

The Valuation of Mortgage Loan Commitments Using Option Pricing Estimates

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Abstract. This paper values mortgage loan commitments in the context of the option pricing theory developed by Black and Scholes [1] and Merton [6]. A valuation model is derived and empirical results are presented.

Introduction

The past decade has been a period of extremely volatile interest rates. Following the change in monetary policy regime implemented in October 1979, numerous studies have documented increased volatility. For example, Roley [10] found Treasury bill rate volatility increased thirty times, Friedman [3] documented increased volatility in the corporate bond markets, and Pesando and Plourde [9] found increased volatility in the Canadian long-term bond markets.

Until recently, mortgage interest rates have been immune to this increased volatility. However, a number of recent changes have led to increased volatility in these markets as well. First, deposit rate ceilings were removed during the 1970s and 1980s. Second, the individual state usury ceilings on residential mortgage loans were preempted by the 1980 DIDMCA.¹ Although the law allowed states to override the federal law by acting before April 1, 1983, only a few states acted. Finally, and probably most importantly, the growth of the secondary mortgage market has caused mortgage rates to track more closely to other capital market rates (see Roth [11]).

As a result of this increased volatility, the traditional mortgage rate commitment extended to potential homebuyers has become a more valuable service provided by financial institutions. With the increased emphasis on fee-based income from financial services and the increased competition resulting from the deregulated environment, proper pricing of such services is of paramount importance to financial lending institutions. Thus, this paper values mortgage loan commitments as put options using the option pricing theory developed by Black and Scholes [1] and Merton [6]. Thakor, Hong and Greenbaum [12] valued commercial loan commitments using the option pricing theory. This paper, although differing significantly in the details, follows along the same lines as this previous work.

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The Model

Fixed-rate mortgage loan commitments are made to individuals by most financial lending institutions. Typically, an applicant, subject to credit approval, receives a commitment from the lending institution for a mortgage loan at a rate fixed today to be taken down by some specified future point in time. This commitment is, in essence, a put option that allows the individual to sell his indebtedness (i.e., sell a mortgage) in return for the proceeds from the loan. A crucial element of this commitment is that the rate on the loan is fixed at $t = 0$; therefore, the lending institution bears the interest-rate risk. The individual can exercise the option depending upon the movement in interest rates between the time the commitment is made and the time at which the commitment expires.

Consider a lender who, at $t = 0$, extends a fixed-rate mortgage commitment binding until $t = T$. The mortgage is for M years and the rate is fixed at the current ($t = 0$) rate r_0 . As a result of this commitment, the borrower has the option to sell his indebtedness at $t = T$ in return for L dollars. The value of this indebtedness, X_t , is a function of the market loan rate in existence at time T when the loan is taken down. This value is uncertain because the future interest rate is uncertain. Likewise, the value of the indebtedness at any time t , X_t , is also unknown because it depends upon X_T .

Thus, the mortgage commitment is a put option that allows the borrower to sell the value of his indebtedness, X_T , at time $t = T$ in return for a loan of amount, L , the strike price. Using the Black-Scholes model, the value of the commitment, at $t = 0$, U_0 , is:

$$U_0 = -X_0 N(-d_1) + L \exp(-rt) N(-d_1 - \sigma\sqrt{T}) \quad (1)$$

where

r = risk-free rate from $t = 0$ to $t = T$

T = time to expiration of the mortgage commitment

$X_0 = E(X_T) \exp(-rt)$ where $E(\cdot)$ is the expected value operator

σ = standard-deviation of the continuously-compounded rate of return on the underlying state variable, X_t over the period $t = 0$ to $t = T$

$d_1 = (\ln(X_0/L) + (r + .5\sigma^2)T)/(\sigma\sqrt{T})$

$N(\cdot)$ = the cumulative normal density function

\exp = the natural number (approximately 2.71828) raised to the argument

X_0 , U_0 , L are defined above

To utilize equation (1), σ needs to be determined. To do this, we need an expression for the underlying state variable, X_t . Let c be the loan payment per unit time. Then the face value of the loan is simply the present value of the loan payments, c , discounted at the current loan rate, r_0 . We have

$$L = \int_0^M c \exp(-r_0 s) ds = c(1 - \exp(-r_0 M))/r_0 \quad (2)$$

or

$$c = r_0 L / (1 - \exp(-r_0 M)) \quad (3)$$

Using equation (3), the value of the indebtedness at any time t is the sum of the loan payments discounted at the contemporaneous mortgage rate, r_t , thus,

$$X_t = \int_0^m c \exp(-r_t s) ds = \frac{R_O L (1 - \exp(-r_t M))}{r_t (1 - \exp(-r_o M))} \quad (4)$$

