Optimal Information Disclosure in Auctions: The Handicap Auction*

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Abstract

We analyze the situation where a monopolist is selling an indivisible good to risk neutral buyers who only have an initial *estimate* of their private valuations. The seller can release (but cannot observe) signals, which refine the buyers' estimates. We show that in the expected revenue maximizing mechanism, the seller allows the buyers to learn their valuations with the highest possible precision, and her expected revenue is the same as if she could observe the additional signals. We also show that this mechanism can be implemented by what we call a "handicap auction." In the first round of this auction, each buyer privately buys a price premium from a menu published by the seller (a smaller premium costs more), then the seller releases the additional signals. In the second round, the buyers play a second-price auction, where the winner pays the sum of his premium and the second highest non-negative bid.

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1 Introduction

In many examples of the monopolist's selling problem (optimal auctions),[1] the seller has considerable control over the accuracy of the buyers' information concerning their own valuations. Very often, the seller can decide whether the buyers can access information that refines their valuations; however, she either cannot observe these signals, or at least, she is unaware of their significance to the buyers. For example, the seller of an oil field or a painting can determine the number and nature of the tests the buyers can carry out privately (without the seller observing the results). Another example (due to Bergemann and Pesendorfer, 2002) is where the seller of a company has detailed information regarding the company's assets (e.g., its client list), but does not know how well these assets complement the assets of the potential buyers. Here, the seller can choose the extent to which she will disclose information about the firm's assets to the buyers. Sometimes, the buyers' valuations become naturally more precise over time as the uncertainty of the good's value resolves, and the seller can decide how long to wait with the sale.

When the buyers' information acquisition is controlled by the seller, that process can also be optimized by the mechanism designer. In this paper, we explore the revenue maximizing mechanism for the sale of an indivisible good in a model where the buyers initially only have an estimate of their private valuations. The valuation estimates are refined by signals (added to the initial estimates) that the seller can costlessly release but cannot directly observe. This model captures the common theme of the motivating examples: the seller controls, although cannot learn, private information that the buyers care about.

Our main result is that in the revenue-maximizing mechanism, the seller will allow the buyers to learn their valuations as precisely as possible, and that her expected revenue will be as high as if she could observe the additional signals.[2] That is, the buyers will not enjoy additional informational rents from learning their valuations more accurately when the access to additional information is controlled by the seller. Besides these surprising findings, an added theoretical interest of our model is that the

[1] Early seminal contributions include Myerson (1981), Harris and Raviv (1981), Riley and Samuelson (1981), and Maskin and Riley (1984).

[2] In this hypothetical situation the buyers may or may not observe (but in either case can ex post verify) the additional signals.

standard revelation principle cannot be applied, yet we are able to characterize the optimal mechanism.

We also exhibit a simple mechanism, dubbed the *handicap auction*, which implements the revenue-maximizing outcome. This auction consists of two rounds. In the first round, each buyer buys a price premium from a menu provided by the seller (a smaller premium costs more). Then the seller releases, without observing, as much information as she can. In the second round, the buyers bid in a second-price auction, where the winner is required to pay his premium over the second highest non-negative bid. We call the whole mechanism a handicap auction because buyers compete under unequal conditions in the second round: a bidder with a smaller premium has a clear advantage.[3]

For a single buyer, the handicap auction simplifies to a menu of buy-options (a schedule consisting of option fees as a function of the strike price), where the buyer gets to observe the additional signal after paying for the option of his choice.

Our model also nests the classical (independent private values) auction design problem as a special case, where the additional signals are identically zero. In this case, the handicap auction implements the outcome of the optimal auction of Myerson (1981) and Riley and Samuelson (1981).

Several papers have studied issues related to how buyers learn their valuations in auctions, and what consequences that bears on the seller's revenue, both from a positive and a normative point of view. One strand of the literature, see Persico (2000), Compte and Jehiel (2001) and the references therein, focuses on the buyers' incentives to acquire information in different auction formats. Our approach is different in that we want to *design* a revenue-maximizing mechanism in which the seller has the opportunity to costlessly release (without observing) information to the buyers. In our model, it is the seller (not the buyers) who controls how much information the buyers acquire.

Information disclosure by the seller has been studied in the context of the winner's curse and the linkage principle by Milgrom and Weber (1982). They investigate whether in traditional auctions the seller should commit to disclose public signals that are affiliated with the buyers' valuations. They find that the seller gains from committing to

[3]The handicap auction can also be implemented as a mechanism where, in the first round, each bidder buys a discount (larger discounts cost more), and then participates in a second price auction with a positive reservation price, where the winner's discount is applied towards his payment.

full disclosure, because that reduces the buyers' fear of overbidding, thereby increasing their bids and hence the seller's revenue. Our problem differs from this classic one in many aspects. Most importantly, in our setting, the signals that the seller can release are private (not public) signals, in the sense that each signal affects the valuation of a single buyer and can be disclosed to that buyer only. The seller will gain from the release of information (which she does not even observe) not because of the linkage principle, but because the information can potentially improve efficiency, and she can charge for the access.

Our motivation is closer to that of Bergemann and Pesendorfer (2002). They consider the task of designing an information disclosure policy for the seller that allows to extract the most revenue in a subsequent auction. Their problem is very different from ours in that the seller is not allowed to *charge* for the release of information. Their model also differs from ours in that the buyers do not have private information at the beginning of the game. Under these assumptions, Bergemann and Pesendorfer (2002) show that the information structure that allows the seller to design the auction with the largest expected revenue is necessarily imperfect: in this structure, buyers are only allowed to learn which element of a finite partition their valuation falls into.

In contrast, in our paper, we design the expected revenue maximizing mechanism where the information structure and the rules of transaction *together* are chosen optimally. The difference may first seem subtle, it is important nevertheless. What we assume is that the seller can integrate the rules of information acquisition into the mechanism used for the sale itself. For example, in our model, the seller can charge the buyers for getting more and more accurate signals (perhaps in several rounds); the buyers could even be asked to bid for obtaining more information.

The idea that "selling" the access to information may be advantageous for the seller can be easily illustrated by an example. Suppose that there are two buyers who are both unaware of their valuation (drawn independently from the same distribution), which the seller can allow them to learn. Then consider the following mechanism. The seller charges both buyers an entry fee, which equals half of the expected difference of the maximum and minimum of two independent draws of the value-distribution. In exchange, she allows the buyers to observe their valuations (after they have paid the entry fee), and makes them play an ordinary second-price auction. The second-price auction will be efficient, and the buyers' ex-ante expected profit exactly equals the upfront entry fee. The seller ends up appropriating the entire surplus by charging the buyers for observing their valuations.[4]

This simple solution — the seller committing to the efficient allocation, revealing the additional signals, and charging an entry fee equal to the expected efficiency gains — only works when the buyers do not have private information to start with. Otherwise (for example, if the buyers privately observe signals, but their valuations also depend on other signals that they may see at the seller's discretion), the auctioneer, as we will show, does not want to commit to an efficient auction in the continuation, so the previously proposed mechanism does not work. We have to find a more sophisticated auction, and this is exactly what we will do in the remainder of the paper.

The paper is structured as follows. In the next section, we outline the model and introduce the necessary notation. In Section 3, we first derive the revenue maximizing auction for the case when the seller can observe the additional signals that refine the buyers' valuations. Then, we show that the same allocation and expected revenue can also be attained by a handicap auction, even if the seller cannot directly observe the additional signals. The results are illustrated by a numerical example. We conclude and remark on extensions in Section 4.

