Duration and Risk

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Abstract

Duration has long been used as a means of managing the risk of bond portfolios. It has also been extended to the analysis of equities. Although it is often been compared with the half-life of an asset, it is more correct to consider duration as the approximate percentage change in price for each 1% change in yield. Given this view, it will be seen that the volatility of an asset and its duration are closely related.

This article uses the duration of a conventional valuation model to estimate the ex ante volatility and total risk of the commercial property market in the United Kingdom. The approach has potential value in estimating the risk of a new property where historic time series information is either limited on not available.

Introduction

Risk is concerned with uncertain outcomes. Measures of risk should, therefore, reflect investors' expectations rather than focus on what has happened in the past. This view, which captures the spirit on modern portfolio theory, is drafted entirely in terms of expectations. Historic measures of risk are only helpful in terms of defining scenarios that may be useful in forecasting expected risk. Unexpected shocks to an economic system can, therefore, give rise to risk measures that may be biased.

Although most of the principal asset markets have long established time series of returns that can be used to estimate risk, this is certainly not the case with real estate. For a new property, a time series of historic returns is clearly not possible. In this case, the problem for investors is how to estimate risk with only limited information about the lease structure of a property. This article draws on research on duration that was undertaken in the late 1970s by Boquist, Racette and Schlarbaum (1975), Lanstein and Sharpe (1978) and Livingston (1978) in order to estimate both volatility and total risk.

This article is structured as follows. The next section briefly covers the main formulae used in estimating duration. The following section extends this to estimate the volatility of property relative to an index of property market movements. Next, the duration of property is estimated using a valuation model that is familiar to most United Kingdom valuers. The monthly volatility is then estimated for each sector of the U.K. property market relative to the Investment Property Databank (IPD) Monthly Index from 1987 to 1998. The duration model is used to estimate total risk on a monthly basis. Next, the duration of a growth explicit model is estimated in order to shed light on the estimation of the inflation flow through rate. The final section is the conclusion.

Duration

Duration is frequently used in the bond market as a means of matching asset liabilities. It measures the sensitivity of the value of an asset to changes in interest rates. It was first developed by Macaulay (1938) and is represented by the value D_t in the following expression:

$$\frac{dV_t}{dy_t} \cdot \frac{1}{V_t} \approx \frac{-D_t}{(1 + y_t)}. \tag{1}$$

Where V_t is the value of the asset and y_t is the discount rate at time t. The whole expression on the right hand side of Equation (1) is referred to as the modified duration D^*. Rearranging gives the growth rate in terms of modified duration as follows:

$$\frac{dV_t}{V_t} \approx -D_t^* dy_t. \tag{2}$$

Estimating Volatility Relative to a Market Index

Following Livingston (1978), we can represent the rate of return of property j over a short period as:

$$R_{jt} = \frac{V_{jt} + a_{jt} + dV_{jt}}{V_{jt}}, \tag{3}$$

where a_{jt} and V_{jt} are the initial income and property value and dV_{jt} is the anticipated change in value at time t.

Substituting from Equation (2) for property j gives:

$$R_{jt} \approx 1 + \frac{a_{jt}}{V_{jt}} - D^*_{jt} dy_{jt}. \tag{4}$$

For small values of t, a_{jt}, V_{jt} and D^*_{jt} can be assumed to be constant. This implies that over time changes in the total return are influenced by changes in yield. As property markets tend to be yield driven this assumption is not unreasonable.

At time t the variance of Equation (4) becomes:

$$\text{Var}(R_{jt}) \approx D^{*2}_{jt} \text{Var}(dy_{jt}). \tag{5}$$

A similar expression also exists for the variance of an index of property market movements R_{mt} such that:

$$\text{Var}(R_{mt}) \approx D^{*2}_{mt} \text{Var}(dy_{mt}). \tag{6}$$

The single index model suggests that the volatility of an investment relative to an index can be expressed as follows:

$$\beta_{jt} = \frac{\text{cov}(R_{jt}, R_{mt})}{\text{Var}(R_{mt})}. \tag{7}$$

This can also be written as:

$$\beta_{jt} = \frac{\rho(R_{jt}, R_{mt})\sigma(R_{jt})\sigma(R_{mt})}{\text{Var}(R_{mt})}. \tag{8}$$

By substitution, we get the following:

$$\beta_{jt} \approx \frac{D^*_{mt} D^*_{jt} \rho(R_{jt}, R_{mt})\sigma(dy_{jt})\sigma(dy_{mt})}{D^{*2}_{mt} \text{Var}(dy_{mt})}. \tag{9}$$

Simplifying gives:

$$\beta_{jt} \approx \frac{D^*_{jt}}{D^*_{mt}} \frac{\rho(dy_{jt}, dy_{mt})\sigma(dy_{jt})}{\sigma(dy_{mt})}. \tag{10}$$

Thus, the volatility of a property relative to a property market index is made up of two components. The first part is the modified duration of the property divided by a similar figure for the property index. The second part is the covariance of changes in the equivalent yield of the property relative to changes in the property market yield. This latter expression can also be interpreted as the volatility of changes in the property yield. We can therefore write Equation (10) as:

$$\beta_{jt} \approx \frac{D^*_{jt}}{D^*_{mt}} \beta_{dy_{jt},dy_{mt}}. \tag{11}$$

Note that Equation (11) provides an estimate of β_{jt} measured relative to a property market index. The justification for this approach is that property investors are frequently concerned with knowing how well their portfolio has performed relative to the property market. The estimate of volatility given in Equation (11) is therefore useful for performance measurement purposes. If, in addition, the property index represents a reasonable proxy for the whole property market, then assuming equilibrium conditions there will be a linear relationship between the expected return on the property market and the market portfolio. Given an estimate of the expected risk premium for both the property market and the market portfolio, this would imply that Equation (11) could also be used to estimate systematic risk in a more general capital market framework.

