SUSTAINABLE GROWTH WITH ENVIRONMENTAL SPILLOVERS: A RAMSEY-KOOPMANS APPROACH

by

Lee H. Endress
James A. Roumasset
Ting Zhou

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James A. Roumasset

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Abstract

In this paper, we apply the canonical approach of Ramsey, Koopmans, and Diamond to the problem of optimal and intertemporally-equitable growth with a non-renewable resource constraint and show that the solution is sustainable. The model is extended to cases involving environmental amenities and disamenities and renewable resources. The solutions equivalently solve the problem of maximizing net national product adjusted for depreciation in natural capital and environmental effects, which turns out to be both sustainable and constant even without technical change.

University of Hawaii at Manoa
Department of Economics
Social Science Building, Room 542
2424 Maile Way
Honolulu, HI 96822

1. Intro: To discount or not to discount?

"Sustainable growth" is commonly modeled as a problem of maximizing an intertemporal utilitarian welfare function, subject to the constraint that consumption or utility growth cannot be negative (e.g. Asheim, 1995). The constrained-utilitarian approach applies positive discounting of future utility, consistent with Koopmans' (1960) demonstration that there does not exist a utility function defined on all consumption streams that satisfies the usual axioms of rational choice and timing neutrality (i.e. without discounting). In an alternative approach (Beltratti et. al., 1993, 1995 and Chichilnisky; 1996),[1] social welfare is modeled as a weighted average of conventional growth and a concern for sustainability.

Both schools of thought reject the possibility of using a zero utility discount rate to represent a social planner's concern for intergenerational equity. This is odd in light of the fact that the pioneers of growth theory advocated exactly that. Ramsey warned that the use of a positive utility discount rate is "ethically indefensible" and reveals "a weakness of the imagination." Koopmans himself (1965) noted that "we welcome equally a unit increase in consumption per worker in any one future decade… Mere numbers cannot give one generation an edge over another…" Moreover, while Heal *et. al.* recognize Koopmans' (1960) nonexistence theorem, they seem to overlook his subsequent solution (1965), which relies precisely on the notion of "intergenerational neutrality," for a specific, non-empty subset of feasible consumption paths, and is captured in turn by the zero utility discount rate. Accordingly, the first objective of the present paper is to return to the canonical approach of Ramsey and Koopmans and explore the extent to which concerns of the sustainability dialog can be captured by that approach.

[1] See also the discussion of this approach in Heal, 1998.

Most of the sustainable growth literature focuses on the trade-off between the accumulation of produced capital and the depletion of natural capital (e.g. Toman *et. al.*, 1995) to the neglect of the Brundtland Commission's original emphasis on the importance of managing inter-linkages between poverty, population growth, and environmental degradation. A second objective of the present paper, therefore, is to help develop these linkages by extending the optimal growth framework to allow for environmental amenities and disamenities.

A third school of thought recommends maximizing "green net national product" (e.g. World Bank, 1997) wherein one subtracts depreciation of natural capital along with that of produced capital in reckoning net national product. Weitzman (1976, 1999) has indeed shown that such a measure is formally equivalent to maximizing a utilitarian welfare function. But while this measure is commonly described as sustainable income, utility discounting permits its maximized value to eventually decline. In other words, maximizing sustainable income is inconsistent with optimal sustainable growth as defined by either of the above-mentioned schools. We will show, however, that this paradox does not arise in the Ramsey-Koopmans framework. Maximizing real national income in the appropriately adjusted model yields an income stream that is not only sustainable but also constant in the optimal program. Moreover, the Weitzman "hypothetical stationery equivalent income" is just the golden rule utility level.

In section 2 below, we present the conditions for optimal and ethically neutral growth in a model with a non-renewable resource and a backstop technology. The maxi-min solution is shown to be a special case of this solution, albeit one which is unlikely to be preferred. In section 3, we extend the model to include environmental amenities and disamenities associated with the use of renewable and non-renewable resources. In section 4, we investigate the relationship between Ramsey-Koopmans optimal growth and sustainable income. Section 5 provides a brief summary and concluding remarks.

2. Intertemporally-neutral optimal growth with a non-renewable resource

2.1 Koopmans' impossibility theorem and his solution

An immediate obstacle to maximizing a utilitarian welfare function without discounting is that the value of such a function is infinite for some feasible consumption streams. Koopmans (1960) went further, proving the impossibility of representing intertemporally-neutral planner preferences, over all consumption vectors, by any utility function.

Building on the earlier work of Ramsey (1928), Koopmans (1965) argued that one way out of the dilemma of a non-existent utility function of all consumption paths is to identify a subset of all feasible paths on which one can define a neutral utility function. Ramsey's criterion for eligibility in the subset is a sufficiently rapid approach of the path to a "bliss point". Koopmans' criterion is less restrictive: "We shall find that in the present case of a steady population growth the golden rule path can take the place of Ramsey's state of bliss in defining eligibility" (Koopmans, 1965, p.500). Specifically, consumption must approach the golden rule consumption level with sufficient rapidity that the area of deficit between the "felicity" of consumption and that of golden rule consumption converges to a finite number, i.e., $V(t) < \infty$ where V is the planner's utility, defined as the area given in equation 3. Using this criterion, Koopmans (1965) demonstrates that each eligible path is superior to each path that is ineligible. Moreover, one can rank eligible paths and determine one that is optimal.

2.2 Sustainability without really trying

Following Koopmans' method, in this paper, we propose an approach to sustainability that is based on a resource management and capital accumulation obtained by setting the rate of time preference, ρ, equal to zero. Instead of maximizing the social utility as a discounted sum, the target is now maximizing the utility function, defined as an infinite sum of the difference between the actual consumption trajectory and the optimal consumption.

In order to introduce natural capital into the Ramsey-Koopmans framework, consider an economy that uses a natural resource (R), in addition to capital (K) and labor (L) to produce a single homogeneous good. Assume that the production technology is constant returns to scale, so that the production functions Q(K,R,L) is homogeneous of degree one. In order to present the argument in its starkest form, we abstract from population growth and technological change and normalize L = 1, such that Q(K,R,L) can be expressed as F(K,R). Following the standard approach, output of production is divided among consumption, gross investment, and the cost of providing the resource as an input to the production process.

Let è be the unit cost of extracting the natural resource and providing it as an input of production. We assume that this cost is a decreasing function of the resource stock X (e.g. Heal, 1976). Produced capital, K, depreciates at the rate δ. The dynamic equation governing capital accumulation is

$$\dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad (1)$$

In this section, the natural resource is assumed to be non-renewable, and the dynamic equation governing the resource stock becomes

$$\dot{X}_t = -R_t \qquad (2)$$

We augment this basic model by incorporating a backstop technology that has a fixed unit extraction cost θ_b. Consider, for example, the case of oil, a non-renewable resource. Oil stocks are drawn down as the economy grows until unit cost, θ, has risen sufficiently to warrant the switch to a superabundant, but high cost, alternative energy source (e.g., coal gasification, nuclear fission/fusion, solar energy). Once the switch has been made, we assume that the backstop technology delivers energy at constant cost, θ_b. Therefore, the locus of unit extraction cost is the lower envelope of curves $\theta(X)$ and θ_b, as shown in *Figure 1*. This case contrasts with the conventional Hartwick-Solow model in which extraction costs are constant up to a specific quantity, after which no amount of the resource is obtainable at any cost.[2]

Social welfare over the feasible and eligible consumption paths makes use of the auxiliary "felicity" function, $U(C_t)$. As in Koopmans (1965), we assume $U_C > 0$, $U_{CC} < 0$ and $\lim_{C \to 0} U(C) = -\infty$, such that periods of very low consumption are avoided as much as possible.