2 The Model

Assume that there are n potential buyers for an indivisible good. The seller's valuation for the good is zero. The valuation of buyer $i \in \{1, \dots, n\}$ is the sum of two random variables, v_i (called "type") and s_i (called "shock"), which are distributed independently (of each other and across i) according to cumulative distribution functions F_i and G_i, respectively.[5] We assume that the support of F_i is $[0,1]$, on which $f_i = F_i'$ exists, and that this distribution exhibits a monotone hazard rate, that is, $(1 - F_i)/f_i$ is weakly decreasing on $[0,1]$. We also assume that the distribution of the shocks is atomless, however, we do not make any restriction on the support of the G_i's. We will use v to denote the vector of types and s to denote the vector of shocks. We will also

[4] This example (when the buyers have no initial private information) has been studied independently by Gershkov (2002), who also obtained the same result.

[5] The reader may find it helpful to think of the shock as a noise with zero mean, so the buyer's type is his expected valuation for the good. However, we will not make this assumption in the formal model.

use the usual shorthand notation for the vector of types of buyers other than i, v_{-i}, and let s_{-i} denote $(s_j)_{j \neq i}$.

The realization of v_i is observed by buyer i. Although neither the seller nor buyer i can directly observe the shock, the seller has the ability to generate signals conditional on s, which only buyer i will observe. In particular, we assume that the seller can allow buyer i to observe his shock, s_i, without the seller learning its value.[6]

All parties are risk neutral. The seller's objective is to maximize her (expected) revenue. Buyer i's utility is the negative of his payment to the seller, plus, in case he wins, the value of the object, $v_i + s_i$. Every buyer i has an outside option of zero utility.

The seller can design any (indirect) mechanism, which can consist of several rounds of communication between the parties (i.e., sending of messages according to rules specified by the seller). The seller can also release signals (without observing them). Transfers of the good and money may also occur as a function of the history. The set of all indirect mechanisms is rather complex, and the standard revelation principle cannot be applied. However, this issue is avoided by the approach that we take in the next section.

3 Results

Our main result is the characterization of the expected revenue maximizing mechanism in the model introduced in Section 2. We will show that an optimal mechanism exists, which can be practically implemented as a "handicap auction" (for a description, see the Introduction or Subsection 3.2 below). We will also show that this mechanism achieves the same expected revenue *as if* the seller could observe the realizations of the shocks. In other words, while the buyers still enjoy information rents from their types, all their rents from observing the shocks can be appropriated by the seller.

In Subsection 3.1, we start with the derivation of the optimal mechanism when the seller can observe the shocks (while the buyers cannot) after having committed to an indirect mechanism. In Subsection 3.2, we show that in our model, the same expected revenue can also be generated by the seller without observing the shocks.

[6] In many applications, the seller may not be able to generate just *any* random signal correlated with s. Therefore, all we assume is that the seller can "show" s_i to buyer i if she so decides.

3.1 The Optimal Mechanism When the Seller Can Observe the Shocks

Let us assume, in this subsection only, that the seller alone can observe (and verify to a third party) the realizations of the shocks after having committed to an indirect mechanism. The Revelation Principle applies, hence we can restrict our attention to mechanisms where the buyers report their types, and the seller determines the allocation and transfers as a function of the types and the shocks. We will analyze truthful equilibria of direct mechanisms that consist of an allocation rule, $x_i(v_i, v_{-i}, s_i, s_{-i})$ for all i, and an (expected) transfer scheme, $t_i(v_i, v_{-i}, s_i, s_{-i})$ for all i. Here, $x_i(v_i, v_{-i}, s_i, s_{-i})$ is the probability that buyer i receives the good, and $t_i(v_i, v_{-i}, s_i, s_{-i})$ is the transfer that he expects to pay, given the reported types and the realization of the shocks.

We will use the tools of Bayesian mechanism design to find the optimal (expected revenue maximizing) auction. The result will provide an upper bound on the expected revenue the seller can achieve in the case when she cannot observe the shocks directly, which is going to be the subject of Subsection 3.2.

If buyer i with type v_i reports type $\hat{v}_i$ then his expected payoff will be

$$\pi_i(v_i, \hat{v}_i) = \mathbf{E}_{v_{-i},s}\left[x_i(\hat{v}_i, v_{-i}, s_i, s_{-i})(v_i + s_i) - t_i(\hat{v}_i, v_{-i}, s_i, s_{-i})\right], \tag{1}$$

where $\mathbf{E}$ stands for expectation. Let $X_i(v_i) = \mathbf{E}_{v_{-i},s}[x_i(v_i, v_{-i}, s_i, s_{-i})]$, and introduce $\Pi_i(v_i) = \pi_i(v_i, v_i)$ for the indirect profit function. Then, (1) can be rewritten as

$$\begin{aligned} \pi_i(v_i, \hat{v}_i) &= \mathbf{E}_{v_{-i},s}\left[x_i(\hat{v}_i, v_{-i}, s_i, s_{-i})(\hat{v}_i + s_i + v_i - \hat{v}_i) - t_i(\hat{v}_i, v_{-i}, s_i, s_{-i})\right] \quad (2) \\ &= \Pi_i(\hat{v}_i) + (v_i - \hat{v}_i)X_i(\hat{v}_i). \end{aligned}$$

Incentive compatibility of the mechanism means that, for all $v_i, \hat{v}_i \in [0, 1]$, we have $\pi_i(v_i, v_i) \geq \pi_i(v_i, \hat{v}_i)$, that is, (1) is maximized in $\hat{v}_i$ at $\hat{v}_i = v_i$. Using (2), we can rewrite incentive compatibility as

$$\Pi_i(v_i) - \Pi_i(\hat{v}_i) \geq (v_i - \hat{v}_i)X_i(\hat{v}_i), \;\; \forall v_i, \hat{v}_i \in [0, 1], \text{ and } i = 1, ..., n. \tag{3}$$

In the following Lemma, we apply standard arguments (see Myerson, 1981) for characterizing incentive compatible mechanisms.

Lemma 1 *Assume that, after having committed to a selling mechanism, the seller can observe the realizations of the shocks. A direct mechanism is incentive compatible if and only if, for all* $i = 1, ..., n$ *and* $v_i \in [0,1]$, X_i *is weakly increasing, and*

$$\Pi_i(v_i) = \Pi_i(0) + \int_0^{v_i} X_i(\nu) d\nu. \tag{4}$$

Proof. By (3) and its counterpart where the roles of v_i and $\hat{v}_i$ are reversed,

$$(v_i - \hat{v}_i) X_i(\hat{v}_i) \leq \Pi_i(v_i) - \Pi_i(\hat{v}_i) \leq (v_i - \hat{v}_i) X_i(v_i).$$

This inequality implies that X_i is weakly increasing and therefore is integrable, and so equation (4) follows.

