Pricing Limited Partnerships in the Secondary Market

Authors Daniel C. Quan and Chang N. Xuan

Abstract

This study investigates the pattern of prices for multiple partnerships sold in the secondary market. In the model, the partnership buyer prefers to purchase the units sequentially since sellers have varying desires to sell. The benefit of a sequential purchase strategy is partially offset by rational sellers who demand higher prices in earlier sales since the possibility of future sales reduces the sellers' eagerness to sell in earlier rounds. If this strategic component is sufficiently large, a pattern of decreasing prices should be observed. Using a panel dataset comprised of 52,679 transactions from eighteen real estate limited partnerships, and after controlling for performance characteristics, the study finds that prices decrease over time, thus indicating a significant strategic component in this market.

Introduction

Syndicated limited partnerships (LPs) provide an accessible and uncomplicated way for middle to upper income investors to enjoy co-ownership of asset pools. Typically limited partners purchase claims to future returns of assets acquired by the general partner who manages the assets day-to-day and represents investors' fiduciary interests. Limited partners favor such an arrangement since their liabilities are limited to the amount of their capital investments, and cash flow distributions are prioritized to guarantee their initial investments and a promised return before the general partner is compensated. The success of this structure has resulted in $132 billion worth of LPs investing in real estate ($69 billion), oil and gas ($32.3 billion), leasing equipment ($7.2 billion), cable television ($1.9 billion) and other miscellaneous areas. By design LPs are illiquid investments with long holding periods.

In 1980, Liquidity Fund initiated a program of buying LPs and effectively creating a secondary market to provide liquidity. The number of firms that buy and sell LPs has since grown; many of them purchasing partnerships to form investment funds exclusively comprised of seasoned LPs have survived the earliest and often the riskiest part of their economic life. Furthermore, in contrast to new partnerships, these LPs provide historical performance records thus allowing investors to better assess future performance. These investments also have a shorter holding period and, depending on the structure, can yield tax benefits for certain investors.

A critical task facing firms that trade in this secondary market is determining an LP's purchase price because the difference between the resale and acquisition price determines profits. Pricing involves determining of the so-called ''break-up'' value, an estimate of the partnership's instantaneous asset liquidation value less liabilities and the general partner's compensation. It is not uncommon for LPs to be purchased at 20% of the break-up value, representing an 80% discount. In this study, the total average discount was 30%.[1] This discount appears to vary considerably, depending on whether the purchases were made in the earlier or the later rounds of purchases. The average discount ranges from 23% to 43%:

This study proposes a strategic model of how these discounts are determined and investigates their pattern when LPs from the same partnership are purchased over time. The model recognizes that a limited partner's decision to sell at a particular price is similar to the task facing bidders in a common value multiple-object sealed-bid auction. Several distinguishing features of this market motivate the modeling approach.

First, limited partners are often uninformed about the true value of their claim on assets. Although general partners provide periodic reports on asset performance, these estimates are often overly optimistic. In particular, for real estate LPs, there is considerable uncertainty about the value of the underlying real estate, and estimates are often based on imprecise appraisals. Even if a reasonable asset value is available, limited partners cannot determine their partnership stakes without detailed information about the general partner's compensation and all other liabilities.

Second, prices will likely reflect strategic behavior among competing sellers because the number of purchased units is often small relative to the total number of LPs outstanding. Buyers often only want to acquire a limited number of units from any given partnership.

When purchased simultaneously, multiple LPs must be purchased at the same price. Given that limited partners are uninformed about the value of their partnership, buyers may wish to price discriminate by purchasing multiple LPs sequentially, thus establishing a pattern of increasing prices, or decreasing discounts. However, by anticipating a sequential purchasing strategy, sellers may demand higher prices in the early rounds as they wait to sell in the later rounds. This suggests that prices may fall over time as more units are sold. Whether a pattern of increasing or decreasing discounts emerges depends largely on which effect dominates. The strength of each effect largely depends on the sellers' private signals and perhaps in the number of outstanding LPs.

To empirically detect a pattern of discounts, a trade-level panel database that records sequential purchases made over time was used. The sample is comprised of 2,424 transactions involving 52,679 individual LPs. After controlling for each partnership's performance, the analysis reveals a strong pattern of decreasing discounts.

Next, the trading process and how the purchasing firm solicits the limited partners is described.[2] The model and empirical results follow.

The Trading Process and Strategic Considerations

The trading process is initiated when the acquiring firm performs a preliminary analysis on all existing LPs to identify subgroups that own desirable and performing assets. A letter sent to each limited partner within each sub-group solicits sellers. No prices are quoted at this stage. Upon receiving a sufficient number of positive responses from a given partnership, the acquiring firm determines the partnership's break-up value and each limited partner's share. Because partnership structures differ in the cash distribution between the limited and general partners, information must be obtained from either the general partner or from 10K reports that partnerships are required to file.

The break-up value is comprised of the acquiring firm's appraisal of the partnership's real estate holdings less liabilities. This value corresponds to the sum payable to the limited partners in accordance with the priorities set forth in the partnership agreement. The final offer price is determined to be a fraction of this break-up value. This process of determining the offer price typically takes two to three weeks.

