An Analysis of Financial and Nonfinancial Prepayment of GNMA Securities with a Varying Coefficient Model

*James B. Kau**
*Thomas M. Springer***

Abstract. **This paper develops a model that examines the financial and nonfinancial prepayment of GNMA securities. A varying coefficient model depicts prepayment as a dynamic process, allowing for changes in factors, reflecting differences in time, debt and borrowers' characteristics. This model provides a means for systematically incorporating hypothesized effects of nonfinancially motivated prepayment while isolating financially induced calls on the debt. Also, the model captures the impact on prepayment of interactions between financial and nonfinancial variables.**

Introduction

The ability of the borrower to prepay the underlying debt, thereby terminating the contract and causing an unscheduled return of principal, has a major impact upon the value of pooled mortgage securities. Optimal or financially induced prepayment, assuming no transaction costs, occurs when the outstanding loan balance is less than the present value of the payment stream. Transaction costs increase the magnitude of the spread between the debt contract rate and the market interest rate that is necessary to make prepayment economically feasible (Dunn and Spatt, 1986). Borrowers do not always prepay when it is optimal and may prepay in the absence of financial optimality. Suboptimal or nonfinancially induced prepayment occurs for reasons other than the relationship between the debt contract rate and the market interest rate. Examples of motivations for suboptimal prepayment include relocation, desire for increased housing stock, divorce, and destruction of the collateral. Pooled debt, such as securitized mortgages, experiences enough variability in suboptimal prepayment and subsequently in value to warrant inquiry.

The purpose of this study is to complement current research to more fully understand the nature of callable debt instruments. Procedures presented in the paper allow for the separate measurement of the impacts of financial and nonfinancial incentives on prepayment.

A prepayment-incentive relationship or, more generally, an incentive gradient is used to explain prepayment behavior. A standard functional form used for empirical studies (Green and Shoven, 1986; Schwartz and Torous, 1989) assumes the equation for the incentive gradient is

*Department of Real Estate, University of Georgia, Athens, Georgia 30602.
**College of Business, Florida Atlantic University, University Tower, 220 SE 2nd Avenue, Fort Lauderdale, Florida 33301.
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$$\ln P(I) = \ln F_0 + gI, \tag{1}$$

which is the natural log of the exponential

$$P^{gI}(I) = F_0 e, \tag{2}$$

where $P(I)$ is the annualized monthly prepayment rate at incentive I, F_0 is the baseline level of prepayments, I is a measure of the incentive to prepay, and g is the incentive gradient, measuring the percentage increase in prepayment as I increases. Previous valuation models (as a standard, see Dunn and McConnell, 1981) have explained call intensity by financial motivation with modifications incorporated to include suboptimal prepayment.

This paper proposes an alternative method for analyzing the variable nature of prepayment. The varying coefficient model (VCM) depicts prepayment as a dynamic process, allowing for changes in factors, reflecting differences in time, debt, and borrower characteristics. Using the exponential prepayment-incentive function as a base, the VCM provides a means for systematically incorporating hypothesized effects of nonfinancially motivated prepayment while isolating financially induced calls on the debt. Since a number of the incentive factors exhibit high secondary relationships with time, the VCM also represents a basis for sharpening existing forecasting tools. Thus, by modifying the exponential form used by Green and Shoven (1986) and Schwartz and Torous (1989), we are able to extract additional information from the data.

Literature Review

Debt prepayment and its implications have received considerable attention in the literature, especially since the decline in lending rates in the early eighties. Dunn and Spatt (1986) examine the underlying theory behind optimal prepayment (refinancing) and define various properties and bounds. Foster and Van Order (1985) include a discussion on prepayment in general and its importance in the valuation process.

Many recent studies, both from academia and from Wall Street, have focused upon the prepayment behavior of mortgage-backed securities (MBSs). Bartholomew, Berk and Roll (1988) examine the influence of age, season, borrower's expectations, borrower heterogeneity and the ratio of the contract interest rate to the market interest rate as a measure of the economic incentive to refinance. Green and Shoven (1986) look at length of tenure and the benefit of refinancing as a percentage of house value. Schwartz and Torous (1989) examine season, age, remaining principal and the difference between the contract interest rate and the appropriate Treasury rate. Quigley (1987) studies non-prepayment and delayed decisionmaking by homeowners, confirming decreased mobility when mortgagors are locked into favorable borrowing rates. Other studies (for example, Peters et al., February, May, 1984) look at many of these same characteristics.

Differences in model specification, theory and empirical results in the studies cited above indicate that the simple exponential function (equation 2) is misspecified. An equally plausible interpretation suggests the possibility that additional specializing arguments are required to measure time varying and debtholder differences in the prepayment-incentive function. For example, variations in financial and nonfinancial prepayment may result from nonhomogeneous characteristics of borrowers, contract rates on debt not independent of maturity, the season of the year for termination and expectations of the time of optimal prepayment given new information on current contract rates. The simple exponential function cannot accommodate these variations. The VCM model presented in section four allows for a dynamic process for changes in the factors of prepayment.

The Data

Data employed consist of prepayments for a sample of twenty-seven GNMA thirty-year, fixed-rate, pass-through securities for 114 months from January 1978 to June 1987.[1] Two corresponding sets of additional data are also used. The first consists of the mortgage-backed, security-specific variables, *SEASON* (X_1) and *AGE* (X_2), which are expected to capture nonfinancial prepayment. The second set of concomitant data is time dependent and designed to explain differences in prepayment caused by variations in past rates on debt contracts, thus accounting for the effects of financially motivated incentives to prepay.[2] The data are summarized in Exhibit 1.