Following Koopmans, the social planner's utility function is expressed as:[3]

$$V = \int_0^\infty [U(C_t) - U(\hat{C})]dt \qquad (3)$$

and the corresponding planner's optimization problem is:

[2] The assumption of rising extraction costs and a backstop technology, e.g. photovolaics, is considerably more realistic than the inverted L-shaped extraction cost schedule that is usually assumed (see Chakravorty, et. al., 1997). An alternative to the backstop assumption would be to follow Hotelling (1931) wherein resource use is truncated on the demand side. In our framework, however, this would require the complication of multiple consumption goods.

[3] To maximize H with respect to the control variable R, implicitly we require $R \geq 0$. Correspondingly, the Kuhn-Tucker condition is $\partial H/\partial R \leq 0$ and the complementary-slackness proviso that $R(\partial H/\partial R) = 0$. But in as much as we can rule out the extreme case of $R = 0$, we postulate that $R > 0$, implying an interior solution.

$$
\begin{aligned}
&\text{Max} \quad V \\
&\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \ddot{a}K_t - \grave{e}(X_t)R_t - C_t, \qquad K(0) = K_0 \\
&\qquad \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0 \ , & t \geq T \end{cases} \\
&\qquad X_t \geq 0 \qquad\qquad\qquad\qquad\qquad\qquad X(0) = X_0
\end{aligned} \tag{4}
$$

where $U(\hat{C})$ is felicity at the golden rule level of consumption and T is the endogenous time for which $\theta(X_t) = \theta_b$, $\forall t \geq T$.

Since $\dot{K}_t = 0$ in the steady state, the golden-rule consumption level, $\hat{C}$, can be found (generalizing from Solow, 1956) by maximizing:

$$C = F(K, R) - \ddot{a}K - \grave{e}_b R, \tag{5}$$

where R is the amount of the backstop resource consumed in the steady state.

The corresponding first order conditions, which comprise the golden rule for capital accumulation and resource management, are:

$$\frac{\partial C}{\partial K} = F_K - \delta = 0 \tag{6}$$

and,

$$\frac{\partial C}{\partial R} = F_R - \theta_b = 0 \tag{7}$$

These conditions yield the golden rule steady state levels, $\hat{K}$ and $\hat{R}$. $\hat{C}$ is now defined as

$$\hat{C} = F(\hat{K}, \hat{R}) - \delta\hat{K} - \theta_b \hat{R} \tag{8}$$

The Hamiltonian for this problem is (for simplicity, the subscripts t's are dropped)

$$H = [U(C) - U(\hat{C})] + \ddot{e}[F(K, R) - \ddot{a}K - \grave{e}(X)R - C] + ø[-R] \tag{9}$$

Incorporating the inequality constraints imposed on the problem, we form the Lagrangian

$$L = H + \phi\{X\} + \hat{o}\{\grave{e}_b - \grave{e}(X)\} \ , \tag{10}$$

such that the complimentary slackness conditions associated with the inequalities are

$$\begin{aligned} \tau\frac{\partial L}{\partial \tau} &= \tau[\theta_b - \theta(X)] = 0 \\ \phi\frac{\partial L}{\partial \phi} &= \phi X = 0 \end{aligned} \tag{11}$$

Application of the maximum principle to this optimal control problem yields the following two efficiency conditions (see Appendix I for details):

$$\eta(C)\frac{\dot{C}}{C} = F_K - \delta \tag{12}$$

$$F_R - \theta(X) = \frac{\dot{F}_R}{(F_K - \delta)} \tag{13}$$

Condition (12) is the Ramsey condition that governs the optimal path of consumption leading to golden rule steady state. In the analogous approach to the "modified golden rule", $\eta(C)\frac{\dot{C}}{C} + \rho = F_K - \delta$, there are two parameters governing the savings and the rate of capital accumulation. The first is the absolute value of the consumption elasticity of the marginal utility, $\eta(C)$. Lower $\eta(C)$ implies a lower social opportunity cost of savings (greater tolerance for intergenerational inequality), more rapid capital accumulation, lower interest rates, and higher growth rates of consumption. The second parameter is the social rate of time preference ρ, which reflects the social valuation of future felicity in term of today's felicity. However, in our setting, by treating all generations equally, $\rho = 0$.[4]

[4] It would be unsound to derive equation (12) by setting $\rho = 0$ in the standard Ramsey condition. Nonetheless, the equation that one gets by doing exactly that turns out to be correct.

Condition (13) is a generalization of Hotelling's Rule. The LHS is the *in situ* marginal value of the resource and the RHS is the marginal user cost.[5] Thus, the optimal consumption trajectory and the optimal motion of the state variables, K and X, are governed by two intuitive conditions, the Ramsey savings rule, with a zero utility-discount rate, and a general-equilibrium Hotelling rule for the case of rising extraction costs.

2.3 Relationship to other approaches

Hartwick (1977) and Solow (1974, 1986) have shown that for a Cobb-Douglas production function of capital and a non-renewable resource with a constant extraction cost, that extracting the resource according to the Hotelling rule and then saving exactly the resource rents thus generated leads to a consumption path that is sustainable and constant over time. The Hartwick-Solow rule has been justified *ex post* as being the highest consumption path that is intergenerationally equitable in the sense of delivering equal consumption to all generations.

This maxi-min consumption path may be generated as a special case of our basic model. Rearranging equation (12), we have:

$$\frac{\dot{C}}{C} = \frac{F_K - \delta}{\eta(C)} \tag{14}$$

As the social aversion to intertemporal inequality, $\eta(C)$, approaches infinity, $\dot{C}/C \to 0$, generating constant consumption for all $t \geq 0$.

As a special case, suppose social planner's preferences are represented by $V = \int_0^\infty [U(C_t) - U(\hat{C})]dt$ where $U(C_t)$ takes the CES form:

[5] Endress and Roumasset (1994). For a partial equilibrium market equivalent of equation (13), see Hansen (1980).

$$U(C_t) = -C_t^{-(\eta-1)}, \qquad \eta > 1 \tag{15}$$

As η gets larger and larger, the initial level of consumption increases, and the consumption trajectory becomes flatter. This is illustrated in *Figure 2*. No matter how high η, the upper bound of consumption remains at $\hat{C}$, so long as η is not infinite. Once η becomes infinite, however, both the upper and lower bound switch to $\bar{C}$ in *Figure 2*, exactly the maxi-min level of intertemporal consumption.