Therefore, the continuously-compounded rate of return on the underlying state is

$$\begin{aligned} \ln(l + R_t) &= \ln(X_t/X_{t-1}) \\ &= \ln(r_{t-1}(1 - \exp(-r_t M)) - \ln(r_t(1 - \exp(-r_{t-1} M))) \end{aligned} \quad (5)$$

Thus,

$$\sigma^2 = \text{var}(\ln(X_t/X_{t-1})) \quad (6)$$

where the argument of the variance is given by equation (5). Using equation (4) and assuming $E(r_t) = r_o$, we have

$$E(X_T) = L$$

and

$$X_O = L \exp(-RT) \quad (7)$$

That is, the value of the indebtedness is simply the present value of the loan amount. Using equation (7),

$$d_1 = -(\ln(\exp(-rt)) + rT + .5\sigma^2 T) / (\sigma\sqrt{T}) = .5\sigma\sqrt{T}$$

Equation (1) becomes

$$U_o = L \exp(-rt)(2N(.5\sigma\sqrt{T} - 1)) \quad (8)$$

Equation (8) is a put option expression for the valuation of a mortgage commitment. The value is linear in the size of the loan and depends explicitly on four variables: r , T , L and σ .

Although the model provides us with an explicit expression for the value of a mortgage commitment, it does suffer from the usual problems of the option pricing theory. The model assumes the value of the indebtedness follows a pure random walk. This has not been established but appears to be a reasonable assumption because of the stochastic nature of interest-rate changes. The model allows for no early exercise of the option and assumes no transaction costs. These requirements are not serious; however, since there may be transactions costs in finding another mortgage if the option is not exercised, then the estimated values may have a slight upward bias. The model also requires the underlying asset to be marketable so that an arbitrage strategy may be employed if it is mispriced. The nonmarketability of the underlying asset may be a more severe problem. However, numerous other applications of the option pricing theory have ignored this issue as well. For example, deposit insurance has been valued by Merton [8] and Marcus and Shaked [5], loan guarantees have been valued by Merton [7] and even education has been valued by Dothan and Williams [2]. Laying aside these potential difficulties, the model is empirically examined in the next section of this paper.

Empirical Findings

The valuation expression for a mortgage commitment derived in the previous section was estimated using two different mortgage rate series. They were the rates on thirty-year mortgage commitments for delivery in thirty days as quoted on a daily basis in the *Wall Street*

Journal by the Federal Home Loan Mortgage Association (Freddie Mac) and the Federal National Mortgage Association (Fannie Mae). These are the rates at which these federal agencies agree to purchase mortgages from lending institutions which will result from commitments actually made. There is only one rate per day quoted on all mortgages to be purchased, however, there can be negotiated rate differentials. These rates were gathered over a twenty-five-month period from August 1985 through August 1987.

The risk-free rate of return used was the average of the dealer bid and ask discount rates on Treasury bills maturing in thirty days as reported on a daily basis in the *Wall Street Journal*. These average rates were adjusted to true continuous time yields for use in the option valuation model. Consistent with the rate series employed, the length of the loan commitments valued was thirty days (i.e., $T = 30/365$) and the maturity of the mortgages was thirty years (i.e., $M = 30$). Equation (6) was estimated over the period of the commitment to obtain a volatility estimate of the underlying state variable, namely the customer's indebtedness.²

The mortgage commitment values were then estimated using equation (8) on a daily basis for both the Fannie Mae and Freddie Mac commitments over the period August 1985 through August 1987.³ The results of the Fannie Mae series are presented graphically in Exhibit 1 for a \$100,000 loan commitment. Exhibit 2 presents summary statistics for the Fannie Mae estimates. The estimated commitment values averaged \$1,354 on a \$100,000 mortgage commitment over the period of analysis. A considerable dispersion in values was found, ranging from \$530 to \$3,147, as shown in Exhibits 1 and 2.

Since there are no observable prices of mortgage commitments, it is difficult to determine the accuracy of these estimates. However, they can be compared with the actual savings that would be realized if one held the commitment versus obtaining a mortgage at the current spot rate at $t = T$. The estimated savings would be:

$$\text{Savings estimate} = \max \left(\begin{array}{l} \text{present value of loan} \\ \text{payments without} \\ \text{commitment} \end{array} - \begin{array}{l} \text{present value of} \\ \text{loan payments} \\ \text{with commitment} \end{array} \right)$$

Since the option model is in continuous time, the savings estimate should be a continuous time estimate. Letting R and r be the mortgage rates with and without the commitment respectively, the savings estimate (see Appendix A) is:

$$\text{Savings estimate} = \max \left(0, \frac{L}{R} (1 - \exp(-RM)) \left[\frac{r}{(1 - \exp(-rM))} - \frac{R}{(1 - \exp(-RM))} \right] \right) \quad (9)$$

Note that if r is less than R , the savings is zero, but if the commitment rate R is less than the mortgage rate r at $t = T$, the commitment has value that increases linearly with L just as in the option valuation model.