The advantage of using Equation (11) to estimate volatility is that it does not rely on a time series of historical data so it is expressed in expectations form. As the duration measure is estimated from available data, the volatility of a property can easily be estimated whenever a valuation is undertaken.

Expressing duration and volatility in this way offers a number of useful insights. For example, it will be evident from Equation (11) that the β of a property depends on the relative size of the duration of the property and the property market as well as the volatility of changes in the property yields. A good example of the importance of this latter point arises in the valuation of over rented property. In this case, the valuer may argue that over an agreed time horizon changes in the market yield will have little influence on the yield appropriate to the property, so that the covariance between yield changes will be close to zero. This would result in a value for β_j that is also close to zero even though the respective duration figures take on positive values. An implication of this finding is that in a capital market framework the appropriate discount rate at which to value the property should be close to the risk free rate of return. It is interesting to note that in the early 1990s it was not uncommon to see over rented properties being valued using the return on long dated gilts.

Estimating Duration

In order to use Equation (11), it is necessary to derive an expression for the duration of a property. From Equation (2), it will be seen that the modified duration of property j at time t can be expressed as follows:

$$-D_{jt}^{*} = \frac{dV_{jt}}{dy_{jt}} \cdot \frac{1}{V_{jt}}. \tag{12}$$

In property terms the property value, V_{jt}, can be estimated from the present value of a term and reversion model commonly used by U.K. valuers. In this case, the term is represented by an initial income stream, a_{jt}, that is fixed for n years at which time it is reviewed to the open market rental value, RV_{jt}. The present value is found by discounting at the equivalent yield rate y_{jt}. For property j, we can write the present value at time t as follows:

$$V_{jt} = a_j \left[\frac{1 - (1 + y_{jt})^{-n}}{y_{jt}} \right] + \frac{RV_{jt}}{y_{jt}(1 + y_{jt})^{n}}. \tag{13}$$

Rearranging gives:

$$V_{jt} = \frac{a_{jt}}{y_{jt}} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1 + y_{jt})^{n}}. \tag{14}$$

This is known as the equivalent yield model. The yield is usually lower than the risk adjusted rate of return reflecting the fact that there is growth in the income stream.

When using this model, it is common practice to set RV_{jt} equal to the current rental value even though it arises n periods in the future. To this extent, there are economic inconsistencies in the way the model is specified. However, in the U.K.

the equivalent yield model is the most common approach used to value property because it incorporates readily available information that is expressed in current day terms. Economic deficiencies in the model, as well as differences in the lease structure, are therefore accommodated in the choice of equivalent yield. These figures are widely publicized with property transactions and are valuable as a source of comparable valuation data. At the index level, time series of equivalent yields are also readily available and form an important part of published information. In a market that is yield driven, it is probably fair to say that the common currency of most professional valuers is the equivalent yield. It will be clear from this brief discussion that equivalent yields embody a lot of information about the lease structure of individual properties, together with expectations of rental value growth and expected returns. It is worth pointing out that although Equation (14) can be shown to be mis-specified in economic terms, there is no guarantee that it will produce valuations that will differ from a model that explicitly allows for growth in rental values. The choice of yield in this model is, however, important. We will return to the relationship between these models when we examine the inflation flow through rate.

Because of the importance of the equivalent yield, valuers are interested in the effect that small changes in yield can have on changes in capital value. It is appropriate, therefore, to examine the duration of property in relation to changes in equivalent yield.

From Equation (14), the first derivative of V_{jt} with respect to y_{jt} can be expressed as:

$$\frac{dV_{jt}}{dy_{jt}} = -\frac{a_{jt}}{y_{jt}^2} - \frac{(RV_{jt} - a_{jt})}{y_{jt}(1 + y_{jt})^n}\left[\frac{1}{y_{jt}} + \frac{n}{(1 + y_{jt})}\right]. \qquad (15)$$

Dividing through by the property value V_{jt} gives the modified duration as follows:

$$D_{jt}^* = \left\{\frac{a_{jt}}{y_{jt}^2} + \frac{(RV_{jt} - a_{jt})}{y_{jt}(1 + y_{jt})^n}\left[\frac{1}{y_{jt}} + \frac{n}{(1 + y_{jt})}\right]\right\} \cdot \frac{y_{jt}(1 + y_{jt})^n}{a_{jt}(1 + y_{jt})^n + (RV_{jt} - a_{jt})}. \qquad (16)$$

Note that for a fully rack rented property in which the rental value, RV_{jt}, is equal to the passing income, a_{jt}, the modified duration reduces to:

$$D^*_{jt} = \frac{1}{y_{jt}}. \tag{17}$$

Thus, in the case of a fully rack-rented property the capitalization factor, or years' purchase, is equivalent to the modified duration.