An alternative to maxi-min welfare is the concept of constrained utility maximization. The main idea, due to Asheim (1988), is to apply a non-declining utility constraint ($\dot{U}(C) \geq 0$) to the maximization of utilitarian welfare. But, as noted by Toman *et. al.* (1995), such an approach does not resolve how the social welfare function should directly reflect concerns about intergenerational equity. Besides the *ad hoc* nature of the utility constraint, constrained optimization cannot provide a full ranking of alternatives, because alternatives that violate the constraint cannot be compared. In the Hartwick-Solow economy, for example, if either the elasticity of substitution between natural capital and produced capital is less than one or the output elasticity of natural capital is greater than that of produced capital (with elasticity of substitution equal to one), the sustainability constraint renders the maximization problem infeasible. In this case, none of the feasible paths can be ranked.[6]

Rather than adding a sustainability constraint or specifying axioms that a "sustainably-correct" social planner's preferences must satisfy (e.g. Beltratti *et. al.*, 1995), our approach follows Ramsey and Koopmans and finds an optimal and intertemporally neutral growth path. In

[6] Even if constrained utility maximization is reformulated as a lexicographic (vector-valued) utility function (see Endress, 1994), the model is still characterized by the rejection of tradeoffs (Dasgupta, 2000). That is, no negative consumption growth, however close to zero, can be justified, even if it affords higher sustainable consumption in the future.

the basic case above and in the cases below wherein the resource generates an environmental amenity or disamenity, we find that the optimal path is sustainable, even though we do not require it to be so.

3. Extensions: Environmental Effects and Renewable Resources

3.1 Fund Pollution

We now turn to the case wherein use of a non-renewable resource, e.g. petro-chemically sourced energy, generates pollution. For simplicity, assume that pollution (E_t) is emitted as a constant proportion of resource use (R_t) and that emission units are set such that the proportion is one. Therefore,

$$E_t = R_t, \quad t < T \tag{16}$$

In the case of fund pollution, emissions are assumed not to accumulate, i.e., emissions, but no stock pollutants, enter into the utility function. Now our maximand becomes,

$$\text{Max} \quad V = \int_0^\infty [U(C_t, E_t) - U(\hat{C}_t, \hat{E})]dt \tag{17}$$

where $U_C > 0,\ U_E < 0;$ and $U_{CC} < 0,\ U_{EE} < 0$, signifying increasing marginal disutility of pollution.

As in the basic case, $\hat{C}$ is associated with a total switch to the backstop technology (e.g. photovoltaics). Use of the primary resource and the corresponding emissions are then both zero so that $\hat{E} = 0$. Golden rule consumption is therefore given by equation (8), with $\hat{K}$ and $\hat{R}$ exactly the same as in the basic model.

An optimum trajectory for consumption and capital accumulation satisfies:

$$
\begin{aligned}
\text{Max} \quad & V = \int_0^\infty [U(C_t, E_t) - U(\hat{C}, 0)] dt \\
\text{s.t.} \quad & \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t) R_t - C_t, \qquad K(0) = K_0 \\
& \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases} \\
& X_t \geq 0 \qquad X(0) = X_0
\end{aligned} \tag{18}
$$

Application of the maximum principle to this optimal control problem gives the following efficiency conditions (See Appendix II for details):

$$
-\frac{\dot{U}_C}{U_C} = F_K - \delta \tag{19}
$$

$$
F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} + \frac{1}{F_K - \delta}\left(\frac{\dot{U}_E}{U_C}\right) - \frac{U_E}{U_C} \tag{20}
$$

Equation (19) appears to be the familiar Ramsey condition. However, since U now has two arguments, the time derivative of U_c will involve a cross term,

$$
-\left(\frac{U_{CC} C}{U_C}\right)\left(\frac{\dot{C}}{C}\right) + \left(\frac{U_{CE} E}{U_C}\right)\left(\frac{\dot{E}}{E}\right) = F_K - \delta \tag{21}
$$

If C and E are separable arguments of the felicity function, equation (21) collapses to the conventional Ramsey savings rule.

Turning to equation (20), the LHS and the first term on the RHS constitute the generalized Hotelling condition (cf. equation (13)). The last term on the RHS is just the marginal damage cost (MDC = $-\frac{U_E}{U_C}$). The remaining term, $\left(\frac{\dot{U}_E}{U_c}\right) = \frac{\partial}{\partial E}\left(\frac{U_E}{U_C}\right)\dot{E} + \frac{\partial}{\partial C}\left(\frac{U_E}{U_C}\right)\dot{C}$, which is positive or negative respectively depending on whether the first or second term dominates. For example, if the marginal damage cost is relatively flat and the income elasticity of environmental quality is high enough, the entire term is negative. This would imply that the optimal pollution tax would be less than the marginal damage cost.

3.2 A Non-Renewable Resource with Environmental Disamenities

Now consider the case of stock pollution, such as greenhouse gases, wherein emissions contribute to the stock of pollution, M, which depreciates at rate x.[7] Our model now becomes:

$$
\begin{aligned}
&\text{Max} \quad V = \int_0^\infty [U(C_t, M_t) - U(\hat{C}, \hat{M})]dt \\
&\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad K(0) = K_0 \\
&\qquad \dot{X}_t = \begin{cases} -R_t, & t < T \\ 0 \;\;, & t \geq T \end{cases} \\
&\qquad \dot{M}_t = \begin{cases} R_t - \xi M_t, & t < T \\ -\xi M_t \;\;, & t \geq T \end{cases} \\
&\qquad X_t \geq 0 \qquad\qquad X(0) = X_0
\end{aligned}
\tag{22}
$$

where $U_C > 0, U_M < 0;$ and $U_{CC} < 0, U_{MM} < 0$, corresponding to the previous section, and where $\hat{M} = 0$ and $\hat{C}$ is the Solow golden rule consumption as before.[8]

Application of the maximum principle leads to a Ramsey condition that is identical to (19), and the expansion of $\dot{U}_C$ will be analogous to equation (21). If C and M are separable, we have the conventional Ramsey savings rule once more.

The generalized Hotelling condition for this case is:

$$
F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} - \left(\frac{1}{F_K - \delta}\right)\left(\frac{U_M - \mu\xi}{U_C}\right), \tag{23}
$$

---

[7] This model is similar to that of Nordhaus (e.g. 1991) albeit with explicit consideration of resource depletion but without the intervening climate model.

[8] In the golden rule steady state, the economy has switched to the backstop resource and the stock of pollution has depreciated to zero. Analogous to consumption in the Koopmans model, the stock of pollution in the optimal trajectory asymptotically approaches its golden rule level but never actually reaches it.

where μ is the shadow price of the pollution stock (see Appendix III). The last term on the RHS is the marginal externality cost for stock pollution. It is smaller than MDC $(-\frac{U_M}{U_C})$ because the shadow price of pollution, μ, is negative.

We now consider the special case of the above for which $\xi = 0$, i.e. the stock pollutant does not depreciate. In this case, the backstop technology is immediately employed and the primary resource is never used, so that the golden rule stock of pollution is zero ($\hat{M} = 0$). This result provides a case in which the strategy of *strong sustainability* is optimal. In its usual justification, strong sustainability is associated with the preservation of natural capital and is defended as an ecological imperative, not derived (see e.g. Pearce and Barbier, 2000). Strong sustainability critics have derided the strategy as being a "category mistake," i.e. not derived from more fundamental objectives, and for denying resource-rich economies a major source of savings and capital formation (Dasgupta and Mahler, 1995). The zero pollution result exemplifies a different approach to strong and weak sustainability than is usually found in the literature. Instead of proposing the strategy as both the objective and the means of optimal growth, our approach separates ends and means. But while ecologically-oriented proponents of preservation suggest that strong sustainability is especially important when natural capital is essential and irreplaceable, our result suggests that strong sustainability is an optimal strategy when natural capital has an abundant and perfect substitute.