Exhibit 1
Characteristics of the Prepayment Data Used to Estimate the Prepayment-Incentive Function[a]

	Mean	Standard Deviation	Minimum	Maximum	Units of Measure
Prepayment rate[b]	10.2265	13.9866	.100	69.900	Annualized percentage
Age[c]	3.8658	2.5840	.020	11.350	Years since origination
AGE (X_2)[d]	16.2443	6.0622	.260	22.610	Annualized percentage
TREND (X_3)	.0348	.0450	−.018	.163	Difference of ratios
NEWMIN (X_4)	.0035	.0090	.000	.053	Ratio
INCENTIVE	.0466	.0942	.000	.534	Ratio

Correlation Matrix

	Prepayment rate	*Age*	*AGE* (X_2)	*TREND* (X_3)	*NEWMIN* (X_4)	*INCENTIVE*
Prepayment rate[b]	1.000					
Age[c]	.022	1.000				
AGE (X_2)[d]	.286	.019	1.000			
TREND (X_3)	.342	.203	.027	1.000		
NEWMIN (X_4)	.521	−.066	.048	.106	1.000	
INCENTIVE	.884	−.060	.279	.268	.470	1.000

[a]N = 2029 observations.
[b]annualized monthly prepayment rates observed for GNMA securities backed by fixed-rate mortgages
[c]observed weighted average maturity for GNMA mortgage pools
[d]annualized monthly prepayment as predicted using a cubic function of *AGE*
Source: Authors' calculations

SEASON (X_1) accounts for the fact that more relocations take place in spring and summer, relative to the rest of the year. This variable takes on the value of 1 if the observed month is from April to August and 0 otherwise. The associated coefficient is expected to have a positive value. *AGE* (X_2) is a two-stage variable that emulates several of the existing prepayment models. Similar to the variables used by Schwartz and Torous (1989) and Green and Shoven (1986), prepayment starts slow, increases to a peak and then declines as a mortgage pool ages. The *AGE* variable used, following the work of Roberts (1987), allows prepayment to increase once again.[3] *AGE* is assumed to model prepayment independent of the influence of changing interest-rate levels. It does this by imitating the existing historical-based mortgage prepayment relationships, for example the Public Securities Association (PSA) model, emphasizing the higher prepayment expected for younger as opposed to more seasoned securities. *AGE* recognizes the mobility of the population,[4] income effects and other demographic factors not explained by other variables (Roberts, 1987). *SEASON* and *AGE* are nonfinancial variables. Yet, the relocation of homeowners and the formation of new households often take place in conjunction with economic reasons for debt prepayment. Job transfers and other causes of relocation associated with *SEASON* and *AGE* are often assumed independent of interest-rate levels, but debtholders may postpone nonfinancial prepayment because of financial considerations. A variable is introduced later in this section to capture the interaction between *AGE* and changes in interest-rate levels.

Expectations or *TREND* (X_3) takes on the values of:

$$\ln[R_t/R_{(t-1)}] - \ln[R_{(t-1)}/R_{(t-7)}] \text{ if } R_{(t-1)} < R_{(t-7)}$$

$$0 \text{ otherwise,}$$

where R is the market interest rate and t is the relevant month.[5] This variable captures the effect of borrower's expectations of future interest rates. If interest rates exhibit an increasing six-month trend, that is $R_{(t-1)} > R_{(t-7)}$, the variable takes a value of zero. If interest rates exhibit a decreasing six-month trend, the variable assumes a nonzero value according to whether the short-term (one-month) percentage change is greater (negative value) or lesser (positive value) than the long-term (six-month) percentage change in interest rates. Finally, if the long-term trend is decreasing, but the short-term trend is increasing, the variable is positive. The greatest positive value occurs when downtrends are perceived to be bottoming out. The expected value of the estimated coefficient is positive.

NEWMIN (X_4) compares the lowest market interest rate observed since the origination of the security with the current market rate. If a new minimum market rate is encountered, the greater the percentage by which it is lower than the previous minimum, the greater the value of this variable. *NEWMIN* is defined as the greater of zero or,

$$\ln[\min(R_{(0-t-1)})/R_t],$$

where t is the current time period.[6] *NEWMIN* accounts for the effect that new lower levels of refinancing rates have on borrowers who failed to prepay previously. This variable has a positive expectation.

The variable *TREND* × *NEWMIN* (X_3X_4) is a financial interaction variable designed to account for differing behavior of debtholders under different interest-rate environments.[7] This variable is expected to augment the understanding of prepayment relationships as explained by *NEWMIN* or *TREND* separately. *AGE* × *TREND* (X_2X_4) is a financial interaction variable designed to measure the impact of debtholders' expectations on nonfinancial prepayment decisions.[8] This variable is expected to better describe the prepayment predicted by *AGE* or *SEASON* separately. Additional interaction effects between the posited variables and the incentive variable, which will be introduced later, are introduced into the model in the "gradient" equation.

To account for the frictions of decisionmaking and refinancing, a two-month lag is used on all independent variables that derive their value from interest-rate levels. Other studies have used lags ranging from one to three months to account for friction in the marketplace. The two-month lag allows for the time required for a borrower to make a decision and for a result to transpire.

VCM and the Estimation Method

The VCM approach uses the standard natural logarithm of the exponential function but introduces systematic parameter changes. The parameters of the prepayment-incentive function ($\ln F_0$ and I) are hypothesized to vary as a result of the interplay of time-varying and debt-conditioning variables that reflect financial and nonfinancial factors. The incentive to prepay, or I, in the VCM model, defined as the benefit of refinancing as a percentage of the debt obligation, specifically measures economically optimal prepayment motivated by favorable refinancing rates. The benefit of refinancing is the present value of the remaining payment stream discounted at the market interest rate less the outstanding principal. A positive relationship exists between I and prepayment levels. For similar purposes, Green and Shoven (1986) use a variable, *Lock-In*, which compares the prepayment benefit to the property value. *Lock-In* fails to separate the effect of the increases in value arising from declining mortgage refinancing rates, from the equity increases due to property appreciation and amortization of the mortgage principal. The incentive variable (I) does not consider the appreciation of the property. It only measures economically optimal prepayment motivated by favorable refinancing rates. By disregarding property appreciation, the variable does not make the implicit assumption that all properties underlying the mortgages behave alike; nor does it account for the influence of equity build-up.[9] The model of Bartholomew et al. (1988) specifies the logarithm of the annualized monthly prepayment rate two months in the future as an additive function of a set of variables. Green and Shoven (1986) and Schwartz and Torous (1989) use proportional hazards models that have the same general specification; however, their formulations recognize a base prepayment level (baseline hazard function). This baseline corresponds to F_0 in equation (2).

