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by

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Working Paper No. 93-19
August 1993

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Macintosh
Filename: 4 Golden Rules
Last Revised: 8/5/93
Submitted to The Economic Record: 8/5/93

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I. Introduction

In a recent review, Ruttan (1992) observes that academic discourse is experiencing a "third wave" of heightened post-war sensitivity about the limits to growth inherent in the finite stocks of natural and environmental resources.[1] In these discussions, the concepts of "sustainable growth" and "sustainable development" have been suggested, reminiscent of "growth with equity" as an objective that gives appropriate weights to possibly conflicting goals. Despite its popularity, however, sustainability has largely eluded any consensus regarding a precise definition. We suggest that greater precision could be achieved by drawing from and extending the standard golden rules from the growth theory literature.

The common concern shared by sustainability proponents is that present consumption is at the expense of future generations due to its depletion of natural and environmental resource stocks. Solow (1986) has proposed that this problem may be appropriately addressed by the criterion of *intergenerational equity,* defined as the maximin level of consumption that can be sustained over all time. Under special assumptions, maximin consumption can be attained by extracting resources according to the Hotelling principle and investing the resource royalties thus derived into

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* We wish to thank the National Science Foundation for financial support and Partha Dasgupta, Joseph Stiglitz, T.N. Srinivasan, Stephen Swallow, Harvey Lapan and anonymous referees for helpful comments on a previous draft.

1 The first such episode was in the late 1940's and early 50's when the manufacturing boom turned the U.S. from a net resource exporter to a net importer. The second wave followed the boom in resource prices which followed the apparently prophetic publication of Meadows et al., *Limits to Growth.* See Goeller and Weinberg (1976), for a debunking of second wave mythology.

capital accumulation (Hartwick 1978). This rule provides a solution to the maximin consumption problem and simultaneously shows that, under the assumptions specified, at least one sustainable consumption path exists.

As discussed by Dasgupta and Heal (1979), however, the maximin welfare criterion leaves countries at the mercy of their initial capital stock. Countries that are capital poor are forever constrained to have lower levels of per capita consumption than more advanced countries that are already capital rich. In addition, the assumptions used in the derivation are quite stringent, including zero population growth in the absence of technological change, no capital depreciation, and output elasticity of capital greater than that of the resource.

Solow (1974) examines intergenerational equity and exhaustible resources under conditions of zero extraction costs. In Hartwick (1978), the rule of investing rents from exhaustible resources is derived in the case of constant, but nonzero, extraction cost. Cairns (1986) extends the Hartwick investment rule to the case where Ricardian differential rents are generated through exploitation of a non-homogeneous resource. In the Cairns model, mineral production costs increase as the quality of ore exploited decreases. Cairns shows that the basic model can be modified to incorporate the possibility that extraction costs increase with cumulative extraction.

In the face of exponential population growth and limited resources, Solow (1974) shows that "... no positive constant consumption per worker is maintainable forever." With respect to the problem of capital depreciation, Hartwick (1978) notes, "... (the) savings investment rule will not provide for the maintaining of per capita consumption constant over time. The current decline in per capita consumption is simply the amount of the produced commodity required to offset the current amount of depreciation in reproducible capital." Solow (1974), Stiglitz (1974), Dasgupta and Heal (1979), and Wan (1989) all contain demonstrations that a necessary condition for maximin consumption is that output elasticity of capital be greater than that of the finite resource.

An alternative representation of intergenerational equity involves suppressing to zero the social rate of time preference in intertemporal welfare maximization. In particular, Ramsey (1928) held that it is "ethically indefensible for society to discount future utilities" (paraphrased by Solow, 1974).[1] "We ought to act as if the social rate of time preference were zero (though we would simultaneously discount future consumption if we expect the future to be richer than the present)." [Ramsey, 1928]

In the Hartwick/Solow model, the optimal trajectory of per capita consumption with a zero social rate of time preference increases indefinitely, and there is no steady state. Both consumption and growth are sustainable. In the present paper, we explore an alternative model. Extraction costs are permitted to rise as a function of the cumulative amount of the non-renewable resource extracted, in contrast to the constant extraction cost assumption in the Hartwick/Solow model, and the output elasticity of capital is not required to exceed that of the resource. Instead, the extraction cost function is assumed to be bounded from above by a backstop technology. The backstop assumption may be thought of as an approximation which may be made arbitrarily close to the actual extraction cost function. This assumption permits the restoration of steady state results for which both golden and modified golden rules can be derived. Also, in contrast to additional restrictive assumptions needed to support maximin consumption, the model in this paper allows for both population growth and capital depreciation.

The paper is organized as follows. The model is described in section II. The modified golden rule is derived and conditions are stated for whether that rule violates a sustainability condition that constrains consumption not to fall below the maximin level. Comparative statics results are also derived showing that steady state consumption decreases as the backstop cost and rate of time preference increase. In section III, a special case is considered where the concern for the future is manifested by setting the rate of time preference to zero and solving for the golden rule. This solution does not violate the sustainability constraint. These and additional concluding remarks are summarized in section IV.