Now we prove that equation (4) and X_i weakly increasing are sufficient for incentive compatibility. If $v_i \geq \hat{v}_i$ then from (4)

$$\begin{aligned} \Pi_i(v_i) &= \Pi_i(\hat{v}_i) + \textstyle\int_{\hat{v}_i}^{v_i} X_i(\nu) d\nu \\ &\geq \Pi_i(\hat{v}_i) + \textstyle\int_{\hat{v}_i}^{v_i} X_i(\hat{v}_i) d\nu = \Pi_i(\hat{v}_i) + (v_i - \hat{v}_i) X_i(\hat{v}_i), \end{aligned}$$

while if $\hat{v}_i \geq v_i$ then

$$\begin{aligned} \Pi_i(v_i) &= \Pi_i(\hat{v}_i) - \textstyle\int_{v_i}^{\hat{v}_i} X_i(\nu) d\nu \\ &\geq \Pi_i(\hat{v}_i) - \textstyle\int_{v_i}^{\hat{v}_i} X_i(\hat{v}_i) d\nu = \Pi_i(\hat{v}_i) + (v_i - \hat{v}_i) X_i(\hat{v}_i). \end{aligned}$$

This establishes that (3) holds. ■

Now we turn to the problem of determining the revenue-maximizing mechanism. Using (4), we can write the expectation (over all types) of buyer i's surplus as

$$\begin{aligned} \int_0^1 \Pi_i(v_i) f_i(v_i) dv_i &= \Pi_i(0) + \int_0^1 \int_0^{v_i} X_i(\nu) d\nu f_i(v_i) dv_i \\ &= \Pi_i(0) + \int_0^1 \int_\nu^1 X_i(\nu) f_i(v_i) dv_i d\nu \\ &= \Pi_i(0) + \int_0^1 X_i(\nu)(1 - F_i(\nu)) d\nu \\ &= \Pi_i(0) + \mathbf{E}_{v,s} \left[x_i(v_i, v_{-i}, s_i, s_{-i}) \frac{1 - F_i(v_i)}{f_i(v_i)} \right]. \end{aligned}$$

On the second line, we applied Fubini's Theorem. On the third line, we substituted $1 - F_i(\nu)$ for $\int_\nu^1 f_i(v_i)dv_i$. Finally, we plugged in the definition of X_i.

The seller's expected revenue equals the difference between the expected social surplus and the sum of the ex ante expectation of the buyers' surpluses,

$$\sum_{i=1}^{n} \mathbf{E}_{v,s}\left[\left(v_i + s_i - \frac{1-F_i(v_i)}{f_i(v_i)}\right) x_i(v_i, v_{-i}, s_i, s_{-i})\right] - \sum_{i=1}^{n} \Pi_i(0). \tag{5}$$

The mechanism design problem is to maximize (5) by choosing the integration constants and the vector of trade probabilities subject to the incentive compatibility constraints. That is, by Lemma 1, the problem is to choose for all i, $\Pi_i(0)$ and for all i and $(v_i, v_{-i}, s_i, s_{-i})$, $x_i(v_i, v_{-i}, s_i, s_{-i})$, so that X_i is weakly increasing and (5) is maximized. The following proposition characterizes the solution to this problem.

Proposition 1 *Assume that, after having committed to a selling mechanism, the seller can observe the realizations of the shocks. In the expected revenue maximizing mechanism,* $\Pi_i(0) = 0$*, and the allocation rule is,*

$$x_i(v_i, v_{-i}, s_i, s_{-i}) = \begin{cases} 1/|M| & \text{if } i \in M \text{ and } v_i + s_i - \frac{1-F_i(v_i)}{f_i(v_i)} > 0 \\ 0 & \text{otherwise} \end{cases}, \tag{6}$$

where

$$M = \left\{ j : v_j + s_j - \frac{1-F_j(v_j)}{f_j(v_j)} = \max_{k=1,\ldots,n} \left\{ v_k + s_k - \frac{1-F_k(v_k)}{f_k(v_k)}, 0 \right\} \right\}.$$

The seller's expected revenue from this mechanism is,

$$\mathbf{E}_{v,s}\left[\max_{j=1,\ldots,n} \left\{ v_j + s_j - \frac{1-F_j(v_j)}{f_j(v_j)}, 0 \right\}\right]. \tag{7}$$

In other words, the seller sets the allocation rule so that the buyer with the largest non-negative shock-adjusted virtual valuation, $v_j + s_j - (1-F_j(v_j))/f_j(v_j)$, will win. The transfers, and therefore the seller's profit, are determined by the incentive compatibility constraints. The proof (below) is standard.

Proof. The proposed allocation rule, (6), together with $\Pi_i(0) = 0$ for all i, pointwise maximizes (5). We will prove that it can be made incentive compatible using an appropriately chosen transfer scheme.

By assumption, $(1-F_i(v_i))/f_i(v_i)$ is decreasing, therefore the shock-adjusted virtual valuation function, $v_i + s_i - (1 - F_i(v_i))/f_i(v_i)$, is increasing in v_i. This implies that for all i, v_{-i}, s_i, and s_{-i}, $x_i(v_i, v_{-i}, s_i, s_{-i}) \geq x_i(\hat{v}_i, v_{-i}, s_i, s_{-i})$ if and only if $v_i \geq \hat{v}_i$, and, after taking expectation with respect to v_{-i} and s, $X_i(v_i) \geq X_i(\hat{v}_i)$ if and only if $v_i \geq \hat{v}_i$. That is, X_i is weakly increasing.

It remains to show that there exists a transfer scheme such that (4) holds, $\Pi_i(0) = 0$, and all types of all buyers participate. Recall that in a mechanism with allocation rule x_i and transfer scheme t_i, the expected profit of buyer i with type v_i is

$$\Pi_i(v_i) = \mathbf{E}_{v_{-i},s}\left[(v_i + s_i)x_i(v_i, v_{-i}, s_i, s_{-i})\right] - \mathbf{E}_{v_{-i},s}[t_i(v_i, v_{-i}, s_i, s_{-i})].$$

Using x_i given in (6), define buyer i's transfer as

$$t_i(v_i, v_{-i}, s_i, s_{-i}) = \int_0^{v_i} X_i(\nu)d\nu - \mathbf{E}_{\tilde{v}_{-i},\tilde{s}}\left[(v_i + \tilde{s}_i)x_i(v_i, \tilde{v}_{-i}, \tilde{s}_i, \tilde{s}_{-i})\right],$$

where $X_i(v_i) = \mathbf{E}_{\tilde{v}_{-i},\tilde{s}}[x_i(v_i, \tilde{v}_{-i}, \tilde{s}_i, \tilde{s}_{-i})]$. Observe that (4) holds, and $\Pi_i(0) = 0$. Finally, all buyers participate because their outside option is zero by assumption, $\Pi_i(0) = 0$, and Π_i is increasing. ■

Remark 1 It is clear from the proof that the claim of Proposition 1 remains true even if the monotone hazard rate assumption is violated, but the virtual valuations are weakly increasing, that is, if $v_i - (1 - F_i(v_i))/f_i(v_i)$ is weakly increasing in v_i for all i.

In the next subsection we show that the same outcome can also be implemented by the seller even if she cannot observe the shocks, as long as she can allow the buyers to observe them.

3.2 The Optimal Mechanism When the Seller Cannot Observe the Shocks: The Handicap Auction

Assume that the seller cannot directly observe the shocks, but she can allow the buyers to learn them. Clearly, in this case, the seller cannot do better than under the assumptions of Subsection 3.1 (where she could observe the shocks after having committed to a mechanism). The main result of the present subsection is that we exhibit a mechanism, called the "handicap auction," which implements the same allocation (with the same expected revenue) as the revenue maximizing mechanism of Subsection 3.1.

