - p} - s - d'$$

Recall that $\underline{x}(d,p)$ and $\overline{x}(d,p)$ are the roots of the equation version of (14). Thus,

$$\{\underline{x}(d,p), \overline{x}(d,p)\} = \left\{ \frac{V + s + d - p \pm \sqrt{(V+s+d-p)^2 - 8(s+d)V}}{2V} \right\} \tag{26}$$

Claim: *(i) If $d = 0$, $p = c$, $x \in \{\underline{x}(0,c), \overline{x}(0,c)\}$ and $x \geq \sqrt{s/V}$, or $d = 0$, $p = V - 3\sqrt{sV} + s$, and $x = \underline{x}(s) = \sqrt{s/V}$, then $\Delta(d', p'; d, p, x) \leq 0$, for all $d' \geq 0$ and $p' \geq c$. (ii) If one or more of these conditions fail, then there are $p' \geq c$ and $d' \geq 0$ arbitrarily close to p and d respectively, such that $\Delta(d', p'; d, p, x) > 0$.*

Proof. : We start with (ii). If $x \in (\underline{x}(d,p), \overline{x}(d,p))$, then (14) holds with strict inequality. Hence,

$$x < \frac{V - 2\frac{s+d}{x} - p}{V - \frac{s+d}{x}}$$

and, for p' close to p,

$$x < \frac{V - 2\frac{s+d}{x} + p' - 2p}{V - \frac{s+d}{x} + p' - p} = y.$$

It follows that, for p' close to p and $d' = d$, $\Delta(d', p'; d, p, x) > 0$. Suppose therefore that $x \in \{\underline{x}(d,p), \overline{x}(d,p)\}$. This implies $\Delta(d, p; d, p, x) = 0$. Therefore, if $d > 0$, then for $p' = p$ and any $d' < d$, $\Delta(d', p'; d, p, x) > 0$. Suppose therefore that $d = 0$. Observe that

$$\left. \frac{\partial}{\partial p'} \Delta(d', p'; d, p, x) \right|_{p'=p} = -1 + V \frac{s+d}{x} / \left(V - \frac{s+d}{x} \right)^2.$$

Thus, when $d = 0$, $\frac{\partial}{\partial p'} \Delta(d', p'; d, p, x)\Big|_{p'=p} < (=) 0$ iff $x > (=) \sqrt{\frac{s}{V}}$. Thus, if $p > c$ and $x \neq \sqrt{s/V}$, or $p = c$ and $x \leq \sqrt{s/V}$, then there is $p' > c$ arbitrarily close to p such that $\Delta(d, p'; d, p, x) > 0$. Therefore, either $p = c$ and $x \geq \sqrt{s/V}$ or $p > c$ and $x = \sqrt{s/V}$. In the latter case it follows from (14) and (26) that $p = V - 3\sqrt{sV} + s$, and $x = \underline{x}(0,p)$.

To prove (i) observe that $\Delta(d', p'; d, p, x)$ is a concave function of p'. When $d = 0$, $p > c$ and $x = \sqrt{s/V}$ then $\frac{\partial}{\partial p'} \Delta(d', p'; d, p, x)\Big|_{p'=p} = 0$. When $d = 0$, $p = c$ and $x \geq \sqrt{s/V}$ then $(\partial \Delta(d', p'; d, p, x)/\partial p')_{p'=p} \leq 0$. Hence, in both cases, $\Delta(d', p'; d, p, x)$ is maximized at $p' = p$ and $d' = 0$,over all $d' \geq 0$ and $p' \geq c$. ■

Now part (i) of the claim implies that any fixed price equilibrium with $d = 0$, $p = c$, $x \in \{\underline{x}(0,c), \overline{x}(0,c)\}$ and $x \geq \sqrt{s/V}$, or $d = 0$, $p = V - 3\sqrt{sV} + s$, and $x = \underline{x}(0,p) = \sqrt{s/V}$ is a full equilibrium since it would be optimal for the principal to ignore any deviation. Part (ii) implies that there are no other equilibria with these beliefs. But since these beliefs make the deviation less attractive than any other beliefs, the implication is that there are no other non-degenerate equilibria.

Finally, the range of s and c for which these equilibria exist is determined by the (in)equalities (14)(holding as an equality), (15), (16) and $x \geq \sqrt{\frac{s}{V}}$. Using (26), it is a routine matter to verify the following facts. The outcome $(d,p,x,f) = (0,c,\overline{x}(0,c),0)$ satisfies these inequalities iff $s \leq \overline{s}$ and $c \leq \overline{c}(s)$. The outcome

$$(d,p,x,f) = (0, V + s - 3\sqrt{sV}, \sqrt{s/V}, \frac{2(p-c)}{2(p-c)+xc})$$

satisfies these conditions iff $s \in [s', s''] \subset [0, \overline{s}]$, where $s' > 0$ solves $V + 2s - 4\sqrt{sV} = 0$ and $s'' < \overline{s}$ solves $V + s - 3\sqrt{sV} = 0$, and $c \leq V + s - 3\sqrt{sV}$.

The outcome

$$(d,p,x,f) = (0,c,\underline{x}(0,c),0)$$

satisfies these conditions iff $s \leq \overline{s}$, $V + s - 2\sqrt{2Vs} \geq c \geq V + s - 3\sqrt{Vs}$ and $V - s - 3c - \sqrt{(V+s-c)^2 - 8sV} \geq 0$. The values of s in this sub-region are such that $s \geq \widetilde{s}$ where $\widetilde{s} > 0$ is the smaller solution of $2V + 4s - 6\sqrt{2sV} = 0$. Thus, for $s \leq \min[\overline{s}, \widetilde{s}]$ the unique equilibrium outcome is $(d,p,x,f) = (0,c,\overline{x}(0,c),0)$. ■

Proof of Proposition 6: Proposition 5 establishes that, for sufficiently small s, $(d,p,x,f) = (0,c,\overline{x}(0,c),0)$ is the unique equilibrium outcome with $x > 0$. Now, from (17) and the equality version of (14) we get

$$\frac{dW(x,f)}{dp} = \left(f(V - c - 2s/x^2) + 2s/x^2\right)\frac{dx}{dp} + 2c - cx - p$$

At $(d,p,x,f) = (0,c,\overline{x}(0,c),0)$, $dW(x,f)/dp = -2s/x\sqrt{(V+s-c)^2 - 8Vs} + 2(1-x)/x$ which is positive for sufficiently small s. ■

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