The firm must also determine the method of buying the desired number of units. Where simultaneous multiple purchases are involved, firms are required to purchase all units at the same price. A sequential purchase strategy will allow them to pay different prices for each purchase and the firm must determine a schedule of prices that they are willing to pay for the number of units they wish to purchase.

The most common motive for selling is investors' dissatisfaction with the underlying assets' performance (Wollack and Donaldson, 1992). This dissatisfaction combined with the observation that most limited partners are unsure of the value of their partnerships suggests that their selling decisions depend crucially on their information sets. And since each limited partner has equal claims on the underlying assets, it appears natural to model each limited partnership as having some common value.

Conceptually, the limited partners submit ''bids'' to sell their LPs to the buyer, and because all partnership claims are identical, the partner willing to sell for the lowest price ''wins.'' Each limited partner must determine a selling price based on incomplete information about the partnerships' common value. Thus, the risk neutral bidder's task is to determine a selling price that would maximize expected profits, defined as the product of the probability of selling (or the probability their selling price is one of the lowest) and their asking price.

The purchase of multiple LPs from competing sellers introduces another dimension to our auction. Since the simultaneous purchase of multiple partnerships must be made at the same price, a buyer may price discriminate and purchase the number of desired units sequentially, paying lower prices to buyers with low signals in earlier rounds and paying higher prices in later rounds. The profitability of this strategy is partially offset by sellers who may set high prices in earlier rounds and sell in later ones if necessary. With such strategies, prices would fall over time as more units are sold. The possibility of later sales works to the buyer's disadvantage, thus, the buyer is discouraged from disclosing the number of units to be purchased. On the other hand, the limited partners recognize that the intention of the buyer may not be truthfully disclosed and, therefore, assumes the buyer may purchase more units than initially announced.

Partnership Pricing Model

In the model, all agents are risk neutral and discounting is ignored.[3] Learning is also ignored. For uninformed sellers, observation of previous sales has considerable informational value for the remaining partners and, therefore, will affect their subsequent selling decisions. Although this raises interesting issues, notably the possibility that the buyer may use this mechanism to negotiate prices in earlier sales to influence later sales, this element is suppressed in the model. Furthermore, since LP units are not traded in a centralized exchange, no mechanism exists whereby such information can be conveyed to the remaining limited partners.[4]

Simultaneous Purchase

First consider a partner's selling price given that an external buyer credibly communicates that his wish to buy one such partnership. There are n partners each holding an equal interest in an asset with a common value v.[5] Let $P(x_i)$ be the price partner i is willing to sell given the signal x_i for $i = 1, 2, \ldots, n$. $P(.)$ is an increasing function of the signal.[6] The signal can be interpreted as the limited partner's estimate of v. $F(x_i|v)$ represents the conditional distribution of the i^{th} partner's estimate conditional on v. $\phi(v|x_i)$ is the conditional posterior density of v given the partner i's signal. $F(.)$ is common knowledge.

A buyer wishes to purchase one out of n LP units. Let the subsequent market valuation of the partnership be the common value v.[7] The payoff to a risk neutral limited partner who decides to sell at P is $P - v$. Because $P(.)$ is an increasing function of the partner signal x, the probability that P is lower than all other partners is equivalent to the probability that the partner has the lowest signal. Thus, the partner's expected profit from selling at P is $P - v$ times the probability that the partner has the lowest signal. Integrate over the distribution of v conditional on private signal x, the limited partner's problem is to determine P, which maximizes the expected profits given the distribution of signals and the partner's uncertainty about v.

$$Max\ E\,[\Pi] = \int (P - v)[1 - F(P^{-1}(x)|v)]^{n-1}\phi(v|x)dv. \qquad (1)$$

Ideally i would like to sell for as much over v as possible. However, i is faced with competition from the other $n - 1$ partners. $[1 - F(P^{-1}(x_i)|v)]^{n-1}$ is the probability that the other partners will not sell for less. If i asks too high a price, other partners can "outbid" i. Thus the optimal selling price entails a surplus bounded by the competitive actions of the other potential sellers.

By maximizing Equation (1) with respect to P and simplifying, a symmetric equilibrium for P must satisfy the following differential equation:

$$P'(x) = \frac{\int (n - 1)(P - v)[1 - F(x|v)]^{n-2}f(x|v)\phi(v|x)dv}{\int [1 - F(x|v)]^{n-1}\phi(v|x)dv}. \qquad (2)$$

Equation (2) does not have a closed-form solution and further simplifications are required in order to solve for the equilibrium. A tractable solution can be obtained if it is assumed, as in Levin and Smith (1991) and Thiel (1988), that (1) $\phi(v)$ is constant for all v or that the partners have diffuse priors, (2) estimation errors are independent of the true value such that the conditional signal density function can be expressed as an unconditional density and (3) each partner's estimate of v is unbiased or $E[x_i] = v$. Under these assumptions, the problem can be expressed as:

$$P'(x) + K_1P(x) + K_2 - K_1x = 0, \text{ where} \qquad (3)$$

$$K_1 = -\frac{n(n - 1)}{\sigma}\int_{-\infty}^{\infty} [1 - F(z)]^{n-2}f(z)^2dz, \qquad (4)$$

$$K_2 = -n(n - 1)\int_{-\infty}^{\infty} z[1 - F(z)]^{n-2}f(z)^2dz, \qquad (5)$$

$z = (x - v)/\sigma$ the normalized signal, and σ is its standard error.

