Public Land Use Constraints: Lot and House Configuration

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***Abstract.* The public sector constrains the size and shape of lots and buildings via zoning ordinances and subdivision regulations. Zoning ordinances utilize setback requirements, open space ratios, minimum lot area and floor-to-area ratios. Subdivision regulations utilize street and sidewalk spacing requirments. This article provides a framework in which one can analyze the precise impact of these control devices. The choice of developers who face these controls is discussed in terms of a rule of thumb and in terms of a model of profit maximization.**

Zoning, subdivision and similar ordinances limit private property rights and constrain choice regarding the configurations of houses and lots. Such ordinances are both widespread and amazingly homogeneous across the United States. Thus, analyzing aspects of one particular set of land use control ordinances allows implications to be drawn for ordinances in a wide variety of cities. This article analyzes the impact of zoning and subdivision ordinances upon the feasible choices for residential lot and building configurations in one particular municipality.

This analysis is but one of the pieces needed for a comprehensive theory of optimal lot and house configuration. Edelson (1975) addresses optimal lot size, but not configuration (*i.e.*, shape), for land in a given development. He considers both varying lot sizes within the development to accomodate buyers with a continuum of incomes and homogeneous small developments aimed at a single income level. Edelson's development costs are primitive and regulatory constraints nonexistent. Colwell and Scheu develop a theory of optimal lot configuration with a peculiar objective function (profit per *lot* is maximized rather than profit for the entire development) with more elaborate development costs and with two of the most important constraints (*i.e.*, minimum depth implicit in the "subdivision regulations" and minimum lot area specified in the zoning ordinance). Cannaday and Colwell (1990) improve upon the objective function (profit per *acre* is maximized) but ignore the constraints altogether. Doiron, Shilling and Sirmans (1992) and later Colwell and Ebrahim (forthcoming) develop models of optimal office building configuration but ignore the lot configuration and any constraints from the public sector. The purpose of this article is to model the constraints in some detail and, for the most part, to ignore the objective function.

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When viewed separately, the constraints tend to appear very complicated; however, many of these complications disappear when all the zoning and subdivision constraints are viewed together. The complications disappear because most of the constraints prove not to be binding in the presence of a few of the other constraints. Of course, some of the constraints that prove not to be binding for single-family housing may be binding for other land uses, or could become binding if other constraints were relaxed slightly. This article provides an analytical structure within which it is easy to see the implications of these constraints both separately and together. Within this structure, it is possible to consider the interaction of the constraints with developer rules of thumb, as well as with sophisticated objective functions.

The remainder of this article is organized into four sections. The first section describes and integrates several types of restrictions found in the typical zoning ordinance. In the second section, important features of the New Streets and Subdivision Ordinance of Champaign, Illinois (1978) are described, and are compared to the zoning constraints. The third section identifies a set of choices defined by a common rule of thumb, and suggests how these choices may be combined with the public sector constraints. Finally, the fourth section offers some conclusions.

The Zoning Ordinance

The zoning laws for Champaign (1980) contain several features that can affect the frontages and depths of residential lots. These features include setback requirements, a minimum open space ratio (OSR_{min}), a minimum lot area (LA_{min}) and a maximum floor to area ratio (FAR_{max}). Setbacks are described first, followed by open space ratios. Setbacks and open space ratios then are considered together. The minimum lot area and the maximum floor to area ratio are covered last.

Setbacks

Setback requirements force the builder to position a structure on a lot so that each exterior wall is at least the specified minimum distance from the nearest property line. Champaign's setbacks for R-2 zoning are 25 feet in front, 10 feet in back and 6 feet on each side (Champaign, 1980:58). Because of these requirements, a lot must have a depth of at least 35 feet and a frontage of at least 12 feet before any structure can be built upon it. Setback specifications therefore influence the size and configuration of a house to be built on a given lot, or the configuration of the lot on which a particular structure can be built.

Some ranges of house configurations (*i.e.*, massing) are considered in the discussion that follows, though there certainly are other possible configurations. For illustrative purposes, we consider L-shaped and rectangular, one-story houses of 2,000 square feet each with a 20'x20' attached garage, while we ignore U-shaped, H-shaped, bi-level, tri-level and two-story designs. The range of house configurations considered involves the least sparing use of the land, and therefore includes the configurations most likely to cause the setback constraints to bind. The only lot configurations considered are rectangular, so lot configurations can be summarized simply by frontage and depth measurements.

The constraint imposed upon lot configuration by the setback requirements, in the context of the house configurations and size considered, is illustrated by the convex thick line in Exhibit 1. The garage is shown by the dark square set back 6 feet from the ‘‘west’’ side and 25 feet from the front. If the house extends 100 feet along the front and 20 feet in depth, as shown by the rectangle shaded with positively sloped lines, then the lot can be no less than 132 feet in frontage and 55 feet in depth. At the other extreme, the house would not require any frontage, beyond that required by the garage and the side setbacks, if it extended for 100 feet directly behind the garage, as shown by the rectangle with horizontal shading. This house configuration would constrain the lot to be no less than 32 feet in frontage and 155 feet in depth. Between these extremes, the houses could assume L-shapes (wrapping around their garages) in order to minimize the required lot sizes. Each of these houses would be flush with the front of the garage. One of these intermediate house configurations is shown by the L-shape with negatively sloped shading, corresponding to a minimum lot frontage of 60 feet and depth of 85 feet. Of course, any rectangular lot configuration with dimensions along the thick convex line in Exhibit 1 is attainable if the dimensions of the house are modified to meet the setback requirements while the gross floor area is held constant.

Open Space Ratios

The most complex component of the zoning ordinance is the open space ratio (*OSR*). This ratio is computed as follows:

$$OSR = \frac{UOS}{GFA}$$

where *UOS* is usable open space, that portion of the lot that is unoccupied by the house, garage and driveway and has no dimension less than 18 feet (Champaign, 1980:30), and *GFA* is the gross floor area, measured with reference to the outside dimensions of the house.

The city of Champaign has a minimum open space ratio of 0.45 (Champaign, 1980: 58).

$$OSR_{min} = 0.45$$

In order to analyze the effect of the *OSR* constraint, we make the same assumptions concerning the garage and house size and configurations that have been utilized in the analysis of the setback requirements (*i.e.*, we consider 2,000 square foot, one story, L-shaped and rectangular houses and $20' \times 20'$ garages). In addition, it is assumed that the driveway is 16 feet wide and is centered on the garage, leaving 2 feet of garage on either side of the driveway.

