Mankiw's Puzzle on Consumer Durables: A Misspecification

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Abstract

Mankiw (1982) shows that consumer durables expenditures should follow a linear ARMA(1,1) process, but the data analyzed supports an AR(1) process instead; thus, a puzzle. In this paper, we employ a more general utility function than Mankiw's quadratic one. Further, the disturbance and depreciation rate are respecified, respectively, as multiplicative and stochastic. The analytical consequence is a nonlinear ARMA(∞,1) process, which implies that the linear ARMA(1,1) is a misspecification. A historical data analysis appears to support the nonlinear model. Since actual data are influenced by historical events, we also carry out a Monte Carlo study to strengthen our point.

Keywords: Utility function, multiplicative disturbance, nonlinear ARMA(∞,1) process, stochastic depreciation, misspecification error

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In his seminal paper, Hall (1978) posits that consumption follows a random walk. Testing his hypothesis using quarterly data on the U.S. consumer nondurables and services expenditures for the period 1948.1-1977.1, he finds that the data support a slight variant of his theory, which permits a brief lag between changes in permanent income and consumption. This implies that once the consumption in a period is controlled for, no other information in that period helps forecast future consumption. The Keynesian economists have challenged Hall's hypothesis by showing empirically that current consumption depends partly on current income in addition to past consumption.

Mankiw (1982) applies Hall's theory to consumer durables expenditures, assuming a quadratic utility function and an additive error term. He shows that if Hall's theory holds, consumer durables expenditures should follow an ARMA(1,1) process, which is reduced to an AR(1) process for nondurables expenditures.[1] Contrary to the theoretical expectation, however, the US quarterly data on consumer durables expenditures support an AR(1) process instead of an ARMA(1,1). It appears that depreciation rates play no role in determining consumer expenditures. This is against both Hall's theory and common intuition. Mankiw attributes this puzzling result to the misspecification of the utility function.

[1] Winder and Palm (1996) and Romer (2001), in their respective contexts, also show that consumer durables expenditures do not follow a random walk.

Based on a Taylor expansion, Mankiw (1985) derives a log linear equation, which relates consumer durables to consumer nondurables. He points out that the log of consumer nondurables follows a random walk, as shown by Hansen and Singleton (1983). This implies that the log of consumer durables also follows a random walk. However, he defines consumer durables as net stock of durables at year-end instead of quarterly expenditures, which is a flow variable. Additionally, this approach is a multivariate alternative with lagged consumption and interest rate as independent variables. Hence, it does not resolve Mankiw's puzzle as observed in the univariate context.

In a major attempt to resolve Mankiw's puzzle, Caballero (1990) hypothesizes that the underlying reason for this puzzle lies in consumers' slow adjustment of their durables expenditures. For empirical analysis, Caballero expands the ARMA(1,1) to ARMA(1,5) and ARMA(1,8) processes to accommodate this slow adjustment. He finds that the sum of moving average effects is statistically significant, although all individual moving average effects are insignificant. However, there is no theory which justifies that insignificant moving average effects in an ARMA(1,5) or ARMA(1,8) combined is equivalent to a significant moving average effect in an ARMA(1,1) model. Therefore, Mankiw's puzzle in earnest still remains unresolved .

In this paper, we attempt to explain Mankiw's puzzle in a way fundamentally different from the previous papers. Our approach starts from respecifying the quadratic utility function and the additive error term in Mankiw (1982). Further, the depreciation rate is assumed to be stochastic over time. Consequently, consumer durables expenditures follow a nonlinear ARMA(∞,1). This implies that the linear ARMA (1,1) is a misspecification. A historical data analysis supports the nonlinear ARMA(∞,1) process. Since actual data are influenced by historical events and an unbiased estimation of this nonlinear model may not be simple, we also carry out a Monte Carlo study to strengthen our point.

In Section I we derive the theoretical model, and in Section II, we show analytically that the misspecification results in a bias. In Section III, we present empirical and Monte Carlo evidence in support of our theoretical model. Section IV shows evidence of Mankiw's model misspecification using historical data and generated data. Section V concludes the paper.

I. The Theory

Mankiw (1982) extends Hall's (1978) utility maximization problem to the case of consumer durables. He applies Hall's first order condition, the renowned Euler equation, to the stock of durables, K_t:

(1) $$E_t U'(K_{t+1}) = \lambda U'(K_t), \text{ where } \lambda = (1+\rho)/(1+r),$$

ρ and r are the rate of subjective time preference and real rate of interest, respectively, both assumed constant over time. E_t is the expectation conditional on all information available in period t; $U(\cdot)$ is the instantaneous utility function, which is strictly concave. He writes the stochastic counterpart of Equation (1) as:

(2) $$U'(K_{t+1}) = \lambda U'(K_t) + u_{t+1}.$$

Assuming that the utility function is quadratic, he shows that:

(3) $$K_{t+1} = a_0 + a_1 K_t + u_{t+1},$$

where a_o and a_1 are constants.