The a priori basis for determining the functional form that relates the parameters of the exponential prepayment-incentive function to the conditioning variables is limited. Given limited information, a VCM with a polynomial structure for parameter changes is posited. Arbitrarily, the polynomials relating $\ln F_0$ and I to the conditioning variables X_1, X_2, X_3, and X_4 are assumed of first and second order, respectively. Even with only four conditioning variables the possible specifications due to the number of different interaction terms are large. Added restrictions based on the intended model uses (see Wallace and Asher, 1972) make the specification more manageable.

Letting i denote time, and j the differing coupon securities, the VCM is

$$\ln P(I)_{ij} = \ln F_0 + gI_{ij} + \xi_{ij} \tag{3}$$

for 2029 observations. The additive error term is ξ_{ij}. Incorporating the assumptions yields two variable coefficient equations,

$$\ln F_0 = B_0^0 + B_1^0 X_1 + B_2^0 X_2 + B_3^0 X_2^2 + B_4^0 X_3 + B_5^0 X_3^2 + B_6^0 X_4 + B_7^0 X_4^2 + B_8^0 X_3 X_4 + B_9^0 X_2 X_3 \tag{4}$$

and

$$g = B_0^1 + B_1^1 X_1 + B_2^1 X_2 + B_3^1 X_2^2 + B_4^1 X_3 + B_5^1 X_3^2 + B_6^1 X_4 + B_7^1 X_4^2 + B_8^1 X_3 X_4 + B_9^1 X_2 X_3. \tag{5}$$

The model as specified in (3), (4) and (5) involves coefficient restrictions across time and securities. Thus pooling is required which leads to heteroscedasticity. This is corrected by generalized least squares.[10] The variable coefficient structure presented in (4) and (5) is substituted into (3) to reparameterize the natural logarithm of the exponential prepayment incentive function, resulting in the VCM.

The advantages of the VCM provided by (4) and (5) combined with the log-linear incentive function hypothesis are apparent. The VCM generates financial and nonfinancial prepayment results but within the context of a functional form that has empirical support. Moreover, the flexibility of the VCM would appear to make any single-variable functional form more useful for financial analysis and prediction. Since the selected financial and nonfinancial incentive characteristics may be projected on the basis of time, the model is functional for both forecasting and financial analysis because the estimates are from cross-sectional and time series data.[11]

Empirical Results

Exhibit 2 presents the estimates for the pooled data based on equations (4) and (5). Coefficients for baseline prepayment and the incentive gradient are quadratic functions of the conditioning variables *AGE* (X_2), *TREND* (X_3) and *NEWMIN* (X_4) and linear functions for *SEASON* (X_1) and the interaction terms, *TREND* × *NEWMIN* (X_3X_4) and *AGE* × *TREND* (X_2X_3). Examination of significant levels of the estimated parameters indicates that the variables are important in shifting baseline prepayment and the incentive gradient from security to security and between financial and nonfinancial variables.

Results for *TREND* are significant and show that baseline prepayment is greater as *TREND* becomes increasingly positive. The negative quadratic term has minor influence within the range exhibited by *TREND*. Estimates on the constant, linear, and quadratic terms for the incentive coefficient, g, are 1.4185, −16.2428, and 58.0995, respectively, and statistically significant. The impact of *TREND*, which measures the impact of

Exhibit 2
Prepayment-Incentive Function Estimated with Coefficients Jointly Conditioned on Selected *SEASON, AGE, TREND,* and *NEWMIN* Variables[a,b]

Conditioning Variable	Constant Coefficient ($\ln F_0$)[c,d]			Incentive Coefficient (g)[c]		
	Constant	Linear(X_k)	Quadratic(X_k^2)	Constant	Linear(X_k)	Quadratic(X_k^2)
SEASON	.0224	.0186		1.4185	.0675	
((X_1)	(2.43)	(2.58)		(3.65)	(.68)	
AGE	.0224	.0192	.0029	1.4185	.2992	−.0162
(X_2)	(2.43)	(1.44)	(4.45)	(3.65)	(1.42)	(−3.22)
TREND	.0224	2.6949	−10.4096	1.4185	−16.2428	58.0995
(X_3)	(2.43)	(4.43)	(−2.36)	(3.65)	(−3.78)	(2.37)
NEWMIN	.0224	33.5294	−474.8681	1.4185	−182.9919	3809.4597
(X_4)	(2.43)	(10.37)	(−5.68)	(3.65)	(−9.83)	(8.22)
TREND × *NEWMIN*	.0224	−100.5098		1.4185	555.5881	
(X_3X_4)	'2.43)	(−4.13)		(3.65)	(4.51)	
AGE × *TREND*	.0224	−.4141		1.4185	1.8743	
(X_2X_3)	(2.43)	(−2.46)		(3.65)	(2.10)	

[a]values in parentheses are estimated student's t-statistics.
[b]the coefficient of determination for the regression used for these parameter estimates was .59.
[c]constant, linear, and quadratic parameters for the indicated conditioning variables on $\ln F_0$ and g (the prepayment incentive function)
[d]the index k takes on values, 1, 2, 3 and 4, indicating *SEASON, AGE, TREND* and *NEWMIN*, respectively.
Source: Authors' analysis

debtholders' expectations of future interest rates, first decreases then increases the prepayment gradient.