The minimum 18 foot dimension for *UOS* constrains lot depth to contain a back yard depth of at least 18 feet deep if *UOS* is to be found in the back yard, where it is most efficiently provided (due to the location of the driveway, which consumes open space, in the front yard). This 18′*UOS* constraint would be binding (the OSR_{min} itself would not) between lot sizes of 120′×38′ and 50′×66′, as shown in Exhibit 2. In this range, a house would have "zero lot lines" at the front and sides and an 18 foot deep back yard; its *OSR* would meet or exceed the minimum required. For example, note that for the extreme case of the rectangular house extending 100 feet "eastward" from the garage the back yard open space measures 18′×120′ (2160 square feet), so the *OSR* = 2160/2000 = 1.08. The L-shaped house illustrated, however, with its open back yard measuring only 18′×50′ (900 square feet) has an *OSR* = 900/2000 = 0.45, just meeting the minimum required.

In order to build on a lot narrower than 50 feet, a subdivider would have to provide additional back yard depth. Between dimensions of 50′×66′ and 20′×165′, the OSR_{min} constraint would bind, as shown in Exhibit 2. In the extreme case within that range (*i.e.*, at 20′×165′), the back yard would have to be 45 feet deep in order to produce the 900 square feet of *UOS* for an *OSR* of 0.45. Each such house would still have "zero lot lines" at its front and sides.

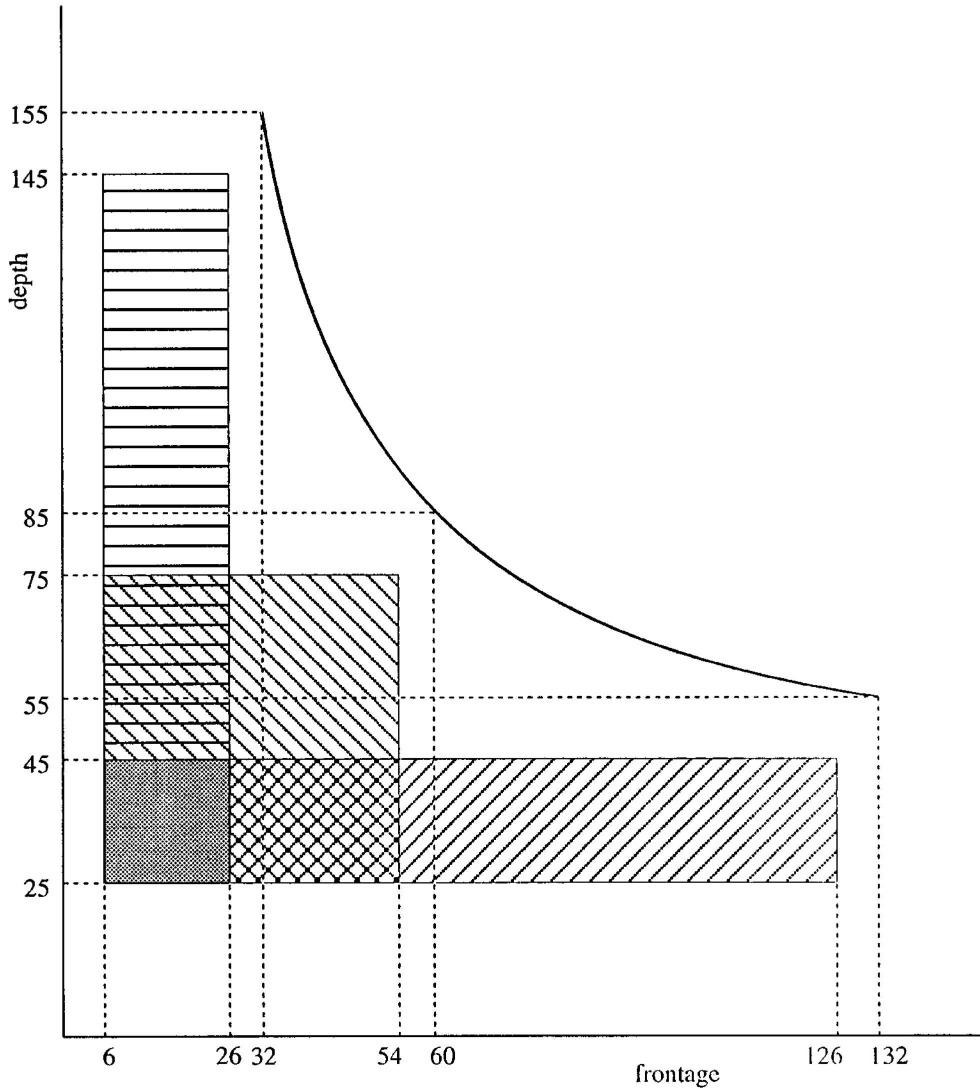
It would seem from Exhibits 1 and 2 that the various setback requirements should always be binding when compared to the minimum open space ratio of 0.45 and the minimum usable open space dimension of 18 feet. In other words, it might appear that a lot owner meeting the setback requirements would, in all cases, automatically satisfy the OSR restriction. Unfortunately, this view does not recognize the complexity of the constraints when setbacks and *OSR*s are considered together.

Setbacks and Open Space Ratios

Considering setback requirements and the minimum open space ratio together produces a complex system of lot configuration constraints. The configuration of the house and lot that minimizes the required lot dimensions changes five times (*i.e.*, six distinctly different configurations) within the relevant range of frontage and depth combinations. This section proceeds by reducing the frontage and observing the resulting depth constrained by both setbacks and open space ratios. We begin with a rectangular house attached to the side of the garage with all the *UOS* in the front yard. Next, we have an L-shaped house flush with the front of the garage and with the *UOS* in the front. Third, we continue with an L-shape but set the house back from the front of the garage—still with the *UOS* in the front. Fourth, the L-shaped house is brought back to be flush with the front of the garage but the *UOS* is found both in the front and back yards. Fifth, an L-shaped house is flush with the garage but all the *UOS* is in the backyard. Finally, a rectangular house is attached to the back of the garage.