With the following identity incorporated into Equation (3),

(4) $K_{t+1} \equiv (1-\delta) K_t + C_{t+1}$, where δ is the depreciation rate,

consumer durables expenditures, C_t, is derived as an ARMA(1,1) process:

(5) $$C_{t+1} = a_o + a_1 C_t + u_{t+1} + (1-\delta) u_t.$$

In our analysis, there are three major respecifications. First, we respecify the utility function as:

(6) $$U(K_t) = \xi + \alpha K_t^{\theta}.$$[2]

Second, we respecify the error term u_t in Equation (2) as multiplicative[3] so that:

(7) $$U'(K_{t+1}) = \lambda U'(K_t) u_{t+1},$$

where

(8) $$u_{t+1} = e^{\left(\varepsilon_{t+1} - \frac{\sigma_\varepsilon^2}{2}\right)} = e^{-\frac{\sigma_\varepsilon^2}{2}} e^{\varepsilon_{t+1}} \text{ with } \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2).$$[4]

Third, we respecify the depreciation rate to be stochastic around a deterministic depreciation rate:

(9) $$\delta_t = \delta + e_t, \; e_t \sim N(0, \sigma_e^2).$$

Substituting the derivatives of Equation (6) for t and t+1 into Equation (7) and using Equation (8), we obtain a stochastic model in a multiplicative form:

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[2] This functional form covers all four utility functions frequently used in macroeconomics.

[3] Rational expectations theory requires an additive disturbance. However, if a log linear form is more appropriate than a linear one for consumption, then the error term is additive in log form. For empirical evidence that the log linear form is a more likely one, see Vu (2005,a).

[4] Note that $E(u_{t+1}) = 1$. Hence, taking expectation of Equation (7) reverts it to the optimization condition

(10) $$K_{t+1} = \psi K_t e^{\nu_{t+1}}\text{, where } \psi = \left(\lambda e^{-\frac{\sigma_\varepsilon^2}{2}}\right)^{\frac{1}{\theta-1}}; \quad \nu_{t+1} = \frac{\varepsilon_{t+1}}{\theta-1}.$$

Replacing the deterministic depreciation rate in Equation (4) with the one in Equation (9), we obtain:

(11) $$K_{t+1} = \left[1-\left(1-\delta_t\right)B\right]^{-1} C_{t+1},$$

where B is the backshift operator.

Reflecting Equation (11) on Equation (10), we obtain:

(12) $$\left[1-k_t B\right]^{-1} C_{t+1} = \psi\left\{\left[1-k_{t-1}B\right]^{-1} C_t\right\} e^{\nu_{t+1}}.$$

where $k_t \equiv 1-\delta_t$.

Premultiplying $\left(1-k_t B\right)$ to both sides of Equation (12) gives:

(13) $$C_{t+1} = \psi\left\{\left(1-k_{t-1}B\right)^{-1} C_t\right\} e^{\nu_{t+1}} - k_t\psi\left\{B\left[\left(1-k_{t-1}B\right)^{-1} C_t\right]\right\} Be^{\nu_{t+1}}.$$

in Equations (1).

Expanding $\left(1-k_t B\right)^{-1}$ within the brackets yields:[5]

(14) $$C_{t+1}=\psi C_t e^{v_{t+1}}\left\{1+\sum_{i=1}^{\infty}\left[\left(k_{t-1}^{i}-k_t k_{t-2}^{i-1}e^{v_t-v_{t+1}}\right)\frac{C_{t-i}}{C_t}\right]\right\}$$

$$\equiv \psi C_t e^{v_{t+1}} Z_{t+1},$$

where

(15) $$Z_{t+1}=1+\sum_{i=1}^{\infty}\left[\left(k_{t-1}^{i}-k_t k_{t-2}^{i-1}e^{v_t-v_{t+1}}\right)\frac{C_{t-i}}{C_t}\right].$$

Taking the logarithms of Equation (14) yields:

(16) $\ln C_{t+1}=Z_{t+1}+(\phi+\ln C_t+v_{t+1})$, where $\phi=\ln\psi$ (a constant).