II. A Modified Golden Rule

Consider an economy that uses three inputs, capital (K), labor (L), and a natural resource (R) to produce a single homogeneous good. Assume that the production technology is constant returns to scale, so that the production function, F(K,L,R), is homogeneous of degree one. Following the standard approach (see e.g., Wan [1989]), output of production is divided among consumption, gross investment, and the cost of providing the resource as input to the production process. Let Θ be the unit cost of the natural resource, which we take to be a decreasing function of the resource stock, D(t). Capital depreciation occurs at the rate δK. Then,

$$F(K,L,R) = \dot{K} + \delta K + \Theta R + C \qquad (1)$$

In the case of oil, for example, oil stocks are drawn down as the economy grows, until the unit cost, Θ, of providing oil as an input to production has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source with unit cost, Θ_b (e.g., coal, solar energy, geothermal, or nuclear fusion). Following Nordhaus (1973, 1979) and Heal (1976), we consider such a superabundant energy source to flow from a backstop technology with constant unit cost. Capital and labor costs alone determine the price of the energy resource; i.e., scarcity rents are no longer significant. In their supply-side study of oil prices, Roumasset, Isaak, and Fesharaki (1983) conducted a sensitivity analysis showing that the assumption of an unlimited backstop technology does not introduce a substantial inaccuracy in the estimation of efficiency prices if the backstop price is set sufficiently high and if total resources available at and below the backstop price are abundant. The test of this condition is that the efficiency prices in the present and projection period of interest (e.g., 50-100 years) are not sensitive to changes in the backstop price of the ultimate resource. On this basis, we take the incorporation of the backstop technology as an empirically sound approach to resource modeling. Moreover, the point at which the extraction cost reaches its ultimate backstop plateau may be made arbitrarily remote. For example, if we take the exhaustible resource to be an all-purpose energy resource, measured e.g. in oil-barrel equivalents, intermediate plateaus can be taken as various grades of oil, coal, nuclear fission, nuclear fusion, etc. In such an application, extraction cost would include conversion costs of the oil-alternative technology as well as the extraction and production costs of the energy source (see, e.g. Nordhaus, 1979). In this way, the quantity of the aggregate resource at which the backstop substitute comes in can be arbitrarily high, limited only by limits on information and estimating capability.

As a related matter, we briefly comment on the issue of optimal sequencing of resource extraction with rising unit extraction costs. Solow and Wan (1976) considered a simple two-period, two-deposit model of an exhaustible resource and showed that it is preferable to fully exploit the low cost deposit first before extracting from the high cost deposit in period one, while deferring some extraction program. Subsequently, Kemp and Long

(1980) developed a more general model wherein it may be preferable to exploit high and low resource deposits simultaneously for the purpose of smoothing consumption over time. Commenting on the work by Kemp and Long, Lewis (1982) derived sufficient conditions under which strict sequencing of extraction (from low cost to high cost) becomes optimal, consistent with the model of Solow and Wan: extracted resources can be converted into productive capital.

A different problem arises in the case of physical mining constraints which prevent optimal sequencing of resource extraction per Solow and Wan (1976). Hartwick (1978) and Cairns (1986) examined situations where sources of varying quality must be exploited at a single time. While yielding important insights related to the generation of Ricardian rents, the Hartwick/Cairns model would add little to our consideration of the steady state golden rules made possible by the existence of a backstop technology, beyond the modifications of trajectories leading to the steady state. We therefore adhere to the simpler model of extraction costs rising with cumulative extraction.

The basic model in the present paper, as represented by dynamic equation (1), is consistent with the sufficient condition established by Lewis (1982). We therefore assume that the backstop resource will not be used until unit extraction cost, Θ, rises to the backstop cost, Θ_b.

Until the transition to the backstop resource, there is a finite constraint on the resource stock:

$$\int_0^{\infty} R(t)dt \leq D_0 < \infty. \tag{2}$$

Assume that the labor force grows at rate n from an initial level of $L(0) = L_0$. Given the homogeneity of F(K,L,R), labor can be factored out to yield a production function of the form, f(k,r), where k is the capital to labor ratio, and r is the resource to labor ratio. The dynamic equation of growth now becomes

$$\dot{k} = f(k,r) - \mu k - \Theta r - c, \qquad \Theta \leq \Theta_b. \tag{3}$$

Here, c is per capita consumption, $\mu = n + \delta$, and Θ_b is the constant unit cost of supplying the backstop resource.

A modified golden rule can be obtained when we include time preference in the model and maximize the conventional measure of social welfare. With ρ as the rate of time preference and U(c) as the utility of consumption of the representative agent, the planner solves

$$\max_{\substack{c>0 \\ r>0}} W = \int_0^\infty \exp(-\rho t)\, U(c)dt \tag{4}$$

$$\text{s.t.} \quad \dot{k} = f(k,r) - \mu k - \Theta r - c \tag{4a}$$

$$\dot{D} = -rL, \qquad \Theta \leq \Theta_b \tag{4b}$$

$$k(0) = k_0, \qquad D(0) = D_0. \tag{4c}$$

At some endogenous time, T, the unit cost, Θ, of the exhaustible resource reaches the backstop cost, Θ_b, and a transition is made to the substitute resource.[2] Because of the inequality constraint on θ, the Hamiltonian, H, must be augmented to form the Lagrangian function, L:

$$L = H + \gamma[\Theta_b - \Theta],$$

where $H = \exp(-\rho t)U(c) + \lambda[f(k,r) - \mu k - \Theta r - c] - \psi[rL]$. (5)