An easy way to interpret this is to recognize that a 1% shift in yields should result in a change in value that is approximately equal to the duration. The relationship is approximate because the duration model assumes that, as yields change, the change in value is linear. In reality, however, this relationship is curvilinear. To illustrate, consider the value of $1 capitalized in perpetuity at 6.5%. The value in this case is $15.38 and the duration, using Equation (17), is 15.38 years. If yields dropped by 1% to 5.5%, the capital value of $1 in perpetuity would now be $18.18. This represents an increase of 18.22% over the original value, which is more than implied by the duration. However, if yields increased by 1% the capital value would drop to $13.33. This represents a drop in value of 13.31%, which is less than that implied by the duration. The average of these two changes at 15.76% is, however, much closer to the percentage change implied by the duration of 15.38 years.

Although it is possible to compensate for these changes by taking into consideration the convexity of the value-yield curve, our interest in volatility is concerned more with the relative change in duration so that accounting for convexity may not make a substantial difference to the overall calculation.

Estimating the Volatility of the U.K. Property Market: 1987–1998

Equation (11) estimates, at time t, the volatility of property relative to an index of property market movements. Given a time series of values, equivalent yields, income and rental value estimates for both the property market and each sector of the property market the model can be used at successive points in time to estimate changes in sector volatility.

The data used in the following analysis is based on the Investment Property Databank (IPD) Monthly Commercial Property Index. This index is published monthly and tracks the performance of the office, retail and industrial sectors of the U.K. commercial property market. The properties included in the index are held mainly by property bonds and unit trusts and there is an obligation that they be valued monthly. The index runs from December 1987 and a summary of the index composition as at December 1998 is given in Exhibit 1.

Using data from the IPD monthly index, Equation (11) can be used to estimate the volatility of each sector of the U.K. property market from December 1987 to

Exhibit 1 | Composition of IPD Monthly Index at December 1998

	Retail	Office	Industrial	All Properties
Total Capital Value £m	3,899.80	2,273.90	1,444.20	7,690.30
Total Rental Value £m	293.80	197.00	135.40	632.30
Number of properties	1,484	621	509	2,672

December 1998. For simplicity, it is assumed that the number of years to the next rent review remains constant at 2.5 years. For all the properties included in the monthly index, IPD publish aggregate time series information covering, amongst other statistics, rental values, income received and equivalent yields. These data are used with Equation (16) to derive the modified duration in each month for both each sector as well as the total property market.

In order to use Equation (11) to estimate the volatility of each sector, changes in the covariance between the sector and market yields are required for each period covered by the data. Strictly speaking, the covariance term included in Equation (11) should be based on investors expectations concerning the change in sector yield relative to changes in the market yield. However, as this is not available we have chosen to estimate a proxy value by estimating the slope coefficient from a time varying regression model using changes in the sector equivalent yield as the dependent variable and changes in the all property yield as the independent variable.[1]

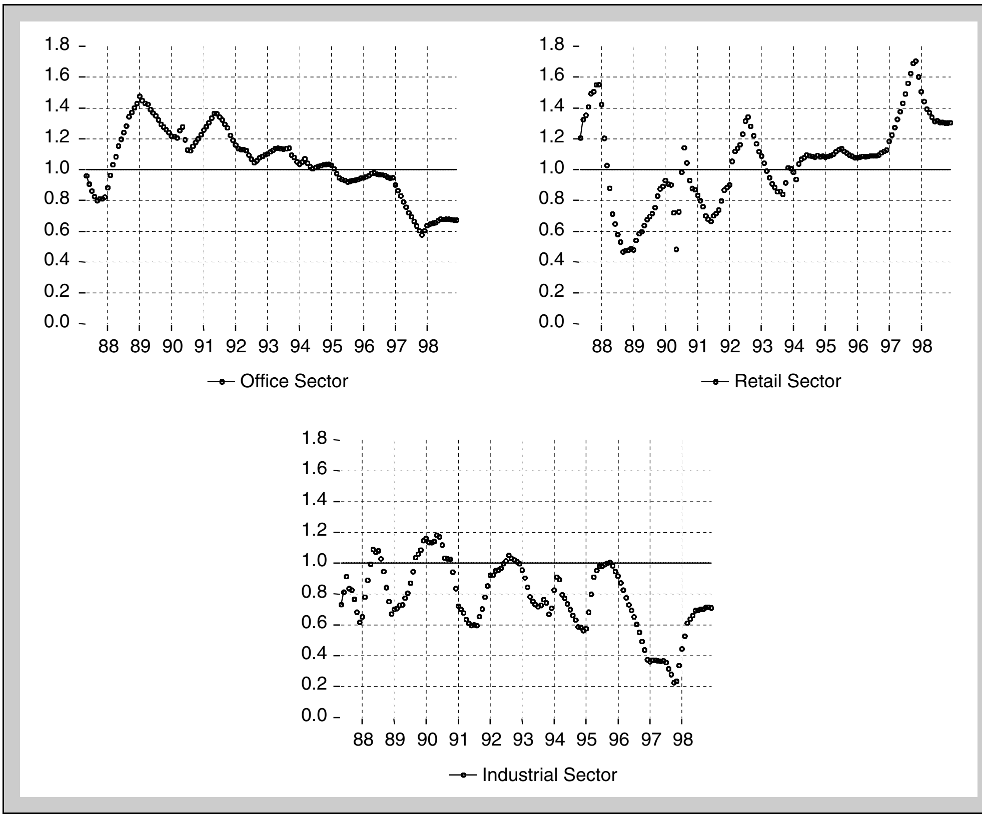
The time-varying regression estimates changes in the slope coefficient on a period-by-period basis and confirms that it is time varying. Although not strictly in expectations form, we could argue that if the time varying slope coefficient represents the aggregate view of all investors in the property market, it can then be used as a proxy for the expected value. Whether it is a reasonable proxy is, however, an empirical issue.