Since we are assuming that the depreciation rate is stochastic, $\delta_t=\delta+e_t$, from Equation (15), $\ln Z_{t+1}\neq 0$ for $\forall\ \delta\in\left(0,1\right]$; and so, $\ln C_{t+1}$ has a complex nonlinear ARMA process that may be described appropriately as a nonlinear ARMA(∞,1). Thus, specifying $\ln C_{t+1}$ as ARMA(1,1) will be a misspecification for both durables and nondurables.

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[5] See Appendix A

As a special case of Equation (15), suppose $\sigma_e^2 = 0$, i.e., the depreciation rate is deterministic, then $\delta_t = \delta \in (0,1]$ in Equation (15). If $\delta = 1$ as in Hall (1978), then $\ln Z_{t+1} = 0$, and nondurables expenditures follow an AR(1) process. However, if $\delta_t = \delta \in (0,1)$ as in Mankiw (1982), then $\ln Z_{t+1} \neq 0$, and durables expenditures follow a nonlinear ARMA(∞,1) process, even for $\sigma_e^2 \in H(0)$, where H is the neighborhood region. Hence, specifying ARMA(1,1) for durables expenditures will be a misspecification.

II. Specification Bias

Rewriting Equation (16) in vectors for notational economy:

(17) $y = \phi + w + x + u$,

where the vectors are of order $T \times 1$, defined as:

$$y = (\ln C_2, \ln C_3, ..., \ln C_{T+1})',$$

$$w = \left(\ln Z_2, \ln Z_3, ..., \ln Z_{T+1}\right)',$$

$$x = (\ln C_1, \ln C_2, ..., \ln C_T)'$$

$$u = (v_2, v_3, ..., v_{T+1})'$$

$l = (1,1,\ldots,1)'$.

If we misspecify the model in Equation (17) as:

(18) $$y \equiv \beta_1 l + \beta_2 x + u ,$$

then the OLS estimates of $\beta_2 (=1)$ can be written as:

(19) $$\begin{aligned} \hat{\beta}_2 &= \left(x'Mx\right)^{-1} x'My \\ &= \left(x'Mx\right)^{-1} x'M(\beta_1 l + w + \beta_2 x + u) \\ &= \left(x'Mx\right)^{-1} x'Mw + \beta_2 + \left(x'Mx\right)^{-1} x'Mu , \end{aligned}$$

where $M = I_T - l(l'l)^{-1}l'$.

Hence, the small sample bias of $\hat{\beta}_2$ will be:

(20) $$\hat{\Delta}_2 = \hat{\beta}_2 - \beta_2 = \left(x'Mx\right)^{-1} x'Mw + \left(x'Mx\right)^{-1} x'Mu .$$

Taking the probability limit of Equation (20) yields:

(21) $$p\lim\left(\hat{\Delta}_2\right) = p\lim\left(\frac{1}{T}x'Mx\right)^{-1} p\lim\frac{1}{T}x'Mw + p\lim\left(\frac{1}{T}x'Mx\right)^{-1} p\lim\frac{1}{T}x'Mu$$

$$= \frac{Cov(\ln C_t, \ln Z_{t+1})}{Var(\ln C_t)} + \frac{Cov(\ln C_t, v_{t+1})}{Var(\ln C_t)} = \frac{Cov(\ln C_t, \ln Z_{t+1})}{Var(\ln C_t)} < 0 ,$$

As shown in Appendix B, $Cov(\ln C_t , \ln Z_{t+1}) < 0$ for durables, and equals 0 for nondurables if $\sigma_e^2 \in N(0)$, where N is the neighborhood region. Hence,

(22) $\Delta_2^d < 0; \quad \Delta_2^{nd} = 0 ,$

where the superscripts *d* and *nd* stand for durables and nondurables expenditures, respectively. From Equation (16), $\ln C_t$ has a unit root, and so, $Var(\ln C_t) \xrightarrow{T\to\infty} \infty$, which implies:

(23) $p\lim(\hat{\Delta}_2) \xrightarrow{T\to\infty} 0 .$

III. Nonlinear ARMA (∞, 1) as Data Generating Process

A. Empirical Evidence

To see the small sample bias, we estimate the AR(1) model in log linear form for five different periods, each containing 40 quarterly observations. The historical data set consists of real expenditures on durables and nondurables from the U. S. National Income and Product Accounts: quarterly, per capita, seasonally adjusted, and chained to 2000 dollars. Nondurables expenditures are defined as combined expenditures on nondurable goods and services. Following Mankiw (1982), we exclude the Korean War period to avoid extra constraints on the theory. The starting points of the five series are ten years apart. As reported in Table 1, the slope estimates for durables expenditures appear to be consistently lower than their counterparts for nondurables, whereas the reverse is the case for the intercepts. This implies that the bias of $\hat{\beta}_2$ for nondurables is smaller than the one for durables.