The complementary slackness condition associated with the inequality constraint is

$$\gamma \bullet \frac{\partial L}{\partial \gamma} = \gamma[\Theta_b - \Theta] = 0$$

Standard application of the maximum principle yields the following efficiency conditions:

$$\frac{\dot{f}_r}{f_r - \Theta} = f_k - \delta, \text{ for } 0 < t < T \tag{6}$$

$$f_r = \Theta_b, \text{ for } t \geq T \tag{6'}$$

$$\frac{-\dot{U}'(c)}{U'(c)} = f_k - (\mu + \rho). \tag{7}$$

Equation (6) is essentially Hotelling's rule in a general equilibrium context.[3] Equation (7), the Ramsey rule, can be simplified by introducing the consumption elasticity of marginal utility,

$$\eta(c) = -c\,\frac{U''(c)}{U'(c)}. \tag{8}$$

Using this definition, equation (7) becomes

$$\eta(c)\,\frac{\dot{c}}{c} = f_k - (\mu + \rho). \tag{9}$$

For a production technology, F, that is constant returns to scale in all inputs, the only possible steady state growth rate is zero (see e.g., Sala-i-Martin [1990]). Therefore, along the steady state path, $\frac{\dot{c}}{c} = 0$, and

$$f_k = (\mu + \rho).$$

Two conditions now define the modified golden rule for growth and capital accumulation when a backstop natural resource, essential to production is available in infinite supply:

$$f_k = (\mu + \rho) \text{ and } f_r = \Theta_b, \qquad t \geq T. \tag{10}$$

There is no presumption in this formulation that the backstop and the steady state are reached simultaneously.

Efficient evolution of the economy toward the modified golden rule steady state growth path is governed by equations (6) and (7). In particular, resource extraction should be governed by Hotelling's rule. Overuse of the resource, counter to this rule, would be inefficient in both the short run and the long run. But, just as important, underuse would be inefficient as well.

We now consider conditions under which the modified golden rule is consistent with the notion of sustainable development. The approach we propose relies on comparison of consumption trajectories rather than capital stocks. If one accepts the premise that consumption is the ultimate goal of economic activity, then this approach goes to the heart of the matter. We may define a consumption trajectory to be *sustainable relative to* c_{min} iff there exists some T such that $c_t \geq c_{min}$, $\forall$ $t \geq T$. In particular, it is interesting to consider the case where $c_{min} \equiv \bar{c}$, the maximin constant level of consumption that could be attained in the absence of a backstop, since $\bar{c}$ has been previously singled out as a possible benchmark for intergenerational equity (Hartwick, 1977; Solow, 1986). Other benchmark levels of c_{min} are possible, such as some agreed upon subsistence level. However, the specification of subsistence level may be somewhat arbitrary.

The computation of $\bar{c}$ can be formulated as a standard optimization problem (see Wan [1989]):

$$\bar{c} = \max_{r \geq 0} c_0 \tag{11}$$

$$\text{s.t. } \dot{c} = 0 \text{ and (4a), (4b), and (4c).}$$

The solution to this problem, if it exists, yields

$$\bar{c} = \bar{c}\,(k_0, D_0). \tag{12}$$

The dependence of $\bar{c}$ on k_0 and D_0 reinforces the idea that economies constrained to maximin per capita consumption are at the mercy of initial conditions. Solow (1974), Dasgupta and Heal (1979), and Wan (1989) derive solutions to versions of problem (11), showing the specific dependence of per capita consumption, $\bar{c}$, on the initial capital stock, k_0, and resource stock, D_0.

In contrast, we advance the following proposition concerning the modified golden rule steady state.

Proposition 1: Given an economy governed by dynamic equation (3), let c^* be the steady state modified golden rule level of per-capita consumption associated with condition (10). Then c^* is independent of the initial per capita capital stock, k_0, and resource stock, D_0.

Proof: The modified golden rule (10) defines the steady state marginal products f_k and f_r, which in turn determine the well-defined steady state values k^* and r^*. Consumption level, c^*, can then be computed as $f(k^*,r^*) - \mu k^* - \Theta_b r^*$.

As noted by Wan (1989), there is no guarantee that a solution exists for production functions other than Cobb-Douglas functions of the form, $f(k,r) = Ak^a r^b$. Even in this case, a necessary condition for existence of a maximin solution is that $b>a$. Moreover, both population growth and capital depreciation must be zero, so that $\mu = 0$. The behavior of extraction costs, as a function of the resource stock, D, may also affect the existence of a maximin solution.

The relationship between c^* and $\bar{c}$ (assuming $\bar{c}$ exists) will depend on the rate of time preference, ρ, and on the backstop cost, Θ_b. The following proposition verifies that, as expected, the steady state level of per capita consumption, c^*, declines with an increase in ρ and with an increase in Θ_b.

Proposition 2: Under the assumptions of proposition 1, let ρ be the rate of time preference and Θ_b be the unit cost of the backstop resource. Then i) $\frac{\partial c^*}{\partial \rho} < 0$, and ii) $\frac{\partial c^*}{\partial \Theta_b} < 0$.