The savings estimates given by expression (9) were calculated and are reported in Exhibit 2 for the Fannie Mae series. The average savings estimate over the sample period was \$1,279, only \$75 less than the option model estimates. This translates into a 5.5% estimation error. The t -statistic reported in Exhibit 2 tests whether the difference between the two sample means is zero. The statistic allows for unequal population variances (see Welch [13]). The null

Exhibit 1
Estimated Mortgage Commitment Values
August 1985–August 1987

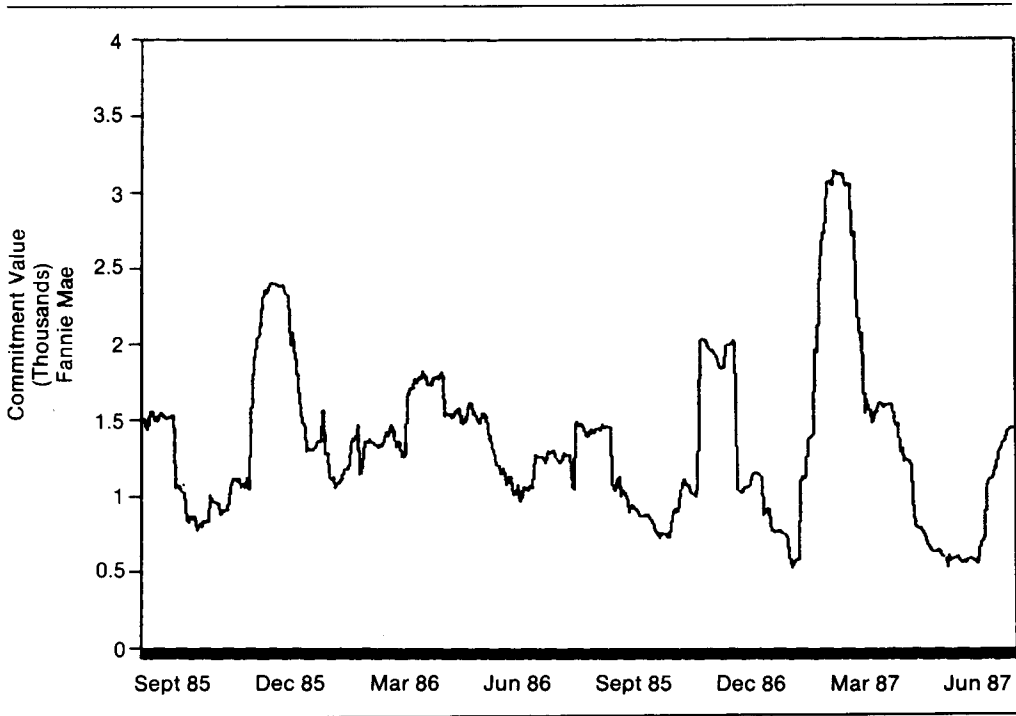


Exhibit 2
Summary Statistics
Mortgage Loan Commitment Estimates
(Fannie Mae)

	Mean	Medium	Standard Deviation	Minimum	Maximum
Option Estimates	\$1,354	\$1,293	\$ 537	\$ 530	\$ 3,147
Actual Saving Estimates	\$1,279	0	\$2,572	0	\$13,744
Difference ¹ (Option estimate minus actual savings estimate)	\$ 75	\$ 939	\$2,320	\$ -10,638	\$ 2,412
	(.65) ²				
% Difference	5.5%				

¹The reported statistics are for the daily differences between the option and savings estimates with over 500 observations.

²The *t*-statistic tests whether the difference between two means with unequal variances is equal to zero (see Welch [13]) and is not statistically significant.

hypothesis is that the difference is zero and the reported statistic is not statistically significant. However, the variability in the savings estimates is much larger. Notice also that since the median is zero, most savings are zero. That is, whenever interest rates decline over the period of commitment, there is no savings from holding the commitment.

Comparative Static Relationships

The option model of equation (8) implies that commitment values are a function of four variables, $U_o = f(L, r, \sigma, T)$. An important issue is the sensitivity of the commitment value estimates to the underlying parameters. Using equation (8), the comparative static relationships for U_o have been derived and are presented in Appendix B.