In general, a *handicap auction* consists of two rounds. In the first round, each buyer i, knowing his type, chooses a price premium p_i for a fee $C_i(p_i)$, where C_i is a fee-schedule published by the seller. The buyers do not observe the premia chosen by others. The second round is a traditional auction, and the winner is required to pay his premium over the price. Between the two rounds, the seller may send messages to the buyers. In our model, the seller will allow every buyer to learn the realization of his shock between the two rounds, and the second round is a second price (or English) auction with a zero reservation price.

We call this mechanism a handicap auction because in the second round, the buyers compete under unequal conditions: a bidder with a smaller premium has a clear advantage. An interesting feature of our auction is that the bidders *buy* their premium in the initial round, which allows for some form of price discrimination. We will come back to the issue of price discrimination in Subsection 3.3.

An interesting alternative way of formulating the rules of the handicap auction would be by using price discounts (or rebates) instead of price premia. In this version, each bidder first has to buy a discount from a schedule published by the seller. Then the buyers are allowed to learn the realizations of the shocks, and are invited to bid in a second price auction with a reservation price r, where the winner's discount is applied towards his payment. The reader can check that a handicap auction can be easily transformed into a mechanism like this by setting r sufficiently high (larger than the highest p_i in the original fee-schedules), and specifying that a discount $d_i = r - p_i$ is sold for a price $C(p_i)$ in the first round. In what follows, however, we will use the original form of the handicap auction.

If there is only a single buyer, then the handicap auction simplifies to a *menu of buy options*: p_i can be thought of as the strike price, and the upfront fee, $C_i(p_i)$, is the cost of the option. In the second round, the buyer can exercise his option to buy at price p_i (there is no other bidder, so the second-highest bid is zero), for which he initially paid a fee of $C_i(p_i)$. We will revisit the single-buyer case later in the context of a numerical example.

First, we state what happens in the second round of the handicap auction.

Lemma 2 *In the second round of the handicap auction (after each buyer learns the realization of his shock), it is a weakly dominant strategy for buyer i with price premium p_i to bid $b_i = v_i + s_i - p_i$.*

Proof. The second round is a second-price auction where buyer i knows that when he wins, he pays the second highest non-negative bid plus his own premium p_i. If he submits a bid $\hat{b}_i > v_i + s_i - p_i$ instead of b_i, then the only occasion when this bid makes a difference is when he wins with $\hat{b}_i$ (which is therefore non-negative) and the second highest bid (or zero, whichever is larger), b_j, is between b_i and $\hat{b}_i$. His profit is $v_i + s_i - b_j - p_i < v_i + s_i - b_i - p_i = 0$, so he ends up worse off. A similar argument shows that i can only miss profitable opportunities by bidding $\hat{b}_i < b_i$. Therefore bidding $b_i = v_i + s_i - p_i$ is indeed a weakly dominant strategy. ■

From now on, we assume that the buyers follow their weakly dominant strategies in the second round. Then the handicap auction can be represented by pairs of functions, $p_i : [0,1] \to \mathbb{R}$ and $c_i : [0,1] \to \mathbb{R}$, for $i = 1, ..., n$, where $p_i(v_i)$ is the price premium that type $v_i \in [0,1]$ chooses (in equilibrium) for the fee of $c_i(v_i) \equiv C_i(p_i(v_i))$. In what follows, let $w_j = v_j + s_j - p_j(v_j)$, introduce an artificial buyer numbered $j = 0$ with $w_0 = 0$, and denote $\max_{j \neq i}\{w_j, 0\}$ by $w_{-i}^{\max}$.

Incentive compatibility of the handicap auction $\{c_i, p_i\}_{i=1}^n$ means that type v_i does not want to deviate and choose $p_i(\hat{v}_i)$ for fee $c_i(\hat{v}_i)$ in the first round. If he deviates, then by Lemma 2, he will bid $v_i + s_i - p_i(\hat{v}_i)$ in the second round. Therefore, if buyer i with type v_i pretends to have $\hat{v}_i$ in the first round while the other buyers behave truthfully (that is, for all $j \neq i$, type v_j buys premium $p_j(v_j)$), then his payoff is,

$$\pi_i^*(v_i, \hat{v}_i) = \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i+s_i-p_i(\hat{v}_i)\geq w_{-i}^{\max}\}}(v_i + s_i - p_i(\hat{v}_i) - w_{-i}^{\max})\right] - c_i(\hat{v}_i). \tag{8}$$

Incentive compatibility of the mechanism means that v_i maximizes $\pi_i^*(v_i, \hat{v}_i)$ in $\hat{v}_i$. Let $\Pi_i^*(v_i) = \pi_i^*(v_i, v_i)$ be the buyer's equilibrium profit function. Introduce

$$Q_i(v_i, \hat{v}_i) = \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i-p_i(\hat{v}_i)+s_i\geq w_{-i}^{\max}\}}\right], \tag{9}$$

the expected probability that type v_i wins the second round after having chosen premium $p_i(\hat{v}_i)$ in the first round, given that all other bidders behave truthfully.

Lemma 3 *A handicap auction* $\{c_i, p_i\}_{i=1}^n$ *is incentive compatible if and only if, for all* $i = 1, ..., n$ *and* $v_i \in [0,1]$,

$$\Pi_i^*(v_i) = \Pi_i^*(0) + \int_0^{v_i} Q_i(\nu, \nu) d\nu \tag{10}$$

and for all $v_i', v_i'' \in [0,1]$ *such that* $v_i' < v_i < v_i''$,

$$Q_i(v_i, v_i') \leq Q_i(v_i, v_i) \leq Q_i(v_i, v_i''). \tag{11}$$

Condition (11) states that a buyer with a given type is weakly more (less) likely to get the good in equilibrium than he would be by imitating a lower (higher) type in the first round. Essentially, this means that lower types should get higher premia in any incentive compatible handicap auction. More precisely, if p_i is weakly decreasing for all i then (11) holds, and the converse is true if the density of s_i has full support on $(-\infty, +\infty)$.

Proof. **[Necessity]** We first prove that incentive compatibility of the handicap auction implies (10) and (11). Incentive compatibility is equivalent to,

$$\text{for all } i \text{ and } \hat{v}_i < v_i, \;\; \pi_i^*(v_i, \hat{v}_i) \leq \pi_i^*(v_i, v_i) \text{ and } \pi_i^*(\hat{v}_i, v_i) \leq \pi_i^*(\hat{v}_i, \hat{v}_i). \tag{12}$$

In the rest of this part of the proof, assume $\hat{v}_i < v_i$. Introduce, for all $x, y \in [0,1]$,

$$\Delta_i(x,y) = \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{w_{-i}^{\max} - s_i + p_i(y) \in (x \wedge y, x \vee y]\}}(x + s_i - p_i(y) - w_{-i}^{\max})\right].$$