For the general case when the buyer wishes to buy k units simultaneously at a unit price, the differential Equation (3) is preserved but for this case:

$$K_1 = -\frac{(n-k+1)(n-k)}{\sigma}\int_{-\infty}^{\infty}[1-F(z)]^{n-k-1}f(z)^2dz. \tag{6}$$

$$K_2 = -(n-k+1)(n-k)\int_{-\infty}^{\infty}z[1-F(z)]^{n-k-1}f(z)^2dz. \tag{7}$$

The general solution to the above problem is:

$$P(x) = x - \frac{1+K_2}{K_1} + \beta\exp(-K_1x). \tag{8}$$

where for signal distributions that are symmetric, $(1 + K_2)/K_1 < 0$ and β is a parameter that is defined by an additional appropriate boundary condition. As pointed out by Levin and Smith (1991), individual rationality requires $\beta \leq 0$. This condition ensures that $P(x)$ cannot be greater than x and rules out the possibility that sellers have negative expected profits. Wilson (1990) showed that if the signal distribution is normal and that the priors are diffuse in the limit, $\beta = 0$ and the symmetric equilibrium pricing strategy is linear. Assuming a uniform signal distribution over the range, $[v - \varepsilon, \bar{v} + \varepsilon]$, it can be shown that $K_1 = -(n - k + 1)/2\varepsilon$ and $K_2 = (n - k + 1)/2 - 1$. Substituting these expressions into Equation (8) produces the following closed-form solution for the symmetric Nash equilibrium pricing schedule for k units:

$$P(x) = x + \varepsilon + \beta\exp\left(\frac{x(n-k+1)}{2\varepsilon}\right). \tag{9}$$

Because $\beta < 0$, $P(x)$ is decreasing in n and increasing in k. For a given $k < n$, as the number of partners increase, the lower will be the equilibrium selling price. Similarly, for a given n, the price paid for the purchase of k units will be lower than the purchase price for $k + 1$ unit. The more units desired, the more likely that each will be chosen. Thus, partners will increase their selling prices accordingly and, therefore, will reduce the discount.

For normally distributed signals, when $k = 1$, the symmetric equilibrium pricing strategy is:

$$P(x) = x + \alpha_n \sigma + \beta \exp(-K_1 x), \tag{10}$$

where:

$$\alpha_n = \frac{\int_{-\infty}^{\infty} t^2 d[F(t)]^n}{\int_{-\infty}^{\infty} t d[F(t)]^n} \quad \text{and} \quad K_1 = -\sigma^{-1} \int_{-\infty}^{\infty} t d[F(t)]^n. \tag{11}$$

and $F(.)$ is the normal distribution function.[8] The intuition from the above expression comes from noting that α_n is convex in n; and P is therefore convex in n. Intuitively, if the nonlinear term is ignored, the curvature of α_n implies that selling prices will be higher for small n because a larger surplus will be demanded by limited partners when they face less competition. As n increases, the surplus is reduced as competition increases and lower selling prices result. However, if n increases beyond the minimum point of α_n, the price will increase, which corresponds to an adjustment for the likelihood of falling victim to the winner's curse. Since the likelihood of being victimized grows as the number of partners increase, the rational seller must add on a premium to compensate for this possibility.

Purchasing Sequentially or Simultaneously?

A buyer who wishes to purchase k LPs must determine not only the offer price but also the best mechanism for the purchase; either sequentially or simultaneously. Each mechanism will yield different profits since the partners' selling strategies will also differ.

If the number of units desired by the buyer is known and credibly communicated to the sellers, the selling price will not be affected by sequential buying. However, the buyer may consider announcing that only one unit will be purchased when, in fact, the buyer wants more. With the possibility that the buyer may want more than one unit, the optimal selling price for the initial sale will be set differently due to the potential of a later sale. In the following model, conditions where such a deceptive practice is optimal are determined and whether it will yield the buyer higher profits.

To demonstrate this effect, consider the case when the maximum number of units a buyer is willing to purchase is two. First, consider the selling strategy when the buyer wishes to buy only one unit. Let q be the probability that the buyer only wants one and $(1-q)$ that the buyer wants two. The probability that the buyer is misleading is p.[9] The probability the buyer wants one unit as disclosed is $q/(q+p(1-q))$. Correspondingly, the probability that there will be an offer to buy another unit is $p(1-q)/[q+p(1-q)]$. Disclosing the true number of units desired is a buyer choice variable and conditions for are derived for truthful disclosure. This is achieved by determining the Nash equilibrium corresponding to the proposed sequential solicitation game. Each partner, conjecturing that the buyer has a certain probability of buying in later rounds, determines the selling price in the first round and expected selling price in the second round if necessary.

