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Financial Planning in a Multinational Firm Facing Flexible Exchange Rates

by

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I. Introduction

Flexible exchange rates pose a continuing risk to firms that operate in international markets. The risks are associated with transactions resulting in accounts receivable and current liabilities and with stocks such as cash balances, inventories, and long-term fixed assets. This paper is concerned with the continuing risks imposed by currency fluctuations on short-run activities of a multinational firm. For example, collection of the receivable resulting from a foreign sale is usually made in sixty or ninety days, and if the sale is denominated in a foreign currency, the receivable is subject to exchange rate risk. Similarly, the dividends paid by a foreign subsidiary to the parent must be converted at the exchange rate at the time of payment. A capital expenditure for the construction of a new plant in a foreign country involves payments in the foreign currency over an extended period of time. While the time schedule of the payments may be known, the exchange rates at which the payments will be made are not known. A survey of management attitude towards these forms of transactions exposure has been given by Rodriguez [7]. The emphasis in this paper is on how to deal with the exchange rate risks.

The firm may act to reduce such risks by hedging with forward exchange contracts or by borrowing and lending in foreign currencies.[1] To cover a 90-day receivable denominated in British pounds, for example, an American firm could sell pounds forward today at the 90-day forward rate and thus be assured of being able to convert pounds to dollars at a specified exchange rate in ninety days. Alternatively, the firm could wait thirty days and sell pounds forward sixty days at whatever forward rate would hold at that time. Even for sales that have not yet occurred, say those taking place six months from now, the firm is able to hedge at any time between the present and the time at which the receivables will be collected.

The hedging of foreign currency transactions involves a significant risk that can have a major effect on profits.[2] For example, if pounds are sold forward ninety days at a forward rate of \$2.25/£ and in ninety days the spot exchange rate is \$2.30/£, the firm will have incurred an opportunity loss of more than two percent on sales. Such hedging decisions must necessarily involve expectations regarding the future evolution of both spot and forward exchange rates, but exchange rate movements are extremely difficult to predict, so a means of estimating and dealing with such movements is needed and will be provided here. Besides the hedging decisi ns, a multinational company must make the usual short-term financial decisions involving borrowing and lending, transfer pricing, dividend payments, etc. These decisions should be made in conjunction with the hedging decisions in order to meet the financial objectives and requirements of the firm.

A number of short-term financial planning models have been developed that deal with limited aspects of this problem.

Rutenberg [8] has presented a deterministic multiperiod model for financial planning in a multinational firm with exchange rates taken as fixed. Krause [3] considered a multi-period financial planning model that incorporates uncertainty as a disturbance term in a state-transition function, but to obtain a solution, he resorts to a quadratic objective function and an additive disturbance in a model without constraints. Wundisch [8] developed a cash-planning model for a multinational firm that incorporates both spot and forward exchange rates, but he does not treat future spot rates as uncertain. Mehta and Inselbag [5] have presented a model of working capital management in a multinational corporation but utilize "anticipated-formal or implicit-changes in the relative values of currencies" rather than uncertain future exchange rates. In the context of a mean-variance portfolio model Lietaer [4] treats hedging decision when exchange rates are uncertain. Jääskeläinen and Salmi [2] present a two-period model for determining hedging, sales, production, transfer price, and borrowing levels. Their solution procedure however is such that it preserves feasibility in any state of the world.

The model presented herein is intended to serve as a guide to the planning of these short-term financial decisions over a set of short time periods, e.g., a month each. To simplify the model, longer-term decisions such as those involving capital investments, production, and sales will be taken to be fixed during the horizon of the short-term financial planning model. The financial planners are assumed to receive deterministic production and sales plans as well as capital investment schedules and to plan subject to these inputs. The decision variables in the planning model are concerned with (a) transfer prices, (b) borrowing and lending, and (c) forward exchange transactions. The model consists of three parts: (1) a stochastic process model that represents the evolution of foreign exchange rates, (2) a cash flow model that characterizes the cash flows possible, given the organizational structure of the firm, and (3) a solution model that transforms the stochastic model into a deterministic equivalent that may be solved. The stochastic characterization of the exchange rates is presented in Section II, the cash flow model follows in Section III, and the solution procedures are given in Section IV.

II. A Stochastic Characterization of Fluctuating Exchange Rates

The levels of foreign exchange rates are determined by both an exogenous process and an endogenous process. The exogenous process involves trade between countries, flows of long- and short-term capital, tourism, governmental transactions, etc. These flows tend to determine the general levels of exchange rates, but the relationships among spot exchange rates and between spot and forward rates is determined by an endogenous arbitrage process. For example, if s_{ij} is the spot exchange rate between countries i and j, measured in units of currency j per unit of currency i, then s_{ji} must equal $1/s_{ij}$ or else investors may exchange one currency for another and gain without any investment. For example, suppose that an investor sells x units of currency i against currency j and simultaneously sells the currency j proceeds against currency i. The sale of currency i yields xs_{ij} units of currency j, and the sale of this quantity yields $(xs_{ij})s_{ji}$ units of currency i. The gain to the investor is $xs_{ij}s_{ji} - x = x(s_{ij}s_{ji}-1)$. If $(s_{ij}s_{ji}-1) >(<) 1$, then $x >(<) 0$ yields a gain to the investor. Since no investment is made in connection with this transaction and no risk is involved, arbitragers will make trades until, in equilibrium,

$$(2.1a) \qquad s_{ji} = 1/s_{ij}$$

This relationship can be extended to any number of currencies so that, for example, $s_{ji} = (1/s_{ik})/s_{kj}$, and hence

$$(2.1b) \qquad s_{ik}s_{kj} = s_{ij}.$$

A forward exchange rate is that rate of exchange at which one currency may be exchanged for another at some specified date in the future. For example, two traders may sign a contract today to sell x units of currency i for currency j at a predetermined rate z_{ij} in ninety days. In ninety days one investor sells the x units of currency for xz_{ij} units of currency j and the other receives the x units of currency i. Using an argument identical to that above, it may be shown that, in equilibrium,

$$(2.2) \qquad z_{ji} = 1/z_{ij} \quad \text{and} \quad z_{ik}z_{kj} = z_{ij}$$

for all currencies. If the first equality were not satisfied, for example, an investor could gain by selling forward x units of currency i against currency j and simultaneously selling forward the proceeds of that transaction against currency i (x would be positive or negative as appropriate).