Table 1. Small Sample Estimation: Historical Data

[Evidence of the bias resulted from estimating Equation (18)]

Estimation Period	Slope estimates ($\hat{\beta}_2$)		Intercept estimates ($\hat{\beta}_1$)	
	Nondurables	Durables	Nondurables	Durables
1955.1-1964.4	1.0116	.94437	-.14429	.35599
1965.1-1974.4	.97349	.94990	.25317	.64213
1975.1-1984.4	.98860	.92973	.11394	.50816
1985.1-1994.4	.96505	.88188	.34479	.89131
1995.1-2004.4	1.0006	.98696	-.03547	.12385

To evaluate whether the bias approaches zero as sample size increases, we estimate the slope and intercept for six different sample sizes with the same starting point, the size increasing from 20 quarterly observations to 200. As shown in Table 2, the bias of $\hat{\beta}_2$ for durables appears to taper off as the sample size increases, as expected in light of Equation (23).

Table 2. Increasing Sample Estimation: Historical Data

[Evidence supporting Equation (23)]

Sample Size	Slope estimates $(\hat{\beta}_2)$		Intercept estimates $(\hat{\beta}_1)$	
	Nondurables	Durables	Nondurables	Durables
1955.1-1959.4	.97997	.75530	.04350	.9275
1955.1-1964.4	1.0166	.94437	-.14429	.35599
1955.1-1974.4	1.0006	.98965	.00007	.07536
1955.1-1984.4	.99895	.99775	.01554	.02421
1955.1-1994.4	99781	.99777	.02604	.02427
1955.1-2004.4	.99837	1.0014	.02092	.00003

However, historical data may have been affected by other exogenous events, which may have obliterated what otherwise would have been a clearer manifestation of the nonlinear ARMA(∞,1) process. Therefore, reconfirmation of the theoretically expected bias through a Monte Carlo experiment will strengthen the evidence from historical data.

B. Monte Carlo Evidence

Using our theoretical model in Equation (14), we generate 232 observations which match the historical data for quarterly expenditures on durables and nondurables for the period 1947.1-2004.4. Theoretically $\ln Z_{t+1} \neq 0$ for durables; hence, the first few generated observations of C_{t+1} are volatile as the new innovations are added, until already cumulated past innovations dominate over the new addition. Thus, we generate a total of 244 observations and remove the first twelve. Since for nondurables, $\ln Z_{t+1} = 0$, volatility is not a problem; thus we generated 232 observations—exactly the same number as in the historical data set.

In generating the time series, we choose the values for C_0 and C_1 and variances of e_t and v_t in such a way that the generated series simulates the historical data as closely as possible. For durables, we follow Mankiw (1982) in assuming that the deterministic component of the depreciation rate is $\delta = 0.05$. For ϕ, we use the estimated growth rate of the historical C_t: approximately 1 percent for durables, and 0.55 percent for nondurables. The generated data simulates the historical data fairly well, as shown in Figure 1 and Figure 2 for durables and nondurables expenditures, respectively.