The above discussion on *TREND* and other results shown in Exhibit 2 are more precisely interpreted by examining the sample data. The estimated incentive gradient coefficients for *TREND* at a very positive (.1) value, at the sample mean (.03483) and at a slightly negative (−.015) value are

$$g = 1.4185 - 16.2428(.1) + 58.0995(.1)^2 = .3752,$$

$$g = 1.4185 - 16.2428(.03483) + 58.0995(.03483)^2 = .9232,$$

$$g = 1.4185 - 16.2428(-.015) + 58.0995(-.015)^2 = 1.6752.$$

Thus, other things equal, securities experiencing a downward trend in interest rates demonstrate an increasing responsiveness to changes in the incentive (I) variable as the decline in interest rates accelerates. For example, if the relevant interest rate six months ago was 12.00%, last month's rate was 11.50% and the current rate is 11.00%, *TREND* has a value of

$$TREND = \ln(11.00/11.50) - \ln(11.50/12.00) = -.00189,$$

with an impact on baseline prepayment of

$$\ln F_0 = .0224 + 2.6949(-.00189) - 10.4096(-.00189)^2 = .0173,$$

and an impact on the incentive gradient of

$$g = 1.4185 - 16.2428(-.00189) + 58.0995(-.00189)^2 = 1.4494.$$

The baseline prepayment (F_0) predicted by *TREND* alone is 1.02%. For the average incentive to prepay ($I = .046$), the prepayment predicted by *TREND*'s effect on g alone, is .07%. The total impact of *TREND* for an average incentive to prepay ($I = .046$) is 1.09%.

If, on the other hand, a downward trend in rates is reversing, the expected prepayment is different. For example, if the interest rate six months ago was 12.00%, last month's rate was 11.50% and the current rate is 11.75%, then *TREND* has a value of

$$TREND = \ln(11.75/11.50) - \ln(11.50/12.00) = .06407.$$

TREND's impact on baseline prepayment is

$$\ln F_0 = .0224 + 2.6949(.06407) - 10.4096(.06407)^2 = .1523$$

and its impact on the incentive gradient is

$$g = 1.4185 - 16.2428(.06407) + 58.0995(.06047)^2 = .5903.$$

The baseline expected prepayment (F_0) is now 1.16%. With an average incentive ($I = .046$), total prepayment increases to 1.20%. Thus, a change in *TREND* from an accelerating decline to an apparent bottoming out results in an increase in baseline prepayment accompanied by a decrease in the responsiveness to economic incentives to prepay.

From Exhibit 2 observe that the prepayment function coefficients conditioned on *NEWMIN* are all significant. The mean value (.003488) indicates that in most situations the new minimum interest rate encountered is not substantially different from the previously encountered minimum. The positive quadratic term suggests that for a decrease in rates greater than .03906 (a 3.9% drop in rates) the incentive gradient increases. For example, if the historic minimum interest rate over the life of the security was 9.50% and the current rate has dropped to 9.00% *NEWMIN* has a value of

$$NEWMIN = \ln(9.50/9.00) = .05407,$$

with an impact on baseline prepayment of

$$\ln F_0 = .0224 + 33.5294(.05407) - 474.8681(.05407)^2 = .4470,$$

and an impact on the incentive gradient of

$$g = 1.4185 - 182.9919(.05407) + 3809.4597(.05407)^2 = 2.6613.$$

For this drop in interest rates (a decline of 5.5%) the expected baseline prepayment based on *NEWMIN* alone is 1.56%. With an average incentive ($I = .046$), total prepayment increases to 1.77%.

If, on the other hand, the new minimum rate is 9.25%, corresponding to a 2.7% decline, the value of *NEWMIN* decreases to

$$NEWMIN - \ln(9.50/9.25) = .02667,$$

with an impact on baseline prepayment of

$$\ln F_0 = .0224 + 33.5294(.02667) - 474.8681(.02667)^2 = .5788,$$

and an impact on the incentive gradient of

$$g = 1.4185 - 182.9919(.02667) + 3809.4597(.02667)^2 = -.7523.$$

Now, the baseline prepayment based on *NEWMIN* alone is 1.78%. For the average incentive to prepay ($I = .046$), the impact of a change in the slope (g) of the prepayment function is a negative .061%. This results in a decrease in total prepayment to 1.72%. As the magnitude of decline in the minimum observed interest rate increases, baseline prepayment increases to a maximum (at $NEWMIN = .0353$) and then decreases. The incentive gradient (g) decreases to a minimum (at $NEWMIN = .0240$) and then increases. At high levels of *NEWMIN*, despite a high value of the incentive gradient g, the expected prepayment for any level of I is near zero.

AGE and *SEASON* are nonfinancial conditioning variables. Results for the prepayment functions on *AGE* show greater baseline prepayment for recently issued securities. The negative sign on the quadratic term for the incentive gradient indicates that older securities tend to be less responsive to changes in incentive. For *SEASON*, the signs on the conditioning variables indicate greater prepayment during the spring and summer. Both interaction terms are statistically significant and act counter to the impact of *TREND* and *NEWMIN* on both the prepayment baseline and the gradient.

In summary, for the baseline coefficient, $\ln F_0$, estimated parameters on the linear term demonstrate that baseline prepayment increases with increased *TREND, NEWMIN, AGE* and *SEASON*, with the quadratic terms on *TREND* and *NEWMIN* indicating a decline for greater values. Parameter estimates on the linear and quadratic terms for g show that the incentive gradient increases for *SEASON*. For *AGE*, g increases at a decreasing rate, then decreases. Estimates for *TREND* and *NEWMIN* indicate that g decreases at a decreasing rate, then increases. These results demonstrate that financial and nonfinancial impacts on the incentive gradient are quite different.

