Applying the Bootstrap Technique to Real Estate Appraisal: An Empirical Analysis

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Abstract. **The purpose of this research is to demonstrate a statistical technique, bootstrapping, and its application in estimating overall capitalization rates in real estate appraisal. This study compares bootstrapped statistics from real estate sample data with the actual sample statistics in order to determine if significant statistical differences are observed. In addition, the issue of how many minimum data points are required for bootstrapping to yield statistically valid proxies is presented.**

Introduction, Purpose and Significance of Study

The majority of real estate appraisals rely on limited transaction data due to the inherent nonhomogeneity of real estate assets as well as the unorganized state of real estate markets. Although commercial databases are available they normally report only composite index data for various geographical markets and are not usable for specific property analysis.

While the appraisal industry has used such statistical techniques as multiple regression and analysis of variance, these methods require a minimum of approximately thirty observations to be statistically valid. Unfortunately, appraisers rarely have that many observations of comparable data and as a result, traditional statistical techniques have been underutilized by the profession.

The purpose of this research is to demonstrate the bootstrap technique as applied to real estate capitalization rate estimation. This study presents a statistical technique that reduces the need for large samples. The bootstrap technique estimates population statistics from a small limited data sample by repetitively resampling, in a random fashion, the small data sample. Then, the parameter in question for the population can be approximated from the bootstrapped sample data. In addition, confidence intervals as well as estimates of bias can also be calculated. These estimates completely summarize the information about the parameters in question and do not require unwarranted assumptions about the data. Therefore, the power of the bootstrap technique lies in its ability to estimate population statistics and confidence intervals from small samples, without assuming anything about the underlying distribution of the population data. Since the distribution of real estate sales data normally is not known, the bootstrap technique provides an alternative method of analysis for the real estate appraiser.

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This study compares bootstrapped statistics from real estate sample data, with the actual sample statistics in order to determine if significant statistical differences are observed. The study uses relatively large samples of verified real estate sales. In addition, this study examines the issue of how many minimum data points are necessary to be bootstrapped in order to yield statistically valid proxies of a small sample of real estate comparables.

The significance of the bootstrap technique when applied to real estate appraisal is unique. It can be used to help the appraiser approximate a range of overall capitalization rates that give a highly accurate statistical estimate based on ex post data.

Sample and Research Design

Ex post data from a total of 167 different income properties from two geographical markets in Texas (Austin and Houston) between the years 1975 and 1983 were used for the analysis of this study. While it may be argued that the data does not represent current transactional data, the purpose of this research is to illustrate the use of the bootstrap technique and not the timeliness of the data. To the extent that the data reflects random verified sales of actual properties, the bootstrap application should be valid. The data collected on each asset included:

1. Property type—apartment, office, shopping center, and industrial;
2. the dates each property was purchased and then sold;
3. the actual verified sales price for the two transaction dates;
4. net operating income for the property in each of the two transaction dates;
5. financing terms established for the asset including the loan-to-value ratio, interest rate, and term of loan.

A total of 167 actual property observations were available in the entire data set. Capitalization rates were calculated for all observations in the data set at the time of purchase and sale thus providing 334 actual transaction observations.

Sixteen subgroups of the data set were created based on the four different property categories from the two geographical markets. There were a total of eight submarkets since information was available about the purchase and sale of each property, sixteen subgroups existed for analysis. Exhibit 1 lists the sixteen subgroups used. For example, subgroup 1 contains fifty-six actual sales transactions of apartment complexes in the Austin market. This subgroup of apartment complexes has a mean cap rate value of 8.8% with a standard deviation of 1.8%. Subsets of the subgroups were then bootstrapped one thousand times beginning with a subset having two observations from the subgroup, the second subset having three observations from the subgroup, and so on, with each subsequent subset of the subgroup having one additional observation until the number of observations in the subset was equal to the number of observations in the subgroup.

Research Hypothesis and Test Criteria

This study tests two different hypotheses in order to address the issue of how accurate the bootstrap technique is in estimating capitalization rates from limited sample data. The

Exhibit 1
List of Sixteen Subgroups Used in the Statistical Analysis

Subgroup	Sample Mean Cap Rates	Sample Std. Dev.	Sample Size
1. Purchase of Austin Apartments	0.088	0.018	56
2. Purchase of Austin Industrial Properties	0.097	0.012	23
3. Purchase of Austin Office Buildings	0.094	0.012	36
4. Purchase of Austin Shopping Centers	0.096	0.010	13
5. Purchase of Houston Apartments	0.084	0.018	12
6. Purchase of Houston Industrial Properties	0.092	0.012	6
7. Purchase of Houston Office Buildings	0.087	0.011	12
8. Purchase of Houston Shopping Center	0.093	0.014	9
9. Sale of Austin Apartments	0.095	0.028	56
10. Sale of Austin Industrial Properties	0.100	0.016	23
11. Sale of Austin Office Buildings	0.116	0.123	36
12. Sale of Austin Shopping Centers	0.095	0.013	13
13. Sale of Houston Apartments	0.079	0.015	12
14. Sale of Houston Industrial Properties	0.109	0.015	6
15. Sale of Houston Office Buildings	0.093	0.024	12
16. Sale of Houston Shopping Centers	0.094	0.030	9

two research hypotheses tested in this study were:

