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VOTING BEHAVIOR AND INFORMATION AGGREGATION IN ELECTIONS WITH PRIVATE INFORMATION[1]

by

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Abstract

We analyze two-candidate elections in which voters are uncertain about the realization of a state variable that affects the utility of all voters. Each voter has noisy private information about the state variable. We show that the fraction of voters whose vote depends on their private information goes to zero as the size of the electorate goes to infinity. Nevertheless elections fully aggregate information in the sense that the chosen candidate would not change if all private information were common knowledge among voters. We also show that the equilibrium voting behavior is to a large extent determined by the electoral rule, i.e., if a candidate is required to get at least x percent of the vote in order to win the election then in equilibrim this candidate gets very close to x percent of the vote with probability close to one.

1 Introduction

It is a central tenet of the formal literature on elections that information is critical in determining voting behavior and election outcomes.[1] As least since Condorcet scholors have appreciated the role of elections as information aggregation devices. This paper analyzes the information aggregation properties of electoral mechanisms and the effects of strategic voting behavior in an environment of private information. As such it builds on our earlier paper (Feddersen and Pesendorfer 1994). In that paper we demonstrated that there is an analog to the winner's curse in elections–the "swing voter's curse".[2] Like bidder's in a common value auction, voter's must condition their candidate choice and their decision to participate not only on their private information but also on the event that their vote matters.

In this paper we analyze two-candidate elections under a variety of quota rules and information environments. The focus is on voting behavior and information aggregation rather than participation. We consider a population of voters. Each voter's payoff depends on his preference type, on a state of nature, and on the elected candidate. Preference types are drawn independently from a given distribution whereas the state of nature is common for all voters. Voters know their preference type but are uncertain about the state of nature. Every voter receives a signal that provides information about the realization of the state of nature. Voting is costless and voters can either vote for the status quo or for the alternative. The alternative is implemented if the fraction of voters voting for it is at least $1 - q$. We analyze symmetric Nash equilibria of this voting game.

As an example consider an election between a candidate who represents the status quo and a candidate who represents an alternative policy. Voters are uncertain about the cost of implementing the alternative policy. If costs are high then a majority of the voters prefers the status quo whereas if these costs are low then a majority prefers the alternative. Newspapers print a series of articles concerning the cost of the alternative. Each newspaper article is a noisy signal about the cost of the alternative: the higher the costs the more negative articles about the alternative apear. Every voter reads one newspaper article. Since voters read different articles they have

[1] Downs 1957; Palfrey and Rosenthal 1985; Austen-Smith and Banks 1994; Calvert 1986.

[2] Milgrom and Roberts (1982) for an analysis of auctions with common values

different private information.

There are three central results in our paper. First we show that the fraction of the voters whose vote depends on their private signal goes to zero as the size of the electorate goes to infinity. Thus in a large election most voters ignore their private information when they cast their vote. We show this under the assumption that the probability of receiving a particular signal conditional on the state of nature is a continuous function of the state of nature.

To provide an intuition for our first result suppose that by sampling the private signals of many voters the true state can be determined with arbitrary accuracy. Further, suppose that the fraction of voters whose vote depends on their private signal stays bounded away from zero. Then as the population size increases the probability distribution over states of nature conditional on being pivotal will converge to a distribution whose mass is concentrated around one particular state. As long as each voter's private information is noisy, i.e., it does not allow him to infer the true state of nature with certainty, voter's beliefs about the state of nature conditional on being pivotal will be essentially independent of his private signal. Hence, for a typical preference type, the optimal vote choice will be independent of the realization of the private signal. Therefore it cannot be the case that the fraction of voters whose vote depends on their private signal stays bounded away from zero as the size of the electorate goes to infinity.

Nevertheless, we demonstrate that in a large electorate information is fully aggregated in equilibrium, i.e., with probability arbitrarily close to one the elected candidate is the candidate that would have been elected if all the private signals were common knowledge. Thus large elections fully aggregate private information. The crucial assumption for this result is that by sampling the private signals of many voters the true state can be determined with arbitrary accuracy. This result may appear paradoxical in light of our first result that the fraction of voters who reveal their private signal through their vote choice goes to zero. While the fraction of the population's signals that are actually revealed in equilibrium goes to zero, the *number* of voters who reveal their signal goes to infinity so that in the limit all information is revealed.

We also show that the equilibrium voting behavior is to a large extent determined by the electoral rule. The electoral rule is given by the fraction q of the votes that the status quo (Q) must obtain to win. Suppose that the expected fraction of voters who prefer candidate Q in *every* state s is smaller than q and similarly the fraction of voters who prefer the alternative (A) in every state s is smaller than $1-q$. Under this condition we show that the fraction of voters who choose the status quo independent of the realization of their private signal must be close to $1-q$ and the fraction of voters who choose the alternative independent of their private signal must be close to q. If, for example, $q = 1/3$ then Q wins the election whenever the number of votes for Q is larger than or equal to $1/3 \cdot n$, where n is the number of voters. For large n our model predicts that in this case Q will receive very close to $1/3$ of the votes irrespective of the realization of s. Note that this is independent of the distribution of voter preference types as long as the above condition is satisfied and independent of the prior distribution over states.

To provide an intuition for this result note that (by our first result) in a large electorate almost all voters ignore their private information. If the fraction of voters who choose the status quo independent of their private information is unequal to q then in a large electorate the election outcome is essentially independent of the state of nature. However, full information revelation implies that the elected candidate must depend on the realized state of nature and hence the fraction of voters who support the status quo irrespective of their private information must be very close to q.

A central assumption for the described results is that the voter's uncertainty is about a *one-dimensional* state variable. Suppose, e.g., that there is uncertainty both about the state variable that influences voters' payoff and about the distribution from which nature selects the electorate. Assume further that voters receive informative signals about both sources of uncertainty. In this case information cannot be aggregated fully by an election. Moreover, the fraction of voters whose vote depends on their private signal does not converge to zero.

We illustrate our results with a series of examples intended both to demonstrate the manner in which our assumptions affect the results and to show the relevance of our model to the previous literature on two candidate elections.

1.1 Related Literature

The effects of imperfect information on vote choice have been widely explored in the formal literature on elections (Grofman 1993; Grofman and Whithers 1993). The central contribution of the literature on the Condorcet Jury Theorem (Ladha 1992; Miller 1986; Young 1988,1994) is the exploration of the information aggregation properties of electoral rules. This literature assumes that agents vote "naively", i.e., each agent behaves as if his choice alone determines the outcome. Since in elections no single voter's decision determines the outcome, naive voting is not generally an equilibrium of the corresponding voting game.[3] We illustrate in section 5.5. how naive voting typically fails to aggregate all private information while strategic voting leads to full information aggregation. Thus our results show that elections are much better information aggregation devices than the Condorcet Jury Theorem suggests.

Another avenue taken in the literature may be labeled the rational expectations approach. McKelvey and Ordeshook (1985) have argued that the process of information revelation through polls will cause all voters to ultimately behave as if they were perfectly informed. While we find that elections aggregate information fully our model predicts significantly different voter behavior from the rational expectations approach. In our model it is the small fraction of voters whose vote depends on their private information who determine the final outcomein equilibrium.

Our approach is related in some respects to the approach taken by Lohmann (1993) and Austen-Smith (1989). Lohmann analyzed the effects of private information on costly participation in political protest movements while Austen-Smith examined the incentives for strategic voting in two-alternative elections. Neither Lohmann or Austen-Smith considered the asymptotic properties of their models.

Our results are also related to the literature on information aggregation in auctions with common values (Milgrom (1979), Wilson (1977)). The result in the auction literature is that the winning bid converges (in probability) to the true value of the object as the number of bidders goes to infinity. Full information revelation in auctions requires that for every possible value V of the object, bidders may receive a signal that allows them to exclude the possibility that the value of the object is below V (see Milgrom (1979), Theorem 1). For full information aggregation in elections

[3]See Austen-Smith and Banks 1994; Myerson 1994b; and Klevorick et. al. 1985 for exceptions.

much weaker assumptions on a single voter's information suffice. Full information aggregation in elections may be achieved even if each individual signal provides very little information. In a related paper Palfrey (1985) shows how in a Cournot model with demand uncertainty private information gets aggregated fully as the number of firms goes to infinity and hence the aggregate outcome is equivalent to the outcome in a world of perfect competition and full information.

2 Description of the Model

We analyze a two candidate election. Candidates are denoted by $j \in \{Q, A\}$. There are $n+1$ voters. Voters have different preferences. The payoff of a voter with preference parameter $x \in X \equiv [-1, 1]$ if candidate j is chosen is

$$u(j, s, x) \tag{1}$$

where $s \in [0, 1]$ is a state variable. Let

$$v(s, x) \equiv u(A, s, x) - u(Q, s, x) \tag{2}$$

We assume that $v(s, x)$ is continuous and strictly increasing in (s, x).