An arbitrage process also works to establish the relationship between spot and forward exchange rates. Consider an investor in country i who has the choice between the following two investments: (a) save x units of currency i for ninety days in a bank in country i at a gross interest rate r_i ($r_i > 1$), or (b) purchase xs_{ij} units of currency j and save it for ninety days in a bank in country j at an interest rate r_j ($r_j > 1$) while simultaneously signing a forward contract to sell $xs_{ij}r_j$ units of currency j at the forward rate z_{ji}. In ninety days the amount of currency i available is xr_i if the first investment is made and $xs_{ij}r_jz_{ji}$ if the second is made. The second is preferred to the first if $xr_i - xs_{ij}r_jz_{ji} = x(r_i - s_{ij}r_jz_{ji}) < 0$. Thus if $(r_i - s_{ij}r_jz_{ji}) <(>) 0$, investors in country i will make investment (b)((a)) and investors in country j will make the converse investment (b)((a)). This will continue until, in equilibrium, the spot and forward rates satisfy

$$z_{ij} = 1/z_{ji} = s_{ij}r_j/r_i \ . \tag{2.3}$$

Consequently, if the interest rate in country j is less than the interest rate in country i, the forward rate z_{ij} will be less than the spot rate s_{ij}. In this case currency j is said to sell at a forward discount or currency i at a forward premium. The relationship in (2.3) tends to be satisfied, but in reality, because of market imperfections such as margin requirements, for example, it may not be exactly satisfied at all times.[3] These imperfections will not be taken into account in the current model nor will arbitrage decisions be considered, since the firm need not consider such decisions if the arbitrage process itself can be assumed to eliminate any gains to arbitrage.

At each point in time the relationships in (2.1a), (2.1b), (2.2), and (2.3) will hold, and the stochastic process representing the movement of exchange rates must be such that these equilibrium relationships are satisfied at every time in the future for all currencies. If they were not satisfied, then as indicated above arbitrage would occur to adjust the exchange rates so that the relationships were satisfied. We now introduce the following notation that will be used for the exchange rates:

s_{ij0} = the spot rate at time zero

z_{ij0g} = the g-period forward rate available at time zero

r_{i0g}/r_{j0g} = the ratio of the g-period interest rates available at time zero

$\tilde{s}_{ijk}$ = the uncertain g-period forward rate available in period k

$\tilde{r}_{ikg}/\tilde{r}_{jkg}$ = the ratio of the uncertain g-period interest rates available in period k.

The stochastic process thus must begin with the initial conditions s_{ij0}, z_{ij0g}, and r_{i0g}/r_{j0g} and generate $\tilde{s}_{ijk}$, $\tilde{z}_{ijkg}$, and $\tilde{r}_{ikg}/\tilde{r}_{jkg}$ subject to the conditions (2.1a), (2.1b), (2.2) and (2.3). To simplify subsequent notation, all exchange rates will be expressed in terms of those involving currency zero (e.g., s_{0j0} or $\tilde{s}_{0jk}$).

Let $\tilde{u}_{ijk} = \ell n(\tilde{s}_{ijk})$ and $\tilde{u}^k = (\tilde{u}_{01k},\ldots,\tilde{u}_{0nk}) = (\tilde{u}_1^k,\ldots,\tilde{u}_n^k)$, where the currencies considered are indexed by the integers $0,1,\ldots,n$. The stochastic process utilized specifies that $\tilde{u}^k$ is generated from $\tilde{u}^{k-1}$ by independent multivariate normal increments $\tilde{y}^k$, so that

$$(2.4) \qquad \tilde{u}^k \equiv \tilde{u}^{k-1} + \tilde{y}^k, \quad k=1,2,\ldots,K,$$

where $\tilde{y}^k \sim N(m_k, W_k,)$, $\tilde{y}^k$ and $\tilde{y}^\ell$ are independent for $k \neq \ell$, and K is the index of the last period accounted for in the model. Thus the mean vector of the k^{th} increment is denoted by m_k and the variance-covariance matrix is denoted by W_k. From (2.4) we have

$$(2.5) \qquad \tilde{u}^k = u^0 + \sum_{\ell=1}^{k} \tilde{y}^\ell \equiv u^0 + \tilde{w}^k,$$

where $\tilde{w}^k \sim N(\sum_{\ell=1}^{k} m_\ell, \sum_{\ell=1}^{k} W_\ell)$. If we let $m_0 = u^0$ and $W_0 = \underline{0}$, $\tilde{u}^k \sim N(m^k, W^k)$, where $m^k \equiv \sum_{\ell=0}^{k} m_\ell$ and $W^k \equiv \sum_{\ell=0}^{k} W_\ell$.

The distribution of $\tilde{u}_{ijk}$ may be determined from the exchange rates relative to zero, since

$$(2.6) \qquad \tilde{u}_{ijk} = \ell n(\tilde{s}_{ijk}) = \ell n(\tilde{s}_{0jk}/\tilde{s}_{0ik}) = \ell n(\tilde{s}_{0jk}) - \ell n(\tilde{s}_{0ik})$$
$$= \tilde{u}_{0jk} - \tilde{u}_{0ik}.$$

Consequently, $\tilde{u}_{ijk} \sim N(m_j^k - m_i^k,\ v^{ij} W^k (v^{ij})^t)$, where v^{ij} is a $1 \times n$ vector with +1 in position j, -1 in position i, and zeroes elsewhere. All exchange rates $\tilde{s}_{ijk}$ thus have lognormal distributions.