Substituting the results from this time varying model into Equation (11) gives our estimate of the volatility of each sector of the property market on a monthly basis. The results are shown in Exhibit 2 for each sector of the U.K. market.

The general trend in volatility for each sector offers some useful insights into the performance of each sector.

The retail sector, for example, shows a sharp decline in volatility between 1987 and 1989, followed by a steady rise up until the end of 1998. If we assume that investors maintained constant expectations concerning the risk premium and the risk free rate of return, then the change in volatility would imply that the expected value of retail properties peaked in 1989 and have since proceeded to decline. By contrast, the volatility of the office sector shows a mirror image of the retail sector.

Exhibit 2 | Duration Estimates of Volatility for the U.K. Property Market



Following the stock market crash in 1987, the expected value of office properties declined up until 1989–90. Since then there has been a gradual recovery. The industrial sector has performed differently to the other sectors. Although there is a lot of fluctuation in volatility, the general trend is downward implying that the expected value of industrial properties has been increasing.

Although the model is drafted in terms of expectations and we have assumed that the expected risk premium and the risk free rate of return remain constant, the interpretation given above generally follows market experience. Investors over this period have preferred the higher returns offered by industrial property and up until the early 1990s felt that office properties were overvalued and were anxious to sell. With the recession in the U.K. in the early 1990s, retail properties also suffered as a result of lower retail sales. It is clear, therefore, that the model is picking up general trends in expectations that can be observed in the marketplace.

Equation (11) is based on expected values and therefore gives an estimate of expected volatility. Testing the validity of the results therefore presents a number of difficulties. The duration measures are based on investors expectations embodied in yield changes. The resulting beta estimates are, therefore, expected values so that there are no empirical data against which this can be tested. As an alternative, we can test to see whether the duration model picks up general trends in volatility by comparing the duration betas with time varying betas that are estimated by regressing sector returns against property market returns. If both models are picking up the same information, then we should expect to see some similarity in the general trend. However, we should not expect this to be exact. The time varying regression betas are estimated from the following:

$$r_t = \alpha_t + \beta_t r_{mt} + \omega_t, \tag{18}$$

where ω_t is a random error term.

The coefficients in this regression have a time subscript implying that they can vary over time. If we assume that information arrives randomly, the evolution of both parameters will follow a random walk. The coefficients for both α_t and β_t can be written as:

$$\alpha_t = \alpha_{t-1} + \lambda_t, \quad \text{where} \quad \lambda_t \sim \text{NID}(0, \sigma_\lambda^2), \tag{19}$$

$$\beta_t = \beta_{t-1} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t \sim \text{NID}(0, \sigma_\varepsilon^2), \tag{20}$$

where λ_t and ε_t are random error terms that are normal and identically distributed with $E(\lambda_t) = E(\varepsilon_t) = E(\lambda_t, \varepsilon_t) = 0$.

In this form, the intercept and slope coefficients are able to pick up changes in market conditions. By reformulating the system in state-space form, Equation (18) can be written as a measurement equation with Equations (19) and (20) as transition equations. See Harvey (1993) for further details.

Our main interest in this case is with the slope coefficient β_t. This is a time varying parameter that measures the volatility of each sector of the property market, at each point in time, based on an historical series of returns. We will compare this with our duration estimate, β_{jt} give in Equation (11) which is based on the expected cash flows for each sector. Both estimates of volatility give single point figures. Because we are comparing historic and expected values, it is almost certain that they will not match on a period-by-period basis. However, we should expect to see both profiles following the same general trend.

The results of comparing both approaches for each sector are shown in Exhibits 3–5.

What is noticeable from these graphs is that for each sector the duration volatility and the time varying regression profiles follow the same general trend. The retail