<u>Proof</u>: i) In the steady state $\dot{k} = 0$, so that

$$c^* = f(k^*,r^*) - \mu k^* - \Theta_b r^*. \qquad (13)$$

Differentiating with respect to ρ we obtain

$$\frac{\partial c^*}{\partial \rho} = f_k \frac{\partial k^*}{\partial \rho} + f_r \frac{\partial r^*}{\partial \rho} - \mu \frac{\partial k^*}{\partial \rho} - \Theta_b \frac{\partial r^*}{\partial \rho}$$

$$= \left[f_k - \mu\right] \frac{\partial k^*}{\partial \rho} + \left[f_r - \Theta_b\right] \frac{\partial r^*}{\partial \rho}. \quad (14)$$

Using the modified golden rule, condition (10), we can rewrite equation (14) as

$$\frac{\partial c^*}{\partial \rho} = \rho \frac{\partial k^*}{\partial \rho} \quad (15)$$

The partial derivative $\frac{\partial k^*}{\partial \rho}$ can be signed by differentiating the two modified golden rule conditions:

$$f_{kk} \frac{\partial k^*}{\partial \rho} + f_{kr} \frac{\partial r^*}{\partial \rho} = 1, \quad (16)$$

$$f_{kr} \frac{\partial k^*}{\partial \rho} + f_{rr} \frac{\partial r^*}{\partial \rho} = 0.$$

Application of Cramer's rule yields

$$\frac{\partial k^*}{\partial \rho} = \frac{f_{rr}}{f_{kk}f_{rr} - (f_{rk})^2}. \quad (17)$$

By assumption, f(k,r) is concave (diminishing returns to scale), so that $f_{rr} < 0$ and $f_{kk}f_{rr} - (f_{rk})^2 > 0$. Hence for $\rho > 0$,

$$\frac{\partial c^*}{\partial \rho} = \rho \frac{\partial k^*}{\partial \rho} < 0.$$

ii) Differentiate equation (13) with respect to Θ_b:

$$\frac{\partial c^*}{\partial \Theta_b} = f_k \frac{\partial k^*}{\partial \Theta_b} + f_r \frac{\partial r^*}{\partial \Theta_b} - \mu \frac{\partial k^*}{\partial \Theta_b} - \Theta_b \frac{\partial r^*}{\partial \Theta_b} - r^*$$

$$= \left[f_k - \mu\right] \frac{\partial k^*}{\partial \Theta_b} + \left[f_r - \Theta_b\right] \frac{\partial r^*}{\partial \Theta_b} - r^*. \quad (18)$$

With the modified golden rule, equation (18) can be written

$$\frac{\partial c^*}{\partial \Theta_b} = \rho \frac{\partial k^*}{\partial \Theta_b} - r^*. \quad (19)$$

To sign $\frac{\partial k^*}{\partial \Theta_b}$ we differentiate the two modified golden rule conditions with respect to Θ_b:

$$f_{kk} \frac{\partial k^*}{\partial \Theta_b} + f_{kr} \frac{\partial r^*}{\partial \Theta_b} = 0, \quad (20)$$

$$f_{kr} \frac{\partial k^*}{\partial \Theta_b} + f_{rr} \frac{\partial r^*}{\partial \Theta_b} = 1.$$

By Cramer's rule and concavity of f(k,r),

$$\frac{\partial k^*}{\partial \Theta_b} = \frac{-f_{kr}}{f_{kk}f_{rr} - (f_{rk})^2} < 0. \qquad (21)$$

Therefore, $\dfrac{\partial c^*}{\partial \Theta_b} = \rho \dfrac{\partial k^*}{\partial \Theta_b} - r^* < 0.$

Assuming the maximin solution, $\bar{c}$, exists, it is reasonable to suspect that it will depend on backstop unit cost, Θ_b, as well as k_0 and D_0, if Θ_b serves as an upper bound to rising unit cost, Θ. We conjecture that when $\bar{c}$ does depend on Θ_b, $\dfrac{\partial \bar{c}}{\partial \Theta_b} < 0$. If the upper bound on rising unit resource cost increases, resource royalties eventually decrease, so that, under the Hartwick savings rule, investment in capital accumulation is not as great as it otherwise would have been. The constant level of per capita consumption, $\bar{c}$, that can be sustained at the reduced rate of capital accumulative must decline.

Figure 1 provides a schematic of the possible relationships between c^* and $\bar{c}$ for different values and ρ and Θ_b. For a fixed value of Θ_b, $c^* = c^*(\rho,\Theta_b)$ is a declining function of ρ. The maximin level of consumption, $\bar{c}(\Theta_b)$, then determines the maximum rate of time preference, $\bar{\rho}(\Theta_b)$, consistent with sustainable consumption. As illustrated in Figure 1, the defining relationship is

$$c^*\left(\bar{\rho}(\Theta_b),\Theta_b\right) = \bar{c}(\Theta_b). \qquad (22)$$

If $\rho \le \bar{\rho}$ for a given backstop unit cost, Θ_b, then the modified golden rule path of per capita consumption, c^*, will satisfy sustainability relative to $\bar{c}$. The case, $\rho > \bar{\rho}$ (i.e., sustainability is not satisfied), leaves the planner with at least two options. The planner could reduce the social rate of time preference to $\bar{\rho}$ or less. (Section III considers the case where ρ=0, yielding a golden rule). Alternatively, the planner could incorporate directly into problem (4) the constraint $c(t) \ge \bar{c}$ for all $t \ge t_0$, while retaining the prevailing rate of time preference, ρ. This forces the consumption trajectory to eventually dominate