Assumption 1 *$v(x, s)$ is continuous and $v(x, s) > v(x', s')$ if $(s, x) \geq (s', x')$, $(s', x') \neq (s, x)$. Furthermore $v(-1, s) < 0$ for all s and $v(1, s) > 0$ for all s.*

Assumption 1 says that the utility difference between candidate A and candidate Q increases in s and x. Furthermore, voters with preference parameters at the boundary of X prefer one of two candidates irrespective of the state s. As an example, suppose that voters are uncertain about the cost of implementing the policies of candidate A whereas the cost of implementing the policy of candidate Q (the status quo) is zero. Let $c(s)$ be the cost of implementing A's policy, where $c(s)$ is strictly decreasing in s, and continuous. Suppose that $0 < Q < A < 1$. Further let $w : \mathbf{R} \rightarrow \mathbf{R}$ be a single peaked continuous function that reaches a maximum at 0. If $v(s, x) = w(A - x) - w(Q - x) - c(s)$ then Assumption 1 is satisfied if $c(1) = 0$.

By $g(s)$ we denote the density that describes the prior beliefs about the state s.

Assumption 2 *g is continuous and $g(s) > 0$ for all $s \in [0, 1]$.*

Each voter is characterized by his ideal point x and by an information service k which provides a signal to the voter that may give him information about the realization of the state s. We assume that there is a finite number of such information services $k = 1, \ldots, K$. Hence $T \equiv [-1, 1] \times \{1, \ldots, K\}$ denotes the type space. Let F be a probability distribution over T and let F_X denote the marginal of F over $[-1, 1]$.

Assumption 3 *F_X allows a density function f_X, where $f_X > 0$.*

Nature selects the electorate by choosing n voters independently according to the probability distribution F. For $G \subset T$ we will write $F(G)$ to denote the probability that a voter is of a type $(x, k) \in G$, i.e., $F(G) = \int_G dF$.

Information Services

Each voter has access to one information service $k \in \{1, \ldots, K\}$. An information service might be thought of, for example, as a newspaper or a TV channel. Each voter takes a random sample of articles or TV programs and updates his beliefs about the state of the world according to this sample. We assume that the samples taken by different voters are independent, that is, conditional on state s being realized, the information that voter i receives from an information service is independent from the information that voter j receives from the same information service.

More precisely we assume that each information service maps the true state s into a (possibly infinite) sequence of zeros and ones. Each voter takes a random sample from his information service. For simplicity, we assume that the sample size is one for each voter. An extension of our model and the results to arbitrary sample sizes is straightforward but complicates the notation and the exposition of the results without adding new insights.

Each information service provides the voter with a signal $\sigma \in \{0, 1\}$. By $p(k, s)$ we denote the probability that a random sample from the information service k results in signal 0 if $s \in [0, 1]$ is realized.

$\beta(s|\sigma, k)$ denotes the posterior probability density function over states s of a voter who receives signal σ from information service k. Therefore,

$$\beta(s|0, k) \equiv \frac{g(s) \cdot p(k, s)}{\int_0^1 g(s) \cdot p(k, s) ds} \tag{3}$$

and

$$\beta(s|1,k) \equiv \frac{g(s)\cdot(1-p(k,s))}{\int_0^1 g(s)\cdot(1-p(k,s))ds} \tag{4}$$

We assume that receiving a signal $\sigma = 1$ implies that higher values of s are more likely, whereas receiving a signal $\sigma = 0$ implies that lower values of s are more likely.

Assumption 4 *$p(k,s)$ is (weakly) decreasing in s for all k.*

Note that this assumption implies that $\beta(s|0,k)/\beta(s|1,k)$ is (weakly) decreasing in s. Moreover, note that $\beta(\cdot|0,k)$ is first order stochastically dominated by $\beta(\cdot|1,k)$ for all k. If $p(k,s)$ is constant for all s then $\beta(s|0,k) = \beta(s|1,k) = g(s)$. More generally, if $p(k,s)$ is constant over some interval $[s_1,s_2]$ then if we condition on this interval, the posterior is equal to the prior for both signals, i.e., $\beta(s|0,k,[s_1,s_2]) = \beta(s|1,k,[s_1,s_2]) = g(s|[s_1,s_2])$

Strategies and Equilibrium

Each voter can choose Q or A. Let $0 < q < 1$ be a fixed parameter. If the number of voters who choose Q is larger than or equal to $(n+1)\cdot q$ then Q is the outcome. Otherwise, A is the outcome. For simplicity we assume in the following that nq is an integer.[4]

A symmetric pure strategy for a voter is a measurable function $\pi : T \times \{0,1\} \to \{Q,A\}$ and a symmetric mixed strategy is a pair of distributions $\bar{\pi}(\sigma), \sigma = 0,1$ on the set $T \times \{Q,A\}$.

By V we denote the expected payoff of a type (x,k) as a function of his private information σ, his action $\tau \in [0,1]$ and the strategy $\bar{\pi}$ used by all other players. Let

$$V(x,k,\sigma,\tau,\bar{\pi})$$

denote this expected payoff.

A voting equilibrium is a distribution $\bar{\pi}^*$ such that

(i) For a.e. (x,k,τ) in the support of $\bar{\pi}^*(\sigma)$, $V(x,k,\sigma,\tau,\bar{\pi}^*) \geq V(x,k,\sigma,\tau',\bar{\pi}^*)$ for all $\tau' \in [0,1]$, $\sigma = 0,1$.

[4]The only change in the analysis when nq is not an integer is that the expression nq must be replaced with "largest integer that is smaller or equal to nq".

(ii) For a.e. (x, k, τ) in the support of $\bar{\pi}^*$, $V(x, k, \sigma, \tau, \bar{\pi}) < V(x, k, \sigma, \tau', \bar{\pi})$ implies that there exists a $\bar{\pi}'$ such that $V(x, k, \sigma, \tau, \bar{\pi}') > V(x, k, \sigma, \tau', \bar{\pi}')$

Condition (i) implies that a voting equilibrium is a symmetric Nash equilibrium. Condition (ii) implies that no voter uses a weakly dominated strategy. We make the assumption of symmetry both because it simplifies our analysis and because we consider large electorates. As Myerson (1994a) points out, in elections and other large games, it is frequently unreasonable to assume that the identity of every player is common knowledge. Rather we can imagine that players know that certain types are possible and might know the probability of each type but do not know the identity or the exact number of players of each type. Such a model of population uncertainty leads us naturally to the assumption of symmetry, i.e., rather than specifying the behavior of an individual player we can only specify how a typical type of player is expected to play. In this model we take the shortcut of assuming symmetry instead of introducing population uncertainty as in Myerson (1994a). Introducing such uncertainty will not affect our results.

In the following, we show that voting equilibria are always pure strategy equilibria that can be characterized by a partition of the set of types into three subsets: those types who vote for Q irrespective of their private information, those types who vote for A irrespective of their private information and the types who vote for A if they receive signal 1 and vote for Q if they receive signal 0. We refer to these latter types as voters who take *informative action*[5]. The second part of Proposition 1 shows that for each information service there is a pair of cutpoints (x^*_{1k}, x^*_{0k}) such that voters with preference parameters below x^*_{1k} who receive a signal from k will vote for Q independent of the signal. Voters with preference parameters larger than x^*_{0k} will vote for candidate A independent of their signal and voters with preference parameters in the interval (x^*_{1k}, x^*_{0k}) will vote for candidate Q if they receive signal 0 and for candidate A if they receive signal 1. The proof is in the appendix.

Proposition 1 *Suppose Assumptions 1-3 hold. Every voting equilibrium can be described by a pure strategy π^*. Furthermore equilibrium strategies can be characterized*

[5]If voters sample more than one signal from their information service then the definition of informative action is more complicated: a voter takes informative action if there exists a pair of signals σ_1, σ_2 such that σ_1, σ_2 are received with strictly positive probability and the voter chooses A if he receives σ_1 and Q if he receives σ_2. With this definition the following results can be extended to the case of arbitrary sample size.

by a partition (T_Q, T_A, T_i) *of* T *where*

(i) $\pi^*(x, k, \sigma) = Q$ *for* $(x, k) \in T_Q$ *and* $\sigma = 0, 1$

(ii) $\pi^*(x, k, \sigma) = A$ *for* $(x, k) \in T_A$ *and* $\sigma = 0, 1$

(iii) $\pi^*(x, k, 0) = Q$, *and* $\pi^*(x, k, 1) = A$ *for* $(x, k) \in T_i$

Moreover, for each k *there is a pair of cutpoints* (x^*_{1k}, x^*_{0k}), $x^*_{1k} \leq x^*_{0k}$ *such that*

(i) $(x, k) \in T_Q$ *if* $x < x^*_{1k}$

(ii) $(x, k) \in T_i$ *if* $x^*_{1k} < x < x^*_{0k}$

(iii) $(x, k) \in T_A$ *if* $x > x^*_{0k}$

3 Equilibrium Behavior in Large Populations

In this section we characterize equilibrium behavior for large population sizes. In Theorem 1 we show that $F(T^i)$ converges to zero as the number of voters goes to infinity. In other words, the fraction of voters who take informative action converges to zero as $n \to \infty$.

Assumption 5 $p(k, s)$ *is continuous on* $[0, 1]$ *for all* k.

Theorem 1 *Suppose Assumptions (1)-(5) hold. Let* $\{(T^n_Q, T^n_A, T^n_i)\}_n$ *be a voting equilibria. Then* $F(T^n_i) \to 0$ *as* $n \to \infty$.