Let $\tilde{P}$ be the equilibrium price the partner is willing to sell in the first round after the buyer's announcement. Conditional on p and q, the LPs' expected surplus function from such a case could be expressed as:

$$\begin{aligned} E[P - v|k = 1] &= \frac{q}{q+p(1-q)} \int (P-v)[1-F(x|v)]^{n-1}\phi(v|x)dv \\ &\quad + \frac{p(1-q)}{q+p(1-q)} \left\{ \int (P-v)[1-F(x|v)]^{n-1}\phi(v|x)dv \right. \\ &\quad \left. + \int (P-v)[1-(1-F(x|v))^{n-1}][1-F(x|v)]^{n-2}\phi(v|x)dv \right\}. \end{aligned} \tag{12}$$

If the buyer only wishes to buy one unit ($q = 1$), the expected payoff for the seller is the same as the one unit purchase case. This is represented by the first term in Equation (12). If, in actuality, the buyer wishes to buy two units sequentially, then the first part of the second term reflects the likelihood of a seller selling in the first round (or analogously having the lowest signal of the n partners). The second part corresponds to the seller's expected surplus in the second round with the remaining $n-1$ partners, given an unsuccessful sale in the first round.

The Appendix shows that the solution to the partner's optimization problem must satisfy the following differential equation:

$$P'(x) + K_1P(x) + K_2 - K_1x = 0, \tag{13}$$

where:

$$K_1 = \frac{1}{\frac{1+t}{n} + \frac{t}{2(n+1)}} \left\{ \frac{t(n-1)}{\sigma} \int [1 - F(z)]^{2n-4} f(z)^2 dz \right.$$
$$- \frac{(1-t)(n-1)}{\sigma} \int [1 - F(z)]^{n-2} f(z)^2 dz$$
$$\left. - \frac{t(n-2)}{\sigma} \int [1 - (1 - F(z)^{n-1}][1 - F(z)]^{n-3} f(z)^2 dz \right\}. \tag{14}$$

and

$$K_2 = \frac{1}{\frac{1+t}{n} + \frac{t}{2(n+1)}} \left\{ \frac{t(n-1)}{\sigma} \int z[1 - F(z)]^{2n-4} f(z)^2 dz \right.$$
$$- \frac{(1-t)(n-1)}{\sigma} \int z[1 - F(z)]^{n-2} f(z)^2 dz$$
$$\left. - \frac{t(n-2)}{\sigma} \int z[1 - (1 - F(z))^{n-1}][1 - F(z)]^{n-3} f(z)^2 dz \right\}. \tag{15}$$

where: $t = p(1 - q)/q$. The general solution is:

$$\tilde{P}(x) = x - \frac{(1 - K_2)}{K_1} + \beta \exp(-K_1 x). \tag{16}$$

Define P_2^n as the seller's price if there were n partners and the buyer wishes to purchase the maximum two units simultaneously. Let $\tilde{P}_1^n$ be the seller's price in the first round given that the buyer has announced that he wants one unit $\tilde{P}_1^n$ corresponds to a partner's selling price in the first round given that the buyer may wish to purchase additional units in later rounds.

The buyer's expected profit function can be viewed as follows:

$$E[\Pi] = p[v - \tilde{P}_1^n + (v - P_1^{n-1})] + (1 - p)\,[2(v - P_2^n)]. \tag{17}$$

the buyer decides to be misleading, $v - \tilde{P}_n^n$ will be received in the first round of solicitation among the n partners and $(v - P_1^{n-1})$ in the second round. Conversely, if the buyer decides to be truthful, then the decision will be to buy two units resulting in a surplus of $2(v - P_2^n)$. It is important to note that $\tilde{P}_1^n \neq P_1^{n-1}$ since sellers recognize that the buyer may be misleading in the first round. The buyer's task is to maximize the expected profit function with respect to p and determine the method of purchasing multiple units. The following is in general true for a $p > 0$:

Proposition 1. If the signal distribution is uniform around v such that $x \in [v - \varepsilon, v + \varepsilon]$ and $n > 2(1 + t)/t$, then $\tilde{P}_1^n > P_1^{n-1}$.

The $n > 2(1 + t)/t$ requires that there are sufficient numbers of limited partners relative to the probabilities p and q.[10]

Note that:

$$\tilde{P}_1^n = x + \varepsilon + \tilde{\beta} \exp\left[\frac{n(n-1)(1+t)}{\varepsilon[2(1+t)(n-1) + tn]} x\right]. \qquad (18)$$

Also note that the price for a single partnership among $n - 1$ partners is the same as the price for purchasing two units among n partners, thus:

$$P_1^{n-1} = P_2^n = x + \varepsilon + \beta \exp\left(\frac{(n-1)x}{2\varepsilon}\right). \qquad (19)$$

Thus, the condition $\tilde{P}_1^n > P_1^{n-1}$ is satisfied if:

$$\frac{n(n-1)(1+t)}{\varepsilon[2(1+t)(n-1) + tn]} < \frac{(n-1)}{2\varepsilon}. \qquad (20)$$

since $\beta < 0$. By substitution, $\tilde{P}_1^n > P_1^{n-1}$ if $n > 2(1 + t)/t$, which clearly holds.