3.3 A Renewable Resource with Environmental Amenities

Next consider the case of a renewable resource, such as a forest, that generates an environmental benefit and enters the utility function as a stock. Representing the growth function as $G(X)$, the dynamic equation governing the resource stock becomes

$$\dot{X} = G(X) - R \tag{24}$$

In the section below, we show that using a Ramsey-Koopmans approach, solution to this problem is well defined and the optimal trajectory leading to such steady state can be identified. The problem at hand is,

$$\begin{aligned}
&\text{Max} \quad V = \int_0^{\infty} [U(C_t, X_t) - U(\hat{C}, \hat{X})]dt \\
&\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \ddot{a}K_t - \grave{e}(X_t)R_t - C_t, \qquad K(0) = K_0 \\
&\qquad\;\; \dot{X}_t = G(X_t) - R_t, \qquad X(0) = X_0 \\
&\qquad\;\; X_t \geq 0
\end{aligned} \tag{25}$$

where $\hat{C}$ and $\hat{X}$ are now defined as their BCH green golden rule values.

Application of the maximum principle yields the following efficiency conditions (See Appendix IV for details):

$$-\frac{\dot{U}_C}{U_C} = F_K - \delta \tag{26}$$

$$[F_R - \theta(X)](F_K - \delta) = \{\dot{F}_R + [F_R - \theta(X)]G_X - \theta_X G(X) + \frac{U_X}{U_C}\} \tag{27}$$

As expected, consumption path is governed by a Ramsey condition (26).

The LHS of equation (27) measures the benefit of extracting a marginal unit today, $[F_R - \theta(X)](F_K - \delta)$, and the RHS is the marginal cost of such extraction, associated with:

1) the potential resource appreciation, $\dot{F}_R$;

2) the forgone utility from using one unit of the resource, U_X / U_C;

3) the marginal user cost (the remaining two terms).

Additionally, steady state is defined by setting $\dot{F}_R = 0$ and $F_K = \delta$,

$$-\frac{U_X(C^*, X^*)}{U_C(C^*, X^*)} = [F_R(K^*, R^*) - \theta(X^*)]G_x - \theta_X G(X^*) \tag{28}$$

The LHS of (28) is the slope of the indifference curve between C & X, as shown in *Figure 3*. Following Heal (1998), the other curve in *Figure 3* is the resource feasibility frontier, representing the maximum level of C for each level of X when optimizing over produced capital K. This curve is hump-shaped because its slope (the RHS of (28)) is positive for smaller values of X (since $F_R - q > 0$, $q_X < 0$ and $G_X > 0$). It continues to be positive until G_X becomes sufficiently negative to reverse the sign. Equation (28) thus represents the tangency between the indifference curve and the resource feasibility frontier. Note that the steady state levels of C* and X* are precisely the green golden rule level of consumption and resource stock, $\hat{C}$ & $\hat{X}$.

Finally, note that the stock pollution case from section 3.2 without depreciation of the pollution stock can be written a special case of the model here, where G(X) = 0. Since M = X_0 – X, we can rewrite the utility function such that U is a positive function of X, instead of a negative function of M.

4. Net National Product

An alternative to the conventional approaches of optimizing sustainably-weighted or sustainably-constrained growth is to maximize “green” or "sustainable" income. Weitzman (1976, 1999) has shown that green national product, defined as net national product minus the depreciation of natural capital, is indeed a linear approximation of the Hamiltonian of a utilitarian welfare function, i.e. maximizing utilitarian welfare and green national product are roughly equivalent. Green national product is also used interchangeably with "sustainable income" (Pearce and Barbier, 2000; World Bank, 1997). Since Weitzman assumes positive discounting, however, this leads to a paradox. The Hamiltonian for utilitarian welfare maximization with positive discounting can decline over time in the transition to the modified golden rule steady state (Dasgupta and Heal, 1979, chapter 13). That is, maximizing sustainable income leads to non-sustainable income and consumption paths.[9] In this section, we show that this paradox disappears in the case of timing neutrality.

Consider a general case that combines the models of sections 3.2 and 3.3 such that both the disamenity, M, and the amenity of the stock, X, are present. The Hamiltonian along the optimum trajectory remains constant; that is, $dH_t/dt = 0$[10], where

$$H_t = [U(C_t, X_t, M_t) - U(\hat{C}, \hat{X}, \hat{M})] + \lambda_t \dot{K}_t + \psi_t \dot{X}_t + \mu_t \dot{M}_t \tag{29}$$

At the steady state, $\dot{K}_t = \dot{X}_t = \dot{M}_t = 0$ and $U(C_t, X_t, M_t) = U(\hat{C}, \hat{X}, \hat{M})$, implying a zero value for H_t. Consequently, $H_t = 0$ for all time t. Thus,

$$U(\hat{C}, \hat{X}, \hat{M}) = U(C_t, X_t, M_t) + \lambda_t \dot{K}_t + \psi_t \dot{X}_t + \mu_t \dot{M}_t \tag{30}$$

The golden rule level of utility, $U(\hat{C}, \hat{X}, \hat{M})$, is the constant green net national product in utility units for all time periods. Thus maximizing green net national product is equivalent to optimizing the Ramsey-Koopmans welfare function and results in sustainable income.

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[9] In the transition to the modified golden rule steady state, total capital stock, consumption, and green national product typically rise and then fall, albeit capital and green national product fall before consumption.

[10] See Appendix V for mathematical derivation. For simple presentation, capital letters with subscript t stands for variables along optimal trajectory. Indeed, this also implies that our solution satisfies the transversality condition for the infinite-horizon, non-discounting problem as examined by Chiang (1992).

This proposition is illustrated using a much simpler case where total capital is taken only to include produced capital, K, and instantaneous utility is defined solely on consumption good, U(C). In *Figure 4*[11], curve (aa) represents the feasibility frontier of the economy at time t = 0. Consumption level, $\overline{C}$, is the maximum attainable level of consumption at time t = 0 if no investment were to take place, and $U(\overline{C})$ is the associated level of utility. The utility-investment pair $U(C_0), \dot{K}_0$ lies on the optimal trajectory to the steady state. As capital is accumulated, the feasibility frontier moves outward and towards the right until maximum attainable consumption reaches the golden rule level, $\hat{C}$, and $\dot{K}_t = 0$, as depicted by curve (bb). The shadow price of capital, illustrated by the slope of the tangency line, decreases monotonically. For the general case considered above with three types of capital, NNP still transitions to $U(\hat{C})$, the shadow price of capital decreases monotonically, and the other shadow prices transition to constant values, albeit not monotonically.

5. Summary and concluding remarks

The literature on sustainable growth has foundered on the question of whether to represent sustainability as an *ad hoc* constraint on the objective function or by restricting the social planner's preferences. Instead of searching for what is optimal and sustainable, we follow the canonical approach of Ramsey, Koopmans, and Diamond and solve for what is optimal and

[11] Figure 4 generalizes Weitzman's (1976) illustration of the hypothetical stationary equivalent consumption of NNP by illustrating the transition to the steady state. Unlike the Weitzman case, however, NNP remains constant under timing neutrality and equal to the steady state consumption level. We also illustrate NNP in utility units to avoid the linear approximation problem.

intertemporally neutral. This frees us to explore the conditions under which such a program results in sustainable utility

We find that optimal and intertemporally neutral growth is sustainable, even in the presence of non-renewable resources. Adding a constraint that restricts growth of consumption or utility to be nonnegative would be not only *ad hoc* but also redundant. The necessary conditions for optimal growth require that the economy save at the rate given by the familiar Ramsey condition and that resource use and conservation conform to a generalized Hotelling condition. The constraint of weak sustainability, which requires that the depletion of natural capital not exceed the accumulation of produced capital, is similarly redundant. Total capital increases along the optimal growth path, albeit at a declining rate.