Figure 1. Data Comparison: Durables Expenditures

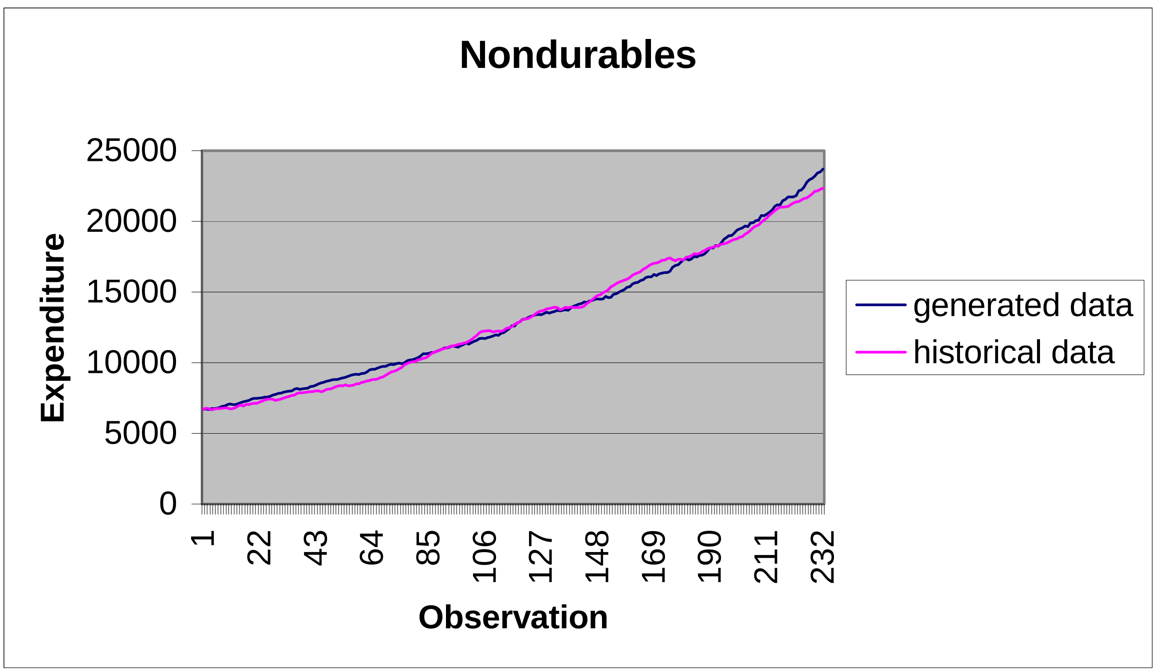
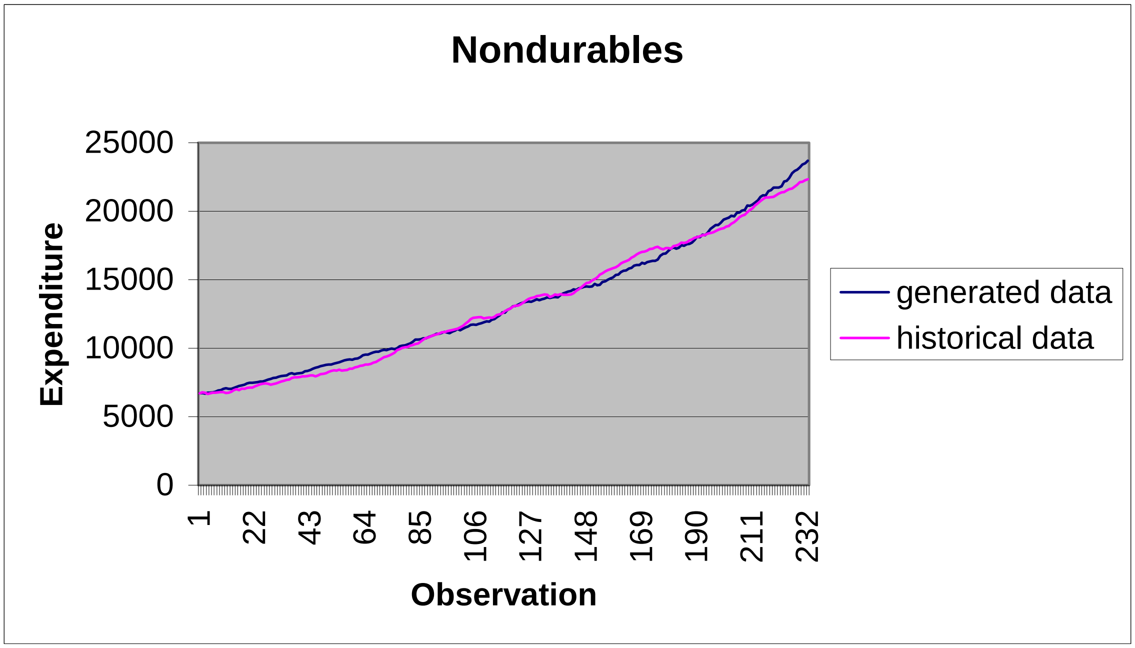


Figure 2. Data Comparison: Nondurables Expenditures



To compare the small sample biases, we iterate estimation of the AR(1) model for $\ln C_t$ 5000 times for different sample periods of size 40. Table 3 shows the means of 5000 slope and intercept estimates for each of the five sample periods. As expected theoretically, the mean slope coefficient for the durables expenditures is smaller than its counterpart for nondurables. The reverse appears to be the case for the mean intercept. These Monte Carlo results are quite consistent with the empirical results using the historical data.

Table 3. Small Sample Estimation: Generated Data

(Evidence supporting the empirical results in Table 1)

Estimation Period	Slope estimates $(\hat{\beta}_2)$		Intercept estimates $(\hat{\beta}_1)$	
	Nondurables	Durables	Nondurables	Durables
1955.1-1964.4	.99019	.93264	.10236	.29672
1965.1-1974.4	.99011	.92942	.20137	.52630
1975.1-1984.4	.97248	.90753	.20351	.81724
1985.1-1994.4	.95823	.91677	.31324	.62837
1995.1-2004.4	.98975	.96258	.16479	.20144

To see whether the bias approaches zero as the sample size increases, we also iterate estimation of the AR(1) model 5000 times for six increasing sample sizes similar to Table 2 for the historical data. Table 4 reports the means of 5000 slope and intercept estimates, which show that the downward bias of the slope estimate tapers off as the sample size increases. This result is also consistent with the results using the historical data.