Exhibit 3 | Comparison of Duration and Time-varying Regression Betas

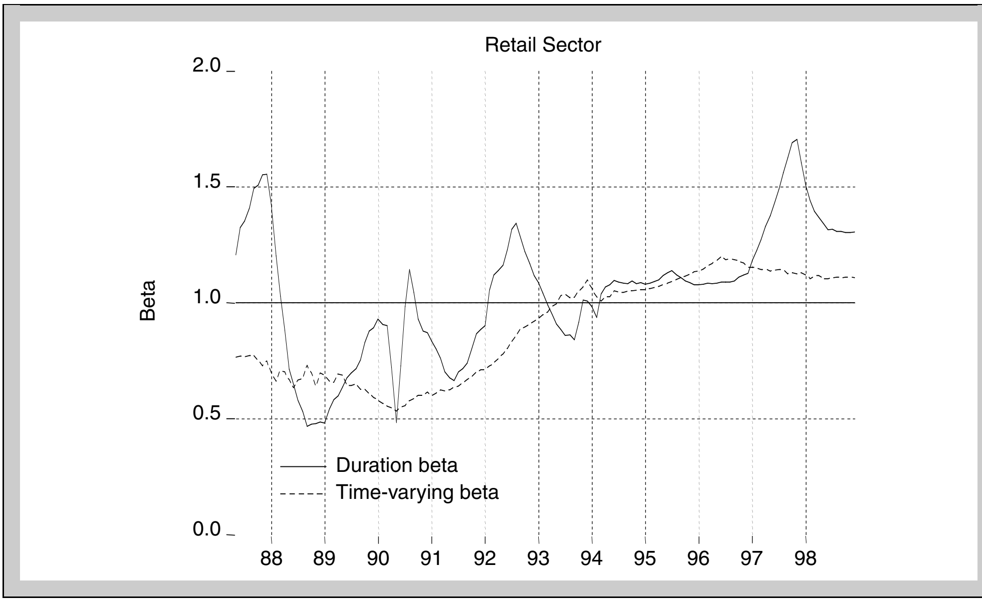


Exhibit 4 | Comparison of Duration and Time-varying Regression Betas

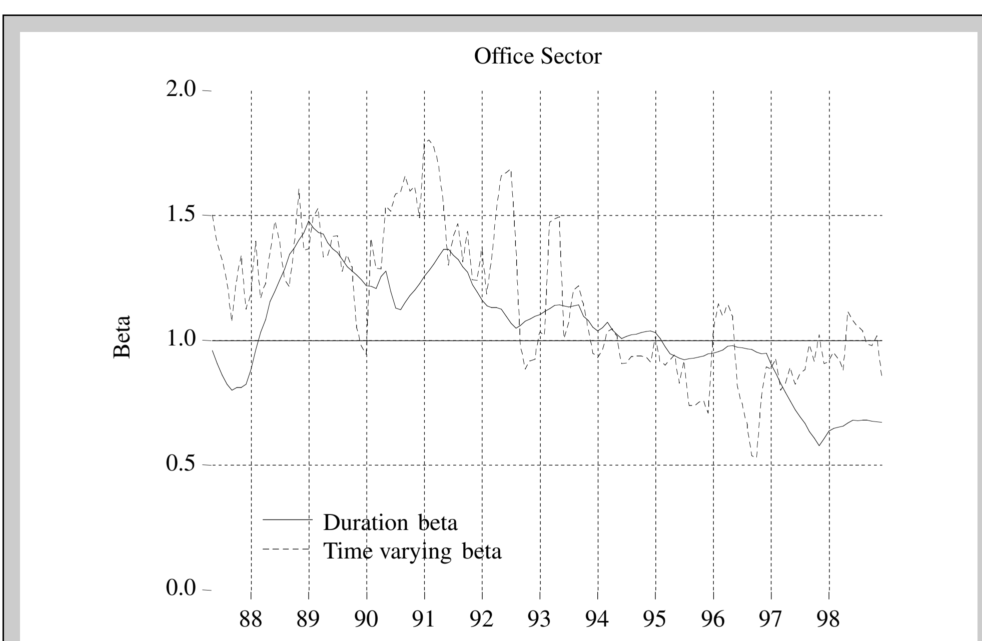
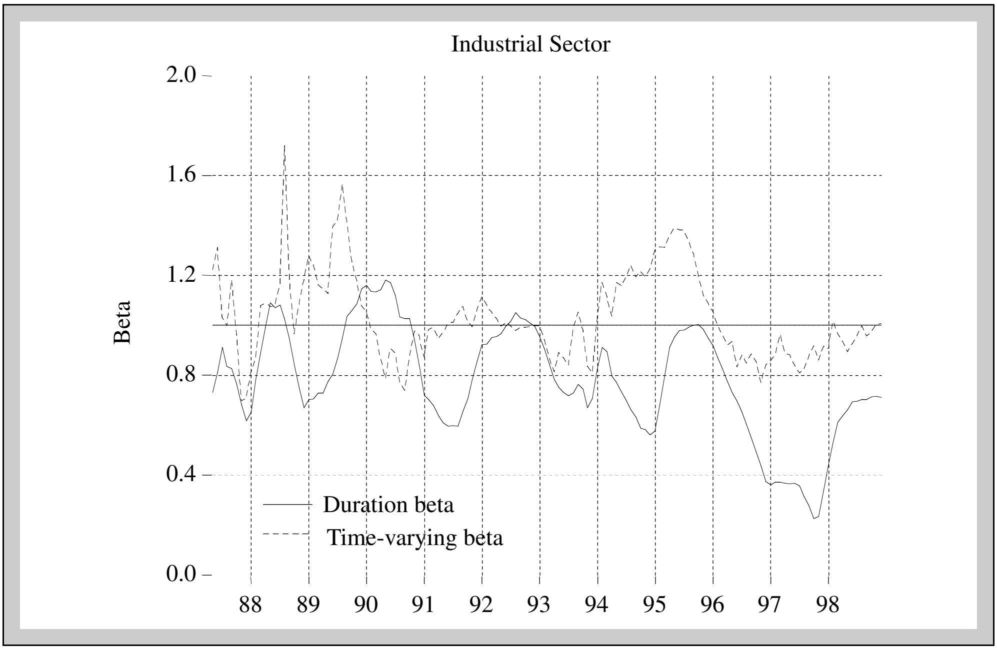


Exhibit 5 | Comparison of Duration and Time-varying Regression Betas



sector shows greater volatility in the duration measure than the time varying measure. This may imply that investors are expecting greater volatility in expected returns than was evident in historic returns. The office sector would seem to imply greater stability in expected returns with the duration volatility being more variable that the time varying regression beta. As far as the industrial sector is concerned, both measures followed a similar variable profile. The statistics for both approaches are summarized in Exhibit 6.