The mean sensitivities (for a loan size of \$100,000) are presented in Exhibit 3. Changes for representative discrete changes in the underlying input parameters are also presented. Exhibit 3 indicates that the estimated commitment values are very sensitive to the volatility and length of the commitment, but relatively insensitive to interest-rate levels.

Discussion

Since a mortgage loan commitment mimics the characteristics of a put option, the option pricing theory can be used to value a commitment. The results indicate that mortgage commitments have considerable value. Their value derives from the fact that a commitment places all of the interest-rate risk with the lender. Thus the commitment is a form of insurance for the holder against adverse interest-rate movements.

A number of implications follow from applying the option pricing theory to mortgage commitments. First, the value of a commitment is linearly related to the size of the loan. This implies that lenders should employ a variable fee structure in the pricing of mortgage commitments. Usually only a fixed fee is charged. Second, the typical charge for a loan commitment is considerably below their estimated values. This may be due to the fact that historically few commitments holders have reneged on loan commitments even when mortgage rates do fall. But with larger interest-rate volatility and better informed borrowers, this trend may not continue. One can easily renege by obtaining a bridge loan or by renegotiating the rate. Third, the comparative static relations indicate the strong sensitivity of the commitment value to the underlying volatility. This suggests that the pricing of commitments should also depend upon the current interest-rate environment.

Exhibit 3
Mean Sensitivities for U_o (with $L = 100,000$)
(August 1985 to August 1987)

FANNIE MAE
— with respect to —

	L	r	σ	T
Mean Sensitivity	.0135	111.28	11381	16383
Mean Change in U_o	\$13.50	\$.11	\$11.38	\$44.88
for a Variable Change of	\$1,000	.001	.001	1 day

The option pricing estimates indicate that commitments are valuable. For comparison, actual savings estimates realized by an option holder were calculated since market prices of commitments are nonexistent. The agreement between the option and savings estimates was quite good, with only a 5% deviation on average. The mean option and savings estimates were not statistically different. This provides important support for the option estimates despite the well-known limitations of the option pricing theory.

Summary

This paper values mortgage loan commitments as put options using the option pricing theory developed by Black and Scholes [1] and Merton [6]. Estimates indicate that commitments have significant value. These estimates are in good agreement with actual savings estimates that would be realized from holding commitments. The results indicate that the value of commitments varies directly with the loan size as well as with interest-rate volatility and suggests that the commitments should be priced accordingly. The model is simple to implement and all parameters can be estimated empirically.

Appendix A

Let R and r be the mortgage rates with and without the commitment respectively. Also, let c and c' be the mortgage payments per unit time with and without the commitment. Then the present value of the payments with (PV_w) and without (PV_{wo}) the commitment are:

$$PV_w = L = \int_0^M c \exp(-Rt) dt \quad (1A)$$

$$PV_{wo} = L = \int_0^M c' \exp(-rt) dt \quad (2A)$$

where M is the loan maturity. The savings from having the commitment is the present value of the reduced payments discounted at the opportunity rate which is the rate, R , that can be obtained with the commitment (if R is less than r). Therefore,

$$Savings = \int_0^M (c' - c) \exp(-Rt) dt = (c' - c)(1 - \exp(-RM))/R \quad (3A)$$

Solving equations (1A) and (2A) for c and c' respectively and substituting these expressions into equation (3A) we get,

$$Savings = \frac{L}{R} (1 - \exp(-RM)) \left[\frac{r}{(1 - \exp(-rM))} - \frac{R}{(1 - \exp(-RM))} \right] \quad (4A)$$

Appendix B

The comparative static relationships for U_o , the mortgage commitment value, have been obtained using equation (8). They are

$$\begin{aligned} \partial U_o / \partial L &= U_o / L > 0 \\ \partial U_o / \partial r &= -TU_o < 0 \\ \partial U_o / \partial \sigma &= LT \exp(rt)(1/2\sigma^2) \exp(-\sigma^2 T/8) > 0 \end{aligned}$$

$$\begin{aligned} \partial U_0 / \partial T = & rL \exp(-rT)(2N(.5\sigma\sqrt{T}) - 1) \\ & + .5L \exp(-rT)(1/2) \exp(-\sigma^2 T/8)/T \cong 0 \end{aligned}$$

Notes

¹The Depository Institution Deregulation and Monetary Control Act of 1980.

²Equation (5) was estimated using each of the annualized mortgage rate series for each day. The variance of this daily series was calculated over the 30-day period of the mortgage commitment. Generally, there were 22 trading days over each 30-day period. These volatility estimates were annualized by multiplying the variance estimates by 250. These annualized volatility estimates were used in equation (8) to value the mortgage commitments.

³The results for the Freddie Mac series are very similar to the Fannie Mae series and are not reported here but are available upon request.

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