For lot dimensions between 132′×55′ and 60′×85′ the constraint is the same, and the house configuration is the same (*i.e.*, L-shaped and flush with front of the garage), as for the setback constraint shown in Exhibit 1. The reason is that the OSR_{min} is

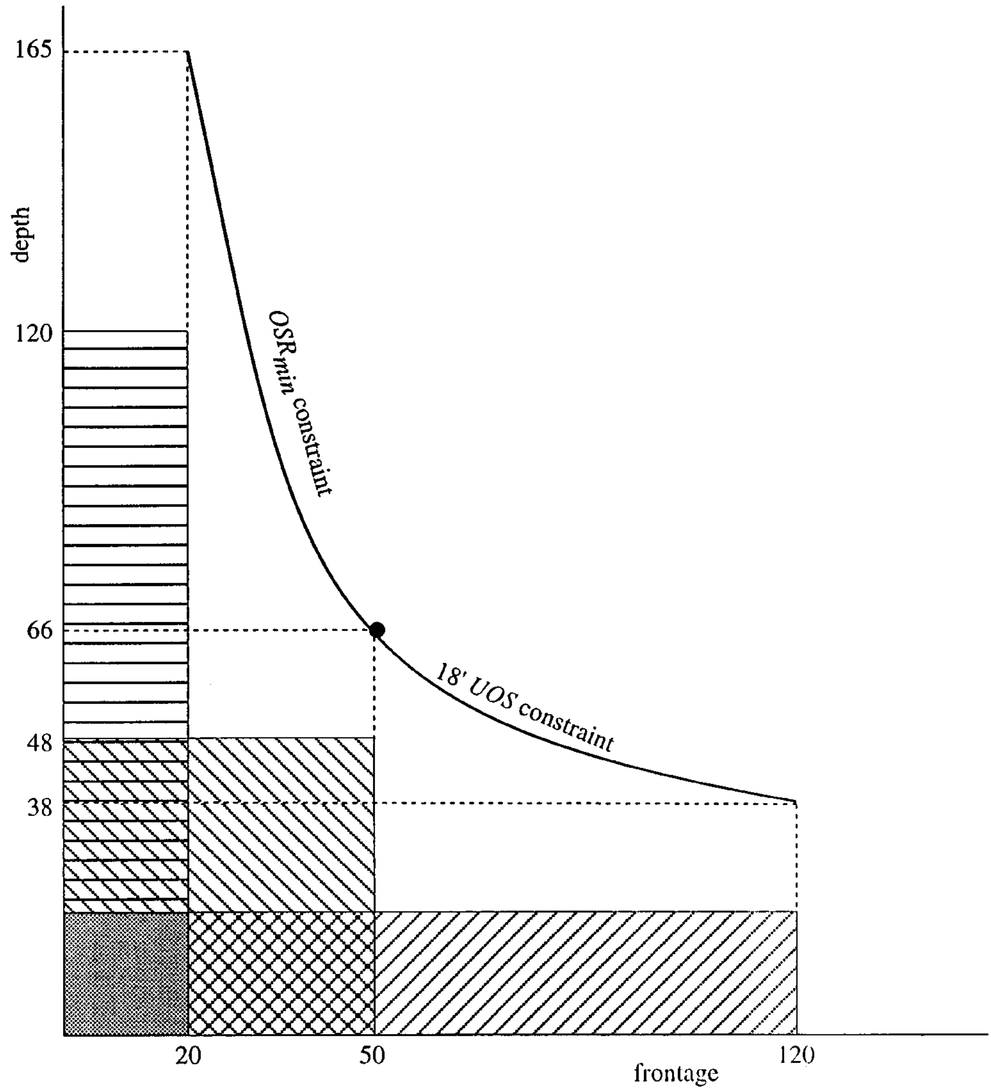
Exhibit 1
The Constraint from Setback Requirements



exceeded, with *UOS* provided by the 25 foot front setback in the presence of sufficient width to offset or overcome the presence of the driveway. At 60′×85′ the *OSR* equals the OSR_{min} (the open front yard area to the "east" of the driveway measures 36′×25′, for the requisite 900 square feet). Any further reductions in frontage require a change in the configuration of the house.

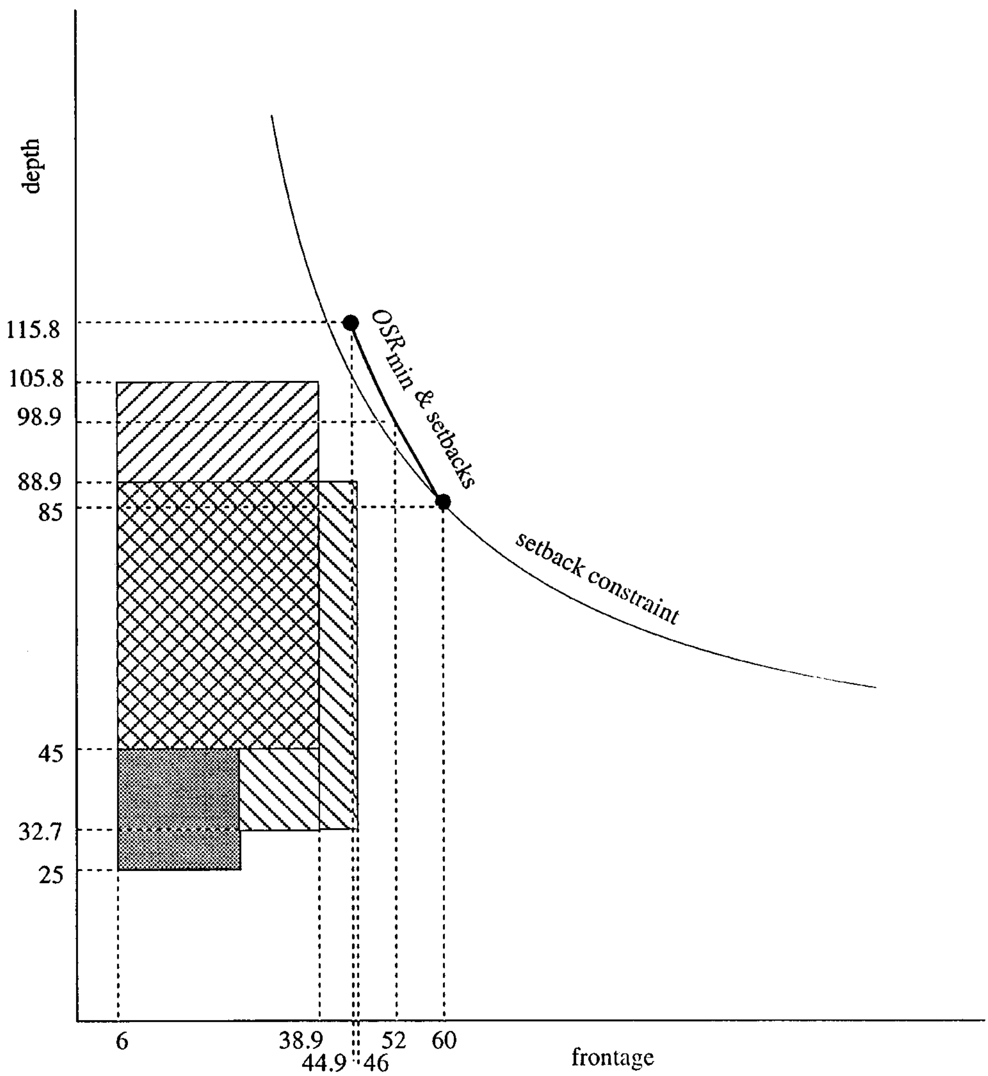
If the house is set back from the front of the garage but the L-shape is maintained, then sufficient additional *UOS* can be created in the front yard to meet the OSR_{min},