An alternative way to assess this version of the VCM is to evaluate the model for each of the securities included. This was done for the indicated conditioning variables at

Exhibit 3[a]
Estimates of the Incentive Function Coefficients Based on the VCM (Tot = Total, Fin = Financial and Non = Nonfinancial Prepayment, respectively)[b]

Coupon	Mat.	Type	$\ln F_0$	g	P	Coupon,	Mat.	Type	$\ln F_0$	g	P
7.5	2007	Tot	1.4377	1.6480	4.54	12	2013	Tot	1.2864	1.6750	3.91
		Fin	.0075	1.3832	1.07			Fin	.1662	.4914	1.21
		Non	1.5982	1.0242	5.18			Non	1.3696	1.5749	4.23
8	2006	Tot	1.2008	2.1666	3.67	12	2014	Tot	1.1482	1.7064	3.41
		Fin	.0075	1.3832	1.07			Fin	.2014	.3411	1.25
		Non	1.3389	1.6446	4.11			Non	1.1139	2.1167	3.36
8	2007	Tot	1.5658	1.3387	5.09	12.5	2010	Tot	1.8182	.4507	6.29
		Fin	.0075	1.3832	1.07			Fin	.1233	.7399	1.17
		Non	1.7376	.6638	5.86			Non	1.9452	.0979	7.03
8	2008	Tot	1.6766	1.0575	5.61	12.5	2013	Tot	1.3277	1.3793	4.02
		Fin	.0075	1.3832	1.07			Fin	.2056	.3390	1.25
		Non	1.8578	.3400	6.51			Non	1.3103	1.7083	4.01
8.25	2008	Tot	1.7086	.9765	5.77	12.5	2014	Tot	1.0664	1.8203	3.16
		Fin	.0087	1.3736	1.07			Fin	.2041	.3411	1.25
		Non	1.8962	.2342	6.73			Non	1.0179	2.2947	3.08
8.5	2006	Tot	.9907	2.5528	3.03	13	2011	Tot	1.7843	.5039	6.10
		Fin	.0075	1.3832	1.07			Fin	.1387	.6619	1.18
		Non	1.1067	2.1306	3.34			Non	1.8924	.2449	6.71
9	2008	Tot	1.7585	.8203	6.03	13	2012	Tot	1.5132	1.0580	4.77
		Fin	.0205	1.3050	1.08			Fin	.2149	.2811	1.26
		Non	1.9449	.0988	7.02			Non	1.5563	1.1290	4.99
9	2009	Tot	1.7876	.7106	6.17	13	2013	Tot	1.2462	1.7439	3.77
		Fin	.0364	1.2131	1.10			Fin	.1606	.5232	1.20
		Non	1.9694	.0300	7.18			Non	1.3170	1.6935	4.03
9.5	2009	Tot	1.8001	.6604	6.24	13	2014	Tot	.9776	1.8690	2.90
		Fin	.0475	1.1476	1.11			Fin	.2319	.2097	1.27
		Non	1.9823	.0064	7.26			Non	.8971	2.4942	2.75
10	2009	Tot	1.8115	.5836	6.29	13.5	2011	Tot	1.7920	.4390	6.12
		Fin	.0707	1.0176	1.12			Fin	.1660	.5226	1.21
		Non	1.9818	.0052	7.25			Non	1.8845	.2667	6.66
11	2010	Tot	1.8047	.5669	6.24	13.5	2014	Tot	1.0835	1.7036	3.20
		Fin	.0866	.9318	1.14			Fin	.2518	.1278	1.29
		Non	1.9623	.0498	7.13			Non	1.0029	2.3210	3.03
11.5	2010	Tot	1.8152	.4587	6.27	14	2012	Tot	1.4583	1.2591	4.55
		Fin	.1233	.7399	1.17			Fin	.1791	.4417	1.22
		Non	1.9418	.1073	7.01			Non	1.5339	1.1845	4.90
11.5	2013	Tot	1.3026	1.6448	3.97	15	2012	Tot	1.6000	.8667	5.15
		Fin	.1662	.4914	1.21			Fin	.2038	.3346	1.25
		Non	1.3889	1.5306	4.30			Non	1.6557	.8776	5.45
						16	2012	Tot	1.4789	1.1585	4.63
								Fin	.1932	.3825	1.23
								Non	1.5282	1.1985	4.87

[a]Coupon is the annual interest paid to investors, which is .5% lower than the contract rate; Maturity (Mat.) is the year when the average loan is fully amortized; $\ln F_0$ is the baseline prepayment; g is the incentive-prepayment gradient; and P is the estimated annualized percentage prepayment.
[b]the sum of Fin and Non does not equal Tot because they share in common a constant term from the estimating equation.
Source: Authors' analysis

sample means. The results, shown in Exhibit 3, represent a generalized relationship between prepayment and the aggregate incentive to prepay (Equation 3). Three separate prepayment functions are generated. The first is total (Tot) with the next two listed items representing financial (Fin) and nonfinancial (Non) prepayment, respectively. The values were obtained using the appropriate conditioning variables in equations (4) and (5).

Results from Exhibit 3 show baseline prepayment to be dominated by nonfinancial relative to financial prepayment. The financial gradient exceeds the nonfinancial at most coupons less than 12%. Above a 12% coupon, the nonfinancial gradient tends to be larger than the financial gradient. Differences between securities are age and coupon related. Securities with coupons above current rates and that have existed in-the-money for a significant period of time, have gradients determined primarily by nonfinancial conditioning variables. Conversely, the securities not as deep or as long in-the-money have incentive gradients (g) that suggest greater responsiveness to financial considerations. Baseline prepayment across all securities appears to be dominated by nonfinancial, i.e., age and season, effects. The R^2 for the generalized VCM is .59 as compared to .14 for a simple constant-coefficient-incentive model. The R^2s for the financial and nonfinancial prepayment-incentive VCMs are .30 and .34, respectively. The advantage of the VCM is, thus, the improved fit; increased reliability of the parameter estimates; and, most important, the increased possibility for functional analysis of prepayment incentive based on financial and nonfinancial motivations.

Specialization of Empirical Results

In this section, an example is used to demonstrate how the results can be utilized in forecasting the changes in prepayment based on knowledge of the conditioning variables for a specific security. The example used for specializing the empirical results is a 12% GNMA maturing in 2013.

Impacts of changes in *TREND, NEWMIN* and *AGE* are analyzed on a partial basis. The value for a specific variable is changed, while others are held at their mean for the 12% GNMA. The variables are changed as follows:

- *TREND* measures expected changes in long-term interest rates. Three scenarios are examined: the average, a slightly positive, and a slightly negative *TREND*.
- *NEWMIN* measures the extent of the decline in interest rates to a new minimum level relative to the previous historic minimum. The mean value of *NEWMIN*, a 50% and a 100% increase from the mean are examined. For the security in question, the impact of moving from a market interest rate of 12% to a new minimum of 11.91% is representative of the mean of *NEWMIN*. Subsequent drops to 11.87% and 11.83%, respectively, reflect increases in *NEWMIN* of 50% and 100% relative to its mean.
- Finally, the impacts of alternative age profiles is shown with age at three, eight and thirteen years, respectively.