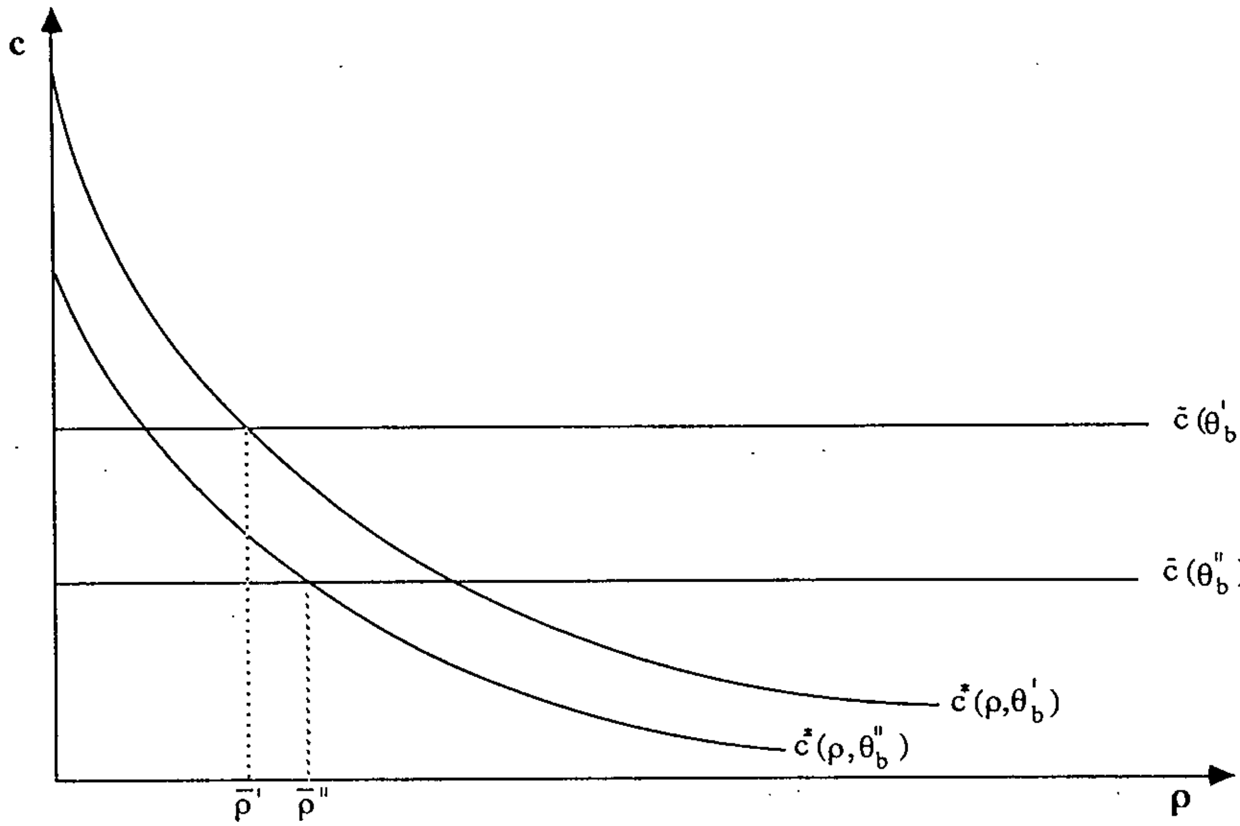

Figure 1. Steady State Consumption and the Rate of Time Preference

the maximin consumption path. The planner may choose $t_0>0$ so as initially to allow $c < \bar{c}$, thereby building up the capital stock to sustain greater consumption in the future.

In Figure 2, we sketch plausible trajectories of per capita consumption leading to modified golden rule growth paths. These trajectories are analogous to those depicted in Diagram 10.3 of Dasgupta and Heal (1979), with the addition of a backstop substitute. With a Cobb-Douglas production function, Dasgupta and Heal (1974, 1979) showed that the consumption trajectory, for the case of an exhaustible resource, will have at most one peak. Moreover, the lower the rate of time preference, ρ, the further in the future will be the peak. Trajectory 2, in Figure 2, satisfies the condition of sustainability relative to maximin consumption, while trajectory 1 does not.

III. Intergenerational Equity and Time Preference: A Golden Rule

A central critique of ecologists and ecological economists to maximizing aggregate discounted welfare is that discounting necessarily prejudices the case against future generations. Some authors have suggested (see e.g., Pearce and Turner [1990]) that the present generation is properly viewed as a steward for the future. Setting $\rho = 0$ in the model introduced above is one way of representing these concerns. Setting the social rate of time preference equal to zero also provides an alternative criterion for intergenerational equity. Instead of requiring consumption in all periods to be equal, this approach simply removes any *a priori* discrimination between generations.