Exhibit 2
The Constraint from Open Space Regulations



up to the point at which frontage is reduced to approximately 45 feet. In Exhibit 3 this range is illustrated by the thick line between the two dots. A typical house in this range is shown by the negatively sloped shading for a lot that is 52′×98.9′. As the frontage shrinks, the house setback from the front of the garage must progressively increase; the setback equals the depth of the garage and the house becomes rectangular at a frontage of 44.9 feet and a depth of 115.8 feet, as illustrated in Exhibit 3. Beyond this point, the house configuration should change in order to allow any further reductions in frontage.

Exhibit 3
Combining Setbacks and Open Space Controls



We are tempted to continue with an L-shape by causing the setback from the front of the garage to exceed the full depth of the garage. However, a marginally more efficient change is to realign the house back to being flush with the front of the garage, thereby obtaining additional *UOS* by creating an 18 foot deep back yard. An example with this configuration is shown in Exhibit 3 as the house with the positively sloped shading. The depth required for this house is only about two inches deeper than the deepest lot outlined in Exhibit 3. This discontinuity is the first, but will not be the largest, encountered in the constraint system. Over the small range of lot configurations, 44.9′×115.8′ to 42′×123′, that use this house configuration, the additional *UOS* gained takes the *OSR* beyond the OSR_{min}. That is, the OSR_{min} is not binding over this range (see Exhibit 4).

For a frontage less than 42 feet, there can be no *UOS* in the front yard because of the assumed driveway placement. In order to get sufficient *UOS* in the back yard to yield the OSR_{min}, the developer must increase the depth of the back yard by nearly three and a half feet. This adjustment creates a discontinuity in the constraint system of almost three and one half feet at a frontage of 42 feet. Between lot sizes of $42' \times 126.4'$ and $32' \times 173.1'$, the houses would continue the L-shaped, flush-front configuration while the depth of the back yard would increase from 21.4 feet to 28.1 feet. A lot of $42' \times 126.4'$ will support a house such as the one with the positively sloped shading in Exhibit 4. At the extreme end of this range (*i.e.*, at $32' \times 173.1'$), the entire house would be in back of the garage, as shown by the negatively sloped shading in Exhibit 4.

Beginning with a shallow, wide lot and a rectangular-shaped house, the house configuration must first change to an L-shape that is flush with the front of the garage as the frontage narrows and the depth increases, so as to meet the setback requirements. Next, as frontage continues to decline, the configuration must change to one for which the front of the house sets back from the garage in order to meet the minimum open space ratio. As the frontage narrows beyond the point where the front of the house is set back the full depth of the garage, the house configuration changes back to the front's being flush with the garage while the back yard depth goes to the 18 foot minimum required for inclusion in usable open space. This configuration continues as frontage declines, until the width of the front yard no longer allows for an 18 foot usable open space minimum. At that point, a substantial increase in back yard depth is required if the minimum open space requirement is to be met as frontage is further reduced. Finally, a house configuration opposite to the initial one is obtained, at which there is no longer any part of the house that is to the side of the garage.

Lot Area and Floor Area Ratios

Zoning controls the minimum lot area (LA_{min}) and the maximum floor to area ratio (FAR_{max}). These constraints are rectangular hyperbolas in frontage and depth space. For example, if $LA_{min} = fd$, where f is frontage and d is depth, then $d = LA_{min}/f$.

Similarly,

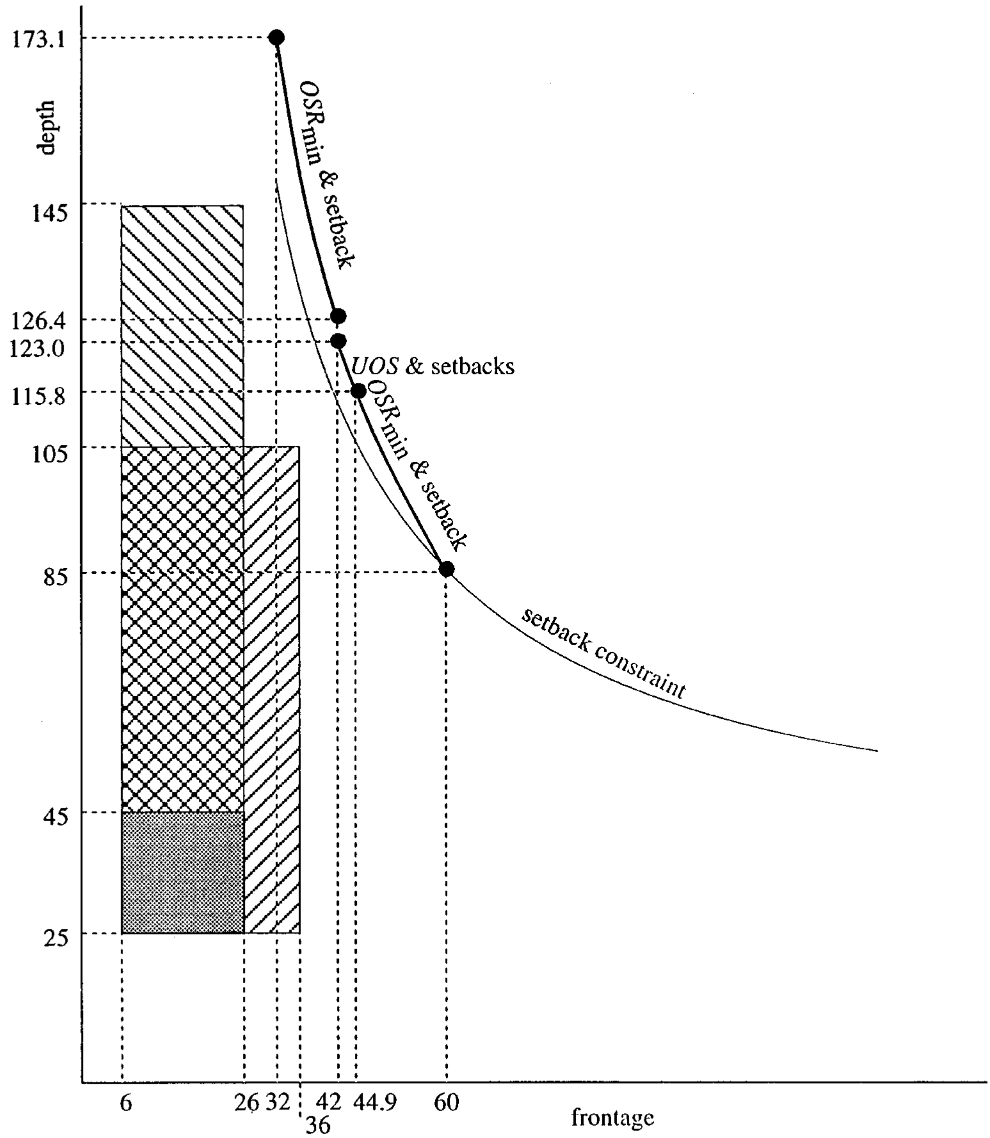
where:

$$FAR_{max} = \frac{GFA}{LA} = \frac{GFA}{fd},$$

$$d = \frac{GFA}{fFAR_{max}}.$$

These constraints are identical when:

Exhibit 4
More Setback and Open Space Control Issues



$$\frac{GFA}{FAR_{max}} = LA_{min}.$$

In Champaign, the FAR_{max} is 0.35 for R-2 zoning (Champaign, 1980). The LA_{min} is 6,500 square feet. Thus, these constraints are equivalent for a house of 2,275 square feet (0.35 $\times$ 6,500). The examples in this paper have been developed in terms of a 2,000 square foot house. With respect to a house of 2,000 square feet, the FAR_{max} rectangular hyperbola is below that of the LA_{min}. For a house of approximately this size, the minimum lot area can be binding but the floor to area ratio cannot (for houses larger than 2,275 square feet, the opposite result holds). Note that in Exhibit 5 the rectangular hyperbola that contains a lot of $65' \times 100'$ (as well as any other lot configuration with 6,500 square feet of area) is above the rectangular hyperbola that indicates all lot configurations with a *FAR* of 0.35, if the house is 2,000 square feet in area. One such lot with $FAR = .35$ has dimensions of approximately $65' \times 87.9'$, as shown in Exhibit 5.

Combining the Zoning Constraints

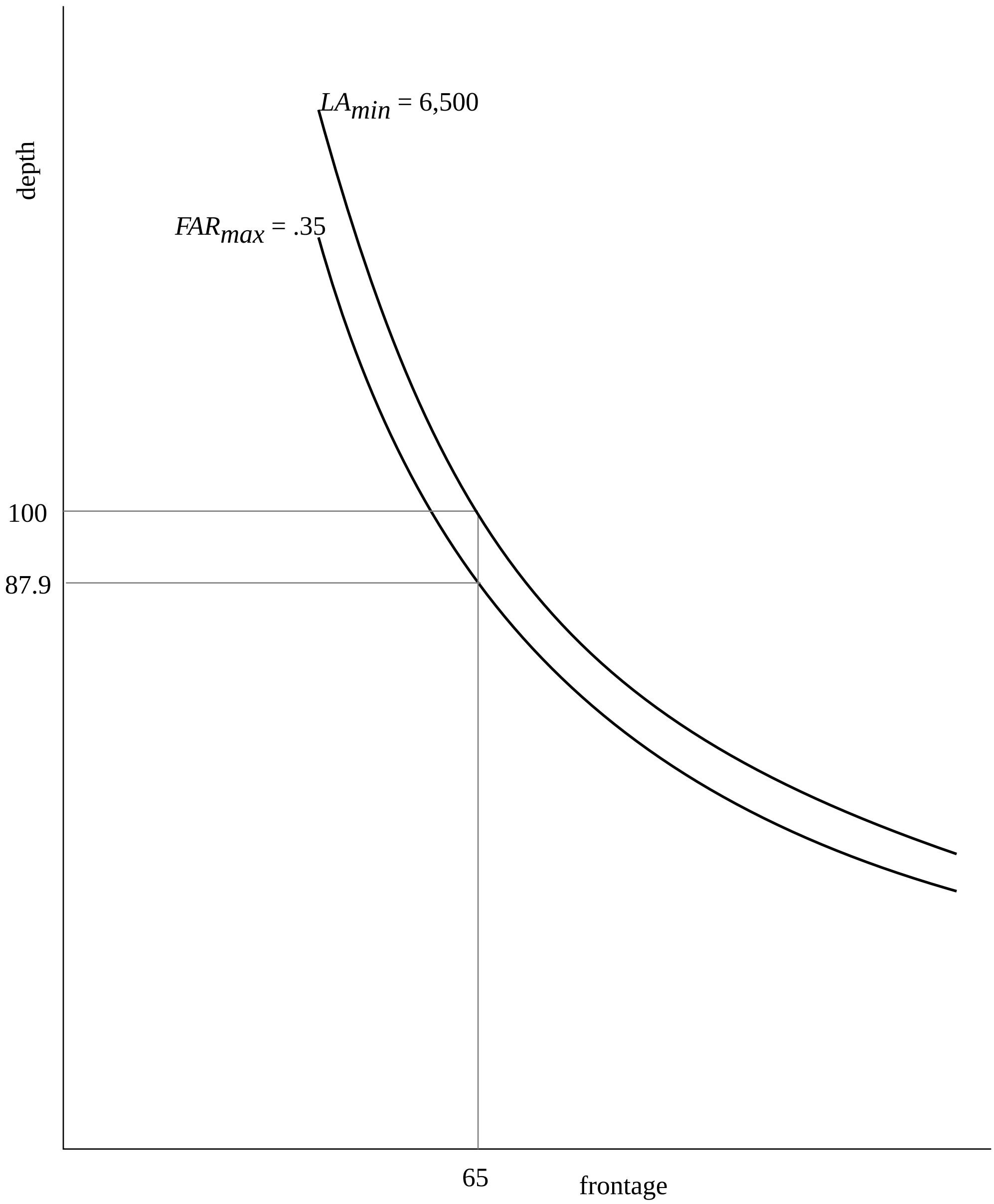
The zoning constraints that bind depend on such factors as the size of the house, as well as on the specifics of the setback requirements, the maximum floor to area ratio, the minimum open space ratio and the minimum lot area. Bringing together the constraints most likely to bind from Exhibits 4 and 5 allows us to visualize which constraints bind, for the house size we have chosen, under the specific features of the Champaign zoning ordinance. This combination is illustrated in Exhibit 6. Note that the constraints intersect for a lot measuring approximately $109' \times 59.6'$. At greater frontage, the setback constraints bind. At lesser frontages, the minimum lot area binds. All the concern about open space ratios and usable open space comes to naught in the face of the specific other constraints. However, with a lower minimum lot area and a higher maximum floor to area ratio, it is conceivable that open space rules could come into play.