Results based on these assumptions are shown in Exhibit 4 and represent incremental effects on $\ln F_0$ and g. These results show that for the 12% GNMA security maturing in 2013, baseline prepayment declines with age whereas the incentive gradient increases,

Exhibit 4
Impact of the Conditioning Variables on Baseline Prepayment ($\ln F_0$) and the Prepayment Gradient (g) for a 12% GNMA MBS Maturing in 2013
(mean values and appropriate shocks to the explanatory variables)

	Conditioning Variables		
	AGE (X_2)	*TREND* (X_3)	*NEWMIN* (X_4)
Mean			
F_0	1.6891	1.6891	1.6891
g	.8010	.8010	.8010
Shock I[a]			
F_0	.4511	1.7794	1.6072
g	3.2456	.3880	1.3052
Shock II[b]			
F_0	.2014	1.8572	1.5385
g	3.2457	.0751	1.7185

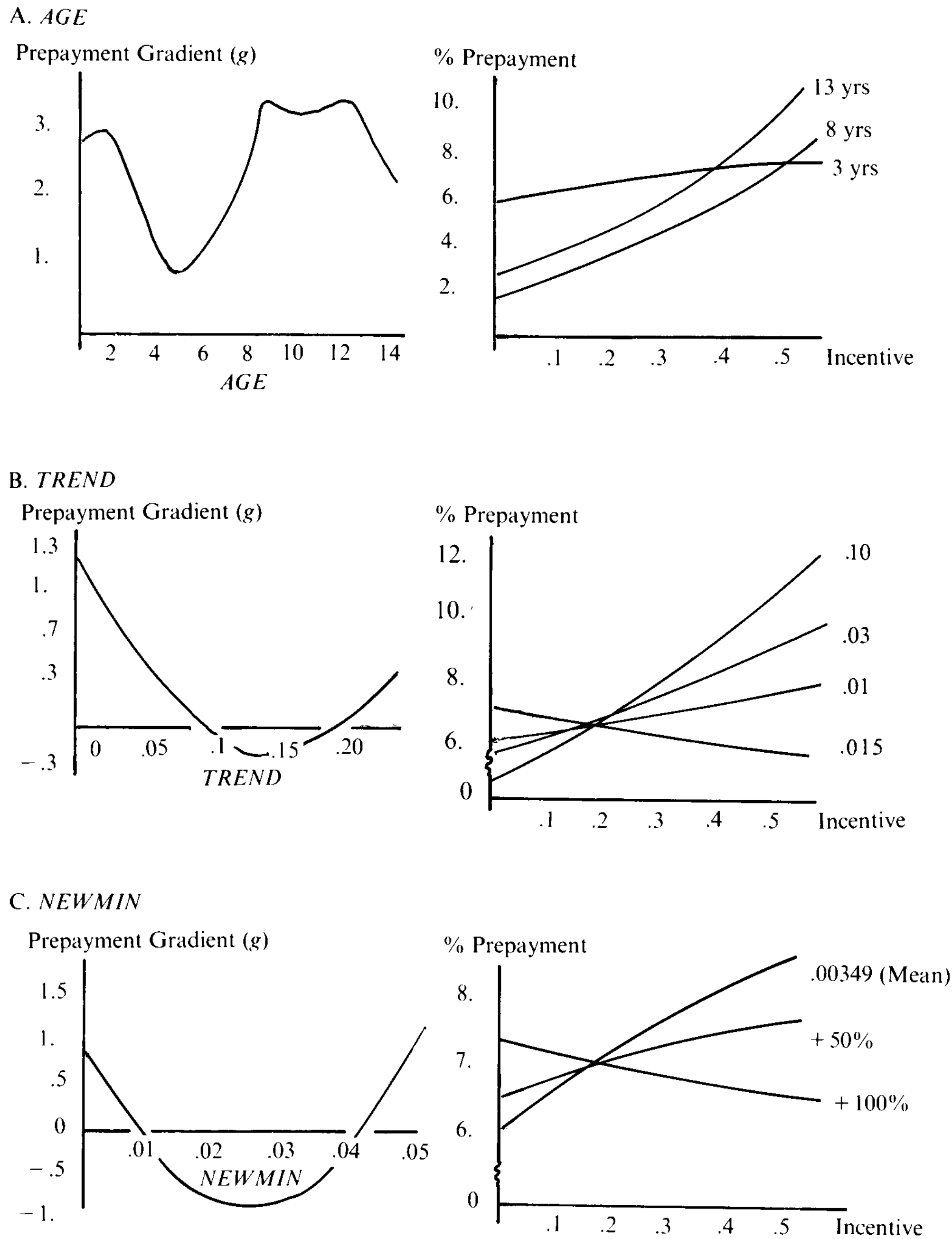
[a]Shock I represents an increase in *AGE* from 3 to 8 years, a change in *TREND* from a mean of .04942 to slightly positive (.01) and a 50% increase in *NEWMIN* from the mean of .00724.
[b]Shock II represents an increase in *AGE* from 3 to 13 years, a change in *TREND* from a mean of .04942 to slightly negative (−.015) and a 100% increase in *NEWMIN* from the mean of .00724.
Source: Authors' analysis

and apparently stabilizes. For *NEWMIN*, new lower interest-rate plateaus increase baseline prepayment at each level but reduce the positive incentive gradient. A slightly negative *TREND*, indicative of an accelerating decline in market interest rates, leads to decreasing baseline prepayment and to an increase in the responsiveness of prepayment with increasing incentive.

In general, the results in Exhibit 4 demonstrate that major impacts on baseline and incentive prepayment do occur with changes in the conditioning variables. This is not surprising because the specification of the structure for the varying coefficients featured possible changes in baseline prepayment and the gradient. Although some of the changed values in Exhibit 4 are far from the means, the results are reasonable. The surface approximated by the polynomial is apparently stable, thus projections or forecasts using the conditioning variables can be viewed with some confidence.