Rewrite $\pi_i^*(v_i, \hat{v}_i)$ as

$$\begin{aligned}
\pi_i^*(v_i, \hat{v}_i) &= \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{\hat{v}_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max}\}}(\hat{v}_i + s_i - p_i(\hat{v}_i) - w_{-i}^{\max})\right] - c_i(\hat{v}_i) \\
&\quad + \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{\hat{v}_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max}\}}\right](v_i - \hat{v}_i) \\
&\quad + \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max} > \hat{v}_i + s_i - p_i(\hat{v}_i)\}}(v_i + s_i - p_i(\hat{v}_i) - w_{-i}^{\max})\right] \\
&= \Pi_i^*(\hat{v}_i) + Q_i(\hat{v}_i, \hat{v}_i)(v_i - \hat{v}_i) + \Delta_i(v_i, \hat{v}_i).
\end{aligned}$$

By similar decomposition,

$$\pi_i^*(\hat{v}_i, v_i) = \Pi_i^*(v_i) - Q_i(v_i, v_i)(v_i - \hat{v}_i) - \Delta_i(\hat{v}_i, v_i).$$

Given this, incentive compatibility of the handicap auction, (12), is equivalent to, for all i and $\hat{v}_i < v_i$,

$$Q_i(\hat{v}_i, \hat{v}_i) + \frac{\Delta_i(v_i, \hat{v}_i)}{v_i - \hat{v}_i} \leq \frac{\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i)}{v_i - \hat{v}_i} \leq Q_i(v_i, v_i) + \frac{\Delta_i(\hat{v}_i, v_i)}{v_i - \hat{v}_i}.$$

Note that $\Delta_i(x, y) \geq 0$ if and only if $x \geq y$, therefore $\Delta_i(\hat{v}_i, v_i) \leq 0 \leq \Delta_i(v_i, \hat{v}_i)$. Hence, incentive compatibility implies

$$Q_i(\hat{v}_i, \hat{v}_i) \leq \frac{\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i)}{v_i - \hat{v}_i} \leq Q_i(v_i, v_i). \tag{13}$$

From this, $Q_i(\nu, \nu)$ is monotone weakly increasing in ν, hence it is integrable, and so (10) follows.

Next, we show that (11) must hold. Assume that $\hat{v}_i < v_i$. If $p_i(\hat{v}_i) \geq p_i(v_i)$ then clearly, $Q_i(\hat{v}_i, \hat{v}_i) \leq Q_i(v_i, \hat{v}_i) \leq Q_i(v_i, v_i)$. Suppose $p_i(\hat{v}_i) < p_i(v_i)$. Introduce

$$\varepsilon_i(x, y) = \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{w_{-i}^{\max} - s_i \in (x - p_i(x) \vee p_i(y), x - p_i(x) \wedge p_i(y)]\}}(x + s_i - p_i(y) - w_{-i}^{\max})\right].$$

Rewrite

$$\begin{aligned}
\pi_i(v_i, \hat{v}_i) &= \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{v_i + s_i - p_i(v_i) \geq w_{-i}^{\max}\}}(v_i + s_i - p_i(v_i) - w_{-i}^{\max})\right] - c_i(v_i) \\
&\quad + \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{v_i + s_i - p_i(v_i) \geq w_{-i}^{\max}\}}\right](p_i(v_i) - p_i(\hat{v}_i)) + c_i(v_i) - c_i(\hat{v}_i) \qquad (14) \\
&\quad + \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{v_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max} > v_i + s_i - p_i(v_i)\}}(v_i + s_i - p_i(\hat{v}_i) - w_{-i}^{\max})\right] \\
&= \pi_i(v_i, v_i) + Q_i(v_i, v_i)(p_i(v_i) - p_i(\hat{v}_i)) + c_i(v_i) - c_i(\hat{v}_i) + \varepsilon_i(v_i, \hat{v}_i),
\end{aligned}$$

$$\begin{aligned}
\pi_i(\hat{v}_i, v_i) &= \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{\hat{v}_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max}\}}(\hat{v}_i + s_i - p_i(\hat{v}_i) - w_{-i}^{\max})\right] - c_i(\hat{v}_i) \\
&\quad + \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{\hat{v}_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max}\}}\right](p_i(\hat{v}_i) - p_i(v_i)) + c_i(\hat{v}_i) - c_i(v_i) \qquad (15) \\
&\quad - \mathbf{E}_{v_{-i}, s}\left[\mathbf{1}_{\{\hat{v}_i + s_i - p_i(\hat{v}_i) \geq w_{-i}^{\max} > \hat{v}_i + s_i - p_i(v_i)\}}(\hat{v}_i + s_i - p_i(v_i) - w_{-i}^{\max})\right] \\
&= \pi_i(\hat{v}_i, \hat{v}_i) - Q_i(\hat{v}_i, \hat{v}_i)(p_i(v_i) - p_i(\hat{v}_i)) + c_i(\hat{v}_i) - c_i(v_i) - \varepsilon_i(\hat{v}_i, v_i).
\end{aligned}$$

By incentive compatibility, (12), $\pi_i(v_i,\hat{v}_i)-\pi_i(v_i,v_i)\le 0\le \pi_i(\hat{v}_i,\hat{v}_i)-\pi_i(\hat{v}_i,v_i)$. By (14) and (15), this is equivalent to

$$Q_i(v_i,v_i)\left(p_i(v_i)-p_i(\hat{v}_i)\right)+\varepsilon_i(v_i,\hat{v}_i)\le c_i(\hat{v}_i)-c_i(v_i)$$
$$\le Q_i(\hat{v}_i,\hat{v}_i)\left(p_i(v_i)-p_i(\hat{v}_i)\right)+\varepsilon_i(\hat{v}_i,v_i). \quad (16)$$

Observe that since $\hat{v}_i<v_i$ and $p_i(\hat{v}_i)<p_i(v_i)$, we have $\varepsilon_i(\hat{v}_i,v_i)\le 0\le\varepsilon_i(v_i,\hat{v}_i)$. Then, $\varepsilon_i(v_i,\hat{v}_i)=\varepsilon_i(\hat{v}_i,v_i)=0$, otherwise (16) implies $Q_i(v_i,v_i)<Q_i(\hat{v}_i,\hat{v}_i)$ contradicting (13). By $\varepsilon_i(v_i,\hat{v}_i)=0$,

$$\mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i+s_i-p_i(\hat{v}_i)\ge w_{-i}^{\max}>v_i+s_i-p_i(v_i)\}}\right]=0,$$

which is equivalent to $Q_i(v_i,\hat{v}_i)=Q_i(v_i,v_i)$. Therefore, $\hat{v}_i<v_i$ implies $Q_i(v_i,\hat{v}_i)\le Q_i(v_i,v_i)$, no matter whether or not $p_i(v_i)\le p_i(\hat{v}_i)$, and the first inequality of (11) holds. Similarly, by $\varepsilon_i(\hat{v}_i,v_i)=0$, $\mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{\hat{v}_i+s_i-p_i(\hat{v}_i)\ge w_{-i}^{\max}>\hat{v}_i+s_i-p_i(v_i)\}}\right]$ is zero, hence $Q_i(\hat{v}_i,\hat{v}_i)=Q_i(\hat{v}_i,v_i)$. Therefore, $\hat{v}_i<v_i$ implies $Q_i(\hat{v}_i,\hat{v}_i)\le Q_i(\hat{v}_i,v_i)$, that is, the second inequality of (11) holds.