The New Streets and Subdivision Ordinance

Champaign's New Streets and Subdivision Ordinance (NSSO) (1978) places restrictions on the location of any public right of way (ROW). Included in the ROW are the relative locations (and the physical requirements) of all streets, sidewalks, and crosswalks, which implicitly impose restrictions on lot dimensions. The requirements for the frequency of streets are as follows:

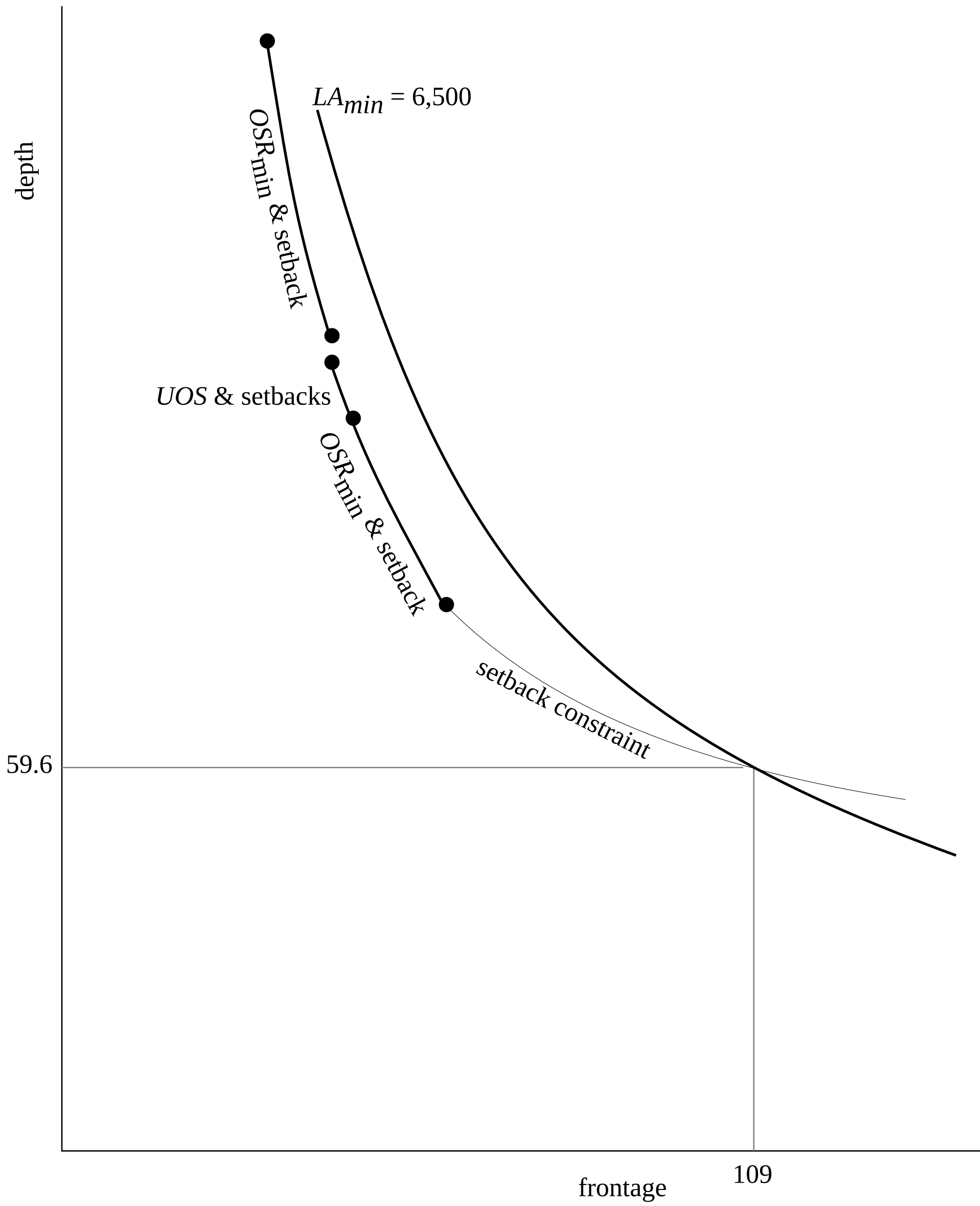
(1) ''No street shall be located less than 260 feet nor more than 1200 feet from any parallel street measured from the center lines of the streets . . .'' (Champaign, 1978:23)
(2) A minimum ROW which includes the street width, the strip of land between the curb and sidewalk (i.e., the parkway), and the width of the sidewalk is 60 feet (Champaign, 1978:25).
(3) Whenever the average length of a block exceeds 800 feet, public crosswalks located down the middle of the block having a ROW of not less than 10 feet may be required (Champaign, 1978:24).

Exhibit 5
Minimum Lot Area and Maximum Floor to Area Ratio



The first of these requirements causes there to be a maximum of 1,200 feet between streets thus constraining both frontage and depth. If there were two 30 foot half ROWs and a 10 foot crosswalk, a lot could have a maximum frontage of 565 feet. The maximum depth would be the same 565 feet. If the block were 800 feet or less, a crosswalk would not be needed, according to the third requirement; therefore,

Exhibit 6
Combining All Zoning Constraints



maximum lot frontage could be 740 feet. The maximum depth would be half that measure, or 370 feet.

The NSSO also puts minimum constraints on the depth of lots, d_{min}. The minimum depth is 100 feet. Note that the first requirement above indicates that the centers of streets can be no closer than 260 feet. Subtracting 30 feet for one half of the ROW on each side of the block and dividing by 2 (*i.e.*, assuming that a lot does not go all the way through a block) yields the 100 foot minimum. We understand that the rationale for this minimum is to separate side streets as they enter a collector, and to reduce accidents from automobiles on adjacent side streets turning into each other as they enter the collector. If the rationale indeed is as described, then the spirit of the law can be met other ways. If adjacent and parallel cul-de-sacs were to enter a larger block from opposite directions, then the minimum depth would seem to be is 35 feet. This depth restriction is not much of a constraint. From the lack of real world examples of this approach, one might infer that the costs involved in getting around the 100 foot minimum might preclude that option, or that regulators would not agree that this design meets the letter of the law.