$\tilde{P}_1^n > P_1^{n-1}$ is consistent with the intuition that the seller will ask for a higher price in the first round if there is a positive probability of selling in the second round. The probability of selling in the second round decreases the cost of not selling in the first, thus, the seller will ask for a higher price in the first round. The above ordering also depends on the relative signals. The buyer benefits from false disclosure when prices of the two units differ; identical signals present no incentive for false disclosure.

The previous result gives rise to an empirical test. An implication of the above ordering result is that prices will decrease over time when LPs are purchased sequentially. This pattern arises because of the repeated game effect in the analysis. In contrast, the absence of strategic considerations will result in a pattern of increasing prices since those who are eager to sell will sell in earlier rounds at low prices while those less eager will sell in later rounds at higher prices.

The model indicates that if future sales are possible, rational sellers would be willing to ask for higher prices in earlier rounds, but as the number of units sold increases or as the buyer's purchases approaches a set limit, sellers would reduce their asking prices thus yielding a pattern of decreasing prices. Since this implication depends on the signal distribution that is unobservable, no formal test of the above model is possible. However, the above ordering effect can be tested if repeated sales of the same partnership units over time can be observed.

Empirical Measure of Discount

Barber (1996) investigated the discount size using cross-sectional data from sales of 112 real estate LPs. He regressed the discount size, defined as the percentage difference between each partnership's sale price and the general partner's (or a third party appraiser) appraised value of each unit, on several partnership performance characteristics. He documents a mean discount of 45% and after including partnership-specific variables to account for cash flow yield, leverage, operating performance and liquidity, his regression model explained 80% of the cross-sectional variation.

The empirical model in this study differs from Barber's (1996) in several ways. Although the same variables are used to control for partnership characteristics, the empirical focus attempts to determine a pattern of discounts when multiple units from the same LP's are sold over time. In addition to the panel nature of the database, the data also differs from Barber in that partnership transactions data was obtained from Liquidity Fund (LF), which records actual trades made over time. The panel database includes information on the number of transactions, the number of partnership units traded in each transaction and the date of each transaction and the price LF paid for each unit. Also made available was LF's internal valuation of each partnership's break-up value, a potentially more reliable estimate of the underlying asset value as compared to Barber's database where the general partner's estimates were used.

Summary statistics are reported in Exhibit 1. The sample contains 3,265 transactions involving the purchase of 66,952 units. The average time span between the first and last purchase was 6.6 years. To see how the discounts may change over time, the first and last percentage discounts (Columns 3 and 5) are averages of the first and last five transactions respectively. Discounts are defined as the percentage of the purchase price to LF's valuation price. The discounts varied considerably over time, from a beginning average discount of 22.8% to a

Exhibit 1 | Descriptive Statistics

Name	First Transaction Date	First Transaction Discount	Last Transaction Date	Last Transaction Discount	Span[a]	Units Sold	No. of Transactions	Discount Difference
Insured Income Properties 1982	1/10/86	29.9	8/3/93	16.8	7.6	4,710	160	−13.1
Insured Income Properties 1983	10/2/86	18.0	5/7/93	18.3	6.6	6,792	199	0.4
Insured Income Properties 1984	3/18/88	13.0	7/20/93	17.6	5.3	6,022	226	4.6
Insured Income Properties 1985	2/18/88	19.1	11/1/93	15.4	5.7	11,309	375	−3.7
JMB Income Properties, Ltd. 10	7/13/87	16.0	5/9/94	65.4	6.8	3,945	281	49.4
JMB Income Properties, Ltd. 11	8/18/87	5.4	10/20/93	62.2	6.2	3,223	223	56.7
JMB Income Properties, Ltd. 12	7/1/87	14.2	9/22/93	54.4	6.2	3,706	263	40.2
JMB Income Properties, Ltd. 13	9/14/88	20.8	3/7/95	48.0	6.5	2,472	117	27.3
JMB Income Properties, Ltd. 9	3/26/85	24.2	11/10/92	52.2	7.6	1,435	124	28.0
MLH Income Realty Partnership 4	2/24/89	37.5	4/29/94	36.3	5.2	2,533	102	−1.2
MLH Income Realty Partnership 5	7/13/89	19.8	1/27/94	22.3	4.5	2,155	123	2.5
McNeil Real Estate Fund 10	6/1/83	26.3	11/8/90	73.1	7.4	3,543	186	46.8
McNeil Real Estate Fund 11	1/19/84	35.7	11/22/88	58.1	4.8	2,219	136	22.4
National Property Investors 6	4/15/86	25.1	12/9/91	48.6	5.7	3,903	149	23.5
Nooney Income Fund Ltd. 11	6/25/84	31.0	10/9/90	46.7	6.3	1,149	99	15.8
Public Storage Properties 7	1/16/85	9.8	3/7/91	25.1	6.1	1,816	86	15.3
Shelter Prop. I LP	4/4/85	24.5	5/13/93	50.0	8.1	1,177	104	25.5
Shelter Prop. II LP	4/11/85	23.5	7/6/94	51.8	9.2	1,320	99	28.3
Shelter Prop. IV LP	3/4/86	27.9	2/1/94	52.5	7.9	1,837	103	24.6
Shelter Prop 5	12/12/85	34.7	6/16/94	51.1	8.5	1,686	110	16.4
Average		22.8		43.3	6.6			20.5
Sum						66,952	3,265	