Impacts of changes in three of the explanatory variables on the prepayment gradient, g, are plotted in Exhibit 5, along with representative structural shifts in the prepayment function. Green and Shoven (1986), Schwartz and Torous (1989) and others have found evidence that decreasing interest rates increase prepayment. The evidence provided by the VCM indicates that as interest rates decrease, baseline prepayment increases dramatically whereas the incentive gradient levels off. These significant shifts in baseline prepayment suggest that the effect of a downward trend in interest rates is reflected in the number of debtholders that move into a positive incentive to prepay. Baseline and incentive results based on the VCM in Exhibits 2 and 5 corroborate the findings of Dunn and Spatt's (1986) general analysis of interest-rate effects, which includes the impact of transaction costs.

Exhibit 5
The Impact of the Conditioning Variables on the Prepayment-Incentive Gradient (*g*)



Source: Author's analysis

The other financial-specific variable, *NEWMIN*, a measure of the impact of a new low in interest rates, has the expected positive sign for baseline prepayment. The values of *NEWMIN* ranged from 0 to .045 with a typical nonzero value of .02. Within this range the empirical results presented in Exhibits 4 and 5C show the gradient as slightly negative, suggesting that debtholders who decide not to prepay when entering a positive incentive situation continue not to prepay regardless of the financial return. The same is true for *TREND*. The financial incentive to prepay has its impact as the debtholder initially moves into the money. Any further financial incentive due to changes in interest rates has little if any impact. *Age* (Exhibit 5A) on the other hand demonstrates both a significant change in baseline prepayment and a positive incentive gradient for newly originated mortgages and for mortgages four years or older, indicating that nonfinancial prepayment does vary with the financial inducement to prepay.

Summary and Conclusions

The VCM has been proposed as a method for introducing financial- and nonfinancial-specific variables into an exponential prepayment-incentive function. An advantage of the VCM is that it permits exploration into the nature of such variables while retaining an interpretable function that is consistent with previous empirical investigations. This facilitates comparisons with the extensive emerging literature on prepayment. In fact many of the estimated prepayment functions are special cases of the VCM with its polynomial structure relating prepayment incentive to the financial and nonfinancial conditioning variables.

Applying the VCM functional form to data on twenty-seven different securities over the time period 1978–83 provided a number of results. First, it is established that a rather simple functional form is appropriate when handled within the context of the VCM framework. The explanatory power of the prepayment-incentive function and the significance levels of the structural parameters are greatly enhanced.

The results show that the conditioning variables reflecting interest-rate trends, new lows in interest rates, age of the security and season can be used to provide explanations of structural differences among securities. Of these results perhaps the most interesting is the positive debtholder response to changes in the explanatory variables regardless of the extent of positive incentive. Correlation between length of time in-the-money and the amount of positive incentive is significant and positive. This further implies differing levels of transaction costs per debtholder; so that for any given change in a conditioning variable the prepayment function shifts for all debtholders. The empirical analysis also supports the hypothesis that interaction exists between financial and nonfinancial incentives to prepay. Other results are consistent with expected behavior from both financial and nonfinancial structural changes.

One important result from this application concerns the use of a simple exponential prepayment function as a tool for projection. As shown, the VCM offers a unique procedure for exploring the impact of financial and nonfinancial conditioning variables on prepayment. The relationship among *TREND, NEWMIN, AGE* and time provides an illustration of how the model can be used in forecasting. Since these variables can be projected on the basis of time, the estimated prepayment function can be used for forecasting changes in value of security portfolios.

Appendix
Characteristics of GNMA Fixed-Rate Mortgage Prepayment Data (June 1987)

Coupon	Maturity Year	Amount Issued ($000,000)	Outstanding Amount ($000,000)	Market Interest Rate[a] Mean	Market Interest Rate[a] Range	Prepayment Rate[b] Mean	Prepayment Rate[b] Max	N
7.50	2007	6977.5	4148.3	12.47	9.15–16.38	4.0	10.1	112
8	2008	4030.3	2709.5	12.47	9.15–16.38	5.0	11.1	112
8	2007	8771.5	5443.0	12.47	9.15–16.38	3.8	10.1	112
8	2006	6538.3	3376.0	1247	9.15–16.38	3.2	10.4	112
8.25	2008	2479.4	1721.0	12.56	9.29–16.38	2.9	10.7	109
8.50	2006	2055.3	883.6	12.47	9.15–16.38	6.0	14.0	112
9	2009	8328.9	6077.8	12.71	9.29–16.38	3.2	11.8	104
9	2008	6482.6	4572.9	12.82	9.29–16.38	2.9	11.2	100
9.50	2009	11,154.7	8187.2	12.97	9.29–16.38	3.2	13.4	94
10	2009	2109.2	1514.6	13.05	9.29–16.38	3.7	18.3	90
11	2010	8321.4	4934.7	13.10	9.29–16.38	6.4	35.6	86
11.50	2010	2233.7	1117.4	13.08	9.29–16.38	8.7	38.4	83
11.50	2013	21,608.5	10,869.5	13.64	9.29–13.78	13.6	48.8	50
12	2013	4999.4	2193.3	12.05	9.29–13.78	15.8	51.2	50
12	2014	7120.8	3506.2	11.85	9.29–13.78	17.2	46.8	41
12.50	2010	4439.6	1665.1	13.08	9.29–16.38	11.9	46.2	83
12.50	2013	4142.4	1641.0	11.93	9.29–13.78	20.1	55.2	44
12.50	2014	3965.5	1604.9	11.79	9.29–13.78	21.7	55.4	39
13	2011	3193.8	1094.4	13.10	9.29–16.38	13.9	48.0	76
13	2012	1485.4	384.1	12.40	9.29–15.59	19.9	62.6	58
13	2013	1954.9	682.8	12.02	9.29–13.78	20.1	55.5	48
13	2014	3214.9	1225.6	11.65	9.29–13.78	25.5	55.1	35
13.50	2011	1303.8	349.5	13.04	9.29–16.38	17.4	52.2	72
13.50	2014	2334.1	820.4	11.65	9.29–13.78	27.9	55.0	35
14	2012	1040.3	134.5	12.88	10.30–16.11	24.2	69.9	55
15	2012	5213.2	961.8	12.62	9.29–16.11	25.8	46.7	62
16	2012	932.5	140.6	13.17	10.30–16.11	26.2	56.3	55

