

# Screening Mortgage Default Risk: A Unified Theoretical Framework

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## Abstract

This study developed a unified framework for theoretically analyzing a set of mortgage attributes that screens borrower types according to their unobservable default risk. In the presence of asymmetric information, a self-selection process is attained, where lower default risk type borrowers choose a mortgage loan with constant over graduated payment, constant over price-level-adjusted payment, adjustable over fixed rate, low over high loan-to-value ratio, and short over long maturity. The study thus examines, among others, various mortgage attributes, which have never previously been considered in the context of mortgage default under asymmetric information. Accordingly, the theoretical predictions produce further grounds for empirical research on mortgage default.

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This study extends the understanding of the way mortgage markets operate. Based on asymmetric information literature, a unified framework was constructed to demonstrate separating equilibria when borrowers' default risk type is unobservable to lenders. Particularly, a separating equilibrium between high and low default risk borrowers is attained, in which the low-risk borrower self-selects by choosing a mortgage with constant payment (CPM) over graduated payment (GPM), CPM over price-level-adjusted payment (PLAM),<sup>1</sup> adjustable-rate (FRM) over fixed-rate (ARM), low over high loan-to-value ratio (LTV), and short over long maturity. The derivations complement the analysis of Brueckner (2000), Posey and Yavas (2001), and Ben-Shahar and Feldman (2003), and generate further grounds for empirical research on mortgage default.

The signaling and screening equilibrium concepts, originally introduced by Spence (1973) and Rothschild and Stiglitz (1976), have been applied in explaining numerous economic phenomena, whose common denominator is the prevalence of asymmetric information among the players in the market. Particularly, in the context of mortgage markets, the models were used to demonstrate the attained separation between high- and low-risk borrowers in terms of both default and prepayment.<sup>2</sup>

Theoretical studies of *prepayment* risk under asymmetric information generate intuitive explanations for the prevalence of most mortgage characteristics: ARM

vs. FRM, long vs. short loan maturity, high vs. low LTV ratio, high-rate-low-point vs. low-rate-high-point, and assumption option vs. due-on-sale clause.<sup>3</sup>

More recently, however, researchers have theoretically analyzed mortgage attributes that function as a screening mechanism for borrowers' unobservable *default* risk. Brueckner (2000) shows that a separating equilibrium exists, where a borrower who is a high (low) default cost—low (high) default risk—self-selects by choosing a high (low) LTV ratio. Posey and Yavas (2001) examine both pooling and separating equilibria in the presence of ARM and FRM contracts, when, once again, default risk is unobservable to lenders. Ben-Shahar and Feldman (2003) show that low (high) default risk borrowers first signal their quality by acquiring a good (bad) credit record and then self-select by choosing a combination of shorter (longer) loan maturity and lower (higher) risk premium rate.<sup>4</sup>

This paper extends the literature on mortgage default under asymmetric information by constructing a simple unified framework that captures the above characteristics and shows their role in separating borrowers' default risk. Moreover, the paper presents a formal analysis of the effect of several additional mortgage attributes on default that have never previously been examined in the literature. Accordingly, the unified framework simultaneously considers a series of default related factors, thereby allowing the comparison among the effects of various mortgage attributes on the corresponding default rates.

The theoretical predictions are generally in line with the empirical evidence of Von Furstenberg (1969), Vandell (1978), Campbell and Dietrich (1983), Cunningham and Capone (1990), Epley, Kartono, and Haney (1996), VanderHoff (1996), Capozza, Kazarian, and Thomson (1997), Deng (1997), Yang, Buist, and Megbolugbe (1998), VanOrder and Zorn (2000), and others. However, some predictions are yet to be empirically tested.

In the next section, the primitives of the model are constructed. Next, the different mortgage attributes that produce separation by default risk type are analyzed. The final section presents concluding remarks.

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## The Model

Consider two types of borrowers who are differentiated by their probability to default on a given mortgage loan. In the context of the model, a borrower defaults due to an exogenous liquidity constraint. The *ex ante* probability of a liquidity crunch to become binding thus differentiates high- and low-risk borrowers.<sup>5</sup> The high- and low-risk borrowers are denoted by  $h$  and  $l$ , respectively.

In a two-period world, a borrower obtains a loan at time zero and at time one either repays the principal plus the accrued interest or defaults. In order to keep the analysis simple, all agents in the model are assumed to be risk-neutral.

The borrower's expected net payoff function (in present value terms) from the loan,  $PV_i$ , is:

$$PV_i = V(L) - \frac{[1 - P^i(L, F, M, C)]V(L)(1 + r) + P^i(L, F, M, C)\alpha}{1 + r_f}, \tag{1}$$

The variables  $L$ ,  $M$ ,  $C$ , and  $F$  are the LTV ratio, the time to maturity of the loan, the dominance of the constant payment (CPM) feature in the loan, and the dominance of the fixed rate (FRM) feature in the loan, respectively.<sup>6</sup> Also,  $V(\cdot)$  is the loan amount,  $P^i(\cdot)$  is the probability of default function of the type  $i$  borrower ( $i = h, l$ ),  $r$  is the interest rate on the loan,  $r_f$  is the risk free rate, and  $\alpha$  is the total cost associated with default.<sup>7</sup>

Equation (1) asserts that a borrower's expected net payoff from the loan is equal to the amount borrowed,  $V(\cdot)$ , net of the present value of the expected cost. The latter equals the full repaid amount,  $V(\cdot)(1 + r)$ , if no default occurs [with probability  $1 - P^i(\cdot)$ ]; plus, the default cost,  $\alpha$ , with probability  $P^i(\cdot)$ —all discounted by one plus the risk-free interest rate,  $r_f$ .