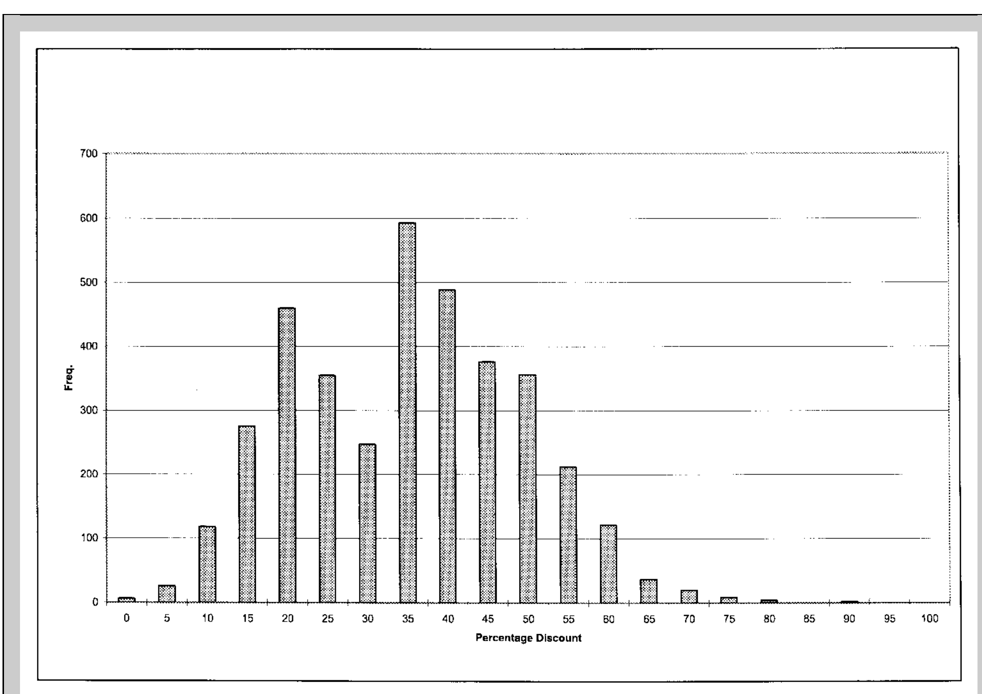
Notes: The discounts are averages of the first and last five transactions from the first and last transactions date respectively.
[a]Span is in years.

final average discount of 43.3%. The overall average discount was 35%. The discount figures are determined by when they are measured. The sample of sales in later transactions is consistent with Barber's (1996) sample. The distribution of discounts is provided in Exhibit 2.

The tabulated discounts in Exhibit 1 can be a misleading indicator of its time-series characteristics in the absence of controlling for specific partnership attributes. The increase in discounts may merely reflect the changing nature of the partnership due to changes in earnings, leverage or the performance of the underlying assets. To control for such effects, factors are incorporated into the model that Barber (1996) found to be significant. The factors include: (1) each partnership's yield; (2) each partnership's leverage; (3) its operating performance as measure by earnings and revenues; (4) a measure unrealized capital gains; and (5) a measure of liquidity.[11]

The same data source as Barber (1996), the publication *Partnership Profiles*, was used to construct the same explanatory variables for the years 1989 to 1993. The yield is the annualized rate of cash distribution for each partnership. Leverage is the ratio of each partnership's secured debt outstanding as a percentage of the partnership's property at cost. Operating performance, a measure of the underlying property's ability to generate revenue, is measured by two variables. The first is

Exhibit 2 | Discount Distribution



gross revenue expressed as a percentage of property at cost, and the second, a measure of earnings, is the ratio of operating surplus over property at cost. Liquidity is measured by the number of partnership units sold. Barber's unrealized capital gains variable was not constructed as his measure requires the general partner's estimate of asset value. The measure used to test for the time varying properties of the discount was the date of each transaction since the date of closing of each partnership's initial offering. Because data from *Partnership Profiles* for 1989 to 1993 was required, the final sample of 2,424 transactions involved the sale of 52,679 limited partnership units from eighteen partnerships.

As in Barber (1996), three variants of the model were estimated. Model 1 uses discounts as the dependent variable; Model 2 uses logged discounts; and Model 3 regresses logged prices on the log of the unit valuations. The OLS pooled results are reported in Exhibit 3.