The model is extended to accommodate environmental amenities and disamenities, resulting in modifications of the Ramsey and Hotelling conditions. In the cases of fund and stock pollution, the Ramsey condition is expanded to include a disamenity term. The Hotelling condition contains an additional term, the "marginal externality cost," which is, however, less than the marginal damage cost for the stock pollution case and ambiguously so for the fund pollution case. This means that the optimal pollution tax may be less than its Pigouvian level even without second-best considerations of public finance.[12]

Another interesting result concerns the case wherein the stock of pollution does not depreciate. In this case the optimal strategy turns out to be not to touch the non-renewable resource and immediately exploit the more costly, but non-polluting backstop. This result shows that the strategy of strong sustainability, which is often advocated on the grounds that natural

[12] See e.g. Bovenberg and Goulder (1996) for a discussion of second-best emission taxes and the "double-dividend" debate.

capital is essential and irreplaceable, turns out to be correct in the opposite case, i.e. where it has a perfect substitute.

The model is further generalized to the case of a renewable resource with a renewable resource. Again the transition to the steady state is governed by Ramsey and Hotelling conditions, and the latter contains a marginal externality cost term that is similar to the stock pollution case.

The Ramsey-Koopmans approach also resolves the paradox between optimizing growth with intergenerational equity and maximizing green national product. Moreover, when intergenerational equity is taken to mean Ramsey-Koopmans intergenerational neutrality, real national income is sustainable and constant in the optimal program, and the Weitzman "hypothetical stationery equivalent income" is actually attained at the golden rule steady state.

Appendix I

The problem is to minimize the difference between actual consumption and optimal consumption, subject to dynamic constraints on capital accumulation and resource use:

$$\text{Max} \quad V = \int_0^{\infty} [U(C_t) - U(\hat{C})]dt$$

$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad K(0) = K_0$$

$$\dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases}$$

$$X_t \geq 0 \qquad X(0) = X_0$$

The Hamiltonian expression for this problem is (for simplicity, the subscripts t's are dropped)

$$H = [U(C) - U(\hat{C})] + \text{ë}[F(K, R) - \text{ä}K - \text{è}(X)R - C] + \text{ø}[-R]$$

The standard first order conditions for this optimal control problem are:

$$(1) \qquad \frac{\partial H}{\partial C} = U_C - \lambda = 0$$

$$(2) \qquad \frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi = 0$$

$$(3) \qquad \frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta]$$

$$(4) \qquad \frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R$$

From the first and the third conditions,

$$(5) \qquad \dot{\lambda} = \dot{U}_C$$

and

$$(6) \qquad \dot{\lambda} = -\lambda[F_K - \delta]$$

Equating expressions for $\dot{\lambda}$ and rearranging yields

$$(7) \qquad -\frac{\dot{U}_C}{U_C} = F_K - \delta$$

or $$\eta(C)\frac{\dot{C}}{C} = F_K - \delta, \qquad \text{where } \eta(C) = -\frac{U_{CC}C}{U_C} > 0.$$

From the second necessary condition:

$$(8) \qquad \dot{\psi} = \dot{\lambda}[F_R - \theta(X)] + \lambda[\dot{F}_R - \theta_X \dot{X}]$$
$$= -\lambda(F_K - \delta)[F_R - \theta(X)] + \lambda[\dot{F}_R + \theta_X R]$$

From the fourth necessary condition,

$$(9) \qquad \dot{\psi} = \lambda\theta_X R$$

Equating expressions for $\dot{\psi}$ and rearranging yields

$$(10) \qquad -(F_K - \delta)[F_R - \theta(X)] + \dot{F}_R = 0$$

or

$$(11) \qquad F_R - \theta(X) = \frac{\dot{F}_R}{(F_K - \delta)}$$

Appendix II

The problem is:

$$\text{Max} \quad V = \int_0^{\infty} [U(C_t, E_t) - U(\hat{C}, 0)]dt$$

$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \delta K_t - \theta(X_t)R_t - C_t, \qquad K(0) = K_0$$

$$\dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases}$$

$$X_t \geq 0 \qquad X(0) = X_0$$

The Hamiltonian for this problem is (for simplicity, the subscripts t is dropped):

$$H = [U(C, E) - U(\hat{C}, 0)] + \lambda[F(K, R) - \delta K - \theta(X)R - C] + \psi[-R]$$

The standard necessary first order conditions for this optimal control problem are:

(1) $$\frac{\partial H}{\partial C} = U_C - \lambda = 0$$

(2) $$\frac{\partial H}{\partial R} = U_E + \lambda[F_R - \theta(X)] - \psi = 0$$

(3) $$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda(F_K - \delta)$$

(4) $$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R$$

From the first equation,

(5) $$\dot{\lambda} = \dot{U}_C$$

and from (3),

(6) $$\dot{\lambda} = -U_C[F_K - \delta]$$

Equating these two expressions for $\dot{\lambda}$ and rearranging gives

(7) $$-\frac{\dot{U}_C}{U_C} = F_K - \delta$$

Differentiating equation (2) with respect to time t,

$$(8) \qquad \dot{\psi} = \dot{U}_E + \dot{U}_C[F_R - \theta(X)] + U_C[\dot{F}_R + \theta_X R]$$

Equating with (4) to obtain:

$$(9) \qquad \frac{\dot{U}_E}{U_C} - (F_K - \delta)[F_R - \theta(X)] + \dot{F}_R = 0$$

Meanwhile,

$$(10) \qquad \frac{\dot{U_E}}{U_C} = (\frac{\dot{U}_E}{U_C}) + \frac{U_E}{U_C} * \frac{\dot{U}_C}{U_C} = (\frac{\dot{U}_E}{U_C}) - \frac{U_E}{U_C}(F_K - \delta)$$

Plugging back into (9),

$$(11) \qquad F_R - \theta(X) = \frac{\dot{F}_R}{F_K - \delta} + \frac{1}{F_K - \delta}(\frac{\dot{U}_E}{U_C}) - \frac{U_E}{U_C}$$

Appendix III

The problem is:

$$\text{Max} \quad V = \int_0^{\infty} [U(C_t, M_t) - U(\hat{C}, 0)]dt$$

$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad K(0) = K_0$$

$$\dot{X}_t = \begin{cases} -R_t, & t < T \\ 0, & t \geq T \end{cases}$$

$$\dot{M}_t = \begin{cases} R_t - \xi M_t, & t < T \\ -\xi M_t, & t \geq T \end{cases}$$

$$X_t \geq 0 \qquad X(0) = X_0$$

The Hamiltonian for this problem is (for simplicity, the subscripts t's are dropped),

$$H = [U(C, M) - U(\hat{C}, 0)] + \text{ë}[F(K, R) - \text{ä}K - \text{è}(X)R - C] + \text{ø}[-R] + \mu(R - \xi M)$$

The standard necessary conditions for this optimal control problem are:

(1) $$\frac{\partial H}{\partial C} = U_C - \lambda = 0$$

(2) $$\frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi + \mu = 0$$

(3) $$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta]$$

(4) $$\frac{\partial H}{\partial X} = -\dot{\psi} = -\lambda\theta_X R$$

(5) $$\frac{\partial H}{\partial M} = -\dot{\mu} = U_M - \mu\xi$$

From the first and the third conditions,

(6) $$\dot{l} = \dot{U}_C$$

and

(7) $$\dot{l} = -l[F_K - d]$$

Equating expressions for $\dot{\lambda}$ and rearranging yields

(8) $$-\frac{\dot{U}_C}{U_C} = F_K - d$$

From the second necessary condition:

(9) $$\dot{y} - \dot{m} = \dot{l}[F_R - q(X)] + l[\dot{F}_R + q_X R]$$

From the fourth and fifth necessary conditions,

(10) $$\dot{y} - \dot{m} = l q_X R + U_M - mx$$

Equating expressions on the RHS of (9) & (10), plugging in the expression for $\dot{l}$, and rearranging yields

(11) $$F_R - q(X) = \frac{\dot{F}_R}{F_K - d} - \frac{U_M - mx}{U_C} * \frac{1}{F_K - d}$$

Appendix IV

The problem is:

$$\text{Max} \quad V = \int_0^{\infty} [U(C_t, X_t) - U(\hat{C}, \hat{X})]dt$$

$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad K(0) = K_0$$

$$\dot{X}_t = G(X_t) - R_t, \qquad X(0) = X_0$$

$$X_t \geq 0$$

The Hamiltonian for this problem is (for simplicity, the subscripts t's are dropped),

$$H = [U(C, X) - U(\hat{C}, \hat{X})] + \text{ë}[F(K, R) - \text{ä}K - \text{è}(X)R - C] + \text{ø}[G(X) - R]$$

The standard necessary conditions for this optimal control problem are:

$$(1) \qquad \frac{\partial H}{\partial C} = U_C - \lambda = 0$$

$$(2) \qquad \frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi = 0$$

$$(3) \qquad \frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta]$$

$$(4) \qquad \frac{\partial H}{\partial X} = -\dot{\psi} = U_X - \lambda\theta_X R + \psi G_X$$

From the first and the third conditions,

$$(5) \qquad \dot{\lambda} = \dot{U}_C$$

and

$$(6) \qquad \dot{\lambda} = -\lambda[F_K - \delta]$$

Equating expressions for $\dot{\lambda}$ and rearranging yields

$$(7) \qquad -\frac{\dot{U}_C}{U_C} = F_K - \delta$$

From the second necessary condition:

(8) $$\dot{\psi} = \dot{U}_C[F_R - \theta(X)] + U_C[\dot{F}_R - \theta_X G(X) + \theta_X R]$$

From the fourth necessary condition,

(9) $$\dot{\psi} = -U_X + U_C \theta_X R - U_C[F_R - \theta(X)]G_X$$

Equating expressions for $\dot{\psi}$ and rearranging yields

(10) $$F_R - \theta(X)(F_K - \delta) = \dot{F}_R + [F_R - \theta(X)]G_X - \theta_X G(X) + \frac{U_X}{U_C}$$

Appendix V

For a general case that combines the models of sections 3.2 and 3.3, consider the following problem:

$$\text{Max} \quad V = \int_0^{\infty} [U(C_t, X_t, M_t) - U(\hat{C}, \hat{X}, \hat{M},)]dt$$

$$\text{s.t.} \quad \dot{K}_t = F(K_t, R_t) - \text{ä}K_t - \text{è}(X_t)R_t - C_t, \qquad K(0) = K_0$$

$$\dot{X}_t = G(X_t) - R_t,$$

$$\dot{M}_t = R_t - \xi M_t,$$

$$X_t \geq 0 \qquad X(0) = X_0$$

$$M_t \geq 0 \qquad M(0) = M_0$$

The Hamiltonian is (for simplicity, the subscripts t's are dropped),

$$H = [U(C, X, M) - U(\hat{C}, \hat{X}, \hat{M})] + \text{ë}[F(K, R) - \text{ä}K - \text{è}(X)R - C] + \psi[G(X) - R] + \mu(R - \xi M)$$

with first order conditions:

(1) $$\frac{\partial H}{\partial C} = U_C - \lambda = 0$$

(2) $$\frac{\partial H}{\partial R} = \lambda[F_R - \theta(X)] - \psi + \mu = 0$$

(3) $$\frac{\partial H}{\partial K} = -\dot{\lambda} = \lambda[F_K - \delta]$$

(4) $$\frac{\partial H}{\partial X} = -\dot{\psi} = U_X - \lambda\theta_X R + \psi G_X$$

(5) $$\frac{\partial H}{\partial M} = -\dot{\mu} = U_M - \mu\xi$$

Now, on the optimal trajectory leading to steady state, consider

(6) $$H_t = [U(C_t, X_t, M_t) - U(\hat{C}, \hat{X}, \hat{M})] + \lambda_t \dot{K}_t + \psi_t \dot{X}_t + \mu_t \dot{M}_t$$

We take the time differentiation, for simple presentation ignoring the t's,

$$\frac{dH_t}{dt} = U_C \dot{C} + U_X \dot{X} + U_M \dot{M} + \dot{\lambda} \dot{K} + \lambda[F_K \dot{K} + F_R \dot{R} - \delta \dot{K} - \theta_X R \dot{X} - \theta(X) \dot{R} - \dot{C}]$$

$$+ \dot{\psi} \dot{X} + \psi[G_X \dot{X} - \dot{R}] + \dot{\mu} \dot{M} + \mu[\dot{R} - \xi \dot{M}]$$

$$= [U_C - \lambda] \dot{C} + [U_M + \dot{\mu} - \mu\xi] \dot{M} + [\dot{\lambda} + \lambda(F_K - \delta)] \dot{K} + [U_X - \lambda\theta_X R + \dot{\psi} + \psi G_X] \dot{X}$$

$$+ \{\lambda[F_R - \theta(X)] - \psi + \mu\} \dot{R}$$

$$= 0$$

where the last step utilizes the above five first order conditions.