Table 4. Increasing Sample Estimation: Generated Data

(Evidence supporting the empirical results in Table 2)

Sample Size	Slope estimates $(\hat{\beta}_2)$		Intercept estimates $(\hat{\beta}_1)$	
	Nondurables	Durables	Nondurables	Durables
1955.1-1959.4	.93968	.88937	.49629	.61174
1955.1-1964.4	.99274	.94181	.06341	.35546
1955.1-1974.4	.99772	.97939	.02186	.13821
1955.1-1984.4	.99929	.98994	.01002	.07803
1955.1-1994.4	.99963	.99492	.00699	.07804
1955.1-2004.4	.99965	.99665	.00672	.03226

The consistency of the estimation results based on the generated data with those based on the historical data appears to give strong evidence for our nonlinear ARMA(∞,1) model as the underlying data generating process for the historical data on durables expenditures.

IV. Mankiw's Puzzle as a Misspecification

In Section III, we have shown the consistency in the estimation results between the historical and generated data, establishing the ARMA(∞,1) as the possible data process that drives the expenditures on both durables and nondurables. This implies that the ARMA(1,1) is a possible misspecification of the ARMA(∞,1), which explains Mankiw's failure to find an ARMA(1,1) process as expected, i.e., Mankiw's puzzle.

In order to strengthen this observation, we test whether Mankiw's estimation result is robust with respect to sample size. We estimate the ARMA(1,1) for six subsamples of different sizes over the period from 1964.1 to 2004.1 as reported in Table 5. The first sample period (1955.1 to 1980.1) is identical with the one in Mankiw (1982). The estimation results are consistent with Mankiw's: the moving average coefficient is negative and insignificant. However, we have inconsistent results across samples. We have a statistically significant moving average coefficient for the second and third sample sizes, but an insignificant moving average for the last two sample sizes. In other words, Mankiw's model is not robust to changes in sample sizes. This can be attributed to the ARMA(1,1) as a possible misspecification of the ARMA(∞,1).

Table 5. ARMA(1,1) Estimation: Historical Data

(Evidence that Mankiw's model is not robust to changes in sample sizes. The first sample period is the same as the one used by Mankiw.)

Sample Size	MA(1)	Standard error	t-statistics
1955.1-1980.1	-.04251	.1006	-.4224
1964.1-2004.1	.1873***	.07827	2.398
1969.1-2004.1	.16133**	.08415	1.917
1974.1-2004.1	.12999	.09131	1.424
1979.1-2004.1	.06349	.1008	.6297
1984.1-2004.1	.01413	.1130	.1251

The ** and *** indicate five and one percent significance levels, respectively

In order to evaluate the empirical results, we generate ten times the data sets driven by ARMA(∞,1) process for the time period from 1954.1 to 2004.1, each time creating six subsamples corresponding to those in Table 5, and estimated Mankiw's ARMA(1,1) model. Table 6 reports the means of the MA coefficient estimates and t-ratios. The results are quite similar to their empirical counterparts in Table 5. The estimation results for the first sample period shows a negative and insignificant moving average as in Mankiw (1982). Further, the statistical significances in Table 6 coincide with those in Tables 5.

Table 6. ARMA (1,1) Estimation: Generated Data

(Evidence supporting the empirical results in Table 5. The first sample period is the same as the one used by Mankiw.)

Sample Size	MA(1)	Standard error	t-statistics
1955.1-1980.1	-.1224	.1102	-1.1477
1964.1-2004.1	.3700***	.07494	4.938
1969.1-2004.1	.2483***	.08423	2.948
1974.1-2004.1	-.1104	.09076	-1.216
1979.1-2004.1	-.1128	.1002	-1.125
1984.1-2004.1	-.1823	.1115	-1.652

The *** indicates one percent significance level.

Given the similar estimation results with respect to sample size found in this section in addition to the consistency in Section III, comments are in order.

First, Mankiw's (1982) conjecture that his puzzling finding on durables expenditures is due to a specification error of the utility function appears to be partly correct. Additionally, misspecification of the disturbance term and the depreciation rate also seem responsible for Mankiw's puzzle. Based upon our results, depreciation rates do play an important role in determining durables expenditures. This is consistent with both Hall's theory and common intuition.