From the distribution of the spot exchange rates the distribution of the forward exchange rates may be determined if the interest rates are given. The g-period interest rates that will exist in period k are also unknown, and the changes in their natural logorithm will be assumed to be given by a normal process. Let $\tilde{\eta}_{ikg} \equiv \ell n(\tilde{r}_{ikg})$ and $\tilde{\eta}^{kg} = (\tilde{\eta}_{0kg}, \tilde{\eta}_{1kg}, \ldots, \tilde{\eta}_{Nkg})$, where $\tilde{r}_{ikg}$ is the country i interest rate in period k for a loan or deposit of g periods. Then $\tilde{\eta}^{kg}$ is generated from $\tilde{\eta}^{k-1,g}$ by

$$(2.7) \qquad \tilde{\eta}^{kg} \equiv \tilde{\eta}^{k-1,g} + \tilde{\delta}^{kg}, \; k=1,\ldots,\bar{K} - g$$

where $\tilde{\delta}^{kg} \sim N(\mu_{kg}, \Sigma_{kg})$ and $\tilde{\delta}^{kg}$ and $\tilde{\delta}^{\ell g}$ are independent for $k \neq \ell$. The changes $\tilde{\delta}^{kg}$ are assumed to be independent[4] of the $\tilde{y}^{\ell}$ for all k and ℓ. Letting $\mu_0^g \equiv (\ell n\, r_{00g}, \ell n\, r_{10g}, \ldots, \ell n\, r_{n0g})$ and $\Sigma_{0g} \equiv \underline{0}$, $\tilde{\eta}^{kg} \sim N(\mu^{kg}, \Sigma^{kg})$ where $\mu^{kg} \equiv \sum_{\ell=0}^{k} \mu_{\ell g}$ and $\Sigma^{kg} \equiv \sum_{\ell=0}^{k} \Sigma_{\ell g}$.

The distribution of the forward exchange rate can thus be determined from

$$(2.8) \qquad \tilde{t}_{ijkg} \equiv \ell n(\tilde{z}_{ijkg}) = \ell n \left(\frac{\tilde{r}_{jkg}}{\tilde{r}_{ikg}} \; \tilde{s}_{ijk} \right)$$

$$= \ell n(\tilde{r}_{jkg}) + \ell n(\tilde{s}_{ijk}) - \ell n(\tilde{r}_{ikg}).$$

Consequently, $\tilde{t}_{ijkg} \sim N(\mu_j^{kg} + m_j^k - m_i^k - \mu_i^{kg},\; v^{ij}\, W^k (v^{ij})^t + \gamma^{ij} \Sigma^{kg} (\gamma^{ij})^t)$ where γ^{ij} is a $1 \times n+1$ vector with a +1 in position $j+1$, a -1 in position $i+1$, and zeroes elsewhere. The forward exchange rates are thus also lognormally distributed.[5]

Two stochastic processes have been defined, one representing changes in interest rates and the other representing changes in spot exchange rates relative to currency 0 which may be thought of as representing the exogenous trade and investment factors previously mentioned. Given those two processes the distribution of any spot or forward exchange rate may be determined from (2.5), (2.6), (2.7), and (2.8). In using this model, a firm need only represent the currencies in which it actually has transactions. Many of those currencies have exchange rates tied to a principal currency such as the dollar, the British pound, or the French franc, so the number of currencies that must be considered will be much smaller than the number of countries with which a firm deals. Also, a number of major European countries have been engaged in a joint float that further reduces the number of currencies that must be represented.[6] For certain other currencies forward markets do not exist,but in those cases borrowing and lending in the foreign currency can often be used for hedging. For other currencies hedging may be impossible and the firm may simply price in its home currency or leave transactions uncovered.

In order to utilize the stochastic model of the exchange rates, one must estimate, subjectively and/or objectively, the parameters in m^k, W^k, μ^{kg}, and Σ^{kg}. A simple procedure for obtaining estimates for the case in which the processes can be assumed to be stationary is to simply use the maximum likelihood estimates of the parameters. If one wishes to incorporate prior information, a conditional multivariate normal distribution for m^k, and a Wishart marginal distribution for W^k, may be used. The posterior marginal distribution of m^k is then a multivariate normal-Wishart distribution. If one is willing to make the assumption that W^k is known, then m^k has a multivariate normal distribution.[7]

III. The Cash Flow Model

A. The Organizational Structure of the Firm

The representation of the cash flows among the components of the firm must capture both the organizational structure of the multinational firm and the variety of financial transactions that can be made. The set of indices of the countries in which the firm operates will be denoted by $N = \{0,1,\ldots,n\}$, where country zero is the home country of the firm. The firm has subsidiaries (or divisions) in a set of countries whose index set is $S \subset N$, and S_j is the index set of the subsidiaries owned directly by subsidiary j, $j \in S$. Direct ownership is used in order to characterize the payment of dividends, so while all subsidiaries could be considered to be subsidiaries of the parent, S_0 will not be identical with S if some subsidiary owns another subsidiary.

Besides the paymen t of dividends a subsidiary may transfer to or obtain funds from other subsidiaries by making intercompany loans, royalty payments, paying management and licensing fees, and by making a variety of other types of transfers.[8] Only the most common types of transfers involving dividends, intercompany loans, royalties, management fees, and licensing fees will be considered here. The following notation will be used to denote the feasible transfer relationships (additional notation is defined in Table 1):

$\beta_j \subset S$ is the index set of the subsidiaries from which subsidiary j may borrow

$\mathcal{L}_j \subset S$ is the index set of the subsidiaries to which subsidiary j may loan

$O_j^* \subset S$ is the index set of the subsidiaries from which subsidiary j may receive royalties, management fees, and licensing fees

$O_j \subset S$ is the index set of the subsidiaries to which subsidiary j may pay royalties, management fees, and licensing fees.

The firm will be assumed to borrow and loan in foreign currencies only from other subsidiaries.

In addition to these means of transferring funds, subsidiaries have some latitude in the choice of the prices at which intracompany transfers are made. If the product transferred has a well-identified market price, the transfer price must be related to the market price. For many intermediate products, however, market prices do not exist, and firms must set transfer prices. The transfer prices directly affect the tax receipts of the countries involved, so there are certain explicit and implicit constraints on the transfer prices.[9] To take into account transactions with clients, both within and outside the firm, the index sets of the subsidiaries and outside clients with which a subsidiary makes transactions will be denoted as follows:

$N_j' \subset S$ is the index set of the subsidiaries to which subsidiary j sells

$N_j^0 \subset S$ is the index set of countries in which there are outside clients to which subsidiary j sells

$M_j' \subset N$ is the index set of countries in which there are subsidiaries from which subsidiary j buys

$M_j^0 \subset N$ is the index set of the countries in which there are outside clients from which subsidiary j buys

$N_j = N_j' \cup N_j^0$ is the index set of countries in which there are clients of subsidiary j

$M_j = M_j' \cup M_j^0$ is the index set of countries for whom subsidiary j is a client

For subsidiary j transfer prices will be associated with each $i \in N_j' \cup M_j'$.