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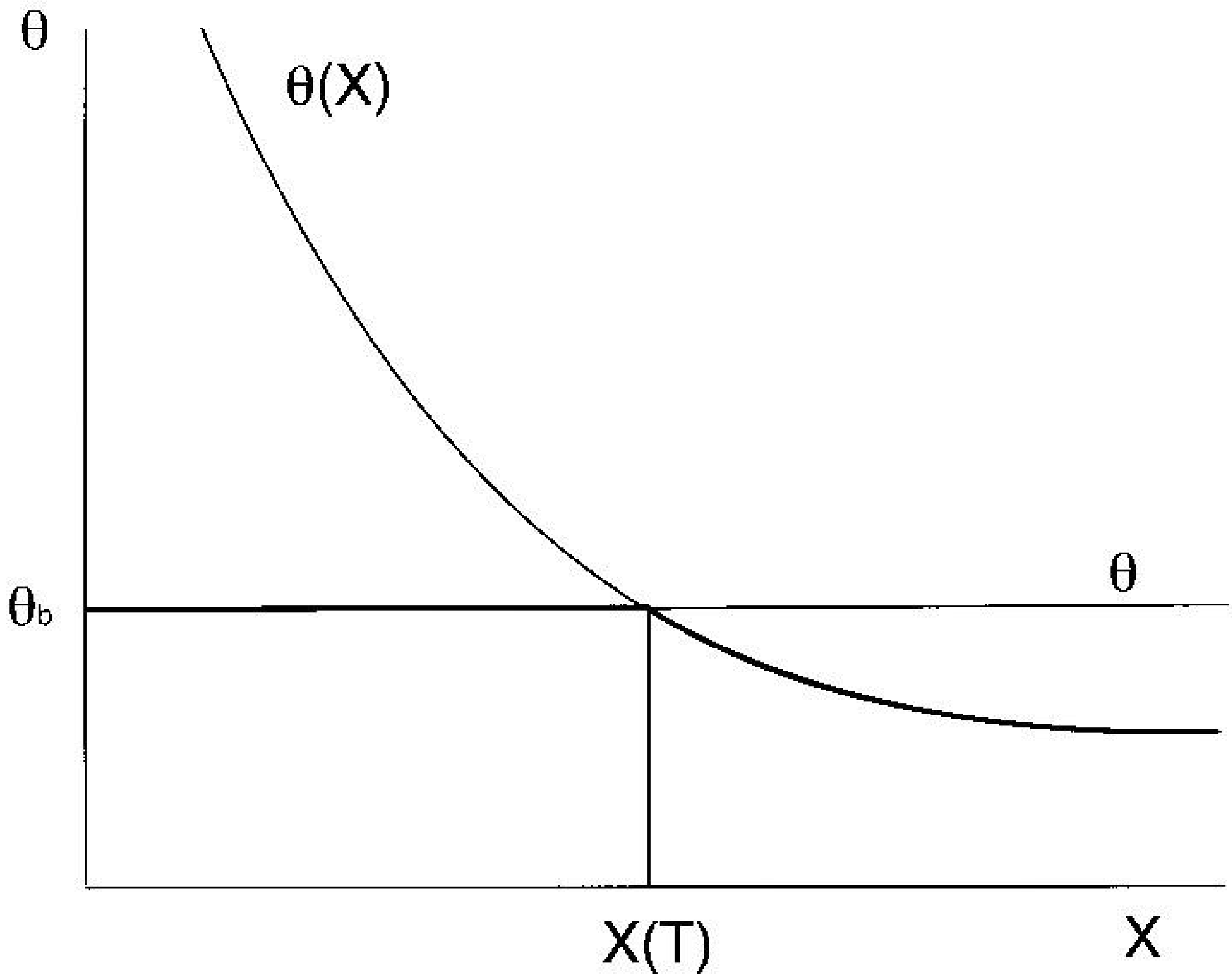