In all cases, the models fit the data reasonably well, and the estimates were consistent with Barber's (1996) estimates. As in Barber, the models achieved the best fit, explaining 82% of the total variation. The coefficient on the years since offering variable captures systematic discount changes due to the timing of these purchases. It can be seen in Model 1 that even controlling for performance factors,

Exhibit 3 | OLS Pooled Results

Dependent Variable	Discount	Log Discount	Log Unit Price
Intercept	50.272	4.185	−0.715
Log unit value			1.01
			(81.54)
Yield	−3.200	−0.120	0.05
	(−38.05)	(−33.78)	(34.32)
Leverage	0.248	0.009	−0.004
	(18.75)	(15.48)	(−17.75)
Revenue	−0.182	0.002	0.004
	(−3.73)	(0.10)	(5.11)
Earnings	0.107	0.002	−0.002
	(4.18)	(1.91)	(−4.95)
No. of trades	0.004	−0.0004	−0.001
	(0.07)	(−1.81)	(−1.26)
Years since offering	1.231	0.013	−0.023
	(10.66)	(2.61)	(−10.23)
Adj. R^2	.6050	.5532	.8263

Note: $N = 2,424$.

there is a strong and significant positive time of transaction effect. The results suggest that, on average, the discount rate increased by 1.23% per year. As a test for nonlinearity, a (years since offering)^2 term was introduced causing the R^2 of the model to improve to .61 and the coefficients (t-Stats) for both terms to change to -2.76 (-3.68) for the year term and 0.29 (5.38) for the square year term respectively. Since both are highly significant, there is strong evidence of a time varying decreasing pattern for discounts, as indicated in the model. As another indication of this effect, the logged prices were regressed on the performance variables and on the years since offering variable. The previous finding of an increasing discount rate corresponds to a negative coefficient on the year variable when the dependent variable is expressed in prices. A rising discount implies that lower prices are paid for each unit over time. This was confirmed in the results since the years variable had the negative sign and was strongly significant, achieving a t-Stat of -10.33.

Conclusion

In this study, a model was developed of the sellers' price setting decisions and the buyer's reaction to the sellers' actions. When future sales are possible, sellers would be more aggressive in setting their asking price since not selling in an earlier round does not preclude future sales. However, as the buyer buys more units, there is a possibility of future sales and this may lead to lower prices. This ''later sale'' effect may result in price decreases as the buyer approaches the limit of unit purchases. This is contrary to the usual intuition that the buyer's ability to purchase sequentially will result in price discrimination whereby units are purchased in earlier rounds at lower prices from those sellers who are most eager to sell. Thus, under price discrimination, prices would increase over time as the number of units is sold. If the later sale effect is strong, a pattern of decreasing prices may be seen. Using a transactions level database, the findings show that there is indeed a decreasing price effect. Although this surely does not constitute a test of the model, it does offer some insight as to factors that may influence the secondary market prices of limited partnerships.

Appendix

The seller's expected profit function given that the buyer has announced that he will only buy one unit is:

$$E[P - v|k = 1]$$
$$= \frac{q}{q + p(1 - q)} \int (P - v)[1 - F(x|v)]^{n-1} \phi(v|x)dv$$
$$+ \frac{p(1 - q)}{q + p(1 - q)} \left\{ \int (P - v)[1 - F(x|v)]^{n-1} \phi(v|x)dv \right.$$
$$\left. + \int (P - v)[1 - (1 - F(x|v)^{n-1})][1 - F(x|v)]^{n-1} \phi(v|x)dv \right\}. \tag{a1}$$

Defining $t = p(1 - q)/q$ and multiplying both sides of the above expression by $[q + p(1 - q)]/q$, Equation (a1) can be expressed as:

$$E[P - v|k = 1]$$
$$= \int (P - v)[1 - F(x|v)]^{n-1}\phi(v|x)dv$$
$$+ t \left\{ \int (P - v)[1 - F(x|v)]^{n-1}\phi(v|x)dv \right.$$
$$\left. + \int (P - v)[1 - (1 - F(x|v)^{n-1})][1 - F(x|v)]^{n-1}\phi(v|x)dv \right\}. \tag{a2}$$

The corresponding first order condition when maximized with respect to P is:

$$(1 + t) \int [1 - F(x|v)]^{n-1}\phi(v|x)dv$$
$$- (1 - t)\int (n - 1)(P - v)x[1 - F(x|v)]^{n-2} \frac{f(x|v)}{P'(x)} \phi(v|x)dv$$
$$+ t \int [1 - F(x|v))^{n-1}][1 - F(x|v)]^{n-2}\phi(v|x)dv$$
$$+ t \int (P - v)(n - 1)[1 - F(x|v)]^{n-2}[1 - F(x|v)]^{n-2} \frac{f(x|v)}{P'(x)} \phi(v|x)dv$$
$$- t \int (P - v)(n - 2)[1 - (1 - F(x|v))^{n-1}][1 - F(x|v)]^{n-3}\frac{f(x|v)}{P'(x)} \phi(v|x)dv = 0. \tag{a3}$$

Collecting terms and rearranging this expression produces:

$$P'(x) = \frac{\begin{gathered}(1+t)(n-1)\int(P-v)[1-F(x|v)]^{n-2}f(x|v)\phi(v|x)dv - t(n-1)\int(P-v)[1-F(x|v)]^{2n-4}f(x|v)\phi(v|x)dv \\ + t(n-2)\int(P-v)[1-(1-F(x|v))^{n-1}][1-F(x|v)]^{n-3}f(x|v)\phi(v|x)dv\end{gathered}}{(1+t)\int[1-F(x|v)]^{n-1}\phi(v|x)dv + t\int[1-(1-(1-F(x|v))^{n-1}][1-F(x|v)]^{n-2}\phi(v|x)dv}. \tag{a4}$$