B. Cash Flow for Subsidiary j

Subsidiary j may receive income from sales to outside clients, sales to other subsidiaries, revenue from lending (minus borrowing costs), dividends, fees, royalties, etc. The taxable profits of a subsidiary will first be specified and then the cash flow developed. The taxable revenue from sales in period k will be denoted by

$$RC_{jk} = \sum_{i \in N_j^0} \delta_{ij}(\tilde{s}_{ijk}) y_{ijk} + \sum_{i \in N_j'} \delta_{ij} (\tilde{s}_{ijk}) t_{ijk} \, x_{ijk}$$

where the sales y_{ijk} to outside clients and the shipments x_{ijk} to other subsidiaries are given by a sales plan or forecast. Accrued sales are assumed to be valued at the exchange rates $\tilde{s}_{ijk}$ at the times of the sales. The firm is able to choose the price t_{ijk} at which goods are transferred within the firm, and the function $\delta_{ij}(\tilde{s}_{ijk})$ is used to indicate the currency in which the sale is made. This pricing decision is assumed to be given, so that if the price is set in currency j, $\delta_{ij}(\tilde{s}_{ijk}) = 1$, and if the price is set in currency i, $\delta_{ij}(\tilde{s}_{ijk}) = \tilde{s}_{ijk}$. Similarly, the import cost of subsidiary j in period k is

$$IC_{jk} = \sum_{i\in M_j^0} \delta_{ji}(\tilde{s}_{jik})y_{jik} + \sum_{i\in M_j'} \delta_{ji}(\tilde{s}_{jik})t_{jik}\, x_{jik}$$

The net revenue RL_{jk} to subsidiary j from all borrowing and lending activities in period k is

$$RL_{jk} = L_{jjk}\ell_{jjk} - B_{jjk}b_{jjk} - \sum_{i\in \beta_j} B_{ijk}\, \tilde{s}_{ijk}b_{ijk} + \sum_{i\in \mathcal{L}_j} L_{ijk}\, \ell_{ijk},$$

where the interest rates for borrowing and lending are denoted by b and ℓ and all loans are made in the currency of the lender and are assumed to have a maturity of one period with interest paid during the period of the loan. That is, the loans are discounted. The variable B_{ijk} is thus the borrowing by j in currency i during period k.[10]

The net effect P_{jk} on profits of fees P_{ijk} paid and received is given by

$$P_{jk} = \sum_{i\in 0_j^*} P_{ijk}\, \delta_{ij}\, (\tilde{s}_{ijk}) - \sum_{i\in 0_j} P_{jik}\, \delta_{ij}\, (\tilde{s}_{ijk}),$$

where P_{ijk} is given by the financial plan. The income D_{jk} from dividends is

$$D_{jk} = \sum_{i\in S_j} D_{ijk}\, \tilde{s}_{ijk},$$

where dividends are assumed to be paid in the currency of i.

The cost incurred by subsidiary j includes production costs and taxes paid on exports and imports[11] and perhaps a value-added tax. The cost of production, distribution, etc. in country j is denoted by f_{jk}, and the effective value-added tax is denoted by v_j. The export tax ET_{jk} is

$$ET_{jk} = \sum_{i \in N_j^0} \delta_{ij} (\tilde{s}_{ijk}) y_{ijk} e_{ij} + \sum_{i \in N_j'} \delta_{ij} (\tilde{s}_{ijk}) t_{ijk} x_{ijk} e_{ij},$$

where e_{ij} is the tax rate per unit of revenue in currency j. The tax is assumed to be paid in period k through estimated tax payments, for example. The import tax IT_{jk} paid by j is

$$IT_{jk} = \sum_{i \in M_j^0} \delta_{ji} (\tilde{s}_{jik}) y_{jik} i_{ji} + \sum_{i \in M_j'} \delta_{ji} (\tilde{s}_{jik}) t_{jik} x_{jik} i_{ji},$$

where i_{ji} is the tax rate in country i. The total cost CC_{jk} of the subsidiary from its production and sales operations is thus

$$CC_{jk} = IC_{jk} + f_{jk}(1+v_j) + ET_{jk} + IT_{jk}.$$

The remaining part of the profit of subsidiary j is the gain or loss from exchange rate variations. In period k the subsidiary may sell (or purchase) forward a quantity q_{ijkg} of foreign currency i against j at the forward rate z_{ijkg} for delivery in period $k+g$.[10] The number of periods g could be 3 for sales on a 90-day payment schedule, for example. To simplify the notation, g is assumed to be the same for all purchases and sales. Forward transactions are assumed to be made only in the currencies of the countries with which j deals or for $j \in M_j \cup N_j$. The gain on a forward transaction is thus $q_{ijkg}(\tilde{z}_{ijkg} - \tilde{s}_{ij,k+g})$.