To give an intuition for Theorem 1 suppose that there is exactly one information service, $p(s)$. Fix a strategy π and suppose that contrary to Theorem 1 a strictly positive fraction of voters takes informative action for all n. For large n the actual vote shares of the two candidates will (with high probability) be very close to the expected vote shares. The probability that a voter is pivotal is maximized in the states S^* for which $|q -$ expected vote share of Q$|$ is minimized. Since $p(s)$ is continuous the expected vote share is a continuous function of s and hence the set S^* is non-empty. If $piv(s)$ is the probability that a voter is pivotal in state s then $piv(s)/piv(s^*) \to 0$ as $n \to \infty$ if $s^* \in S^*$ and $s \notin S^*$. As a consequence, the distribution over states conditional on the event that a voter is pivotal converges to a distribution that has all its mass concentrated on the states in S^*. Furthermore, we show that continuity of $p(s)$ implies that $p(s) = p(s')$ for all $s \in S^*$. This in turn implies that the probability distribution over states conditional on being pivotal and conditional on a voter's private information converges to the prior over states conditional on being in the set S^*. This distribution is independent of a voter's private information. Thus in the limit as $n \to \infty$ the fraction of voters who take informative action must shrink to zero contradicting the assumption that a (uniformly) positive fraction of voters takes informative action.

Proof: The probability that a voter is pivotal in state s is given by:

$$piv^n(s) = \begin{pmatrix} n \\ qn \end{pmatrix} \cdot t^n(s)^{qn} \cdot (1 - t^n(s))^{n-qn}$$

where $t^n(s) = \int_{T_Q^n} dF + \int_{T_i^n} p(k, s) dF$ is the probability that an agent votes for Q given the pure strategy equilibrium (T_Q^n, T_A^n, T_i^n).

Suppose, contrary to the theorem, $F(T_i^n) > \epsilon > 0$ along any subsequence. Since $p(k, s)$ is continuous there exists a set of states S^n such that $piv^n(s)$ is maximized for any $s \in S^n$.

First we claim that S^n is an interval and that for all $s, s' \in S^n$ $t^n(s) = t^n(s')$. Note that by Assumption 4 $t^n(s)$ is (weakly) decreasing in s. Thus it is sufficient to show that $t^n(s) = t^n(s')$ for all $s, s' \in S$. Suppose to the contrary that $s, s' \in S^n$ and $t^n(s) = t > t^n(s') = t'$. Since $t^{qn} \cdot (1-t)^{n-qn} = t'^{qn} \cdot (1-t')^{n-qn}$ and since $t^{qn} \cdot (1-t)^{n-qn}$ is a single peaked function of t this implies that for $t^\lambda = \lambda t + (1-\lambda)t'$, $0 < \lambda < 1$, $t^{\lambda qn} \cdot (1-t')^{n-qn} > t^{\lambda qn} \cdot (1-t')^{n-qn}$. Since p_k is continuous for all k, t^λ is attained for some value of $s'' \in (s, s')$ and hence we have a contradiction.

Second, for any k, if there is a positive measure of types $(\cdot, k)$ in the set T_i^n for all n (along some subsequence) then $p_k(s) = p_k(s')$ for all $s, s' \in S^n$. This directly follows from the fact that $t^n(s) = t^n(s')$. Conversely, if $s \notin S^n$ then there is a k such that there is a positive measure of types $(\cdot, k)$ in the set T_i^n and $p(k, s) \neq p(k, s')$ for $s' \in S^n$.

Third, we claim that as $n \to \infty$ the probability distribution over states conditional on one voter being pivotal coverges (weakly) to a probability distribution which has all the mass concentrated on points in S^n, the set for which $piv^n(s)$ is maximized.

Suppose $S^n = [s_l^n, s_h^n]$. Since the measure of T_i^n is bounded below by ϵ, for every $\eta > 0$ (small enough) we can find a δ (independent of n) such that $|t^n(s_l^n) - t^n(s)| < \delta/2$ for $s \in [s_l^n - \eta, s_h^n + \eta]$ and $|t^n(s_l^n) - t^n(s)| > \delta$ for $s \in [0, s_l^n - 2\eta] \cup [s_h^n + 2\eta, 1]$. But this implies that for any ϵ and any η there is an n such that

$$\frac{piv^n(s)}{piv^n(s')} < \epsilon$$

for all $s \in [s_l^n - \eta, s_h^n + \eta]$ and $s' \in [0, s_l^n - 2\eta] \cup [s_h^n + 2\eta, 1]$. But this in turn implies that

$$\frac{\int_{[s_l^n - 2\eta, s_h^n - 2\eta]} piv^n(s)}{\int_0^1 piv^n(s)} > 2\eta/(2\eta + \epsilon(1 - 2\eta))$$

where ϵ can be chosen arbitrarily small (for a given η) if n is chosen sufficiently large. In particular, setting ϵ so that $2\eta/(2\eta + \epsilon(1 - 2\eta)) > 1 - 2\eta$ implies that if a voter is pivotal then

$$Pr\{s \in [s_l^n - 2\eta, s_h^l + 2\eta]\} > 1 - 2\eta$$

for n sufficiently large. Since η was arbitrary we have established the third claim.

To summarize, we have shown that (along an appropriate subsequence) $piv^n(s)$ converges to a distribution that has all the mass concentrated in an interval $[s_l, s_h]$. Consider an information service k and a subsequence such that there is a positive measure of types $(\cdot, k)$ in the set T_i^n. We also know that $p(k, s) = p(k, s')$ if $s, s' \in (s_l, s_h)$. Such a set of types exists by the hypothesis that the theorem is violated. In the final step of the proof we show that a positive measure of the types $(\cdot, k) \in T_i^n$ is not behaving optimally and thereby obtain the desired contradiction.

To prove this final step it is sufficient to show that for any type who receives the information service k the probability distribution over states conditional on being pivotal and receiving signal $\sigma = 1$ converges (weakly) to the probability distribution over states conditional on being pivotal and receiving the signal $\sigma = 0$. Clearly, since voters' preferences are strictly ordered by preference parameters and since the distribution F_X is absolutely continuous (i.e., has no mass points) this implies that the measure of voters who have access to information service k and are in the set T_i^n has to converge to zero.

To see the final step let $h^n(s|piv, k, \sigma)$ be the probability distribution over states conditional on a voter being pivotal and conditional on receiving signal σ from information service k. Let s_h, s_l be a limit point of the sequence (s_l^n, s_n^h). By the argument above we know that

$$h^n(s|piv, k, \sigma) = \frac{piv^n(s) \cdot \beta(s|k, \sigma)}{\int_0^1 piv^n(s) \cdot \beta(s|k, \sigma)} \to \beta(s|k, \sigma, s \in (s^h, s^l))$$

i.e., the probability distribution over states conditional on being pivotal converges to the posterior of the voter receiving signal σ from information service k conditional on the state being in the set (s^h, s^l). This follows since the prior distribution does not have a mass point (Assumption 2). But note that $p(k, s) = p(k, s')$ for all $s, s' \in (s_h, s_l)$ by the above argument. Hence we have that

$$\beta(s|k, \sigma, s \in (s^h, s^l)) = g(s|s \in (s^h, s^l))$$

which is independent of σ and hence proves the claim and the theorem.□

In the following we analyze the expected vote share of candidate Q as a function of the electorale rule. Theorem 2 says that the expected vote share of candidate Q will be very close to q, i.e., the observed voting behavior is to a large extent determined by the electoral rule. If, for example, the electoral rule states that Q will be implemented whenever the number of votes for Q is larger than or equal to $1/3 \cdot n$ then in a large electorate Q will receive very close to $1/3$ of the votes *irrespective of the realization of* s. On the other hand, if for the same electorate and the same candidates the rule states that Q will be implemented if $2/3$ of the electorate vote for Q then Q will receive very close to $2/3$ of the votes. Note that this does not say that the chances of Q actually winning the election are unaffected by the electoral rule. In fact, we will also show (Theorem 3) that even though the vote shares of both candidates are always close to q and $1-q$ the chances of winning will differ substantially for different q.

Assumption 6 *$p(k, s)$ is strictly decreasing in s for all k.*

Assumption 6 ensures that in every voting equilibrium some voters take informative action and implies that if we sample the signals of many voters we can determine the true value of s with arbitrary accuracy. More precisely, the predicted state conditional on observing n private signals converges in probability to the true state of nature as $n \to \infty$.

Remark: If Assumption 6 holds then every preference type has access to an information service that discriminates between any pair of states with probability 1. For the following results it is sufficient that every preference type has access to an information service that discriminates between any pair of states with strictly positive probability. Thus we could weaken Assumption 6 somewhat without affecting the following results.

Let P^j denotes the set of types who prefer candidate j irrespective of the state s. Then P^Q denote the set of types that prefer candidate Q to A in state $s = 1$ and let P^A denote the set of types that prefer A to Q in state $s = 0$. $F(P^Q)$ is a lower bound on the expected fraction of voters who will vote for Q and $F(P^A)$ is a lower bound on the fraction of voters who vote for A.

Theorem 2 *Suppose Assumptions (1)-(6) hold and suppose that $F(P^Q) < q$, $F(P^A) < 1 - q$. Consider a sequence of symmetric voting equilibria (T_Q^n, T_A^n, T_i^n). Then for all η there is an $\bar{n}$ such that for $n > \bar{n}$*

$$q \in [F(T_Q^n) - \eta, F(T_Q^n) + \eta]$$

In the following we give an intuition for Theorem 2. Assumption 6 implies that every information service discriminates between any pair of states. For large n this implies that, conditional on being pivotal voter beliefs put almost all mass on the neighborhood of one state. If the uncertainty conditional on being pivotal does not vanish as n goes to infinity then the private signal provides useful information for a whole interval of preference parameters. But then the fraction of voters who ignore their signal cannot go to 1 as is implied by Theorem 1.