1. The mean capitalization rates between the subgroup data and the bootstrapped data is not statistically different at the 0.05 level.
2. No statistical difference exists between the bootstrapped data variances and the subgroup variances of capitalization rates.

The Z-test was used to determine the statistical difference between various mean values of capitalization rates of subgroup and observed mean values of bootstrapped samples. A 95% confidence level was selected in calculating the critical Z-value of 1.64. Equation (1) was used in determining the calculated Z-value for each portfolio comparison made.

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_1}}} \tag{1}$$

In order for the Z-test to yield valid results, no statistically significant differences should exist between the variances of the sample subset and the bootstrapped sample. Therefore a chi-square test was also used to check for statistically significant differences between the bootstrapped variances and the sample variances.

The bootstrap technique may also be used to establish an objective confidence interval in market data. A confidence interval is based on observations of a sample and so constructed that there is a specified probability that the interval contains the unknown true value of a population parameter. Using the bootstrapped mean, standard deviation and appropriate Z-value, confidence intervals can be constructed to provide a bounded estimate of capitalization rates based on very small samples.

Hypothesis 2 tests the difference between the bootstrapped subset variances and the sample subgroup variance using the chi-square test.

Research Results

Exhibit 2 presents one of the sixteen subgroups from the data analyzed. There were a total of thirty-six actual observations in this subgroup. The mean cap rate for these thirty-six observations was 9.41% with a standard deviation of 2.1%. Each of the sixteen subgroups were bootstrapped beginning with two randomly selected observations through

Exhibit 2
Capitalization Rates Based on the Purchase of Austin Office Buildings

(1) No. of Obs.	(2) Mean	(3) Bootstrap Variance	(4) Bootstrap Standard Deviation	(5) Full Confidence Interval	(6) Z	(7) Chi-Square Test
2	0.0938	0.0098	0.0992	0.0123	−0.0584	1.567
3	0.0944	0.0077	0.0875	0.0109	−0.0700	2.013
4	0.0938	0.0069	0.0830	0.0103	−0.0702	2.237
5	0.0937	0.0063	0.0795	0.0099	−0.0810	2.443
6	0.0939	0.0053	0.0730	0.0090	−0.0300	2.898
7	0.0940	0.0052	0.0720	0.0089	−0.0062	2.976
8	0.0943	0.0048	0.0694	0.0086	0.0523	3.200
9	0.0940	0.0046	0.0675	0.0084	−0.0151	3.381
10	0.0941	0.0042	0.0652	0.0081	0.0053	3.632
11	0.0938	0.0041	0.0637	0.0079	−0.0624	3.806
12	0.0941	0.0038	0.0619	0.0077	0.0008	4.028
13	0.0940	0.0038	0.0613	0.0076	−0.0200	4.104
14	0.0940	0.0037	0.0606	0.0075	−0.0170	4.203
15	0.0940	0.0035	0.0593	0.0074	−0.0255	4.386
16	0.0940	0.0035	0.0593	0.0073	−0.0243	4.389
17	0.0939	0.0033	0.0570	0.0071	−0.0414	4.741
18	0.0941	0.0033	0.0572	0.0071	0.0130	4.713
19	0.0940	0.0032	0.0565	0.0070	−0.0103	4.833
20	0.0940	0.0031	0.0558	0.0069	−0.0106	4.595
21	0.0941	0.0029	0.0541	0.0067	−0.0038	5.268
22	0.0942	0.0030	0.0550	0.0068	0.0317	5.105
23	0.0941	0.0029	0.0536	0.0066	−0.0059	5.364
24	0.0940	0.0028	0.0533	0.0066	−0.0220	5.434
25	0.0940	0.0027	0.0523	0.0065	−0.0143	5.644
26	0.0941	0.0026	0.0514	0.0064	0.0045	5.838
27	0.0942	0.0027	0.0518	0.0064	0.0268	5.756
28	0.0942	0.0026	0.0508	0.0063	0.0281	5.986
29	0.0940	0.0026	0.0507	0.0063	−0.0248	6.002
30	0.0940	0.0025	0.0496	0.0061	−0.0318	6.268
31	0.0940	0.0025	0.0501	0.0062	−0.0142	6.137
32	0.0940	0.0024	0.0495	0.0061	−0.0240	6.305
33	0.0942	0.0023	0.0483	0.0060	0.0228	6.613
34	0.0941	0.0023	0.0478	0.0059	0.0077	6.756
35	0.0939	0.0022	0.0472	0.0058	−0.0387	6.931
36	0.0941	0.0023	0.0478	0.0059	0.0086	6.762