Based on this analysis, the average betas for each approach are statistically distinguishable from each other at the sector level. Exhibit 7 shows the correlation coefficient between each set of betas.

Exhibit 6 | Comparison of Duration and Time-varying Regression Betas: May 1987–Dec 1998

Sector	Duration Beta	Std. Dev.	Time-varying Regression Beta	Std. Dev.
Retail	1.046	0.283	0.895	0.219
Office	1.043	0.220	1.156	0.285
Industrial	0.778	0.221	1.035	0.275

Exhibit 7 | Correlation Matrix of Duration and Time-varying Regression Betas

	Duration Betas			Time-varying Regression Betas		
	Dur-Ret	Dur-Off	Dur-Ind	TV-Ret	TV-Off	TV-Ind
Dur-Ret	1.000	−0.917	−0.422	0.581	−0.476	−0.346
Dur-Off		1.000	0.471	−0.724	0.601	0.302
Dur-Ind			1.000	−0.532	0.349	0.315
TV-Ret				1.000	−0.796	−0.130
TV-Off					1.000	−0.030
TV-Ind						1.000

The correlation between betas for each method of analysis is, with the exception of the industrial sector at 0.315, strongly positive. The analysis also shows a strong negative correlation of −0.917 between the duration beta estimates for the office and retail sectors. The correlation using the time varying regression betas is −0.796. The expected return on these sectors should, therefore, be negatively correlated. This could have important implications for long-term investors in these sectors.

Over the period analyzed, the ranking of the betas is different for each approach. The duration method shows a ranking of retail-office-industrial whereas the time varying regression method gives a ranking of office-industrial-retail. This, however, may just be a difference of ordering when comparing ex ante with ex post methods of analysis.

Estimating Total Risk

Earlier we showed that at time t the duration model assumes a linear relationship between changes in both value and yield. However, for large changes in yield, the model does not accurately reflect changes in value. It is possible to take this into consideration by estimating the convexity of this relationship. Writing the change in value for property j as the first two terms of a Taylor expansion gives the following:

$$dV_{jt} \approx \frac{dV_{jt}}{dy_{jt}} dy_{jt} + \frac{1}{2} \frac{d^2V_{jt}}{dy_{jt}^2} (dy_{jt})^2. \tag{21}$$

Dividing through by V_{jt} and substituting D_{jt}^* for the modified duration and C_{jt} for convexity gives the following:

$$\frac{dV_{jt}}{V_{jt}} \approx -D^*_{jt} dy_{jt} + \frac{1}{2} C_{jt}(dy_{jt})^2, \quad (22)$$

where $C_{jt} = \dfrac{d^2 V_{jt}}{dy^2_{jt}} \dfrac{1}{V_{jt}}$.

Assuming a fully rack rented property, the percentage change in value can be written as:

$$\frac{dV_{jt}}{V_{jt}} \approx -\frac{1}{y_{jt}} dy_{jt} + \frac{1}{y^2_{jt}} (dy_{jt})^2. \quad (23)$$

Taking convexity into consideration improves our calculations and knowing the distribution of yield changes it would be possible to simulate a distribution for the percentage change in value. However, our interest in looking at convexity is in the effect that it could have on total risk. This is particularly important with large changes in yield. However, the average change in yield for the IPD index is only 0.026% pm. With such a small value, the effect of convexity will only influence the third decimal place in the growth calculations. Thus, as long as yield changes are relatively small, it is likely that convexity will not have a great influence on our estimate of total risk. As this greatly simplifies the calculations we shall, for practical purposes, assume that it can be ignored.

In order to provide an estimate of total risk we will make a further simplification and assume that the property is fully rented so that the current income is equal to the rental value. Given these simplifications, the total risk can be written as:

$$\mathrm{Var}(g_{jt}) = D^{*2}_{jt} \mathrm{Var}(dy_{jt}). \quad (24)$$

The average duration for the IPD All Property Monthly Index is 12.8 years and the variance of the change in yields is 0.0064. Substituting these into Equation (24) and taking the square root gives an average standard deviation of 1.024% per month. This compares with the standard deviation estimated from the published capital growth rates of 0.944%.

The model described in Equation (24) estimates the variance of the capital growth of the IPD All Property Monthly Index at a single point in time, *t*. The model can, however, be used each month to develop time varying estimates of total risk. In order to test how good these estimates are they need to be compared with an alternative method of estimating the total risk over time. The most obvious approach is to use a GARCH model to estimate the conditional variance of the IPD All Property monthly capital growth. See Bollersle and Wooldridge (1992) for details.