For a sale that is not hedged, the gain or loss per unit of the foreign currency is $(\tilde{s}_{ijk+g}-\tilde{s}_{ijk})$ which is the difference between the spot exchange rate, $\tilde{s}_{ijk+g}$ at the time of collection of the receivables, and the spot exchange rate $\tilde{s}_{ijk}$ at which sales were recorded on the books. If, for example, a sale to an outside client were completely covered $(q_{ijkg} = y_{ijk})$, the gain would be $q_{ijkg}\ (\tilde{z}_{ijkg}-\tilde{s}_{ijk})$, which depends on the difference between the forward and spot rates. Only exports and imports are assumed to be subject to exchange rate gains or losses. The net gain NG_{jk} in period k then is

$$
\begin{aligned}
NG_{jk} = & \sum_{i \in N_j^0} (\delta_{ij}(\tilde{s}_{ijk}) - \delta_{ij}(\tilde{s}_{ij,k-g}))y_{ijk-g} \\
& + \sum_{i \in N_j'} (\delta_{ij}(\tilde{s}_{ijk}) - \delta_{ij}(\tilde{s}_{ij,k-g}))t_{ijk-g}\ x_{ijk-g} \\
& - \sum_{i \in M_j^0} (\delta_{ji}(\tilde{s}_{jik}) - \delta_{ji}(\tilde{s}_{ji,k-g}))y_{jik-g} \\
& - \sum_{i \in M_j'} (\delta_{ji}(\tilde{s}_{jik}) - \delta_{ji}(\tilde{s}_{ji,k-g}))t_{jik-g}\ x_{jik-g} \\
& + \sum_{i \in M_j \cup N_j} q_{ij,k-g,g}(\tilde{z}_{ij,k-g,g} - \tilde{s}_{ijk}).
\end{aligned}
$$

The profits tax PT_{jk} of the firm is then

$$
\begin{aligned}
PT_{jk} = & T_j(RC_{jk} + RL_{jk} + P_{jk} + D_{jk} - CC_{jk} + NG_{jk}) \\
& - \tau_j D_{jk} - \tau_{ji} D_{jik}
\end{aligned}
$$

where T_j is the profits tax rate. The term $\tau_j D_{jk}$ is a dividend rebate that reflects the fact that dividends received may not be taxed at the corporate profits tax rate, and $\tau_{ji} D_{jik}$ represents any taxes on dividends paid out by j. The rate τ_{ji} may be negative if country j taxes dividends paid out and may be positive if a rebate is given.

The cash flow CF_{jk} of the subsidiary may now be written as

$$CF_{jk} = RC_{j,k-g} - IC_{j,k-g} + RL_{jk} + P_{jk} + D_{jk} - ET_{jk-g} - IT_{jk-g}$$

$$- f_{jk}(1 + v_j) + NG_{jk} - PT_{jk} - \sum_{j \in S_i} D_{jik} - A_{jk} - \sum_{i \in S_j} A_{ijk}\tilde{s}_{ijk}$$

$$+ \sum_{i \in \beta_j} B_{ijk}\,\tilde{s}_{ijk} - \sum_{i \in \mathcal{L}_j} L_{ijk} + B_{jjk} - L_{jjk},$$

where the last four terms reflect the borrowing and lending of the subsidiary. The cash outlays for long-term capital investments, short-term financing of changes in current assets and current liabilities and changes in any long-term financing are denoted by A_{jk}. If such a payment must be made in a currency i, then it is represented by the term A_{ijk} which must be multiplied by the appropriate exchange rate. The cash balance CB_{jk} for subsidiary j at the end of period k is then

$$CB_{jk} = CB_{jk-1} + CF_{jk}.$$

IV. The Decision Model

A decision model must be based on the cash flow model of the previous section, the uncertain nature of the foreign exchange rates as modeled in section II, and the particular objectives of the multinational firm. Due to the fairly complex nature of the stochastic process generating the exchange rates and their effects upon the many components of the cash flow model, a practical solution procedure is difficult to implement. An overview of possible solution procedures will be considered first and then two rather simple decision models will be presented. These models lead to optimization problems that may be converted to linear programming problems.

The model developed in the previous sections involves the choice of a sequence of decisions over several periods,where the decisions beyond the first period are conditional on the occurrence of random variables, the exchange rates, in **earlier** periods. The appropriate solution procedure for such a model is dynamic programming, but for a model as complex as that presented herein the solution using dynamic programming would be virtually impossible. One alternative approach is to restrict the model to two periods and apply programming with recourse or another solution procedure. For example, Jääskeläinen and Salmi [2] have developed a two-stage model in which "the first stage variables must be chosen in a way that preserves feasibility in every state of the world." Were that approach adopted here, the decision in the first-stage would involve the transfer prices, the forward covering, dividends, and borrowing and lending in foreign currencies, while the second-stage decisions would involve borrowing or lending as needed to satisfy the constraints on cash balances. But programming with recourse would be difficult to extend beyond two periods for essentially the same reasons applicable to dynamic programming.

To arrive at a solution to the model, one essentially has the choice between a static solution procedure, such as programming with recourse, and a dynamic procedure that takes into account the adaptive nature of decisions. One simple dynamic procedure would be to take the expectation of the objective function and the constraints, provide "fat" in the constraints, and use linear programming to determine the values of all decision variables. Such a procedure ignores the adaptive nature of decisions, however. An approach to incorporating the conditional nature of the decisions is to use decision rules for the conditional decisions. That is, a decision in period 3, for example, could be expressed as a linear function of an exchange rate in period two. The program then would determine the optimal linear decision rule, and the actual level of the decision variable, viewed from time zero, would be a random variable. A form of the linear decision rule approach with deterministic equivalent constraints will be applied below.

One remaining difficulty in the solution of the model is the treatment of risk. The basic reason for hedging is a desire to reduce exposure to currency fluctuations. The conceptually most appropriate way to take risk into account is to employ a multiperiod utility function with dividend withdrawals in each period as the arguments. This would result in a nonlinear objective function that would further complicate the solution. A frequently used approach (see [3] [4]) is to use a special form of the utility function that can be expressed in terms of the mean and variance of the dividend withdrawals. The determination of the mean and variance of dividends is possible but difficult in this case, but the resulting objective function would in any event be nonlinear. An alternative approach to taking risk into account is to utilize constraints on the hedging variables. That is, *a priori* limits on hedging could be set with the forward transactions restricted to be less than or equal to the foreign exchange exposure for example. Such limits will be employed herein.

The decision variables that may be chosen by the firm are: (1) the transfer prices t_{ijk}, (2) the amounts borrowed B_{ijk}, (3) the amounts loaned L_{ijk}, (4) the dividends paid D_{ijk}, and (5) the levels of forward currency sales q_{ijkg}. The appropriate objective function depends on the objectives and goals of the particular multinational firm. Some firms may be more concerned with cash flow than with profits, while others may only value the profit or cash flow of the parent company. One objective function that may be used is the maximization of a weighted sum of the expected profits of all the subsidiaries during all the time periods, i.e., maximize

$$(4.1a) \qquad \sum_{j \in S} \sum_{k=1}^{K} E(\tilde{s}_{j0k}) \rho_{jk} \Pi_{jk},$$

where period K is the last one in the time horizon chosen, the weights ρ_{jk} are nonnegative and may reflect time discounting, $E(\tilde{s}_{j0k})$ is the expected exchange rate that converts profits to the parent company's currency, and Π_{jk} is profit.