Let s^* be the state that is most likely to occur if a voter is pivotal. Now we only have to show that at s^* a q-fraction of voters prefers Q and a $1 - q$ fraction of voters prefers A. If $q >$ expected vote share of Q at s^* then it must be the case that $s^* = 1$, since by Assumption (6) this will be the unique state in which

$$|q - \text{expected vote share of } Q|$$

is minimized. But then all voters should behave as if state $s^* = 1$ occured and hence the fraction of voters voting for Q should be $1 - F(T^A) > q$ by the assumption that $F(P^Q) < q$, $F(P^A) < 1 - q$. This contradicts the hypothesis that $q >$ expected vote share of Q. A similar argument can be made for $1-q >$ expected vote share of A

Proof: Consider $\hat{x}^n$ which is defined as the preference parameter for which

$$\int_0^1 v(s, \hat{x}^n) \frac{g(s)piv^n(s)}{\int_0^1 g(s)piv^n(s)} ds = 0$$

i.e. the preference parameter for which a voter prior to receiving the private signal is indifferent between Q and A. (Note that since at $x = -1$ the voter always prefers Q and at $x = 1$ the voter always prefers A continuity implies the existence of $\hat{x}^n$.) By Assumption 6 $(\hat{x}^n, k) \in T_i^n$ for all k. Moreover, continuity of u implies that there is an open set around $\hat{x}^n$ such that all voters with preference parameters in that open set take informative action.

Since T_i^n has strictly positive measure $piv^n(s)$ is strictly increasing if $t^n(s) > q$ and strictly decreasing if this inequality is reversed by Assumption 6. Let s^{*n} be such that $piv^n(s)$ is maximized at s^{*n}. Note that s^{*n} is unique (by Assumption 6).

Let $\phi^n(s) = piv^n(s)/\int_0^1 piv^n(s)$. Consider a convergent subsequence $s^{*n} \to s^*$. First we claim that for all $\eta > 0$, $\phi^n(s) \to 0$ for all s such that $|s - s^*| > \eta$, i.e., ϕ^n converges to a point mass at s^*. Suppose to the contrary that $\phi^n(s)$ stays bounded away from zero for some s with $|s - s^*| \geq \eta > 0$. Since by Assumption 6 $\phi^n(s)$ is monotonically decreasing in the distance from s^{*n} this implies that for every n there is an interval of states $[\hat{s}^n, \hat{s}^n + \delta]$ ($\delta > 0$ independent of n) such that $0 < \epsilon < \phi^n(s) < R < \infty$ for all $s \in [\hat{s}^n, \hat{s}^n + \delta]$. Let

$$h^n(s|k, \sigma) = \frac{\phi^n(s) \cdot \beta(s|k, \sigma)}{\int_0^1 \phi^n(s) \cdot \beta(s|k, \sigma)}$$

Hence $h^n(s|k, 0) = \frac{\phi^n(s)g(s)p(k,s)}{\int_0^1 \phi^n(s)g(s)p(k,s)}$ and $h^n(s|k, 1) = \frac{\phi^n(s)g(s)(1-p(k,s))}{\int_0^1 \phi^n(s)g(s)(1-p(k,s))}$.

By Assumption (6), $h^n(\cdot|k, 1)$ strictly first order stochastically dominates $h^n(\cdot|k, 0)$. Moreover, $\epsilon < \phi^n(s) < R < \infty$ for $s \in [\hat{s}^n, \hat{s}^n + \delta]$ for all n implies that there is an $\epsilon' > 0$ such that

$$\int_0^{\hat{s}^n+\delta/2} (h(s|piv, k, 0) - h(s'|piv, k, 1))ds > \epsilon' > 0$$

for all n and for all k. Since $v(s, x)$ is strictly increasing in s this in turn implies that for every (x, k)

$$\int_0^1 v(s, x)(h^n(s|k, 1) - h^n(s|k, 0))ds > \eta'$$

for some $\eta' > 0$, for all n. But then by continuity of v there is a neighborhood of preference parameters $\hat{X}^n$ around $\hat{x}^n$ such that $x \in T_i^n$ for all $x \in \hat{X}^n$. Moreover, since ϵ' is independent of n the size of this neighborhood can be chosen independent of n and hence $F(T_i^n)$ stays bounded away from zero for all n contradicting Theorem 1. Thus we have shown that for all $\eta > 0$, $\phi^n(s) \to 0$ for all s such that $|s - s^*| > \eta$, i.e., ϕ^n converges to a point mass at s^*. Clearly this implies that also h^n converges to a point mass at s^* for all (σ, k).

Now suppose that $q \geq F(T_Q^n) + \eta$ for all n along some subsequence. Then $q > t^n(s), \forall s$ and hence for n sufficiently large $piv^n(s)$ is maximized at $s^* = 0$. By the argument above this implies that h^n converges to a point mass at $s^* = 0$ for all (σ, k). But then $q \geq F(T_Q^n) + \eta$ contradicts the Assumption that $F(P^A) < 1 - q$. (For $q \leq F(T_A^n) - \eta$ an analogous contradiction can be obtained.) □

4 Full Information Revelation

In this section we ask under what conditions elections fully aggregate private information. Consider the hypothetical situation in which the state is common knowledge among voters. The resulting election outcome will serve as a benchmark for our definition of full information revelation. An election outcome is fully revealing if the election under private information leads to the same outcome as the election outcome when the state is common knowledge among voters.[6]

Definition 1 *A choice $j \in \{Q, A\}$ is called a mistake for s and a population $(x_1, \dots, x_n)$ if (i) for $j = Q$ fewer than qn voters prefer Q to A or if (ii) $j = A$ more than qn agents prefer Q to A.*

Theorem 3 *Suppose Assumptions 1-6 hold. Then for all $\epsilon > 0$ there is an $\bar{n}$ such that for $n > \bar{n}$ the probability that a mistake occurs in any voting equilibrium with n voters is less than ϵ.*

From the proof of Theorem 2 we know that conditional on being pivotal a voter's beliefs put almost all the weight on the neighborhood of one state, i.e., with probability $1 - \epsilon$ the voter believes that $s \in [s^* - \epsilon, s^* + \epsilon]$. But (since $t^n(s)$ is monotonically

[6]Alternatively, we could use as a benchmark the situation in which all the private signals are common knowledge among voters. Note, however, that Assumption 6 and the law of large numbers imply that in a large electorate knowing all signals is almost equivalent to actually knowing the true state of nature.

decreasing) this can only be the case if for $s < s^* - 2\epsilon$ candidate Q gets chosen with probability close to one and for $s > s^* + 2\epsilon$ candidate A gets chosen with probability close to one. From Theorem 2 we know that at s^* with probability close to one a the fraction of voters who prefer Q is very close to q. Therefore, for $s > s^* - 2\epsilon$ the probability of a mistake being made is arbitrarily close to zero for large n. An analogous argument shows that for $s < s^* + 2\epsilon$ the probability of a mistake is arbitrarily small for large n. Hence mistakes can only occur (with large probability) if the state is in the interval $[s^* + 2\epsilon, s^* - 2\epsilon]$. But for small ϵ this does not occur too often.

Proof: If $F(P^Q) > q$ then since all voters whose type is in $F(P^Q)$ will vote for Q, candidate Q will be chosen with probability close to one for large n. Moreover, this choice satisfies full information revelation because voters in P^Q prefer candidate Q in every state s. A similar argument shows that the Theorem is satisfied if $F(P^A) > 1-q$.

If $F(P^Q) < q, F(P^A) < 1 - q$ then from Theorem 2 and the proof of Theorem 2 we know that for all $\epsilon > 0$ there is an $\bar{n}$ such that for $n > \bar{n}$

(i) If a voter is pivotal then $Pr\{s \in [s^{*n} - \epsilon, s^{*n} + \epsilon]\} > 1 - \epsilon$.

(ii) $q \in [F(T_Q^n) - \epsilon, F(T_Q^n) + \epsilon]$.

But this implies that for every $\alpha > 0$ there is an $\bar{n}$ such that if $n > \bar{n}$ and $s < s^{n*} - \alpha$ then the probability that the number of voters who prefer candidate Q to candidate A is larger than $q \cdot n$ with probability $1 - \alpha$. Similarly, if $s > s^{n*} + \alpha$ then the number of voters who prefer candidate Q to candidate A is smaller than qn with probability $1 - \alpha$.