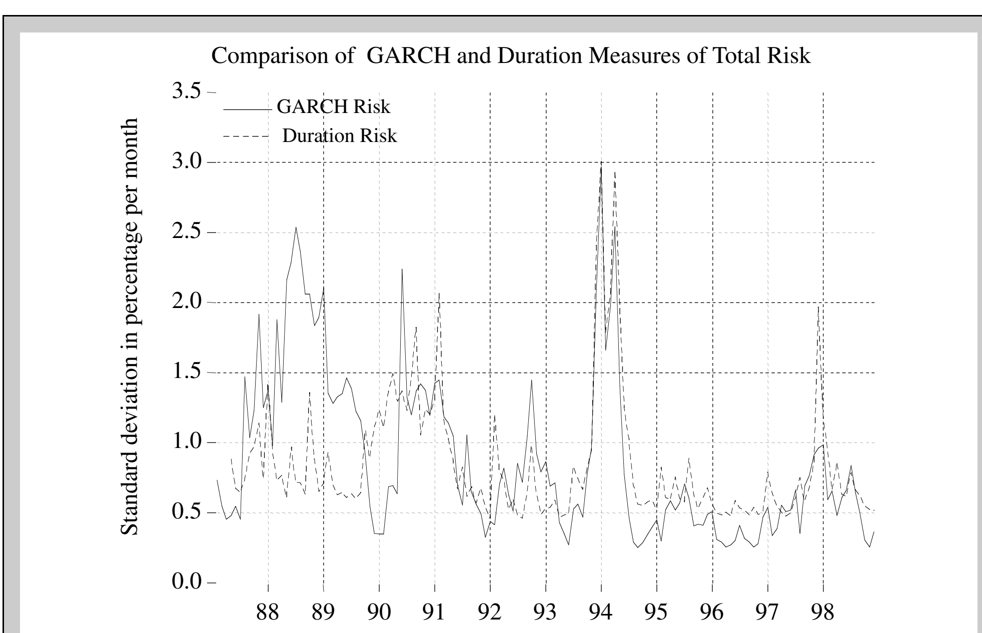
The results of this analysis are shown in Exhibit 8. The profile developed using Equation (24) can be considered to represent an estimate of expected total risk whereas the GARCH model measures realized total risk.

The period after 1990 shows strong positive correlation of 0.83 between the two sets of figures. From 1987 to 1990, there is little correlation between the figures. Given the basis on which the two models have been developed, the differences in risk could reflect differences between what investors expected and what was realized. The period between 1987 and 1990 immediately followed the stock market collapse in the U.K. The duration measure of total risk shows little change in volatility until the early 1990s when the property market started to collapse. By contrast, the realized risk up to 1990 was much higher so there was clearly a difference between what investors expected to happen and what actually happened.

The Inflation Flow Through Rate

The duration model derived in this article has been based on equivalent yields and has enabled us to avoid the problem of defining the inflation flow through

Exhibit 8 | Comparison of GARCH and Duration Measures of Total Risk



rate. As an alternative, however, it is possible to estimate the duration of a valuation model that separates the equivalent yield into components of both total return, rental value growth as well as the inflation flow through rate. For a fully rack rented property subject to periodic rent reviews Ward (1988) and Hamelink, MacGregor, Nanthakumaran and Orr (1998) have shown that at time t the Macaulay duration for property j can as be written as,

$$D_{jt} = \frac{(1 + r_{jt})}{r_{jt}} - \frac{p(1 + r_{jt})^p}{(1 + r_{jt})^p - 1} + \frac{p(1 + r_{jt})}{(1 + r_{jt})^p - (1 + g_{jt})^p} \left[(1 + r_{jt})^{p-1} - (1 + g_{jt})^{p-1} \cdot \frac{\delta g_{jt}}{\delta r_{jt}}\right], \tag{25}$$

where p is the rent review period, r_{jt} is the required return, g_{jt} is the growth in rental value and $\frac{\delta g_{jt}}{\delta r_{jt}}$ is the sensitivity of the growth in rental value to changes in the interest rate. This is also known as the inflation flow through rate. For annual reviews $p = 1$ and Equation (25) simplifies to:

$$D_{jt} = \frac{1 + r_{jt}}{r_{jt} - g_{jt}} \cdot \left[1 - \frac{\delta g_{jt}}{\delta r_{jt}}\right]. \tag{26}$$

In this form, it is easier to see the effect that the inflation flow through rate has on the duration. If $\frac{\delta g_{jt}}{\delta r_{jt}}$ is equal to zero, then changes in the interest rate will have no effect on changes in the rental value growth rate. This would happen if changes in both rental value growth and rates of return were independent of each other. In this case, the rate at which inflation passes into the rental system will be equal to one.

The problem with using Equation (25) to calculate duration is that it is difficult to estimate $\frac{\delta g_{jt}}{\delta r_{jt}}$. In a study on the effect of the inflation flow through rate, Hamelink et al. (1998) proxy this figure by using the cross correlation coefficient between their estimate of expected returns and rental value growth. However, as the inflation flow through rate is expressed in terms of differences, it is not clear whether the correlation coefficient estimated by Hamelink et al. is measuring this factor correctly. To give an example, using IPD All Property monthly data from January 1987 to December 1998 the correlation coefficient between realized total returns and rental value growth is 0.51. Using first differences, the correlation coefficient drops to 0.14.