The weakness of the expected profit objective function is that it does not provide an incentive to move funds to the parent company that then can be distributed to shareholders. An alternative objective function is to maximize the expected, discounted sum of the dividends paid to the parent company.

$$(4.1b) \qquad \sum_{j \in S_0} \sum_{k=1}^{K} E(\tilde{s}_{j0k}) \rho_{jk} D_{j0k}$$

This objective function is consistent with the dividends-withdrawls financial planning models.

The decision variables are subject to a number of constraints imposed by management in order to insure the financial stability and reasonableness of the decisions. Each of these will be presented in turn. Transfer prices must fall within a range of reasonable values in order to satisfy the letter and spirit of tax laws and to avoid uncomfortable government scrutiny. It is thus required that

$$t_{ijk} \leq t_{ijk} \leq \bar{t}_{ijk}\,, \tag{4.2}$$

where k ranges from 1 to K, the combinations of i and j are those appropriate to the firm's organizational structure and sales plan, and the limits $\underline{t}_{ijk}$ and $\bar{t}_{ijk}$ are fixed constants set by management.[12] Similar notation will be used for other fixed constants set by management. The amounts borrowed and loaned must be nonnegative and may have upper bounds, so the constraints

$$0 \leq B_{ijk} \leq \bar{B}_{ijk}, \tag{4.3}$$

$$0 \leq L_{ijk} \leq \bar{L}_{ijk}, \tag{4.4}$$

are employed. The dividends paid must not be negative, and it may be desirable to keep them above some stated minimum. The constraints

$$\underline{D}_{ijk} \leq D_{ijk} \leq \bar{D}_{ijk} \tag{4.5}$$

are thus used, where $\underline{D}_{ijk}$ may be zero.

In planning hedging activities within a multinational company, one faces the problem of whether or not each subsidiary should be able to hedge all its own foreign currency exposure or only the exposure associated with outside clients. If the former approach is used, the firm will hedge its own internal transactions. The hedging limits will be expressed quite generally and the appropriate modifications to prevent hedging of internal transactions can easily be made. The currency i claims CL_{ijh} to be collected in period k are

$$CL_{ijk} = \begin{cases} y_{ijk-g} + t_{ijk-g} + \sum\limits_{i \in S_j} \hat{D}_{ijk} & \text{if } \delta_{ij}(\tilde{s}_{ijk-g}) = \tilde{s}_{ijk-g},\ i \in N_j, \\ \sum\limits_{i \in S_j} \hat{D}_{ijk} & \text{otherwise} \end{cases}$$

where $\hat{D}_{ijk}$ is the planned (or projected) dividends. The currency i obligations OB_{ijk} are

$$OB_{ijk} = \begin{cases} y_{jik-g} + t_{jik-g}\, x_{jik-g} + B_{ijk-1} + A_{ijk} & \text{if } \delta_{ji}(\tilde{s}_{jik-g}) = 1, \\ B_{ijk-1} + A_{ijk}, & \text{otherwise} \end{cases}$$

for $i \in M_j \cup B_j$. If only external transactions are to be covered, then only the y_{ijk-g} and y_{jik-g} terms appear above. The net exposure in currency i is then $CL_{ijk}-OB_{ijk}$, and if the firm wishes to follow a policy of not speculating, then the forward sales for period k, $\sum\limits_{h+g=k} q_{ijhg}$, must be such that

$$\text{(4.7a)} \qquad \min\{0, CL_{ijk}-OB_{ijk}\} \le \sum_{h+g=k} q_{ijhg} \le \max\{0, CL_{ijk}-OB_{ijk}\}.$$

Unless one can determine on an <u>a priori</u> basis if the net exposure is positive or negative, this constraint is nonlinear. An alternative formulation of the hedging constraint is to define each hedging variable as

$$q_{ijhg} = q^{+}_{ijhg} - q^{-}_{ijhg} \ ; \ q^{+}_{ijhg},\ q^{-}_{ijhg} \ge 0,$$

and use the following constraints

$$\text{(4.7b)} \qquad \sum_{h+g=k} q^{+}_{ijhg} \le CL_{ijk},$$

$$\text{(4.7c)} \qquad \sum_{h+g=k} q^{-}_{ijhg} \le OB_{ijk}.$$

Management will probably want or have to set lower limits on the respective cash balances and profits. These are accomplished through the constraints

(4.8) $$CB_{jk} \geq \underline{CB}_{jk},$$

(4.9) $$\Pi_{jk} \geq \underline{\Pi}_{jk} ,$$

where $\underline{\Pi}_{jk}$ may be zero. Due to the effect on income taxes, the model cannot easily accommodate negative profits unless negative profits can be applied against profits in other time periods. If it seems desirable or inevitable that certain Π_{jk} be negative, the cash flow model could be altered appropriately to reflect this, and then constraints of the form

(4.10) $$\Pi_{jk} \leq \overline{\Pi}_{jk} ,$$

with $\overline{\Pi}_{jk} \leq 0$, would replace (4.9) for the required value of j and k.

In many cases it will be desirable to limit a subsidiary's dividends by its profit. This may be accomplished through

(4.11) $$D_{ijk} \leq \Pi_{ik}.$$

This constraint would be replaced by

$$\sum_k D_{ijk} \leq \sum_k \Pi_{ik},$$

if this limitation only exists over a time span which is longer than one period (here k is summed over the periods comprising the indicated time span).

The decision variables and the objective function and constraints in which they interact have now been presented. The difficulty remains, however, that the stochastic nature of the exchange rates prevents the straightforward determination of the optimal values of the decision variables through solution of an ordinary deterministic optimization problem. For example, forward covering decisions in period k should take into account the financial results of the preceeding periods, which are known in period k but unknown at the time a solution to the planning problem is sought. In terms of the planning model, it is therefore impossible to guarantee that all the constraints, on the cash balance CB_{jk} and profit Π_{jk}, for example, can be satisfied. As in other stochastic programming models, it is therefore necessary to reinterpret the constraints, and the objective function as well, in a way that allows for a well-defined optimal solution to be found.