Next we show that (for large n) for $s > s^{*n} + \alpha$ the probability that A is chosen is larger than $1 - \alpha$ and for $s < s^{*n} - \alpha$ the probability that A is chosen is smaller than α. Let $w^n(m, s)$ denote the probability that in a voting equilibrium with n voters m voters choose candidate Q if the state is s. Note that by item (i) above it follows that

$$\frac{w^n(qn, s^{*n} + \alpha)}{w^n(qn, s^{*n})} < \epsilon$$

for n sufficiently large. Since $t^n(s)$ is increasing in s this implies that for all $m < qn$

$$\frac{w^n(m, s^{*n} + \alpha)}{w^n(m, s^{*n})} < \alpha$$

for n sufficiently large. And hence for all $s > s^* + \alpha$

$$\sum_{m<qn} w^n(m, s) < \alpha \sum_{m<qn} w^n(m, s^*) \leq \alpha$$

This implies that A will be chosen with probability larger than $1 - \alpha$. An analog argument shows that or $s < s^{*n} - \alpha$ the probability that A is chosen is smaller than α.

Thus the only time when there is a probability larger than 3α that a mistake is made is if $s \in [s^* - \alpha, s^* + \alpha]$. But since the probability distribution over s is continuous the probability that s is in this interval can be made arbitrarily small for small α and hence the Theorem follows. □

5 An Example

In this section we present an example both to illustrate how our model works and to link our results to the literature on elections and information aggregation. The example relates our work to a model analyzed by Lohmann (1993). Lohmann analyzes a model that is similar to this example although she assumes perfect information about the location of voter preference parameters. Our Proposition 1 applied to this example replicates her results. In addition we illustrate the asymptotic properties of the model and the consequences of differentially informed voters. Second, we show that if the function $p(k, s)$ is not continuous our convergence result (Theorem 1) may fail. Finally, we demonstrate, how Theorems 2 and 3 fail if Assumption 6 does not hold.

Suppose that candidates correspond to positions in a one-dimensional policy space. Furhter, let $0 < Q < A < 1$, $Q + A = 1$ (candidates are located symmetrically around $1/2$) and $u(j, s, x) = -(j - s - x)^2$. In this example the state acts as a shift parameter that shifts both candidate's policies by a factor s. Voter preference parameters are distributed uniformly. Hence $f_X(x) = 1/2$. In this example

$$v(x, s) = (Q^2 - A^2) + 2(A - Q) \cdot x + 2(A - Q)s$$

Thus if W is the payoff difference of voting for A and voting for Q given the strategy of all other players and E denotes the expectation operator then

$$W(x, k, \sigma, \pi^*)/(A - Q) = -1 + 2x + 2E(s|k, \sigma, piv) \tag{5}$$

5.1 Illustration of the results

Suppose that there is one information service and $p(s)$ satisfies Assumption 6. Since Assumptions 1-6 are satsified an equilibrium can be characterized by a pair of cutpoints (x_1, x_0) such that only the voters with preference parameters in the interval (x_1, x_0) make their vote dependent on their signal. All voters with preference parameters below x_1 vote for candidate Q irrespective of their signal and all voters with preference parameters above x_0 vote for candidate A irrespective of their signal.

Furthermore, as $n \to \infty$, $x_1 - x_0 \to 0$, i.e., the two cutpoints converge to one point. In particular, by Theorem 2 and the fact that f_X is the uniform distribution $x_\sigma \to 2q - 1$ for $\sigma \in \{0, 1\}$. Hence in the limit both cutpoints coincide at the point $2q - 1$ which is the preference parameter of the expected median voter. Finally, Theorem 3 shows that if $s > 3/2 - 2q, q \leq 3/4$ then candidate A will be chosen with probability close to one and if $s < 3/2 - 2q, q \geq 1/4$ then candidate Q will be chosen with probability close to one.

5.2 More and less informed voters

Suppose that there are two information services, $k = 1, 2$ and that service 1 provides voters with better information than information service 2. More precisely, suppose that $\beta(s|1, 0)$ is first order stochastically dominated by $\beta(s|2, 0)$ and conversely $\beta(s|1, 1)$ first order stochastically dominates $\beta(s|2, 1)$. In this case our Proposition 1 shows that there are two pairs of cutpoints (x^*_{11}, x^*_{10}) and (x^*_{21}, x^*_{20}) where the set of voters who receive information from service 1 and whose vote depends on their signal are in the interval (x^*_{11}, x^*_{10}) and voters who receive information from service 2 and whose vote depends on their signal are in the interval (x^*_{21}, x^*_{20}).

By Equation 5 $x^*_{k\sigma}$ satisfies

$$1 + 2x^*_{k,\sigma} + 2E(s|k, \sigma, piv) = 0 \tag{6}$$

By construction $E(s|1, 0, piv) < E(s|2, 0, piv)$ and $E(s|1, 1, piv) > E(s|1, 2, piv)$ and hence $x^*_{11} < x^*_{21}$ and $x^*_{10} > x^*_{20}$. Thus voters who receive information from service 1 are more willing to use their information than voters who receive information from service 2.

5.3 An Example where Theorem 1 does not hold.

In this section we give an example that demonstrates how Theorem 1 fails when Assumption 5 is violated. Suppose that $p(s) = 1 - \epsilon$ if $s < 1/2$ and $p(s) = \epsilon$ if $s > 1/2$. Thus p is not continuous at $1/2$ and, even if all available information is aggregated, voters can only learn if the state is above or below $1/2$. Furthermore suppose that $q = 1/2$. The (unique) equilibrium is for all voters who receive signal 0 and have preference parameter smaller than $1/4 - \epsilon/2$ to vote for candidate Q. Similarly, if a voter receives signal 1 and has a preference parameter larger than $-1/4 + \epsilon/4$ he votes for candidate A. The equilibrium strategies in this example are independent of n. To see why the prescribed strategies are an equilibrium note that since the situation is symmetric it is equally likely to be pivotal if $s < 1/2$ as if $s > 1/2$. The distribution over states conditional on being pivotal is uniform over all states. Thus, conditional on being pivotal and receiving signal 0, the expected state is $1/4 + \epsilon/2$, and conditional on being pivotal and recieving signal 1, the expected state is $3/4 - \epsilon/2$. Moreover, this is independent of n. A simple calculation using Equation (5) then shows that the described strategies actually form an equilibrium.

Note that for this information service it is not the case that $p(s) = p(s')$ for all $s \in S^{*n}$ since $p(s)$ is not continuous. As a consequence, the signal is informative *conditional on being pivotal* and private information remains valuable for all n. This example shows how in the absence of Assumption 5, Theorem 1 may fail.

5.4 An Example where Theorem 1 holds but Theorem 3 does not.

In the following example Theorem 1 holds while Theorem 3 is violated, i.e., the voting equilibrium does not aggregate information fully. Suppose there are two information services: Suppose $p(1, s) = 1 - s$ and $p(2, s) = 1/2$, i.e., information service 2 is not informative. Further suppose that $q = 1/2$. The distribution F is such that all voters with preference parameters $x \in [-1, -\epsilon] \cup [\epsilon, 1]$ have access to information service 1 with probability 1, while all voters with preference parameters $x \in (-\epsilon, \epsilon)$ have access to information service 2 with probability 1. We can distinguish two cases:

Case 1. $\epsilon > 1/6$. In this case the equilibrium all voters with positive preference parameters vote for candidate A and all voters with negative preference parameters vote for candidate Q.

To see that this is an equilibrium note that $E(s|\sigma = 1) = 2/3$ and $E(s|\sigma = 0) = 1/3$ for all voters who get an informative signal. Since $1 - 2x - 2E(s|1, piv) = 1 - 2x - 2(2/3) > 0$ for $x < -1/6$ no voter takes informative action whenever $\epsilon > 1/6$. In case 1, irrespective of the state, each candidate has a 50% chance of winning the election. Therefore, with probability 1/2 a mistake is made.

Observe also that the same equilibrium strategies are an equilibrium for any q as long as $q + 1/6 \leq 1/2 + \epsilon$ and $q - 1/6 \geq 1/2 - \epsilon$. Therefore, if $q \neq 1/2$ then $F(T_Q^n) = 1/2 \neq q$.

Case 2. $\epsilon < 1/6$. In this case there are two intervals of voter preference parameters for which the equilibrium strategies prescribe a vote according to the signal received. These intervals are $[-x(n), -\epsilon]$ and $[\epsilon, x(n)]$ with $x(n) > \epsilon$. Theorem 1 implies that $x(n) \to \epsilon$ as $n \to \infty$.

In Case 2 some private information will be revealed. But the probability that a mistake is made is bounded away from zero for all n. Intuitively, as $n \to \infty$ the two intervals of voters who take informative action shrinks too quickly to provide sufficient aggregate information for the "right" choice. Note that due to the symmetry of the situation the probability distribution over states conditional on a voter being pivotal has $s = 1/2$ as a modal point. On the other hand if this distribution puts all the mass on a very small neighborhood around 1/2 then no voter who has an informative signal is willing to take informative action, since voters with preference parameters below $-\epsilon$ will vote for Q irrespective of their signal and voters with preference parameters above ϵ will vote for A irrespective of their signal. Hence the limit distribution over states conditional on a voter being pivotal cannot be the Dirac measure at the point 1/2 but rather is a nondegenerate distribution with modal point 1/2. But this implies that for states around 1/2 there is a positive probability of making a mistake for all n. Moreover, this neighborhood of states for which a mistake is possible does not shrink as the number of voters goes to infinity.

The hypothesis of Theorem 1 is satisfied in this example and hence the set of voters who take informative action converges to zero in measure (in case 1 it is identically zero). However, the assumptions for Theorem 3 are not satisfied since one of the information services does not provide an informative signal. Note, that in this example the true state of nature could be inferred in a large electorate if all voters revealed their private information.