In general, however, maximizing undiscounted aggregate welfare presents a technical problem that was recognized by Ramsey in his classic paper on optimal savings (see Ramsey [1928]). With $\rho = 0$, the integral in equation (4) becomes infinite, so that there is no way to discriminate among

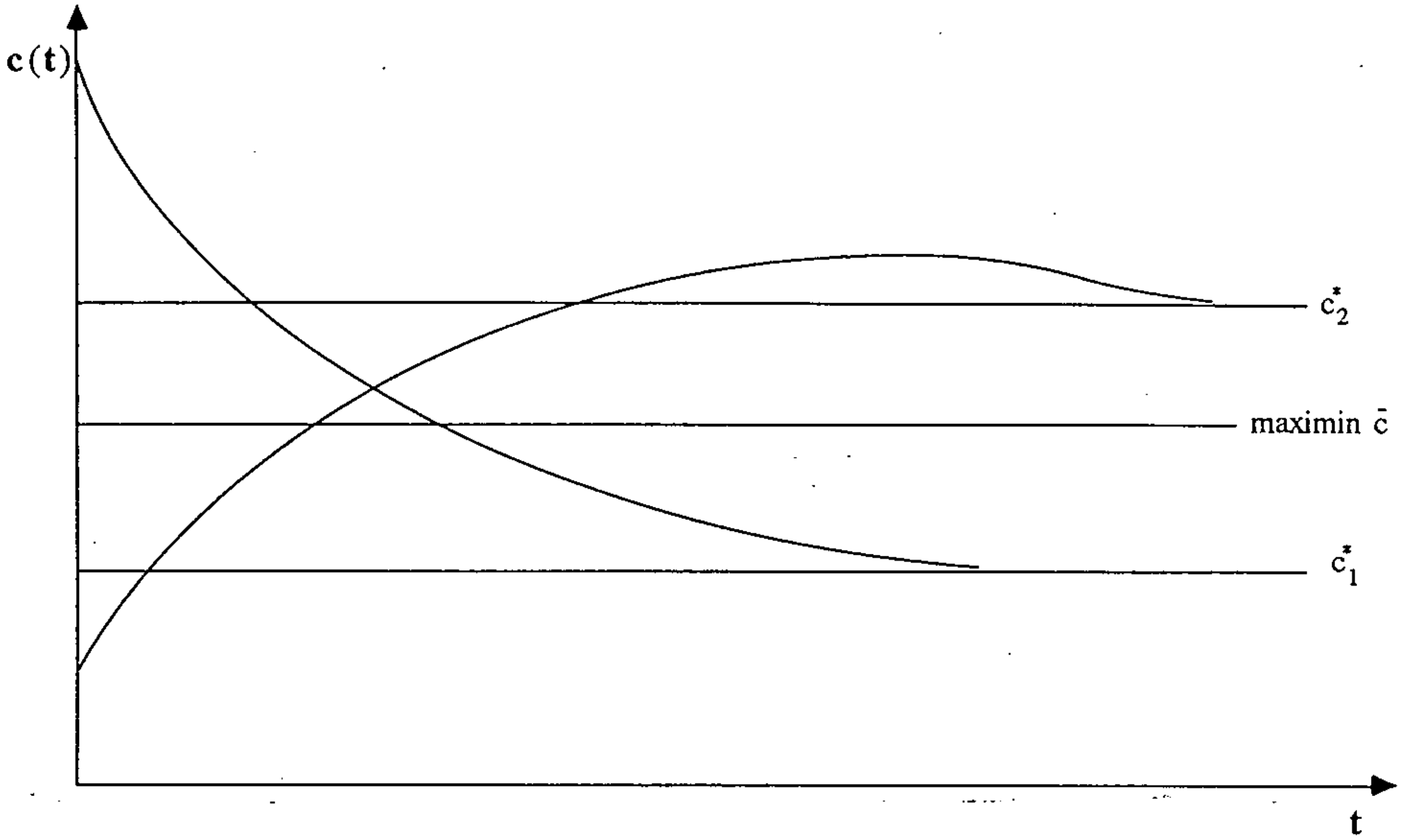

Figure 2. Plausible Consumption Trajectories Leading to the Modified Golden Rule

alternative paths of sustainable consumption; all such paths produce infinite social welfare. Ramsey cleverly tackled this problem by incorporating into the integrand a postulated bliss point. Koopmans (1965) refined this approach by showing that the golden rule path for capital accumulation could be taken as the Ramsey bliss point, i.e., the comparison path against which to measure utility as the economy evolves over an infinite time horizon.

In the resource context, an analogous bliss point can be derived from the dynamic equation of growth, equation (3). Once the shift to the backstop has been made, the strict concavity of f(k,r), and the conditions,

$$f_k(0,0) = \infty,\ f_r(0,0) = \infty,\ \lim_{k \to \infty} = 0,\ \lim_{r \to \infty} = 0, \tag{23}$$

will be sufficient to guarantee the existence of a steady state for which $\dot{k} = 0$. Equation (3) then becomes

$$c = f(k,r) - \mu k - \Theta_b r. \tag{24}$$

The golden rule is now obtained as the set of first order conditions for maximizing steady state per capita consumption. The symbol "ˆ" designates golden rule levels:

$$f_k(\hat{k}, \hat{r}) = \mu, \qquad f_r(\hat{k}, \hat{r}) = \Theta_b. \tag{25}$$

Golden rule levels of per capital consumption and utility are then given by

$$\hat{c} = f(\hat{k}, \hat{r}) - \mu \hat{k} - \Theta_b \hat{r} \quad \text{and} \quad \hat{U} = U(\hat{c}). \tag{26}$$