To keep the resulting models relatively simple, the approach adopted here is to replace the objective function by its expected value, and to apply the constraints with each left-and right-hand side replaced by their expected value. This may require minor changes in some constraints, such as replacing (4.8) by $E\{CB_{jk}\} \geq \underline{CB}^*_{jk}$ (with $\underline{CB}^*_{jk} > \underline{CB}_{jk}$), in order to provide appropriate safety factors, but it has the advantage that the resulting models can then be readily solved to yield useable results.

Two different linear models, based on different ways of specifying values for the decision variables, will be considered. The second model is a generalization of the first one, so only for the second will it be shown that the deterministic equivalent of the stochastic dynamic decision problem is a linear program. The first model uses a zero-order decision rule in which the values of all decision variables are chosen simultaneously,and no ability to alter these values during the time horizon of the model is presumed. All the decision variables are then constants and will be denoted by appending a final subscript of zero to the previous notation. Thus the decision variables are then t_{ijk0}, B_{ijk0}, L_{ijk0}, D_{ijk0}, and q_{ijkg0}.

The second model uses a pseudo-linear decision rule to select the value of each decision variable. That is, each decision variable is expressed as a linear function of the exchange rates appropriate to it. The model is therefore adaptive in that the specific values of the decision variables will only become specified at the time the decisions dependent on them are made. The specific rules for the original decision variables are

(4.12) $$t_{ijk} = t_{ijk0} + t_{ijk1}\,\tilde{s}_{ijk} + t_{ijk2}\,\tilde{s}_{jik},$$

(4.13) $$B_{ijk} = B_{ijk0} + B_{ijk1}\,\tilde{s}_{ijk} + B_{ijk2}\,\tilde{s}_{jik},$$

(4.14) $$L_{ijk} = L_{ijk0} + L_{ijk1}\,\tilde{s}_{ijk} + L_{ijk2}\,\tilde{s}_{jik},$$

(4.15) $$D_{ijk} = D_{ijk0} + D_{ijk1}\,\tilde{s}_{ijk} + D_{ijk2}\,\tilde{s}_{jik},$$

(4.16) $$q_{ijkg} = q_{ijkg0} + q_{ijkg1}\,\tilde{z}_{ijkg} + q_{ijkqz}\,\tilde{z}_{ikg}$$

The decision rules are termed pseudo-linear, since $\tilde{s}_{jik} = 1/\tilde{s}_{ijk}$, and one has, for example,

$$t_{ijk} = t_{ijk0} + t_{ijk1}\,\tilde{s}_{ijk} + t_{ijk2}/\tilde{s}_{ijk}.$$

The pseudo-linear decision rule reduces to the zero-order decision rule simply by setting the second and third coefficients, such as t_{ijk1} and t_{ijk2}, to zero.

The expressions (4.12) - (4.16) must be substituted into the objective function and the constraints (4.1) - (4.11). Then the objective function may be replaced by its expected value and the left- and right-hand sides of each constraint may be replaced by their respective expected values. The resulting optimization problem is a deterministic equivalent for the dynamic stochastic programming problem originally developed. It is not difficult to see that it is a linear program. This follows from the fact that the objective function and all the constraint functions of the stochastic model are linear functions of the original decision variables and these have now been replaced by the decision rules (4.12) - (4.16) that are linear functions of the coefficients, such as $t_{ijk\ell}(\ell=0,1,2)$, that must be chosen. Thus, for example, the deterministic equivalents for the constraints (4.2) are the linear constraints

$$(4.17) \qquad \underline{t}_{ij} \le t_{ijk0} + t_{ijk1}\, E\{\tilde{s}_{ijk}\} + t_{ijk2}\, E\{\tilde{s}_{jik}\} \le \overline{t}_{ijk}.$$

The deterministic equivalents for both the first and second models have now been demonstrated to be linear programs. The only remaining difficulty is the computation of some of the expected values that appear in the linear program. For example, in (4.17) we need $E\{\tilde{s}_{ijk}\}$. But since $\ell n(\tilde{s}_{ijk}) = \tilde{u}_{ijk} \sim N(m_j^k - m_i^k,\ v^{ij}W^k(v^{ij})^t)$, straightforward computation yields

$$E\{\tilde{s}_{ijk}\} = \exp[\ \tfrac{1}{2}(v^{ij}W^k(v^{ij})^t) + (m_j^k - m_i^k)\,].$$

Some of the similar expectations required are more tedious, but they all can be carried out without undue difficulty. For example, the last part of the net gain NG_{jk} involves terms of the form $q_{ij,k-g,g}\, \tilde{s}_{ijk}$. One of the three terms in the computation of $E\{q_{ij,k-g,}\, \tilde{s}_{ijk}\}$ requires the value of $E\{\tilde{z}_{ji,k-g,g}\tilde{s}_{ijk}\}$. Now

$$\begin{aligned}
\ell n(\tilde{z}_{ji,k-g,g}\, \tilde{s}_{ijk}) &= \ell n(\tilde{s}_{ji,k-g}\, \tilde{r}_{ik-g,g}/\tilde{r}_{jk-g,g}) + \ell n(\tilde{s}_{ijk}) \\
&= (-\tilde{u}_{ij,k-g} + \tilde{\eta}_{ik-g,g} - \tilde{\eta}_{jk-g,g}) + \tilde{u}_{ijk} \\
&= (\tilde{\eta}_{ik-g,g} - \tilde{\eta}_{jk-g,g}) + (\tilde{u}_{0jk} - u_{0j,k-g}) \\
&\qquad - (\tilde{u}_{0ik} - \tilde{u}_{0i,k-g}) \\
&= (\tilde{\eta}_{ik-g,g} - \tilde{\eta}_{jk-g,g}) + \sum_{\ell=k-g+1}^{k} (\tilde{y}_j^{\ell} - \tilde{y}_i^{\ell}) \\
&\sim N(\mu_i^{k-g,g} - \mu_j^{k-g,g} + \sum_{\ell=k-g+1}^{k} (m_{\ell j} - m_{\ell i}), \\
&\qquad \gamma^{ik}\, \Sigma^{k-g,g}(\gamma^{ik})^t + v^{ji}\, (\sum_{\ell=k-g+1}^{k} W_\ell)(v^{ji})^t),
\end{aligned}$$