5.5 Strategic versus Naive Voting

In Theorem 1 we show that as the number of voters goes to infinity, the fraction of voters who take informative action goes to zero. Thus only a very small fraction of voters reveal their private information. On the other hand we show that if the information services are perfectly informative, i.e., if with large samples the true state of nature can be determined with arbitrary accuracy, then as Theorem 3 shows, the election outcome will typically be the same as if all voters were perfectly informed. In this section we want to compare the election outcome when voters behave strategically to the election outcome when voters behave "naively." A voter behaves naively if he behaves as if his choice alone determines the outcome. More precisely, a voter of type (x, k) with signal σ votes naively if he votes for Q if $\int v(x, s)\beta(s|\sigma, k)ds < 0$ and if he votes for A if $\int v(x, s)\beta(s|\sigma, k)ds > 0$. Clearly, if voters behave naively, then more of their private information will be revealed and hence there is the potential that under naive behavior the probability that a mistake is made is smaller.

In the following example we show that under naive voting the probability of a mistake is bounded away from zero while (by Theorem 3) under strategic voting the probability of a mistake converges to zero as $n \to \infty$. Thus if the electorate is sufficiently large then under strategic voting the probability of a mistake is smaller than under naive voting. Our model thus gives results that are in the spirit of the results found in Austen-Smith and Banks (1994) in the context of pure common values.

Proposition 2 *Suppose the electorate is as described in the example in section 6. Suppose further there is one information service and $p(s) = (1 - s)$. The prior is such that $\int_{[0,1]} sg(s)ds \neq 1/2$ and $q = 1/2$. Then there exists an ϵ such that under naive voting the probability that a mistake is made is bounded below by ϵ.*

Proof: We can use Equation (5) to show that under naive voting a voter with preference parameter x will vote for candidate Q if

$$-1 + 2x + 2E(s|\sigma) > 0 \tag{7}$$

Let x^0 be such that $1-2x^0-2E(s|0) = 0$ and let x^1 be such that $1-2x^1-2E(s|1) = 0$. Note that full information revelation requires that the expected vote share of either candidate in state $s = 1/2$ is equal to $1/2$. If the expected vote share of candidate Q in state $1/2$ were, for example, strictly bigger than $1/2$ then for large n and for $s \in [1/2, 1/2 + \epsilon]$ (for small ϵ) the election outcome would be such that candidate Q is chosen with large probability if $s \in [1/2, 1/2 + \epsilon]$. This clearly violates full information revelation since a majority of voters would typically prefer candidate A if $s > 1/2$. Moreover, since $p(s)$ is strictly decreasing, voters could infer from the individual signals that the state s is indeed above 1/2. In order for the expected vote share of Q to be 1/2 if $s = 1/2$ it must be the case that $x^0 + x^1 = 0$, i.e., x^0 and x^1 must be symmetric around 0. But this implies that $E(s|0) + E(s|1) = 1$. Let $E(f(s)) = \int_0^1 f(s)g(s)ds$ denote the (unconditional) expectation of $f(s)$. Then

$$E(s|0) = E(s(1-s))/E(1-s)$$

and

$$E(s|1) = E(s^2)/E(s)$$

Hence

$$E(s|0) + E(s|1) = \frac{E(s) - E(s^2)}{1 - E(s)} + \frac{E(s^2)}{E(s)} = 1$$

Note that this implies that

$$E(s)(1 - 2E(s)) = E(s^2)(1 - 2E(s))$$

But since $E(s^2) < E(s)$ for any prior s with full support it follows that $x^0 + x^1 = 0$ if and only if $E(s) = 1/2$. Thus in this example naive voting will lead to a fully revealing election outcome if and only if $E(s) = 1/2$. □

6 Uncertainty about the Distribution of Voters' Preferences

In the previous section the distribution from which voter types are drawn is common knowledge. This assumption ensures that for any sequence of strategy profiles the expected vote share of a candidate converges in probability to the actual vote share. In this section we relax this assumption and introduce uncertainty about the distribution of preferences. We assume that nature not only chooses a state s but also chooses the distribution according to which the electorate gets selected. In such a world voters cannot predict the empirical distribution of preferences accurately even in a very large electorate.

We show that by introducing this second source of uncertainty both Theorems 1 and 3 are no longer valid, i.e., the fraction of voters who use their private signal stays bounded away from zero and the election outcome is not equivalent to the outcome if private signals are common knowledge.

Let $\alpha \in [0,1]$ be a parameter that characterizes the distribution according to which nature selects the electorate. To simplify the analysis we assume that the uncertainty about the distribution takes a very simple form. Voters are uncertain about the expected fraction of partisans, i.e., voters who choose either candidate Q or candidate A irrespective of the state. Let δ_x be a Dirac measure on the point $x \in [-1,1]$. Let F be a measure that satisfies Assumption 3. The distribution H_α is given by

$$H_\alpha = (1-\epsilon)F + \epsilon(\alpha\delta_1 + (1-\alpha)\delta_{-1})$$

Thus H_α has $\epsilon(1-\alpha)$ mass at -1 and $\epsilon\alpha$ mass at $+1$. Note that by Assumption 1 voters with preference parameters at -1 always vote for candidate Q and voters with preference parameter 1 always vote for candidate A.

In the first stage of the game, nature chooses both s and α independently. By $b(\alpha)$ we denote the density that describes the prior beliefs about the state α.

Assumption 7 *b is continuous and $b(\alpha) > 0$ for all $\alpha \in [0,1]$.*

After choosing the state (s,α) nature selects an electorate by taking n independent draws from the distribution H_α. Finally, in addition to the signal σ, every voter receives a signal $\rho \in \{0,1\}$ that provides information about the realization of α. By $r(\alpha)$ we denote the probability that the signal is 0 if α is realized. Again, we assume that conditional on α being realized the signal voters' receive are independent. We assume that receiving a signal $\rho = 1$ implies that higher values of α are more likely, whereas receiving a signal $\rho = 0$ implies that lower values of α are more likely.

Assumption 8 *$r(\alpha)$ is strictly decreasing and continuously differentiable. Moreover, $K = 1$ and $p(1,s) \equiv p(s)$ is strictly decreasing and continuously differentiable.*

The assumption that there is exactly one information service is made to simplify the proof of Theorem 4 but is not necessary for the result. Assumption 8 implies that if we sample the private signals of many voters then we can predict the state (s, α) with arbitrary accuracy.

Note that if a voter learns his preference parameter x and if $-1 < x < 1$ then this does not provide the voter with information about the realization of α. This is the case since the likelihood of observing $x, -1 < x < 1$ is independent of α. The only voters who get information about the realization of α by observing their preference parameter are voters with $x = -1$ and $x = +1$. However, these voters are partisans and will always vote for A (in the case of $x = 1$) or Q (in the case of -1) by Assumption 1. Therefore we do not need to consider the inference problem of these voters.

A pure strategy is now also a function of ρ, i.e., $\pi^* : T \times \{0, 1\} \times \{0, 1\} \to \{Q, A\}$. Again, we define a voting equilibrium to be a Nash equilibrium of the game in which no voter chooses a weakly dominated strategy. Proposition 2 is the analog of Proposition 1 for the model in with aggregate uncertainty about the distribution.

Proposition 3 *Suppose Assumptions 1,2,3,7,8 hold. Every voting equilibrium can be described by a pure strategy π^*. Furthermore equilibrium strategies can be characterized by a pair of partitions $(T_{Q\rho}, T_{A\rho}, T_{i\rho})$, $\rho = 0, 1$ of T where*

(i) $\pi^*(x, k, \sigma, \rho) = Q$ for $(x, k) \in T_{Q,\rho}$ and $\sigma = 0, 1$

(ii $\pi^*(x, k, \sigma, \rho) = A$ for $(x, k) \in T_{A\rho}$ and $\sigma = 0, 1$

(iii) $\pi^*(x, k, 0, \rho) = Q$, and $\pi^*(x, k, 1, \rho) = A$ for $(x, k) \in T_{i\rho}$

Proof : The proof is analog to the proof of Proposition 1.

The following Theorem shows that the set of voters who use their private signal σ stays bounded away from zero in measure and furthermore that equilibrium outcomes are not equivalent to the outcomes that would be achieved if all the private information were common knowledge. This latter result will be shown to hold for a typical utility function $v(x, s)$. To make this precise, consider the set of decreasing real valued functions on $[0, 1] \times [-1, 1]$ denoted by P and endow it with the topology of uniform convergence. We say that a property holds for a *generic* utility function if it holds for all $v \in O \subset P$ where O is open and dense.

Theorem 4 *Suppose Assumptions 1,2,3,7,8 hold and $H_0(P^Q) < q$, $H_1(P^A) < 1-q$ for all α. Consider a sequence of voting equilibria $(T^n_{Q\rho}, T^n_{A\rho}, T^n_{i\rho})$, $\rho = 0, 1$. Then*

(i) There is an $\eta > 0$ such that $H_\alpha(T^n_{i\rho}) > \eta$ for $\rho = 0, 1$ for all n and all α.

(ii) There exists an $O \subset P$, where O is open and dense, such that for every $v \in O$ there exists an $\eta > 0$ such that for all n the probability that a mistake is made is bounded below by η.

Proof: see Appendix.