[**Sufficiency**] We now show that (10) and (11) imply that the handicap auction is incentive compatible. Let $U_i(v_i,s_i)$ be the expected equilibrium profit of type v_i with shock s_i in the second stage. Clearly, $\Pi_i^*(v_i)\equiv\mathbf{E}_{s_i}\left[U_i(v_i,s_i)\right]-c_i(v_i)$. From the incentive compatibility of the second round it routinely follows (see also equation 4 and the proof of Lemma 1) that, for all $s_i\le\hat{s}_i$,

$$U_i(v_i,\hat{s}_i)-U_i(v_i,s_i)=\int_{s_i}^{\hat{s}_i}\mathbf{E}_{v_{-i},s_{-i}}\left[\mathbf{1}_{\{v_i+\sigma-p_i(v_i)\ge w_{-i}^{\max}\}}\right]d\sigma,$$

where $\mathbf{E}_{v_{-i},s_{-i}}\left[\mathbf{1}_{\{v_i+s_i-p_i(v_i)\ge w_{-i}^{\max}\}}\right]$ is the probability that, in equilibrium, type v_i observing shock s_i wins the second round.

By Lemma 2, buyer i with type v_i who pretends to have type $\hat{v}_i$ in the first stage and observes s_i before the second stage will bid $b_i=v_i+s_i-p_i(\hat{v}_i)$ in the second stage, "as if" he had type $\hat{v}_i$ and observed $s_i+v_i-\hat{v}_i$. His probability of winning and expected payment will be the same as if he had a type-shock pair $(\hat{v}_i,s_i+v_i-\hat{v}_i)$. Hence, his expected profit in the second round will be $U_i(\hat{v}_i,s_i+v_i-\hat{v}_i)$.

This noted, we can rewrite $\pi_i^*(v_i, \hat{v}_i)$, with $\hat{v}_i < v_i$, as

$$\begin{aligned}
\pi_i^*(v_i, \hat{v}_i) &= \mathbf{E}_{s_i}\left[U_i(\hat{v}_i, s_i + v_i - \hat{v}_i)\right] - c_i(\hat{v}_i) \\
&= \mathbf{E}_{s_i}\left[U_i(\hat{v}_i, s_i) + U_i(\hat{v}_i, s_i + v_i - \hat{v}_i) - U_i(\hat{v}_i, s_i)\right] - c_i(\hat{v}_i) \\
&= \Pi_i^*(\hat{v}_i) + \mathbf{E}_{s_i}\left[\int_{s_i}^{s_i+v_i-\hat{v}_i} \mathbf{E}_{v_{-i},s_{-i}}\left[\mathbf{1}_{\{\hat{v}_i+\sigma-p_i(\hat{v}_i)\geq w_{-i}^{\max}\}}\right] d\sigma\right] \\
&= \Pi_i^*(\hat{v}_i) + \int_0^{v_i-\hat{v}_i} \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{\hat{v}_i+x+s_i-p_i(\hat{v}_i)\geq w_{-i}^{\max}\}}\right] dx.
\end{aligned}$$

In the last line, we replaced σ by $s_i + x$ and changed the order of integration. Similarly,

$$\pi_i^*(\hat{v}_i, v_i) = \Pi_i^*(v_i) - \int_{\hat{v}_i-v_i}^{0} \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i+y+s_i-p_i(v_i)\geq w_{-i}^{\max}\}}\right] dy.$$

Incentive compatibility of the handicap auction now becomes, for all i, $\hat{v}_i \in [0, 1)$, and $v_i \in (\hat{v}_i, 1]$,

$$\int_0^{v_i-\hat{v}_i} Q_i(\hat{v}_i + x, \hat{v}_i)dx \leq \Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i) \leq \int_{\hat{v}_i-v_i}^{0} Q_i(v_i + y, v_i)dy. \qquad (17)$$

From condition (11), $Q_i(\hat{v}_i + x, \hat{v}_i) \leq Q_i(\hat{v}_i + x, \hat{v}_i + x)$ for $x \in [0, v_i - \hat{v}_i]$. Therefore

$$\int_0^{v_i-\hat{v}_i} Q_i(\hat{v}_i + x, \hat{v}_i)dx \leq \int_0^{v_i-\hat{v}_i} Q_i(\hat{v}_i + x, \hat{v}_i + x)dx = \Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i),$$

so the first inequality of (17) holds. From (11), $Q_i(v_i + y, v_i + y) \leq Q_i(v_i + y, v_i)$ for $y \in [\hat{v}_i - v_i, 0]$, so

$$\Pi_i^*(v_i) - \Pi_i^*(\hat{v}_i) = \int_{\hat{v}_i-v_i}^{0} Q_i(v_i + y, v_i + y)dy \leq \int_{\hat{v}_i-v_i}^{0} Q_i(v_i + y, v_i)dy,$$

and the second inequality of (17) holds, too. Therefore, the handicap auction is incentive compatible. ■

In Lemma 1, we characterized incentive compatible mechanisms under the assumption that the seller can observe the shocks, while in Lemma 3, we characterized incentive compatible handicap auctions for the case when she cannot. Note that the necessary and sufficient conditions for incentive compatibility (in particular, the monotonicity conditions on X_i and Q_i, respectively) are not the same in the two cases. We will comment on the consequences of this fact in the next subsection.

Lemma 3 can be used to derive the handicap auction that maximizes the objective of the mechanism designer. In particular, we can easily determine the expected revenue maximizing handicap auction. In the next proposition we do just that; moreover, we claim that this handicap auction achieves the same expected revenue as if the seller could observe the realization of the shocks.

Proposition 2 *Assume that the seller cannot observe the realizations of the shocks, although she can allow the buyers to observe them. The seller can implement allocation rule (6) with expected revenue (7) via a handicap auction* $\{c_i, p_i\}_{i=1}^n$*, where*

$$p_i(v_i) = \frac{1 - F_i(v_i)}{f_i(v_i)}, \tag{18}$$

and $c_i(v_i)$ *is defined by*

$$c_i(v_i) = \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i+s_i-p_i(\hat{v}_i)\geq \max_{j\neq i} w_j\}}(v_i + s_i - p_i(v_i) - \max_{j\neq i} w_j)\right] - \int_0^{v_i} \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{\nu+s_i-p_i(\nu)\geq \max_{j\neq i} w_j}\right] d\nu. \tag{19}$$

Proof. If, for all $j = 1, ..., n$ and $v_j \in [0, 1]$, type v_j of buyer j purchases a price premium $p_j(v_j) = (1 - F_j(v_j))/f_j(v_j)$ in the first round, then buyer i will win in the second round if and only if, for all j,

$$v_i + s_i - \frac{1 - F_i(v_i)}{f_i(v_i)} \geq \max\left\{v_j + s_j - \frac{1 - F_j(v_j)}{f_j(v_j)}, 0\right\}.$$

This is so because in the second round, every buyer j bids $v_j + s_j - p_j(v_j)$. Hence the allocation rule is indeed the same as (6), provided that all buyers behave "truthfully," i.e., every buyer j with type v_j chooses $p_j(v_j)$ for a fee $c_j(v_j)$ defined in (19).

We can easily check that the handicap auction defined by (18) and (19) satisfies the hypotheses of Lemma 3. First, p_i is weakly decreasing by the assumption of monotone hazard rate. Second, the fee-schedule, (19), is equivalent to (10), as

$$\Pi_i^*(v_i) \equiv \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{v_i+s_i-p_i(v_i)\geq \max_{j\neq i} w_j\}}(v_i + s_i - p_i(v_i) - \max_{j\neq i} w_j)\right] - c_i(v_i).$$

Also note that $\Pi_i^*(0) = 0$ for all i. By Lemma 3, (18) and (19) define an incentive compatible handicap auction.