so that

$$\begin{aligned}
E\{\tilde{z}_{ji,k-g,g}\, \tilde{s}_{ijk}\} = \exp[\, &\tfrac{1}{2}(\gamma^{ij}\Sigma^{k-g,g}(\gamma^{ij})^t \\
&+ v^{ji}\, (\sum_{\ell=k-g+1}^{k} W_\ell)(\gamma^{ji})^t) + (\mu_i^{k-g,g} - \mu_j^{k-g,g}) \\
&+ \sum_{\ell=k-g+1}^{k} (m_{\ell j} - m_{\ell i})].
\end{aligned}$$

V. Summary

The short-term financial planning model presented here incorporates the necessary interrelationships among exchange rates and the conditional nature of decisions through the linear decision rules. The resulting optimization model is complex if risk preferences are taken into account and difficult to optimize because of uncertainty in the constraints. Since the planning model would likely be rerun each period, a solution procedure that can be easily implemented yet captures to some extent the effects of future decisions is needed. The use of linear decision rules and linear deterministic equivalent constraints provides such a solution procedure. By adjusting the right-hand sides of the constraints much of the realism that one would wish to incorporate in the model can be captured. Otherwise, one must await the development of more efficient multistage optimization techniques.

Table 1: Notation

$\tilde{s}_{ijk}$,	period k spot exchange rate in currency j per unit of currency i
$\tilde{z}_{ijkg}$,	a g-period forward rate available in period k, in currency j per unit of currency i
q_{ijkg},	g-period forward sale in period k of currency i against currency j
y_{ijk},	sales revenue in currency i, $i \in N_j^0$
y_{ijk-g},	cash flow in currency i from sales in period k-g collected in period k, $i \in N_j^0$
x_{ijk},	units sold to subsidiary i, $i \in N_j'$
x_{ijk-g},	units delivered to subsidiary i in period k, $i \in N_j'$
t_{ijk},	transfer price set for units x_{ijk}, $i \in N_j'$
$\delta_{ij}(x_{ijk}) = 1$	if the transactions are priced in currency j
$= \tilde{s}_{ijk}$	if the transactions are priced in currency i
B_{ijk},	borrowing by j in currency i, $i \in \beta_j$
b_{ijk},	interest rate on B_{ijk} in j, $i \in \beta_j$
L_{ijk},	lending by j in currency j to i, $i \in \mathcal{L}_j$
ℓ_{ijk},	interest rate on L_{ijk}, $i \in \mathcal{L}_j$
B_{jjk},	home country borrowing
b_{jjk},	interest rate on B_{jjk}
L_{jjk},	home country lending (savings)
ℓ_{jjk},	interest rate on L_{jjk}
P_{ijk},	fees paid by i to j
D_{ijk},	dividends paid by i to j, $i \in S_j$
v_j,	value-added tax rate in j
e_{ij},	export tax rate on shipments from j to i
i_{ji},	import tax rate on shipments from i to j
T_j,	profits tax rate
τ_j,	tax rebate on dividends received
τ_{ij},	tax rebate on dividends paid out to j, $i \in S_j$
CL_{ijk},	currency i claims of j due in period k
OB_{ijk},	currency i obligations of j due in period k

A_{ijk}, net cash outlays in currency i for changes in current accounts, (not involving sales, capital outlays, and long-term financing)

A_{jk}, total net cash outlays in currency j for changes in current accounts (not involving sales, capital outlays, and long-term financing)

$\hat{D}_{ijk}$, planned dividend payments from i to j in period k, $i \in S_j$.

Footnotes

*The work was undertaken while the authors were visiting professors at the Institut D'Administration des Entreprises, Université de Droit, D'Economie, et des Sciences d'Aix-Marseille, Aix-en-Provence, FRANCE.

1. Firms have other means of hedging using swaps or abri loans, for example. The focus of this paper will be on the two principal means of hedging: forward contracts and borrowing and lending.

2. The model considered here deals with the real cash effect of hedging and foreign exchange rate fluctuations and not with "accounting profits" which may be computed using a variety of policies. For example, the Phillips Petroleum Company has the following policy [6,p.29]: "Net unrealized translation losses are charged against income; net unrealized translation gains are credited to a reserve for exchange losses. The same accounting applied for recording unrealized gains and losses on foreign exchange contracts."

3. One significant market imperfection during 1974 involved the risk that forward exchange contracts would not be fulfilled because of bank failures caused by foreign currency speculation.

4. This assumption may be relaxed with little additional complication.

5. The above development is for arbitrary g, but clearly the g-period interest rates must be related to the g+k-period interest rates. This relationship is referred to as the term structure of interest rates and may be taken into account by letting $\tilde{r}_{jkg} = (\tilde{r}_{jk1})^g \tilde{\epsilon}_{jkh}$, where the $\tilde{\epsilon}_{jkg}$ are mutually independent and have lognormal distributions.

6. A number of countries impose multiple exchange rates. In these two-tier systems, a commercial exchange rate with a relatively fixed rate is maintained by the government, while a financial exchange rate is allowed to move with market forces.

7. A more realistic assumption is that the process generating the exchange rates is non-stationary.

8. The taxes include tariffs, duties, border adjustments, etc.

9. The constraints may be explicitly given by tax laws or may be implicitly inferred from the monitoring activities of the tax authorities.

10. The loans between subsidiaries are such that $B_{ijk} \equiv L_{jik}$ and $b_{ijk} = \ell_{jik}$.

11. The taxes include tariffs, duties, border adjustments, etc.

12. If transfer prices should not change very rapidly, constraints such as $.9t_{ijk} \leq t_{ijk+1} \leq 1.1t_{ijk}$ may be in order. Or it may be specified that $t_{ijk+1} = t_{ijk}$ for certain k.