[a]from the FHLBB series of average effective interest rates on existing homes. Prior to September 1983, both FRMs and ARMs are included; after this time, only FRMs are included.
[b]the annualized monthly prepayment rate for a GNMA security of the given coupon and maturity year
The contract interest rate on the underlying fixed-rate mortgages exceeds the coupon rate by .5%.
Source: Federal Home Loan Mortgage Corporation, Federal Home Loan Bank Board

Notes

[1]The observed behavior is the prepayment rate of GNMA thirty-year, fixed-rate, pass-through securities of various coupons backed by mortgages on single-family homes. The prepayment rate is defined as the percentage of outstanding loans that prepay any given month, expressed as an annual percentage. Each security must have $200 million outstanding to be included as an observation. The time-to-maturity is measured as the weighted average maturity (WAM) of the loan pool. The data consists of time series of twenty-seven securities of sixteen different coupons. The data are provided by the Federal Home Loan Mortgage Corporation. The Appendix contains summary information on individual pools.

[2]The aggregated nature of the prepayment data requires the use of a national average interest-rate series. The preferred interest-rate series is the Federal Home Loan Bank Board (FHLBB) national average for newly originated fixed-rate mortgages on existing homes across all lenders. The fixed-rate series proxies refinancing rates; however, it is incomplete. By using the fixed-rate series in combination with the FHLBB series for all loans (FRM and ARM), an interest-rate series is constructed. To correct for the mixture of FRM and ARM data, a dummy variable is used in the model to account for the effect of including ARMs in the interest-rate series. This variable takes on a value of 1 if the series includes all mortgage loans and 0 if the FRM series is the observation. The estimated coefficient for the variable was .00211 with a t-value of 3.21.

[3]This is done by using a two-stage process which forces a cubic relationship. A shortcoming to this variable is that prepayment can become negative rather than being bounded at zero. A strength of this specification is that the data chooses the appropriate prepayment peak. In stage one, prepayment (P) is predicted from the cubic relationship,

$$P = \alpha_1 T + \alpha_2 T^2 + \alpha_3 T^3,$$

where T is the age of the underlying debt as a percent of its term and the α's are regression coefficients. This relationship forces predicted prepayment to increase, fall off and then to increase once again, as suggested by Roberts (1987) and Bartholomew et al. (1988). Prepayment (P) via the age effect is predicted from its cubic relationship with the data,

$$P = 295.2396\ T - 2058.2628\ T^2 + 2738.0612\ T^3.$$

The predicted values from stage one become the AGE (X_2) variable in the VCM model. Prepayments are expected to peak at approximately four years of age. The expected prepayment rates then decline for about eight years at which time they accelerate rapidly. This function explodes beyond the range of the data, thus limiting its applicability to mortgages of all ages.

[4]Because the data consist of FHA and VA loans, which are assumable, individual loans may not be terminated when a borrower moves. Thus, the *AGE* variable only partially captures the effect of population mobility.

[5]The *TREND* variable recognizes the borrower's motivations to prepay given the expectations of future market interest rates. If the *TREND* is upward, any economic benefits are deteriorating and the optimal time to prepay is in the past. A downward *TREND*, which is decelerating or apparently bottoming out ($TREND > 0$) may indicate the most favorable conditions for prepayment, since if the *TREND* reverses as indicated, the economic benefits of prepaying will begin to diminish. An accelerating downward *TREND* ($TREND < 0$) indicates a situation where the debtholder will delay the decision to prepay to take advantage of better conditions in the future. Bartholomew et al. (1988) propose the variable in a general form. We arbitrarily decided to use six months as the period of long-term *TREND* measurement.

[6]*NEWMIN* measures the percentage drop in the historical minimum rate over the life of the

security. This variable is intended to measure the impact of a new minimal refinancing rate on the pool of borrowers who remain. These remaining borrowers are often referred to as holdouts. The incentive to hold out may or may not relate to transactions costs.

[7] $TREND \times NEWMIN$ has a nonzero value when a downward trend in interest rates is either accelerating ($TREND \times NEWMIN < 0$) or decelerating ($TREND \times NEWMIN > 0$) and a new historical minimum interest rate is encountered. Borrowers differ in their responses to lower interest-rate levels if a trend is perceived to be slowing down or if significantly lower rates are anticipated in the near future. For instance, people may choose to postpone prepaying (refinancing) at a new minimum market rate when it is expected that rates will fall further.

[8] $AGE \times TREND$ has a nonzero value when a downward trend in interest rates is either accelerating or decelerating. The expected prepayment as a security ages is affected by the interest-rate environment that security experiences. $AGE \times TREND$ accounts for this shifting of the *AGE* relationship.

[9] Ideally, a prepayment model will include current and past property values. Due to the aggregate nature of the data, estimates of property value for the properties underlying individual mortgages or for an index reflecting the geographic composition of the GNMA security are not available. Although regional differences in prepayment behavior of mortgage-backed securities are evident (see Waldman et al., 1985), the geographical distribution of the mortgages underlying this database is unknown. While the results are biased due to this missing information, we contend that the generalization of the results is not compromised.

[10] The individual pools are weighted by the number of loans per pool to correct for heteroscedasticity. The Durbin-Watson statistic detected autocorrelation which was removed by adjusting the age observations with a standard correlation transformation.

[11] Brown et al. (1975) proposed a similar scheme for relationships that may change over time. In a slightly different context, models for parameter change have been specified with random coefficients (see Rosenberg, 1973; Singh et al., 1976). Random coefficients imply a more complex error structure.

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