It remains to show that the seller's expected revenue in this handicap auction is equal to that in the optimal mechanism of Proposition 1 (where the seller could observe the shocks). Note that the expected payoff of buyer i with type v_i in the mechanism of Proposition 1 is given by (4) with allocation rule (6) and $\Pi_i(0) = 0$. The expected payoff of buyer i with type v_i in the proposed handicap auction is given by (10) with premium schedule (18) and $\Pi_i^*(0) = 0$. Therefore, $\Pi_i(v_i) = \Pi_i^*(v_i)$ for all v_i. Since the allocation rules in the two mechanisms coincide, the total social surplus is the same in both cases. The seller's expected revenue is just the difference of the total surplus and the buyers' payoff, therefore, it must also be the same. ■

Remark 2 If the support of each shock is $(-\infty, +\infty)$ then, as we remarked before the proof of Lemma 3, p_i must be weakly decreasing for the handicap auction to be incentive compatible. Also observe that the optimal allocation rule in Proposition 1 is unique (i.e., allocate the good to the buyer with the highest non-negative shock-adjusted virtual valuation). Therefore, if the seller can achieve the same revenue in a handicap auction without observing the shocks, then the allocation rule must be the same. Hence, the premium in this handicap auction must equal the hazard rate, (18), and the monotone hazard rate assumption is necessary to guarantee the same revenue.

3.3 Discussion

From the seller's perspective, the premium–fee schedule offered in the first round of the handicap auction works as a device to discriminate among buyers with different value estimates. When a buyer decides to participate in the handicap auction, he knows his type (expected valuation), which tells him whether he is more or less likely to win. Therefore, in the first round, a buyer with a high type chooses a small price premium for a large fee in order not to pay much when he wins. Using analogous reasoning, low types choose large price premia, which are cheaper, but make winning more expensive.

It is interesting to observe that in the optimal handicap auction, two buyers with the same actual valuation (same $v_i + s_i$) do not have the same probability of winning. The buyer with the larger v_i will choose a smaller price premium, bid higher in the second round, and will be more likely to win. This shows that the auction does not achieve full ex post efficiency, even under ex ante symmetry of the bidders and conditional on the object being sold.[7]

In order to better explain our main result (Proposition 2), consider a setup where the buyers are ex ante symmetric (the v_i's are identically distributed), and the shocks are mean zero random variables. Let us compare the optimal allocation rule in the case when the seller can observe the shocks (as in Subsection 3.1) with that of the revenue maximizing auction when *nobody* (neither the seller nor the buyers) can observe them. In the latter case, the seller should allocate the good to the buyer with the largest non-negative virtual value-estimate, $v_i - (1 - F(v_i))/f(v_i)$. If the seller can observe the shocks, then, in the optimal mechanism, the good will be allocated more efficiently, as the winner will now be the buyer with the highest non-negative shock-adjusted virtual valuation, $v_i + s_i - (1 - F(v_i))/f(v_i)$, according to equation (6).[8] According to Proposition 2, the seller, by controlling the release of the shocks and without actually observing them, can implement the same allocation, and surprisingly, can appropriate the increase in efficiency.[9]

One may suggest that the way the seller can appropriate all rents from the additional information is that in the handicap auction, she essentially charges the buyers a type-dependent up-front fee equal to the "value" of the information they are about to receive. This intuition may be appealing, but it overly simplifies the workings of the mechanism. First, the value of the additional information to the participants is not well-defined because it depends on the rules of the selling mechanism. This value could be different if the seller chose a mechanism different from the handicap auction. Another argument is that we showed, the seller may not always be able to extract all rents for the additional information via a handicap auction. This is so if the virtual value-estimates are monotone increasing, but the type-distributions do not exhibit

[7]In contrast, in the classical setup with deterministic valuations, the optimal auction of Myerson (1981) and Riley and Samuelson (1981) is efficient conditional on sale, provided that the buyers are ex ante symmetric.

[8]It is easy to see that if $v_i - (1 - F(v_i))/f(v_i) < v_j - (1 - F(v_j))/f(v_j)$, but, by adding the shocks to both sides the inequality is reversed, then $v_i + s_i > v_j + s_j$. Therefore, an allocation based on the shock-adjusted virtual valuations "pointwise" improves efficiency. (This may not be true if the F_i's are not identical.)

[9]If the buyers' ex ante type-distributions are not identical then, as the seller gets to observe the signals, the efficiency of the optimal mechanism may only improve in ex ante expectation. Still, there will be some efficiency gain, which will be fully extracted by the seller even if she cannot observe the additional signals.

monotone hazard rates (compare Remarks 1 and 2). The allocation rule in the optimal mechanism when the seller can observe the shocks will be based on the shock-adjusted virtual value-estimates, but the corresponding premium functions, the p_i's, would not be weakly decreasing. Therefore, the former allocation rule is not implementable via a handicap auction when the seller cannot observe the shocks.

3.4 Determining the Optimal Handicap Auction: A Numerical Example

It may be useful to compute a numerical example not only for illustrative purposes, but also, to see how a seller may be able to compute the parameters of the optimal handicap auction (the price premium–fee schedule) in a practical application.

The optimal $p_i(v_i)$ is given by (18), and $c_i(v_i)$ is given by (19). Supposing that p_i is differentiable, we can rewrite (19) as

$$c_i(0) = \mathbf{E}_{v_{-i},s}\left[\mathbf{1}_{\{s_i - 1/f_i(0) \geq \max_{j\neq i} w_j\}}(s_i - 1/f_i(0) - \max_{j\neq i} w_j)\right], \tag{20}$$

$$c_i'(v_i) = -p_i'(v_i)\,\mathbf{E}_{v_{-i},s_{-i}}\left[\Pr[s_i \geq \max_{j\neq i} w_j - v_i + p_i(v_i)]\right]. \tag{21}$$

Note that (21) is just the first-order condition of $v_i \in \arg\max_{\hat{v}_i} \pi_i^*(v_i, \hat{v}_i)$, where $\pi_i^*(v_i, \hat{v}_i)$ is given by (8).

We will consider the following setup. The types are distributed independently and uniformly on $[0,1]$, and the shocks are distributed independently according to a standard logistic distribution.[10]

First, assume that there is a single buyer, that is, $n = 1$. As we mentioned it earlier, the handicap auction with a single buyer can be thought of as a *menu of buy options*, represented by $C_1(p_1)$, where p_1 is the strike price and $C_1(p_1)$ is the fee of the option. In the first round, the buyer chooses a price p_1 and pays $C_1(p_1)$; in the second round (after having observed s_1), he has the option to buy the good at price p_1.