The welfare criterion for this problem can be written as

$$J = \int_0^\infty [U(c) - \hat{U}]dt, \tag{27}$$

which is bounded above when both the production technology and the utility function are concave (see Burmeister and Dobell [1970]). The criterion J, therefore, has a maximum, and the first order conditions show that this maximum will be attained along the growth path of per capita consumption governed by the equation,

$$\eta(c)\,\frac{\dot{c}}{c} = f_k - \mu. \tag{28}$$

Moreover, the dynamic equations of the marginal product, f_r, for this problem are identical to equation (6) and (6'). Burmeister and Dobell (1970) show that, in general, maximizing the welfare criterion, J, is equivalent to maximizing discounted social welfare with time preference, ρ (such as solving the problem represented by equation (4)), and then letting ρ tend to zero. The golden rule path for capital accumulation and resource management can therefore be defined by the conditions,

$$f_k = \mu \text{ and } f_r = \Theta_b. \tag{29}$$

We may now compare the golden rule to the maximin rule as alternative standards of intergenerational equity. One plausible scenario is illustrated in Figure 3. By definition, the golden rule path yields the maximum possible level of steady state consumption per capita. Hence, maximin justice necessarily implies a level of constant per capita consumption less than or equal to that rendered by the golden rule steady state. As the figure shows, this may result in large and sustained (and therefore infinite) losses in the future in order to raise consumption in the present and near future by small increments.

IV. Concluding Remarks

We believe that the results above help to illuminate the nature of sustainable consumption. Relative sustainability is linked to the idea of eventually meeting or exceeding the maximin level of per capita consumption, $\bar{c}$, or some other level of per capita consumption, $\tilde{c} < \bar{c}$, chosen by the planner.

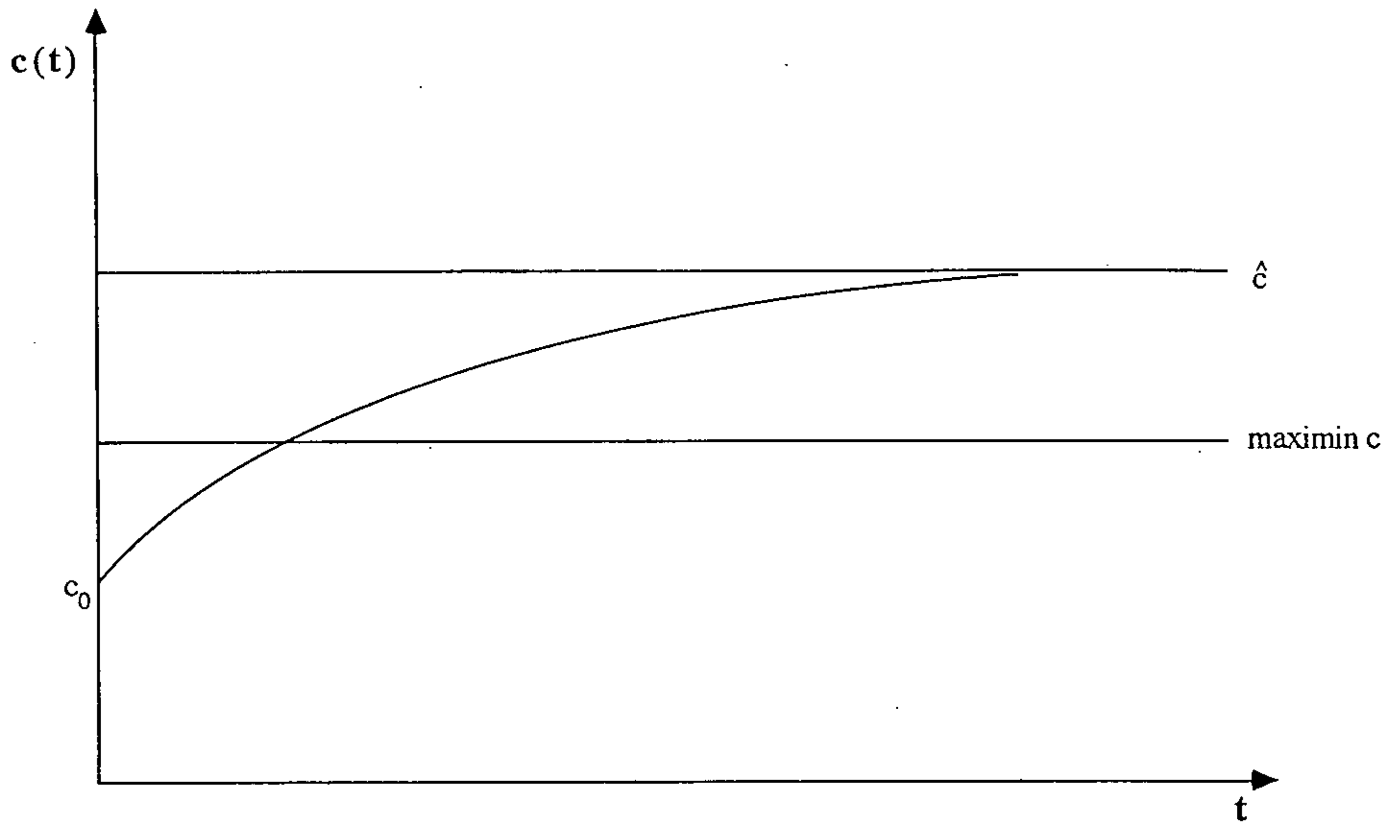

Figure 3. Golden Rule Turnpike vs. Maximin Path as Alternative Trajectories of Sustainable Consumption

The existence of a superabundant backstop natural resource admits the possibility of deriving golden rules that govern efficiency in both capital accumulation and resource management. These golden rules can be conveniently related to the issue of sustainability through comparative analysis of per capita consumption trajectories. While the notion of a backstop technology as a basis for resource management remains controversial, we submit that the existence *per se* of a backstop is not the critical issue. Virtually limitless and renewable resource substitutes do exist (e.g. solar energy). The critical issue is at what unit costs can they be made effective substitutes for non-renewables.