Subgroup Mean 0.0941
Subgroup Variance 0.0004
Subgroup St Dev 0.0210

the total number of observations in the subgroup. For example, the subgroup presented in Exhibit 2 begins with two observations. Hence, two actual observations were drawn from the sample set of thirty-six. These two observations were then bootstrapped one thousand times to yield a mean of 9.38% and standard deviation of 9.92% for the bootstrapped sample of one thousand observations. The next bootstrapped sample was calculated on the basis of three actual observations of the thirty-six data points within the subset. This process was continued until all thirty-six actual observations were used in the final bootstrapped sample. This final bootstrapped sample had a mean cap rate value of 9.41% with a standard deviation of 4.78%.

Column 1 (Exhibit 2) indicates the number of observations randomly taken from the sample subgroup and then used to create a bootstrapped example of one thousand observations. Column 2 presents the mean value of the bootstrapped sample while column 3 lists the bootstrapped sample variance. Column 4 is the square root of column 3, while column 5 reports the full confidence interval about the mean. For example, the two observation bootstrapped samples yield a mean cap rate of 9.38% with ±0.615% as the confidence interval.

Column 6 reports the calculated Z-value based on the difference between the bootstrapped sample mean and the subgroup mean reported at the bottom of Exhibit 2. The Z-scores reported in column 6 indicate that, statistically, there is no significant difference between the subgroup mean and the thirty-five different bootstrapped samples.

Finally, column 7 reports the chi-square calculated value that tests for significant differences between the subgroup variance and the various bootstrapped samples. Again,

Exhibit 3
Summary Data: Sixteen Subgroup Capitalization Rates

(1) Subgroup	(2) Subgroup Mean	(3) Subgroup Std. Dev.	Bootstrap at 10 obs. (4) Mean	(5) Confidence Interval
	%	%	%	%
1. Austin Apts—Purchase	8.78	1.83	8.76	0.94
2. Austin Industrial—Purchase	9.74	1.24	9.76	0.78
3. Austin Office—Purchase	9.41	2.1	9.41	0.81
4. Austin Shopping Ctr—Purchase	9.57	30.93	9.62	0.73
5. Houston Apts—Purchase	8.42	1.79	8.6	0.92
6. Houston Industrial*—Purchase	9.15	1.2	*9.14	0.90
7. Houston Office—Purchase	8.67	1.09	8.69	0.73
8. Houston Shopping Ctr—Purchase**	9.27	0.01	9.42	0.83
9. Austin Apts—Sale	9.5	2.8	9.53	1.15
10. Austin Industrial—Sale	9.99	1.58	9.98	0.87
11. Austin Office—Sale	11.62	12.33	11.83	2.53
12. Austin Shopping Ctr—Sale	9.49	1.27	9.53	0.77
13. Houston Apts—Sale	7.94	1.46	8.0	0.85
14. Houston Industrial—Sale*	10.89	1.5	10.57	0.89
15. Houston Office—Sale	9.26	2.5	8.88	1.02
16. Houston Shopping Ctr—Sale**	9.37	3.0	9.28	1.23

*based on 6 observations
**based on 9 observations

in each of the thirty-five different comparisons made, no chi-square value exceeded the critical chi-square value at the 0.05 level, thereby suggesting no significant differences between the variances of the bootstrapped samples compared to the variance of the subgroup.

Similar statistical results were also found for the remaining fifteen subgroups. That is, no statistical differences existed between mean cap rates of subgroup data and mean cap rates of the bootstrapped data. This was also true with regard to the chi-square test.

Exhibit 3 presents summary data on all sixteen subgroups analyzed. Column 1 identifies the specific subgroup analyzed. Column 2 reports the calculated arithmetic mean for the subgroup while column 3 presents the standard deviation. Columns 4 and 5 detail the bootstrapped mean and confidence interval at the ten-observation level. That is, ten actual observations were selected randomly and then bootstrapped one thousand times to provide the information in columns 4 and 5.