While the frontage and depth maxima are so large as to seldom be binding for urban building lots, the minimum depth from the NSSO may prove to be one of the more important constraints. Exhibit 7 shows the feasible lot configurations as constrained by the minimum depth from the NSSO, along with what appeared to be the binding constraints from the zoning ordinance, as shown in Exhibit 6. It is clear from Exhibit 7 that the zoning setbacks do not bind in this context. Rather, the only constraints that appear to bind are zoning's minimum lot area and the NSSO's minimum depth. Recall that these are the two constraints considered by Colwell and Scheu (1989) in their study of optimal lot size and configuration.

A Rule of Thumb

Land developers often simplify their decision processes into a few general guidelines, or rules of thumb. For example, a typical rule of thumb used by developers in subdividing an area zoned R-2 single family residential is 3.8 to 4.5 lots per acre (Gettel, 1976). Focusing on the space costs of front streets and ignoring the space costs of side streets, one can easily develop a function for such a rule. Each lot will require the following area:

$$f(30 + d)$$

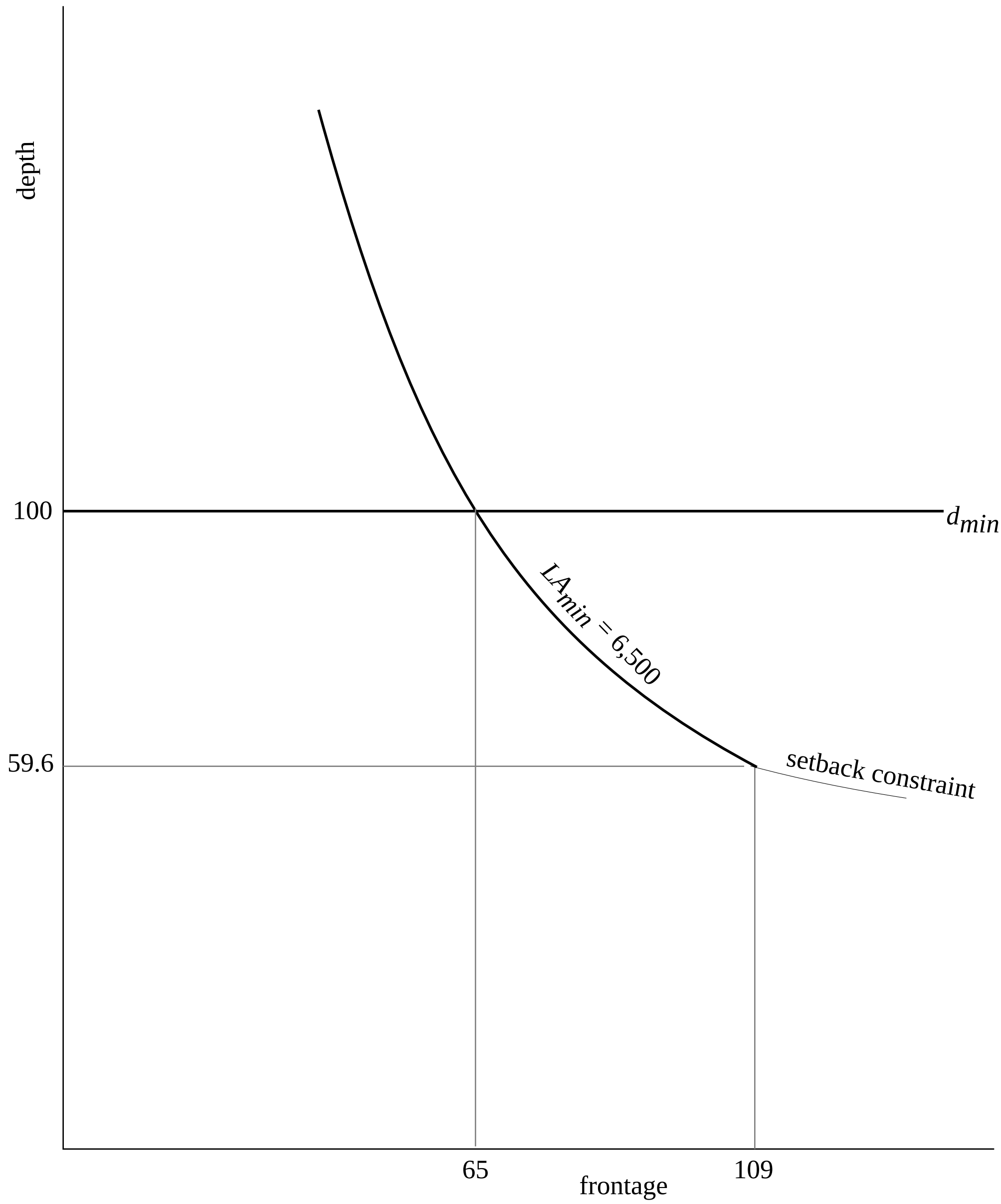
where 30 is the depth in feet of one half a road right-of-way. If there are n lots per acre, then $nf(30 + d) = 43560$, the number of square feet per acre.

Solving for the depth yields:

$$d = \frac{43560}{nf} - 30.$$

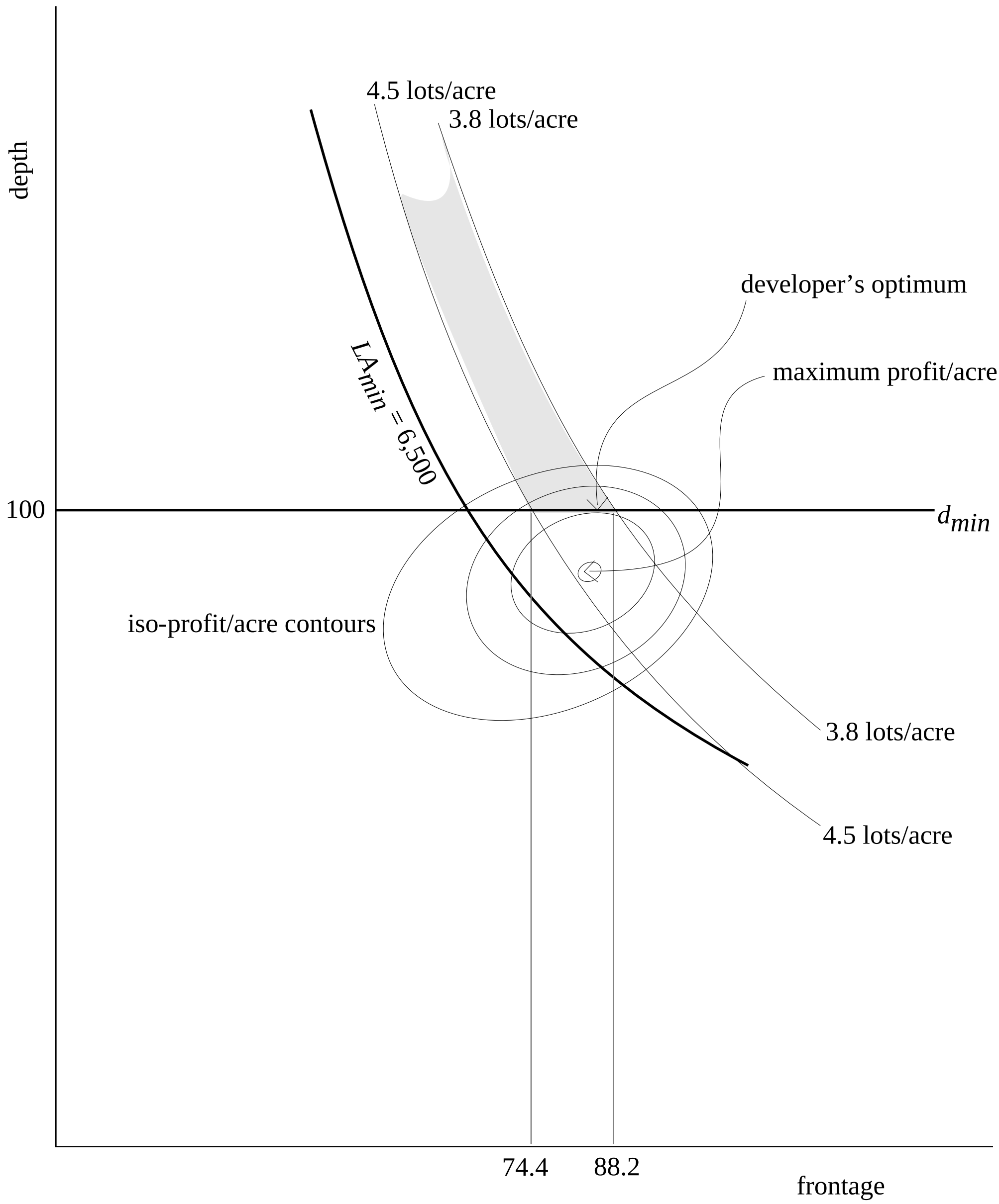
This function is a rectangular hyperbola that has been shifted downward by 30 feet, so it intersects the true rectangular hyperbolas that represent minimum lot area and maximum floor to area ratio constraints. Exhibit 8 shows what the rule of thumb functions would look like for 4.5 lots per acre and for 3.8 lots per acre.

Exhibit 7
Subdivision Regulations and Binding Zoning Constraints



If a developer chooses lot configurations that produce between 4.5 and 3.8 lots per acre and yet meet the public land use constraints, then configurations must be chosen from those shaded in gray in Exhibit 8. If the developer is also bound by the minimum depth from the NSSO, then a frontage between 74.4 feet and 88.2 feet must be selected, while choosing 100 feet of depth, as seen in Exhibit 8. Exhibit 8 shows

Exhibit 8
Rules of Thumb vs. Maximizing Developer Profit



several hypothetical iso-profit/acre contours that are consistent with such a choice. These contours reflect both the implicit prices or ''constraints'' from the market for building lots and the costs of development. For example, if the market detests certain lot configurations, and the implied house designs, there could be a valley in the profit/acre surface from which the iso-profit/acre contours are derived. Note that one of the contours in Exhibit 8 is tangent to the minimum lot constraint from below. The point of tangency would represent the developer's optimal choice; however, the iso-profit/acre contours are not observable. Thus, the rule of thumb might provide the developer with appropriate guidance.

Conclusion

Public land use constraints from the zoning ordinance and from the subdivision regulations can influence the lot and house configurations we see around us. Why do we impose these constraints on ourselves? One answer is that constraints are useful, because the positive externalities associated with yard size and the costs of enforcing private constraints (*i.e.*, covenants) are very high. Thus, we shift the money costs of enforcement to the general taxpayer and shift the psychic costs of enforcement to an uninvolved bureaucracy.

For single-family housing, the constraints that would tend to influence house configuration (the open space ratio and setbacks) tend not to be binding. Instead, the minimum lot area, the maximum floor to area ratio and/or the minimum depth appear more likely to be binding. Nevertheless, a small reduction in the minimum lot area and a small increase in the maximum floor to area ratio can bring these other factors into play. In addition, there may be creative street patterns that render the minimum depth constraint relatively ineffective.

A developer may use rules of thumb, such as the number of lots per acre, to provide a sense of the right order of magnitude for lot design. When experience also suggests that public constraints are binding, the combination of the rules of thumb with the public constraints limits the developer's decision to a very narrow range of possibilities.

This look at constraints does not exhaust the impact on lot and house configurations of, say, subdivision ordinances. For example, requirements that streets be concrete (rather than blacktop or dirt), that storm sewers be pipe (rather than ditches), or that there must be curbs and sidewalks all increase the cost of frontage and modify choice of frontage relative to depth.

The constraints tell us nothing by themselves. It is only when the constraints are considered in the context of lot value and development cost per acre that decisions regarding configuration are made. This article has provided a methodology with which an analyst can examine great complexities in the constraints. While it is possible to consider the market issues at the same time, this is done only in the most casual way in this article.

References

Cannaday, R. E. and P. F. Colwell, Optimization of Subdivision Development, *The Journal of Real Estate Finance and Economics*, 1990, 3, 195–06.

Champaign, City of, Illinois, *New Streets and Subdivision Ordinances*, Council Bill No. 78-01, January 3, 1978, Article 4, 1–48.

Champaign, City of, Illinois, *Amended Zoning Ordinance of 1965*, Revised Draft, 1980, 40–65.

Colwell, P. F. and M. S. Ebrahim, A Note on the Optimal Design of an Office Building, *Journal of Real Estate Research*, forthcoming.

Colwell, P. F. and T. F. Scheu, Optimal Lot Size and Configuration, *Journal of Urban Economics*, 1989, 26, 90–09.

Doiron, J. C., J. D. Shilling and C. F. Sirmans, Do Market Rents Reflect the Value of Special Building Features? The Case of Office Atriums, *Journal of Real Estate Research*, 1992, 7:2, 147–55.

Edelson, N. M., The Developers Problem, or How to Divide a Piece of Land Most Profitably, *Journal of Urban Economics*, 1975, 2:4, 349–65.

Gettel, R. E., *Real Estate Guidelines and Rules of Thumb*, New York: McGraw-Hill, 1976.

Scheu, T. F., Site Valuation and Optimal Development, Dissertation, Urbana-Champaign, IL: University of Illinois, 1985.

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