Let us represent the menu of buy options as a pair of functions, $c_1(v_1)$ and $p_1(v_1)$, $v_1 \in [0,1]$. In the uniform-logistic example, the expected revenue maximizing strike

[10] The cdf of the standard logistic distribution is $G_i(s_i) = e^{s_i}/(1+e^{s_i})$, $s_i \in (-\infty, +\infty)$.

price-schedule is given by (18),

$$p_1(v_1) = 1 - v_1.$$

The equations characterizing the fee-schedule, (20) and (21), become,

$$\begin{aligned} c_1(0) &= \int_1^\infty (s_1 - 1)\frac{e^{s_1}}{(1+e^{s_1})^2} ds_1 = \ln(1+e) - 1, \\ c_1'(v_1) &= 1 - \frac{e^{1-2v_1}}{1+e^{1-2v_1}}. \end{aligned}$$

By integration, we get an explicit expression for $c_1(v_1)$ as

$$c_1(v_1) = \frac{1}{2}\ln(1+e) - 1 + v_1 + \frac{1}{2}\ln\left(1 + e^{1-2v_1}\right).$$

We can express the cost of the option as a function of the strike price as

$$c_1 = C_1(p_1) = \frac{1}{2}\ln\left[(1+e)\left(1+e^{2p_1-1}\right)\right] - p_1.$$

This (downward-sloping) schedule is depicted as the top curve in Figure 1.

If the buyer has a higher estimate then he will choose to buy an option with a lower strike price at a higher cost. For example, if the buyer has the lowest estimate, $v_1 = 0$, then he buys the option of getting the good at $p_1 = 1$, which costs $c_1 = \ln[(1+e)/e] \approx 0.3133$ upfront, and yields zero net surplus. In contrast, the highest type, $v_1 = 1$, buys a call option with zero strike price at a cost of about 0.8133.

Now we turn to the case of many buyers, $n > 1$, in the uniform-logistic example. We will compute the optimal handicap auction represented by $\{c_i, p_i\}_{i=1}^n$. As in the case of $n = 1$, in the revenue-maximizing mechanism, $p_i(v_i) \equiv 1 - v_i$.

Instead of analytically deriving $c_i(v_i)$ for different numbers of buyers, we carry out a more practical Monte Carlo simulation. What we describe below is also the method that a seller could use in order to determine the parameters of an optimal handicap auction in practice.

We take 100,000 random draws from the joint distribution of (s, v_{-i}), and compute $w_j = v_j + s_j - p_j(v_j)$ for all j. Then we determine $c_i(0)$ from (20), where the expectation is estimated by the sample mean. We compute $c_i(v_i)$ recursively, $c_i(v_i + \text{step}) = c_i(v_i) + \text{step} * c_i'(v_i)$, where step$= 1/100$. The derivative can be estimated

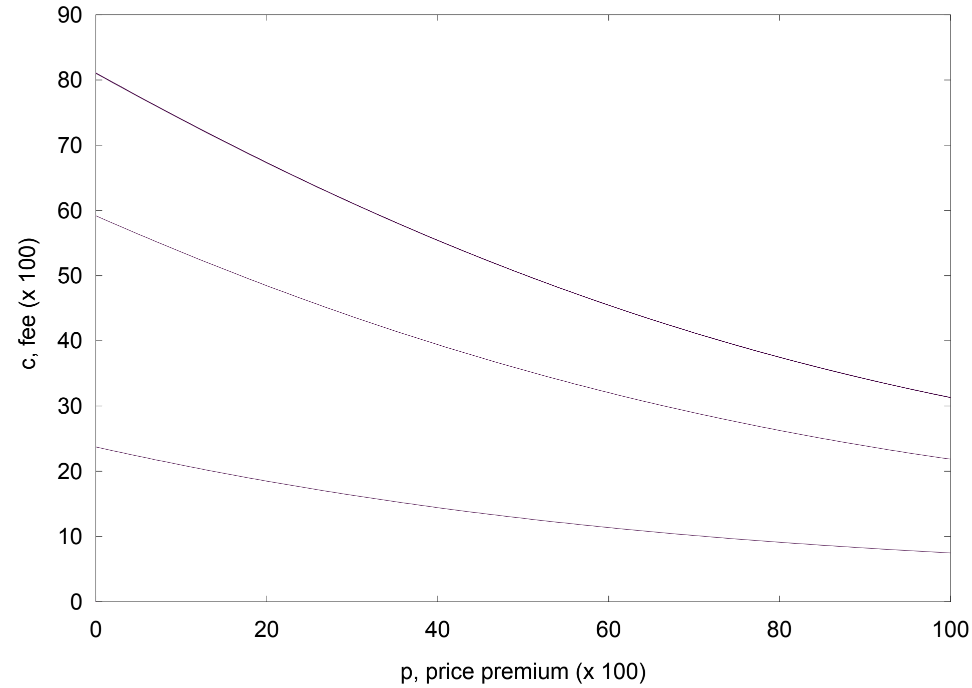

Figure 1: Fee schedules in the revenue maximizing handicap auction (uniform-logistic setup; schedules from top to bottom for n=1, 2, and 5)

from (21), rewritten as

$$c_i'(v_i) = \mathbf{E}_{s_i, s_{-i}, v_{-i}} \left[\mathbf{1}_{\{v_i + s_i - p_i(v_i) \geq \max_{j \neq i} w_j\}} \right].$$

From $c_i(v_i)$ and $p_i(v_i)$ we compute $C_i(p_i) \equiv c_i(p_i^{-1}(p_i))$.

The results of a (typical) simulation are shown in Figure 1. The top curve shows $C_i(p_i)$ for the case of $n = 1$. There are actually two (almost identical) curves superimposed on each other: one graphs the formula that we derived before, the other is the result of the Monte Carlo experiment. The curve in the middle is $C_i(p_i)$ for $n = 2$, and the one in the bottom is $C_i(p_i)$ for $n = 5$. As n increases, $C_i(p_i)$ shifts down and flattens out.

4 Conclusions

In this paper, we analyzed an auction model where the seller could decide how accurately the buyers learned their private valuations. In particular, in our setting, the buyers only knew an initial estimate of their private valuation, and the seller had the ability to release (without observing) independent signals that were added to the buyers' estimates to determine their ex post valuations. In other words, the buyers' valuations were initially uncertain, but the seller could allow them to resolve this uncertainty. We derived the expected revenue maximizing mechanism.

In the optimal mechanism, the seller allows the buyers to learn their valuations with the highest precision and obtains the same expected revenue as if she could observe the additional signals (which she can release, but cannot directly observe). The buyers do not enjoy any additional information rents from the signals whose disclosure is controlled by the seller.

The outcome of this mechanism can be implemented via a "handicap auction." In the first phase of this mechanism, the seller publishes a price premium–fee schedule for each buyer; each buyer chooses a price premium and pays the corresponding fee. Then the seller allows the buyers to learn their valuations with the highest precision. In the second phase, the buyers bid for the good in a second-price sealed-bid auction with a zero reservation price, knowing that the winner will pay his premium over the price. For a single buyer, the handicap auction simplifies to a menu of buy-options.

Interestingly, our main result extends to *general adverse selection models*, as shown in our related paper Eső and Szentes (2002). In that paper, we consider a setup where a principal controls the precision of the agent's information regarding his own type (productivity, ability, etc.), by being able to release, without observing, signals that refine the agent's estimate.[11] In the principal's optimal contract the agent learns his type with the highest precision, yet no information rents will be left with the agent for the additional signals.[12] Again, the one who controls the flow of information appropriates the rents of information.

[11] This is the case, for example, when the principal is the employer of the agent, and decides about the extent of the agent's learning the details of the task, etc.

[12] The optimal contract, however, may be a lot more complicated than the handicap auction.

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