Second, Caballero's (1990) failure to find statistical significance of the individual moving average coefficients, though the sum of the moving average coefficients is significant, may be attributed as well to the misspecification of the nonlinear ARMA(∞,1).

Finally, the inconsistent empirical results elsewhere in the literature, e.g., Hall (1978), where nondurables expenditures follow an I(1) process and Ermini (1988), where they follow an IMA(1,1) may be also attributable to a misspecification of the nonlinear ARMA(∞,1) herein discussed.

V. Conclusion

Since Mankiw's paper in 1982, several attempts have been made to address the inconsistency between the theoretical model and empirical results as observed by Mankiw, but none of them has resolved the issue satisfactorily.

In this paper, we have modified the model in a fundamental way. As a result, we find that the true underlying data generating process is a complex nonlinear ARMA(∞,1) process. Therefore, the standard linear ARMA(1,1) process might be a misspecification. The historical data analysis and the Monte Carlo experiment appear to support our model as the data generating process. Our approach is strictly univariate as in Mankiw (1982) and produces the empirical and Monte Carlo results consistent with the model derived. This seems also to confirm Mankiw's conjecture that his puzzle may be due to misspecification of the utility function.

In this research, we limit ourselves to resolving Mankiw's puzzle on consumer durables in particular and giving some insight into on the existing conflicting results in consumption literature in general. A multivariate approach would be interesting though more challenging, which is beyond the scope of this paper.

References

Caballero, R.J., 1990, “Expenditure of Durable Goods: A Case for Slow Adjustment,” *The Quarterly Journal of Economics,* Vol. 105, No. 3, 727-743.

Ermini, L., 1988, "Temporal aggregation and Hall's model of consumption behavior," *Applied Economics,* 1988, 20. 1317-1320.

Hall, R. E., 1978, “Stochastic Implications of Life Cycle-Permanent Income Hypothesis: Theory and Evidence,” *The Journal of Political Economy,* Vol. 86, No. 6, 971-987.

Hansen, L. P.; Singleton, K. J., 1983, “Stochastic Consumption, Risk Aversion, and Temporal Behavior of Asset Returns,” *The Journal of Political Economy,* Vol. 91, No.2, 249-265.

Mankiw, N. G., 1982, “Hall’s Consumption Hypothesis and Durable Goods,” *Journal of Monetary Economics,* 10 (1982) 417-425.

Mankiw, N. G., 1985, “Consumer Durables and the Real Interest Rate,” *The Review of Economics and Statistics,* Vol. 67, No. 3 (August 1985), 353-362.

Romer, D., 2001, *Advanced Macroeconomics,* McGraw-Hill Higher Education, Boston, 363-365.

Vu, B.T., 2005 (a), "An Alternative Approximation to Consumer Durables Expenditures," Department of Economics, University of Hawaii at Manoa.

Winder, C. C. A.; Palm, F. C., 1996, “Stochastic implications of the life cycle consumption model under rational habit formation,” *Recherches economiques de Louvain,* Vol. 62, Issue 3-4, 403-EOA

Appendix A. Proof for Equation (14)

From equation (13):

(A1) $$C_{t+1} = \psi\left\{\left[1-k_{t-1}B\right]^{-1}C_t\right\}e^{\nu_{t+1}} - k_t\psi\left\{B\left\langle\left[1-k_{t-1}B\right]^{-1}C_t\right\rangle\right\}Be^{\nu_{t+1}}$$

$$= \psi\left\{\left[1-k_{t-1}B\right]^{-1}C_t\right\}e^{\nu_{t+1}} - k_t\psi\left\{B\left\langle\left[1-k_{t-1}B\right]^{-1}C_t\right\rangle\right\}e^{\nu_t}$$

Since

(A2) $$\left[1-k_{t-1}B\right]^{-1}C_t = \left(1+k_{t-1}B+k_{t-1}^2B^2+k_{t-1}^3B^3+\cdots\right)$$

$$= C_t + k_{t-1}C_{t-1} + k_{t-1}^2C_{t-2} + k_{t-1}^3C_{t-3} + \cdots,$$

$$B\left[1-k_{t-1}B\right]^{-1}C_t = B\left(C_t + k_{t-1}C_{t-1} + k_{t-1}^2C_{t-2} + k_{t-1}^3C_{t-3} + \cdots\right)$$