To provide an intuition for the proof of part (i) of Theorem 4 suppose for (s, α) the expected vote share of candidate Q is q. Then we can decrease s and simultaneously increase α so that the expected vote share stays unchanged. Conditional on being pivotal a voter believes that one of the states has occured for which the expected vote share of candidate Q is q. Since now there is a whole interval of s for which (for the appropriate choice of α) this is true, the beliefs over states conditional on being pivotal do not converge to a degenerate distribution. But then the private information of voters provides useful information and hence the set of voters who take informative action does not converge to zero in measure.

To provide an intuition for part (ii) note that there is a function $\mathbf{s} : [0, 1] \to [0, 1]$ with the following property: full information revelation requires that for $s < \mathbf{s}(\alpha)$ the Q is chosen with probability close to one in state (s, α) and for $s > \mathbf{s}(\alpha)$ A is chosen with probability close to one in state (s, α). Thus for states $(\mathbf{s}(\alpha), \alpha)$ the expected vote share of Q must be close to q for a large electorate if full information revelation is to hold since otherwise close to $(\mathbf{s}(\alpha), \alpha)$ (but uniformly bounded away form $(\mathbf{s}(\alpha), \alpha)$) a mistake is made with high probability. This follows from the fact that the derivative of the expected vote share of A with respect to s is uniformly bounded above for all n. We show that equilibrium strategies allow too few degrees of freedom to achieve the required vote share for all states $(\mathbf{s}(\alpha), \alpha)$ for a generic choice of v.

Note that as an alternative to the introduction of uncertainty about the distribution of voter preference parameters we could allow s to be a two dimensional variable. As long as there does not exist a function $a(s_1, s_2)$ such that $v(x, s_1, s_2) = v'(x, a(s_1, s_2))$ a result similar to the one given in Theorem 4 will hold. Thus full information revelation crucially depends on the fact that uncertainty is about a one dimensional state variable.

7 Conclusion

In this paper we present three principal results. First, we show that voters have an incentive to behave strategically in two candidate elections and that, in contrast to the assumption made in the literature on the Condorcet Jury Theorem, the fraction of voters who take informative action decreases to zero as the size of the electorate goes to infinity. Perhaps surprisingly, we find that elections nevertheless aggregate almost all of the private information and choose the correct candidate from the perspective of a fully informed decisive coalition of voters. In fact, we show that in a model with strategic voters information aggregation works better than in a model with naive voters. In contrast to earlier models voters no longer have a weakly dominant strategy to vote for their favorite candidate and voting behavior is to a large extent determined by the electoral rule.

This model has focused on the incentives of voters. A more general framework would endogenize candidate position taking. In our model elections reveal more information than simply which candidate is preferred by a majority. This information may create an incentive for post-election strategic action by winning candidates. For example, the winning candidate might claim a mandate based on the election results and implement a policy different than his or her announced policy. If the policy implemented by the winning candidate depends on the magnitude of his or her electoral victory then the effect may be to change an election into something like a signalling game between voters and winning candidates. We leave it for future work to explore the implications of private information and common values for the interaction between voter and candidate behavior.

8 Appendix

Proof of Proposition 1: Existence of an equilibrium follows from Milgrom and Weber (1985), Theorem 1 (pg. 626). Note, however, that Milgrom and Weber's result does not imply existence of an equilibrium in which no voter uses a weakly dominated strategy. However, consider the following perturbed payoff function for a type:

$$V^{\alpha}(x,k,\sigma,\tau,\bar{\pi}) \equiv (1-\alpha)V(x,k,\sigma,\tau,\bar{\pi})+\alpha\int[\tau u(Q,s,x)+(1-\tau)u(A,s,x)]\beta(s|k,\sigma)ds$$

i.e., with probability α the player's payoff only depends on his own strategy. Note that for every α there exists an equilibrium. Moreover, the convergence result of Milgrom and Weber (Theorem 2, pg 627) implies that if $\bar{\pi}^{\alpha}$ is an equilibrium for the payoff function V^{α}, then $\bar{\pi}^* = \lim \bar{\pi}^{\alpha}$, $\alpha \to 0$ is an equilibrium for the original game. Suppose $\bar{\pi}^*$ is a weakly dominated strategy. Let $D \subset supp\bar{\pi}^*$ be the set of types such that every $(x,k,\tau) \in D$ is weakly dominated by some (x,k,τ'). We claim that D has measure zero. Suppose that D has positive measure. Since F_X has no mass points this implies that for a.e. $(x,k,\tau) \in D$ we have $\int u(Q,s,x)\beta(s|k,\sigma)ds \neq \int u(A,s,x)\beta(s|k,\sigma)ds$. But this implies that (x,k,τ') results in a different payoff than (x,k,τ) against the strategy in which all other players randomize equally between Q and A. But the construction of $\bar{\pi}^*$ implies that (x,k,τ') must result in a lower payoff than (x,k,τ) against this strategy for almost every $(x,k,\tau) \in D$. Hence D cannot have strictly positive measure.

Given any voting equilibrium $\bar{\pi}^*$ consider the set of preference parameters $G \subset T$ such that for each $(x,k) \subset T$ the voter type (x,k) is indifferent between candidate A and candidate Q for some signal $\sigma \in \{0,1\}$. By Assumption 1 the set G can at most have $2 \cdot K$ elements, i.e., G is a finite set. Further note that since the set G is a finite subset of T and since F_X does not have mass points, $F(G) = 0$, i.e., G has measure zero. Moreover, for any $(x,k) \notin G$ there exists a neigborhood N such that for $(x',k) \in N$ the player will vote for candidate j with probability 1. This follows from the fact that u is continuous. Thus, if we ignore points in the set G we can represent the voting equilibrium by a pure strategy $\bar{\pi}^*$ on $T\backslash G$, i.e., by a measurable function $T\backslash G \times \{0,1\} \to \{Q,A\}$. Since play on the set G does not affect any players payoff any extension of $\bar{\pi}^*$ to all of T will constitute a voting equilibrium in pure strategies.

Finally, we have to show that any pure symmetric voting equilibrium can be described by a partition of the set T into measurable subsets (T_A, T_Q, T_i) where $\pi(\sigma, x, k) = Q$ for $(x, k) \in T_Q$, $\pi(\sigma, x, k) = A$ for $(x, k) \in T_A$ and $\pi(x, k, 1) = A$, $\pi(x, k, 0) = Q$ for $(x, k) \in T_i$. Note that $\beta(\cdot|1, k)$ first order stochastic dominates $\beta(\cdot|0, k)$ (Assumption 3) and $v(s, x)$ is an increasing function of s (Assumption 1). Thus any voter who chooses Q after receiving a signal of 1 must also choose Q after receiving a signal of 0 from the same information service. Conversely, a voter who receives a signal 0 and chooses candidate A must also choose candidate A after receiving signal 1.

Let $piv(s, \pi^*)$ be the probability that a voter is pivotal in state s. To prove the final part of the proposition we first show that $piv(s, \pi^*) > 0$ for all s. Note that by Assumption 1 there is an $\epsilon > 0$ such that $u(Q, s, x) - u(A, s, x) > 0$ for all $s \in [0, 1]$ and any $x \in [-1, -1+\epsilon]$. Since $f_X > 0$ it follows that $F(T_Q) > 0$. A similar argument shows that $F(T_A) > 0$. But then there is a positive probability that qn voters choose Q and $n - qn$ voters choose A (ignoring one of the voters) and hence there is a positive probability that a voter is pivotal. Now define

$$W(x, k, \sigma, \pi^*) = \int_0^1 v(s, x) \frac{\beta(s|\sigma, k) piv(s, \pi^*)}{\int_0^1 \beta(s|\sigma, k) piv(s, \pi^*)} \tag{8}$$

Clearly, for $x \in [-1, -1+\epsilon]$, $W(x, k, \sigma, \pi^*) < 0$ and for $x \in [1-\epsilon, 1]$, $W(x, k, \sigma, \pi^*) > 0$. By continuity of W in x there is an $x^*_{k\sigma} \in [-1, 1]$ such that $W(x, k, \sigma, \pi^*) = 0$. The point $x^*_{k\sigma}$ is unique because W strictly increasing in x (by Assumption 1). Further note that receiving signal 1 implies higher values of s are more likely than when receiving signal 0, i.e., from Assumption 3 and the fact that $\beta(s|1, k)$ first order stochastically dominates $\beta(s|0, k)$ it follows that $x^*_{1k} \leq x^*_{0k}$ for all k. □

Proof of Theorem 4:

Part (i): Suppose that contrary to the Theorem $H_\alpha(T^n_{i\rho}) \to 0$ along some subsequence. Let $\gamma(\alpha|\rho)$ denote the posterior density over α conditional on receiving signal ρ and let $piv(s, \alpha)$ denote the probability of a voter being pivotal in state (s, α). By an argument given in the proof of Theorem 2, $F(T^n_{i\rho}) \to 0$ implies that conditional on a voter being pivotal the distribution over states s converges to a point mass at some state s^*, i.e.,

$$\phi(s|\rho) = \int_0^1 \left(\frac{piv^n(s,\alpha)}{\int_0^1 \phi^n(s)} \right) \gamma(\alpha|\rho) d\alpha$$

converges to a point mass at s^* for some s^*. But since $\gamma(\alpha|\rho)$ is bounded above and below by Assumptions 7 and 8 for $\rho = 0, 1$ this implies that $\phi^n(s|\rho'), \rho' \neq \rho$ also converges to a mass point at s^* as $n \to \infty$. Hence the probability distribution over states s conditional on being pivotal and receiving signal ρ' converges to a distribution that has all its mass concentrated at s^*. Therefore,

$$|F(T^n_{Q0}) - F(T^n_{Q1})| \to 0, F(T^n_{i\rho'}) \to 0, \rho' \neq \rho \tag{9}$$