Sample Size Accuracy

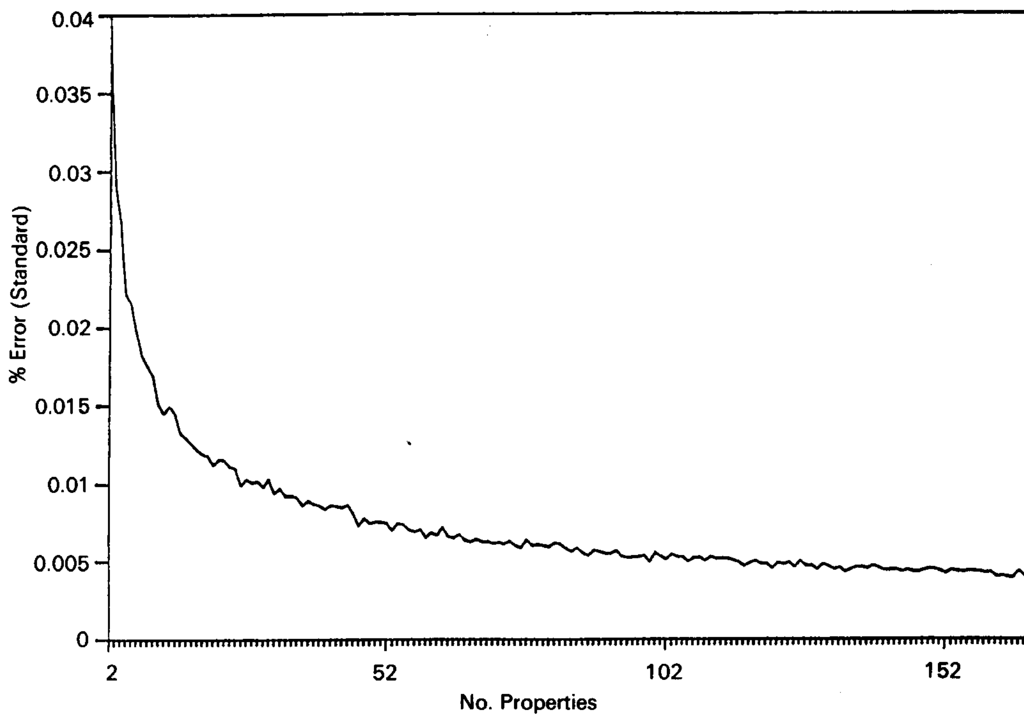
The results of the analysis presented thus far suggest that the bootstrap technique may be of some value to the appraiser who has a limited number of actual observations. For example (refer to Exhibit 2), if the appraiser had four observations to determine a representative cap rate, bootstrapping these four observations would have yielded a mean bootstrapped cap rate of 9.38% and a confidence interval of ±0.515%. That is, the appraiser could assert, with a 95% degree of confidence, that the actual mean cap rate had a value of 8.865% and 9.895%. Clearly, the size of the confidence interval decreases as the number of actual verified observations increase. For example, based on the data in Exhibit 2, the size of the confidence interval is reduced by more than 21% by increasing the number of observations in the sample from two to three. If the sample can be increased from two to six, then the magnitude of the confidence interval is reduced by over 51%. Hence, the greater the number of observations obtained, the smaller the confidence interval. Bootstrapped accuracy thus improves with a greater number of observations.

Exhibit 4 is a graph that summarizes the typical relationship between the percentage of standard error in the variance and the number of properties used to draw from for a bootstrapped sample. Exhibit 4 shows the magnitude of standard error ranging from two actual observations that were bootstrapped all the way up to all 167 observations in the entire data set. It is important to note the relatively rapid decrease in the percentage of standard error that occurred over the first 10 observations (approximately 67%).

Summary, Conclusions and Implications

At first glance the results of the bootstrap technique appear paradoxical. It suggests that from a very small sample of given information, one can derive a good approximation of the actual population parameters. It is as if a method of creating the equivalent of a statistical hologram has been discovered. However, the reader should be cautioned at this point. The

Exhibit 4
Variance of the Mean



results of the bootstrap sample exhibit good parametric properties only when the underlying data sample exhibits similar good parametric properties. This research indicates that the underlying sixteen sample subsets were indeed good statistical samples in all the subsets.

The primary purpose of this research was to demonstrate the bootstrap technique as applied to real estate capitalization rate estimation. The bootstrap technique is especially suited to real estate because it does not need large samples to generate estimates of population distribution parameters.

The relative effectiveness of the bootstrap technique was demonstrated by comparing the statistical parameters of sixteen subgroups to the parameters of bootstrapped data. Statistically, it was found that there was no significant difference in the sixteen-subgroup sample measures of the mean and variance to the bootstrapped parameters for each subgroup.

To the extent that ex post data can be used to estimate capitalization rates when estimating value, this study suggests that fairly accurate estimates of distribution parameters can be obtained by using the bootstrapped technique. The implication for appraisers is obvious. The bootstrap technique would allow for better estimates of historical capitalization rates from properties that have already transacted. This would provide a basis for estimating an overall capitalization rate in estimating value that would tend to be more accurate than relying on a limited sample of observations.

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