Applying the assumptions that the priors are diffuse and that the estimation errors are independent of v, the above expression simplifies to:

$$P'(x) = \frac{\begin{gathered}(1+t)(n-1)\int\left(\frac{P-x}{\sigma}+z\right)[1-F(z)]^{n-2}\int(z)^2dz \\ - t(n-1)\int\left(\frac{P-x}{\sigma}+z\right)[1-F(z)]^{2n-4}f(z)^2\,dz \\ + t(n-2)\int\left(\frac{P-x}{\sigma}+z\right)[1-(1-F(z))^{n-1}][1-F(z)]^{n-3}f(z)^2\,dz\end{gathered}}{(1+t)\int[1-F(z)]^{n-1}f(z)dz + t\int[1-(1-F(z))^{n-1}][1-F(z)]^{n-2}f(z)dz}. \tag{a5}$$

where and $z = (x - v)/\sigma$ and σ is the standard deviation of the random signal x. The denominator of Equation (a5) is $[1 + t]/n + t/2(n - 1)$. Inverting Equation (a5) the above expression yields the desired equilibrium relationship as expressed in Equations (13)–(15).

Endnotes

1 In another study, Barber (1996) documents an average discount of 45%.

2 A good source for understanding this secondary market is Wollack and Donaldson (1992).

3 The assumption of risk neutrality is a strong one. Termed the ''declining price anomaly'' or the ''afternoon effect,'' it is well known that risk aversion will result in declining prices in multiple-object auctions (McAfee and Vincent, 1993). Our risk neutrality assumption allows us to focus on repeated game effects of sellers.

[4] Note that even if the buyer agrees to announce the details of prior sales, such information is not credible since the buyer would have an incentive to distort such information to influence future selling prices.

[5] For tractability reasons, the case when each limited partner holds more than one unit each was not considered.

[6] Only pricing equilibriums where the price is an increasing function of the signals were considered.

[7] This assumption implies that the buyers' resale market is efficient and the LP's true common values are revealed. This appears reasonable since investors in seasoned partnership funds participate frequently and are often informed about the historical payoff of each partnership which comprise the fund.

[8] See Levin and Smith (1991) or Wilson (1990).

[9] It is important to note that we do not derive the equilibrium value for p, the probability that the buyer does not disclose the total number of units he plans to buy. However, the result on the pattern of discounts still holds if p is interpreted as the sellers' probability of a later round sale. In this case, p can come about from exogenous demand considerations. This interpretation for p conforms with LF's practices since they routinely make repeated purchases over time, presumably in response to changing market demand for the units.

[10] This not a restrictive assumption for ''reasonable'' parameter values. For example, if $p = 2/3$ and $q = 1/3$, $n > 6$ is required or that there are at least six limited partners.

[11] Since these factors are used to control for partnership specific factors and are not the main focus of the study, interested readers are referred to Barber (1996) for his justification to include these factors.

References

Barber, B., Forecasting the Discounts of Market Prices from Appraised Values for Real Estate Limited Partnerships, *Real Estate Economics*, 1996, 24:4, 471–92.

Levin, D. and J. Smith, Some Evidence on the Winner's Curse: Comment, *American Economic Review*, 1991, 81:1, 370–75.

McAfee, R. and J. McMillan, Search Mechanisms, *Journal of Economic Theory*, 1988, 44, 99–123.

McAfee, R. and D. Vincent, The Declining Price Anomaly, *Journal of Economic Theory*, 1993, 60:1, 191–212.

Myerson, R., Optimal Auction Design, *Mathematics of Operations Research*, 1981, 6:1, 58–73.

Riley, J. and W. Samuelson, Optimal Auctions, *American Economic Review*, 1981, 71, 381–92.

Thiel, S., Some Evidence on the Winner's Curse, *American Economic Review*, 1988, 78, 884–95.

Weber, R., Multiple-Object Auctions, In Engelbrecht-Wiggans et al., (Eds.), *Auctions, Bidding, and Contracting: Uses and Theory*, 1983.

Wilson, R., Strategic Analysis of Auctions, In R. Aumann and S. Hart, (Eds.), *Handbook of Game Theory,* Amsterdam: North-Holland, 1990.

Wolfson, M., Empirical Evidence of Incentive Problems and Their Mitigation in Oil and Gas Tax Shelter Programs, In J. Pratt and R. Zeckhauser, (Eds.), *Principals and Agents: The Structure of Business*, Harvard Business School Press, 1985.

Wollack, R., and B. Donaldson, Limited Partnerships: How to Profit in the Secondary Market, Dearborn Financial Publishing Inc., 1992.

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Daniel C. Quan, Cornell University, Ithaca, NY 14853 or dq22@cornell.edu.

Chang N. Xuan, China Securities Regulatory Commission, Peoples Republic of China, Beijing 100032 or cxuan@yahoo.com.