$$= C_{t-1} + k_{t-2}C_{t-2} + k_{t-3}^2C_{t-3} + k_{t-4}^3C_{t-4} + \cdots,$$

substituting Equation (A2) into Equation (A1) gives

$$C_{t+1} = \psi\left(C_t + k_{t-1}C_{t-1} + k_{t-1}^2C_{t-2} + \cdots\right)e^{\nu_{t+1}} - k_t\psi\left(C_{t-1} + k_{t-2}C_{t-2} + \cdots\right)e^{\nu_t}$$

$$= \psi C_t e^{v_{t+1}} \left[1 + k_{t-1} \frac{C_{t-1}}{C_t} + k_{t-1}^2 \frac{C_{t-2}}{C_t} + ... - k_t \frac{C_{t-1}}{C_t} e^{v_t - v_{t+1}} - k_t \frac{k_{t-2} C_{t-2}}{C_t} e^{v_t - v_{t+1}} - \cdots \right]$$

$$= \psi C_t e^{v_{t+1}} \left[1 + \left(k_{t-1} - k_t e^{v_t - v_{t+1}} \right) \frac{C_{t-1}}{C_t} + \left(k_{t-1}^2 - k_t k_{t-2} e^{v_t - v_{t+1}} \right) \frac{C_{t-2}}{C_t} + \cdots \right],$$

$$= \psi C_t e^{v_{t+1}} \left\{ 1 + \sum_{i=1}^{\infty} \left[\left(k_{t-1}^i - k_t k_{t-2}^{i-1} e^{v_t - v_{t=1}} \right) \frac{C_{t-i}}{C_t} \right] \right\}$$

which yields Equation (14) in the text.

Appendix B. **Proof for** $Cov\left(\ln C_t, \ln Z_{t+1}\right) < 0$

For two variables p and q, we can write

(B1) $$\frac{\Delta q_t}{\Delta p_t} \equiv \frac{q_t - \mu_p}{p_t - \mu_q} = \frac{(q_t - \mu_p)(p_t - \mu_q)}{(p_t - \mu_q)^2}$$

Since we can rewrite (B1) as

(B2) $$\left(\frac{\Delta q}{\Delta p} \right) (p - \mu_p)^2 = (q - \mu_q)(p_t - \mu_p),$$

(B3) $$E\left(\frac{\Delta q_t}{\Delta p_t} \right) Var(p_t) = Cov(p_t, q_t).$$

However, if $dq_t / dp_t > 0$ for all p_t, $\Delta q_t / \Delta p_t > 0$ which implies $E(\Delta q_t / \Delta p_t) > 0$. Hence, to show $Cov(p_t, q_t) < 0$, it suffices to show $dq_t / dp_t < 0$.

In view of (B3), to prove $Cov(\ln C_t, \ln Z_{t+1}) < 0$, we only need to prove

$\frac{d \ln Z_{t+1}}{d \ln C_t} < 0$. Since

(B4) $$\frac{dZ_{t+1}}{dC_t} = -\left[\sum_{i=1}^{\infty} \left((k_{t-1}^{i} - k_t k_{t-2}^{i-1} e^{v_t - v_{t+1}}) \frac{C_{t-i}}{C_t^{\,2}} \right) \right],$$

(B5) $$\frac{d \ln Z_{t+1}}{d \ln C_t} = \frac{dZ_{t+1}}{dC_t} \frac{C_t}{Z_{t+1}} = -\left[\sum_{i=1}^{\infty} \left((k_{t-1}^{i} - k_t k_{t-2}^{i-1} e^{v_t - v_{t+1}}) \frac{C_{t-i}}{C_t} \right) \right] \frac{1}{Z_{t+1}}.$$

If $\delta = 1$, i.e., $k_t = 0$, (B5) become zero, which implies that $Cov(\ln C_t, \ln Z_{t+1}) = 0$ for nondurables. However, if $\delta_t \in (0,1)$,i.e., in the case of durables, (B5) can be expressed as

$$\frac{\ln Z_{t+1}}{\ln C_t} = -\frac{Z_{t+1} - 1}{Z_{t+1}} \quad .$$

Suppose that

$$p\lim\left(\frac{\ln Z_{t+1}}{\ln C_t} \right) = -\frac{p\lim\left(\frac{Z_{t+1}}{T} \right) - p\lim\left(\frac{1}{T} \right)}{p\lim\left(\frac{Z_{t+1}}{T} \right)} = -1 < 0$$

which implies that $Cov(C_t, Z_{t+1}) < 0$ for durables.

B.2. Proof for $Cov(\ln C_t, v_{t+1}) = 0$

Since v_{t+1} is not contained in ln C_t, $\partial \ln C_t / \partial v_{t+1} = 0$, hence $Cov(\ln C_t, v_{t+1}) = 0$.