An important implication of the analysis in this paper is that, given a backstop technology, the conditions for existence of the modified golden rule are less stringent than conditions for the existence of a maximin consumption path. In particular, existence of the modified golden rule does not require that the elasticity of output with respect to man-made capital exceed the elasticity with respect to natural resources. Moreover, population growth and capital depreciation are both allowable.

The modified golden rule yields a steady state level of per capita consumption, c^*, that depends on the prevailing social rate of time preference, ρ, and the unit cost of the backstop resource, Θ_b but not on initial capital and resource stocks. This result is in stark contrast with maximin constant per capita consumption (when it exists) which is at the mercy of initial conditions. The consumption level, c^*, declines with an increase in ρ. For a given backstop unit cost, Θ_b, there is a maximum social rate of time preference, below which the modified golden rule yields a level of per capita consumption that exceeds the maximin level. Thus for $\rho \leq \bar{\rho}$, the modified golden rule satisfies sustainability relative to maximin consumption. The golden rule is a special case of the modified golden rule where the social rate of time preference is set to zero. The golden rule also solves the problem of finding than consumption trajectory which is sustainable relative to the highest possible c_{min}. That is, the golden rule gives the *eventually sustainable* maximin.

An objection might be raised to the strong dependence of the modified golden rule and golden rule levels of consumption, c^*, on the backstop unit cost, Θ_b. A high backstop cost might threaten the dominance of c^* (modified golden rule) or $\hat{c}$ (golden rule) over the maximin level $\bar{c}$. We suggest, however, that the backstop unit cost, Θ_b, serves as an upper bound on rising extraction costs for the primary resource, so that a high Θ_b will depress $\bar{c}$ as well as c^* or $\hat{c}$.

Moreover, the model could be readily extended to allow for a composite resource along the lines of Nordhaus (1979). This would require specifying the extraction costs of oil, coal, natural gas, uranium and other resources and the conversion costs of each and of solar radiation into usable energy. In this way a minimum cost schedule can be calculated that extends thousands of years into the future. Even though this extraction/conversion cost function may also be unbounded (e.g. even photovoltaic cells would face a rising rental cost for the space they occupy), it can always be approximated by a rising extraction/conversion cost function and an arbitrarily high backstop price. Thus the critical issue is not whether a backstop technology exists, but how high and how far into the future the analyst calculates the rising function.

As a final remark, we recommend that policy discussions not put undue emphasis on results arising from long run steady state conditions. Despite the existence of golden rules under the assumptions set forth in this paper, the concern of most relevance to policy is what happens in the interim on the way to the steady state. In the final analysis, it is the trajectory to the steady state, rather than long run sustainability, that captures concern for the future.

Endnotes

1. As cited in the same paper (1974), Solow notes his ambivalence toward the maximin rule.

2. Equivalent formulations of the problem expressed by (4) are possible. Heal (1976) tackles a similar problem by solving two separate problems and piecing the resulting solution together. In the notation of the present paper, the two problems are:

$$\text{i)}\quad \max \int_0^\infty \exp(-\rho t)u(c)dt$$

$$s.t.\quad \dot{k} = f(k,r) - uk - \Theta_b r - c$$

$$D = -rL, \qquad \Theta \leq \Theta_b \ ,$$

$$\text{ii)}\quad \max \int_0^\infty \exp(-\rho t)u(c)dt$$

$$s.t.\quad \dot{k} = f(k,r) - uk - \Theta_b r - c\,.$$

Heal shows that any optimal path must link the two solutions together.

Alternatively, let r_1 denote per capita use (at time t) of the primary natural resource, and let r_b denote per capita use of the backstop resource (at time t). Then assuming that the two natural resources are prefect substitutes, the problem may be written,

$$\max \int_0^\infty \exp(-\rho t)u(c)dt$$

$$s.t.\quad \dot{k} = f(k,r_1 + r_b) - uk - \Theta r_1 \Theta_b r_b - c$$

$$\dot{D} = -r_1 L, \qquad \Theta \leq \Theta_b\,.$$

3. Given that the unit cost, Θ, is a function of the resource stock, D, one might expect that the derivative of Θ with respect to D would appear in necessary condition (6). The appearance of such a derivative would, in fact, occur in the case of a more general total cost function, $\tilde{\Theta}(r,D)$, depending on both the resource extraction rate, r, and resource stock, D. In this more general case, covered by Fisher (1981), pp. 28-33, the necessary condition becomes,

$$\dot{f} = (f_b - \partial)(f_r - \tilde{\Theta}_r) + \tilde{\Theta}_D - r\tilde{\Theta}_{Dr}$$

When the cost function, $\tilde{\Theta}$, can be written less generally as

$$\tilde{\Theta}(r, D) = r\Theta(D),$$

the last two terms on the right hand side cancel each other; i.e., $\tilde{\Theta}_D = r\tilde{\Theta}_{Dr}$. The remaining expression is equivalent to (6).

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