There also remains the issue of what is the most appropriate interval over which to measure the correlation. Hamelink et al. (1998) use annual data over the period from 1972 to 1996 and estimated the correlation coefficient to be 0.8 from which they argued that the average duration of property is in the region of five years. This is similar to results suggested by Hartzell, Shulman, Lanetieg and Leibowitz (1988). However, many funds are valued quarterly or monthly and it may be that the inflation flow through rate differs with the reporting interval.

The high inflation flow through rate estimated by both Hamelink et al. (1998) and Hartzell et al. (1988) implies that every 1% change in return is almost matched by a similar change in rental values. This suggests that we should observe hardly any change in yields. Although it is true to say that this does happen in some periods it is also evident that yields do change over time. Equation (24) also shows that the volatility of yields is an important component in explaining the total risk of property. If changes in yield were always close to zero this would imply that changes in property values would have hardly any volatility. The use of a low estimate for the duration would merely exacerbate the problem. The evidence does not, however, support this view.

An alternative approach to estimating the inflation flow through rate is to use the duration model we propose. As this is based solely on equivalent yields, it does not require partial derivatives of the returns and growth rate to be estimated separately. If we assume that the equivalent yield fully accommodates the inflation flow through rate, then the duration estimated using this model should be the same as the duration using Equation (25). We can, therefore, set Equation (16) equal to Equation (25) and solve for the appropriate value of $\frac{\delta g_{jt}}{\delta r_{jt}}$.

Two modifications are needed to Equation (16) before this equality holds true. Firstly, Equation (16) estimates modified duration, D^*_{jt}, whereas Equation (25) estimates Macaulay duration, D_{jt}. The difference can easily be accommodated by multiplying Equation (16) by $(1 + y_{jt})$. Secondly, Equation (16) estimates the duration of a reversionary freehold that has n years unexpired until the next review. Equation (25) assumes that the property is valued at the date of the review. This can, however, be taken into consideration by setting $n = 0$ in Equation (16). Taking these changes into consideration it will be seen that the Macaulay duration of the equivalent yield model can be written as:

$$D_{jt} = \frac{1 + y_{jt}}{y_{jt}}. \tag{27}$$

To estimate $\frac{\delta g_{jt}}{\delta r_{jt}}$ we first choose a value for r_{jt} and g_{jt} and estimated the capital value for a given rent review period. The same capital value is then used to

Exhibit 9 | Estimated Inflation Flow Through Rates Assuming 10% Expected Rate of Return

	Rent Review Pattern		
g	1 Year	5 Years	25 Years
0.00%	0.000	0.000	0.000
2.00%	0.018	0.056	0.254
4.00%	0.036	0.107	0.415
6.00%	0.055	0.155	0.519
8.00%	0.073	0.200	0.588

estimate the equivalent yield. Substituting this into Equation (27) gives the Macaulay duration that has made an allowance for the inflation flow through rate. To find what this is we solve for $\frac{\delta g_{jt}}{\delta r_{jt}}$ in Equation (25) so that it has the same duration as Equation (27).

To give an example, assume that r_{jt} is 10%, g_{jt} is 4% and the rent review period is five years. The value of an initial income stream of $1 in perpetuity will, therefore, be $15.50. Based on this figure the equivalent yield is 6.45%. Using Equation (27), the Macaulay duration is estimated to be 16.50 years. We next solve for the value of $\frac{\delta g_{jt}}{\delta r_{jt}}$ in Equation (25) that will equate to our estimated duration of 16.5008 years. In this case, the inflation flow through rate is 0.107. Assuming a required return of 10%, Exhibit 9 summarizes inflation flow through rates for a range of growth rates and review patterns.

You will see that these figures, particularly for the five-year review pattern, are much lower than suggested by both Hamelink et al. (1998) and Hartzell et al. (1988). They are, however, more in line with the observed correlation between the difference in the total returns and the rental value growth rates reported above. With the exception of zero growth, any combination of total return and rental value growth shows that the inflation flow through rate increases with the rent review period. Thus, the longer the period between rent reviews, the more likely it is that changes in return will be matched by changes in rental value growth.

Conclusion

It is possible to estimate both the volatility of property relative to the property market as well as total risk of a property investment using a duration model based on information that is readily available and known to the valuer. As no historic time series data is involved, the approach described offers many benefits over other more traditional models of risk assessment. The immediate benefit is that it uses information developed by the valuer to enable risk estimates to be made for new properties that have no historic data. The success of this approach does, however, depend on the ability of professional valuers to make forecasts of changes in yields in addition to estimating the distribution from which the changes are derived. This, however, may not pose too great a problem as valuers spend a great deal of time collecting and using yields as the basis for estimating current values. It should, therefore, be a relatively simple matter to capture a valuer's expectations concerning the distribution of yields for any individual property. The volatility measure is based on expectations and offers potential in a number of areas such as estimating expected returns, asset allocation, risk monitoring and performance measurement.

We have also offered some comments on the inflation flow through rate, as this is an important part of understanding duration. We pointed out that although this may not be constant, it is part of a system that explains both duration and total risk so it is important to ensure that there is consistency in the estimation of each component.

Endnote

1 This process involves the use of a Kalman filter. See Harvey (1993) for further details.

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