Now let

$$t^n(s,\alpha) = (1-\epsilon)[r(\alpha)(F(T^n_{Q0})+p(s)F(T^n_{i0})+(1-r(\alpha))(F(T^n_{Q1})+p(s)F(T^n_{i1}))]+\epsilon(1-\alpha) \tag{10}$$

$$piv^n(s,\alpha) = \begin{pmatrix} n \\ qn \end{pmatrix} \cdot t^n(s,\alpha)^{qn} \cdot (1 - t^n(s,\alpha))^{n-qn} \tag{11}$$

and

$$piv^n(s|\rho) = \int_0^1 piv^n(s,\alpha)\gamma(\alpha|\rho) d\alpha$$

Clearly s^* is bounded away from 0 and 1 by the hypothesis of the Theorem. ($H_\alpha(P^Q) < q, H_\alpha(P^A) < 1 - q$, hence if, e.g., $s^* \leq \epsilon$ then the expected vote share of Q is larger than $q + \eta$ which in turn implies that $s = 1$ is the state for which voters are most likely to be pivotal). Therefore the monotonicity and continuity of $t^n(s)$ and the fact that conditional on being pivotal the distribution over states converges to a point mass at s^* implies that for each $\delta > 0$ there exists an n such that

$$\frac{piv^n(0|\rho)}{piv^n(s^*|\rho)} < \delta, \frac{piv^n(1|\rho)}{piv^n(s^*|\rho)} < \delta \tag{12}$$

Thus the relative likelihood of being pivotal in state s^* and states $s = 0, 1$ must be small.

By 10 and 11

$$\frac{piv^n(s|\rho)}{piv^n(s'|\rho)} = \frac{\int_0^1 (c^n_1 + v^n(s) + \epsilon(1-\alpha))^{nq}(1 - c^n_1 - v^n(s) - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho) d\alpha}{\int_0^1 (c^n_1 + v^n(s') + \epsilon(1-\alpha))^{nq}(1 - c^n_1 - v^n(s') - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho) d\alpha}$$

for some c_1^n and some v^n where $v^n(s) \to 0$ as $n \to \infty$. Let $\eta_0^n = v^n(s^*) - v^n(0)$, $\eta_1^n = v^n(1) - v^n(s^*)$. Clearly, $\eta_i^n \to 0$, $i = 1, 2$.

$$\frac{piv^n(0|\rho)}{piv^n(s^*|\rho)} \geq \frac{\int_{\eta_0^n}^1 (c_1^n + v^n(s^{*n}) + \epsilon(1-\alpha))^{nq}(1 - c_1^n - v^n(s^*) - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho)d\alpha}{\int_0^1 (c_1^n + v^n(s^{*n}) + \epsilon(1-\alpha))^{nq}(1 - c_1^n - v^n(s^*) - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho)d\alpha}$$

But this implies that if $c_1^n + \epsilon/2 \leq q$ along some subsequence then $\frac{piv^n(0)}{piv^n(s^*)}$ is bounded away from zero for all n. (Note that $x^q(1-x)^{1-q}$ reaches a maximum at $x = q$ and is monotonically increasing for $x < q$ and monotonically decreasing for $x > q$).

$$\frac{piv^n(1|\rho)}{piv^n(s^{*n}|\rho)} \geq \frac{\int_0^{1-\eta_1^n} (c_1^n + v^n(s^{*n}) + \epsilon(1-\alpha))^{nq}(1 - c_1^n - v^n(s^{*n}) - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho)d\alpha}{\int_0^1 (c_1^n + v^n(s^{*n}) + \epsilon(1-\alpha))^{nq}(1 - c_1^n - v^n(s^{*n}) - \epsilon(1-\alpha))^{n(1-q)}\gamma(\alpha|\rho)d\alpha}$$

But this implies that if $c_1^n + \epsilon/2 \geq q$ along some subsequence then $\frac{piv^n(1)}{piv^n(s^*)}$ is bounded away from zero for all n. Thus we have shown that along any subsequence either $\frac{piv^n(0|\rho)}{piv^n(s^*|\rho)}$ or $\frac{piv^n(1|\rho)}{piv^n(s^*|\rho)}$ stays bounded away from zero contradicting 12. Hence it cannot be the case that $H_\alpha(T_{i\rho}^n) \to 0$ which completes the proof of part (i).

Part (ii): Define $x(\alpha)$ by $H_\alpha(x) = q$. Note that $x(\alpha)$ is a continuous and strictly increasing function. Consider the equation

$$v(x(\alpha), s) = 0$$

Since v is strictly increasing there exists a strictly decreasing and continuous function $\mathbf{s}(\alpha)$ such that

$$v(x(\alpha), \mathbf{s}(\alpha)) = 0$$

for all $\alpha \in [0, 1]$. (This follows from $H_\alpha(P^Q) < q, H_\alpha(P^A) < 1 - q$ for all α.) Full information revelation requires that for $s < \mathbf{s}(\alpha)$ Q is chosen with probability close to one in state (s, α) and for $s > \mathbf{s}(\alpha)$ A is chosen with probability close to one in state (s, α). Let P' denote the set of strictly decreasing functions $\mathbf{s}$ and endow it with the topology of uniform confergence. Note that any strictly increasing continuous function $\mathbf{s} : [0, 1] \to [0, 1]$ can be generated by some v. Moreover any open set $O \subset P$ generates an open set $O' \subset P'$. Now observe that

$$t^n(s, \alpha) = B_1 + B_2 r(\alpha) + B_3 r(\alpha)p(s) + B_4 p(s) + B_5 \alpha \tag{13}$$

for some constants $B_1, \ldots, B_5$. For each choice of $B = (B_1, \ldots, B_5)$ we can define a map $\zeta_B : [0, 1] \to [0, 1]$ such that $t^n(\zeta_B(\alpha), \alpha) = q$ for all $\alpha \in [0, 1]$. In other words $(\zeta_B(\alpha), \alpha)$ is the pair such that if the behavior of the electorate is given by the constants $(B_1, \ldots, B_5)$ then if the state is $(\zeta_B(\alpha), \alpha)$ the expected vote share of Q is q and the expected vote share of A is $1 - q$.

To prove the second part of the Theorem we will show that for a generic choice of $\mathbf{s}$ (i) either ζ_B does not exist or (ii) there exists an η and an α such that

$$|\mathbf{s}(\alpha) - \zeta_B(\alpha)| > \eta$$

for all $B \in \mathbf{R}^5$. Suppose for the moment that this claim is true. Then there exists a combination of (s, α) such that a q fraction of the voters prefers Q in a typical electorate. On the other hand, the equilibrium vote share of Q is bounded away from q for all n. Hence there is a neighborhood of (s, α) such that a mistake is made with probabilty close to one. Moreover, since $p(s)$ is continuously differentiable also $t^n(s)$ is continuously differentiable and the derivative of $t^n(s)$ is uniformly bounded for all n. Hence we can find an ϵ neighborhood (where ϵ is independent of n) of (s, α) such hat a mistake is made with probability close to one *for all* n if the state is in this neigborhood. Hence the second part of the Theorem follows.

To prove the claim consider six points $(\alpha_1, \ldots, \alpha_6)$. Define the set S as follows.

$$S = \{(s_1, \ldots, s_6) : \exists B \in \mathbf{R}^5 \text{ with } \zeta_B(\alpha_i) = s_i, i = 1, \ldots, 6\}$$

Since ζ_B is a continuous function of the parameters $(B_1, \ldots, B_5)$ it follows that S is contained in a 5-dimensional manifold. Now consider the set of functions $\mathbf{s}$ that satisfy

$$(\mathbf{s}(\alpha_1), \ldots, \mathbf{s}(\alpha_6)) \notin clS$$

Let Z denote this set. We claim that (i) Z is dense and (ii) Z is open. To see (i) suppose that $\mathbf{s} \notin Z$. Consider the ball $||\mathbf{s}' - \mathbf{s}|| < \epsilon$. Clearly the set

$$T = \{(s_1, \ldots, s_6) : s_i = s'(\alpha_i) \text{ for some } s' \text{ with } ||\mathbf{s}' - \mathbf{s}|| < \epsilon\}$$

is an open subset of $\mathbf{R^6}$. Since S is a 5-dimensional manifold there exists an $\mathbf{s}'$ with $||\mathbf{s} - \mathbf{s}'|| < \epsilon$ and $\mathbf{s}' \in Z$ which proves (i). Z is open since clS is a closed set and hence also (ii) follows. Note that for $\mathbf{s} \in Z$ we have that $\max_i |\zeta_B(\alpha_i) - \mathbf{s}(\alpha_i)| > \eta$ for some $\eta > 0$ uniformly over all choices of parameters $(B_1, \ldots, B_5)$ and hence the Theorem follows. $\square$

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