Figure 1

Marginal Extraction Cost Locus
is the Lower Envelope of
$\theta(X)$ and θ_b

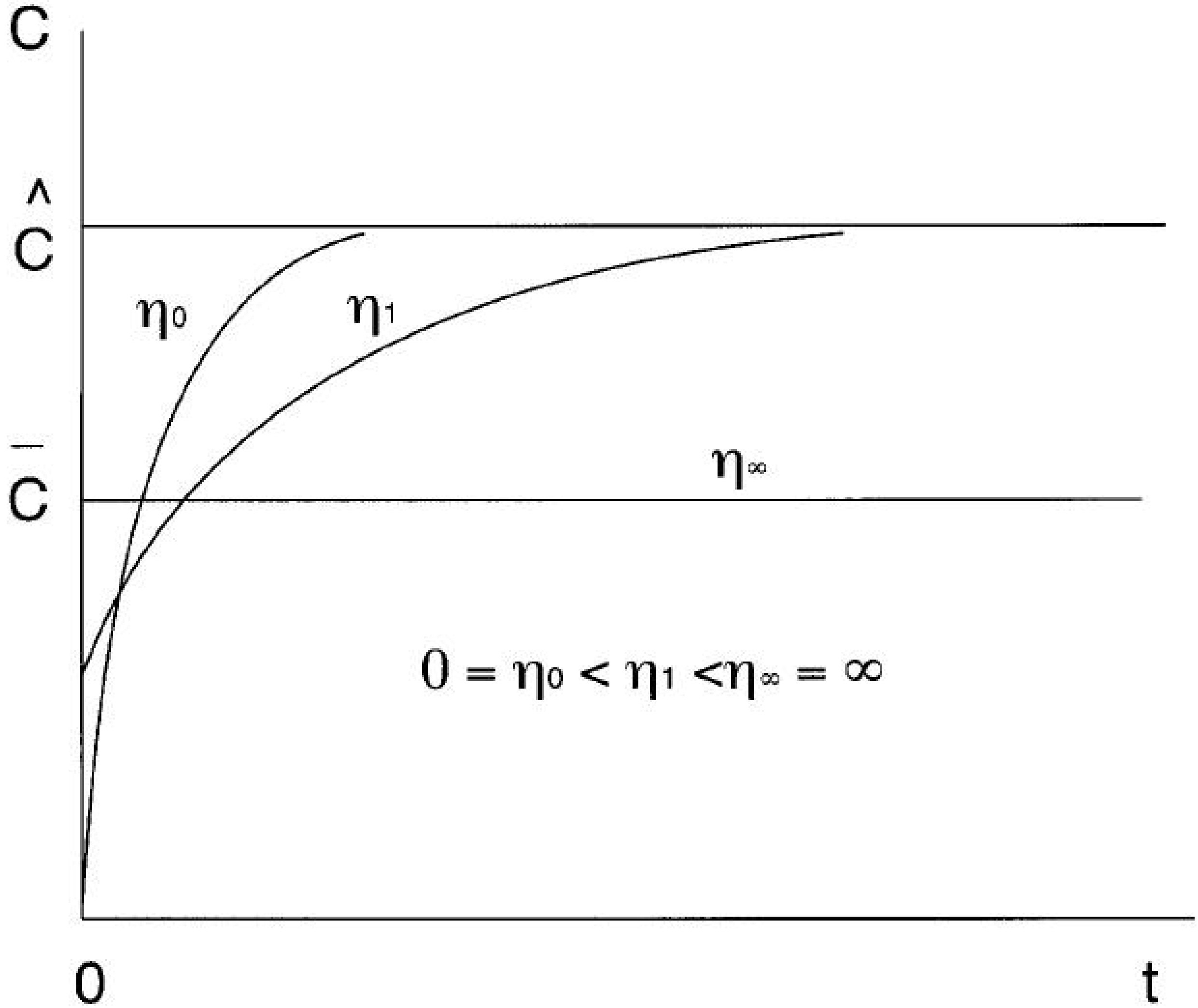

Figure 2

Social Impatience and the
Optimal Consumption Trajectory

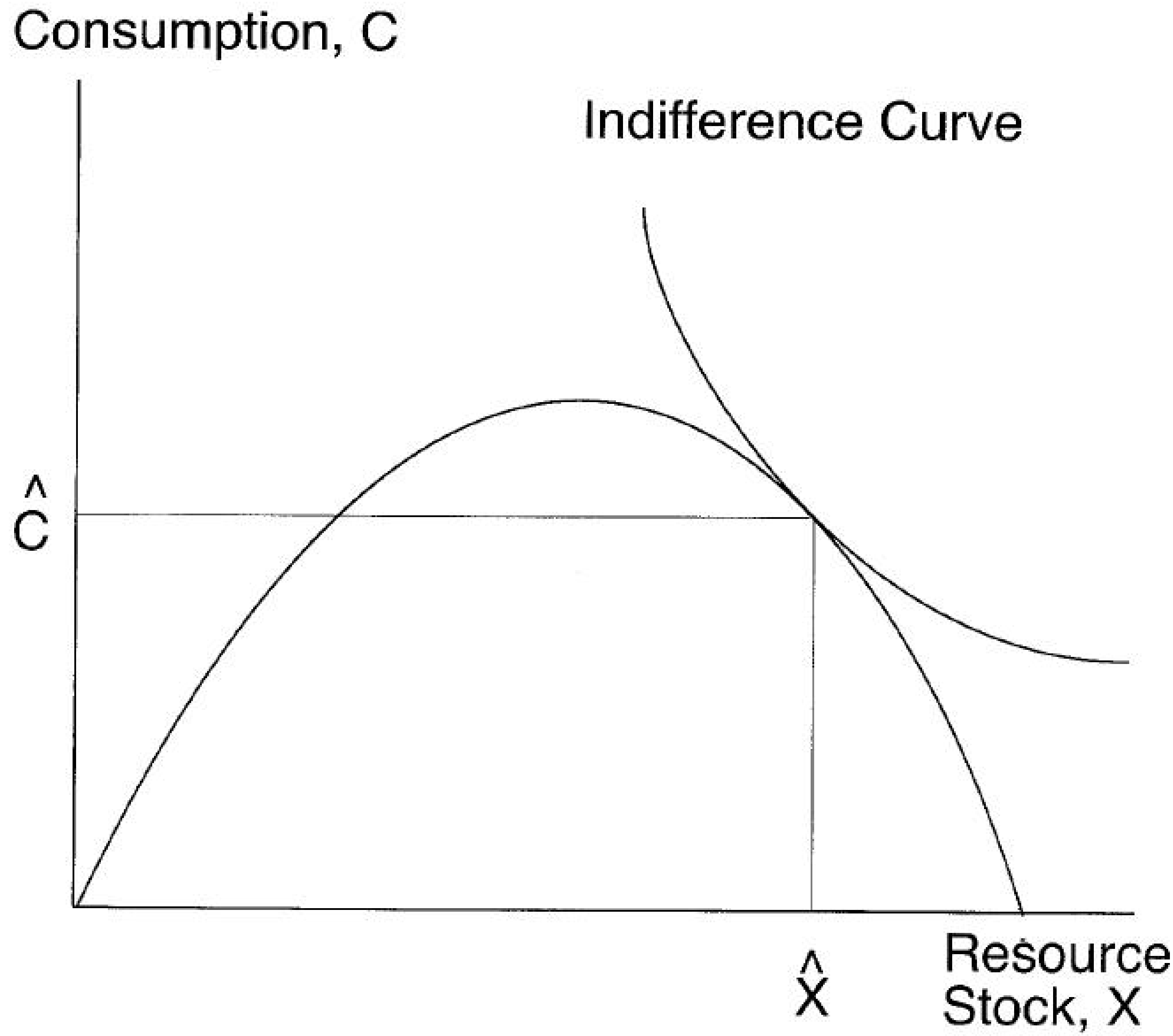

Figure 3

Ramsey-Koopman-Heal Green
Golden Rule Steady State
with Environmental Amenities

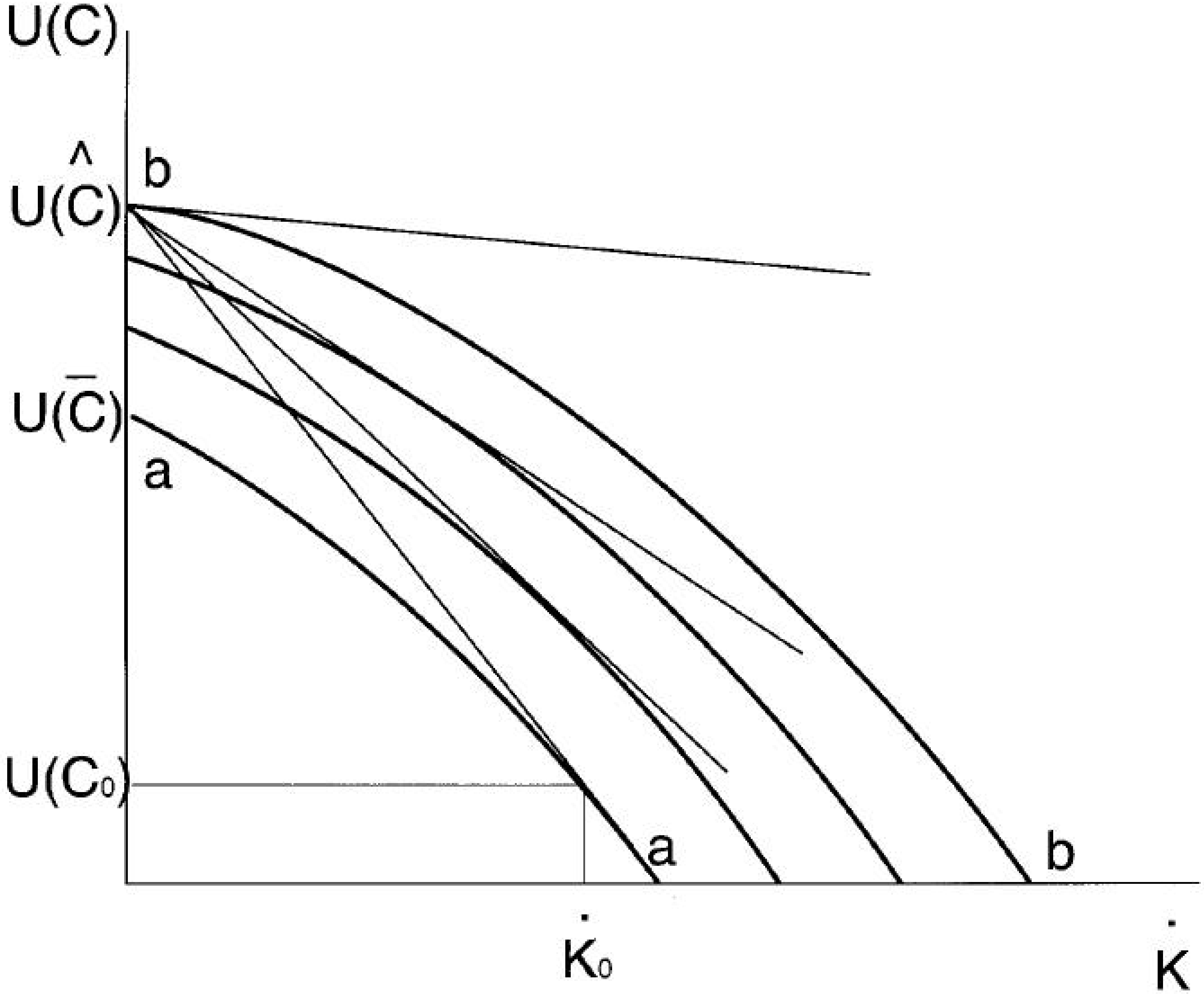

Figure 4

NNP Remains Constant at
its Golden Rule Level