Notice that increasing the LTV ratio,  $L$ , raises the amount borrowed for two reasons: first, by definition, the greater is  $L$ , the greater is the amount of the loan for any given asset value. Also, a greater  $L$  yields a less stringent initial liquidity constraint, thereby allowing the borrower to purchase a more valuable asset. Thus, it is posited that the partial derivative of  $V(\cdot)$  with respect to  $L$ ,  $V_L$ , is positive.<sup>8</sup> That is:

$$V_L > 0. \tag{2}$$

However, the greater the level of  $L$ , the more likely it is that a liquidity crunch will arise when repayments are due and that the borrower will thus eventually experience a default.<sup>9</sup> This implies that the partial derivative of the default probability,  $P^i(\cdot)$ , with respect to  $L$ ,  $P_L^i$ , is positive (that is, increasing  $L$  raises the amount of the loan, which, in turn, raises the amount that is due on the loan and, thereby, the likelihood of defaulting on the repayments because of liquidity crunch):

$$P_L^i > 0. \tag{3}$$

Additionally, in contrast to the fixed-rate mortgage (FRM), the adjustable-rate mortgage (ARM) is subject to potential dramatic changes in the interest rate that is required on the loan balance. This, in turn, increases the probability that the borrower will eventually experience a default due to liquidity crunch.<sup>10</sup> Therefore, it is posited that the probability of default drops as the FRM feature,  $F$ , dominates. That is:

$$P_F^i < 0. \quad (4)$$

In addition, the shorter the loan maturity,  $M$ , the greater becomes the required periodical payment, resulting in a greater potential for liquidity crunch and, in turn, a greater probability of default.<sup>12</sup> Thus, it is posited that the partial derivative of  $P^i(\cdot)$  with respect to  $M$ ,  $P_M^i$ , is negative; that is:

$$P_M^i < 0. \quad (5)$$

Finally, according to conventional wisdom, the more dominant the constant payment feature,  $C$ , within the FRM class [on the account of the graduated payment (GPM) or the price-level-adjusted payment (PLAM) features], the greater becomes the default probability. In other words, the better the mortgage payments match the borrower's generally increasing repayment capability (in the form of either a GPM or a PLAM), the smaller the probability of default.<sup>13</sup> Therefore, the partial derivative of  $P^i(\cdot)$  with respect to  $C$ ,  $P_C^i$ , is assumed to be positive:<sup>14</sup>

$$P_C^i > 0. \quad (6)$$

Optimality conditions also require a set of assumptions regarding second partial derivatives.<sup>15</sup> The first requirement is that the partial derivative of the marginal change in the probability of default with respect to an increase in LTV does not rise.<sup>16</sup> That is:

$$P_{LL}^i \leq 0. \quad (7)$$

Similarly, it is assumed that the partial derivative of the marginal change in the amount of the loan with respect to LTV does not rise:<sup>17</sup>

$$V_{LL} \leq 0, \tag{8}$$

and that the partial derivative of the marginal change in the default probability with respect to maturity does not fall,<sup>18</sup> i.e.,

$$P_{MM}^i \geq 0. \tag{9}$$

Finally, it is posited that the partial derivative of the marginal change in the default probability with respect to the dominance of the constant payment element of the loan does not drop, i.e.,

$$P_{CC}^i \geq 0 \tag{10}$$

and that twice differentiating the default probability with respect to the dominance of the FRM feature of the loan is non-decreasing, i.e.,

$$P_{FF}^i \geq 0.^{19} \tag{11}$$

Now, consider a competitive risk-neutral lender. The lender's expected profit from the mortgage loan,  $\pi(\cdot)$ , is formed similar to the expected payoff function of the borrower. However, while the borrower's payments equal the lender's income, the borrower's default cost,  $\alpha$ , is assumed to be greater than the lender's income under default,  $\beta$ .<sup>20</sup> Hence, equivalently to Equation (1), the present value of the lender's expected profit from offering the loan to borrower  $i$  is:

$$\pi^i = -V(L) + \frac{[1 - P^i(L, F, M, C)V(L)(1 + r) + P^i(L, F, M, C)\beta]}{1 + r_f}, \tag{12}$$

where  $\beta(\cdot)$  is the lender's income if default occurs. That is, the lender lends  $V(L)$  at time zero, expecting to receive in the future  $V(L)(1 + r)$  if no default occurs, and  $\beta$  otherwise.

To eliminate both moral hazard with respect to default on the part of the borrower and arbitrage profits on the part of the lender, it is assumed that  $\alpha$  and  $\beta$  sustain:

$$\beta < V(L)(1 + r_f) \leq V(L)(1 + r) < \alpha. \quad (13)$$

In Equation (13), the default, on one hand, is, in general, more costly than full repayment for borrowers and, on the other hand, generates lower return to lenders than would have been produced by an alternative investment in the risk-free asset.

Now, in order to examine the characteristics of the borrower's indifference curves in the  $(r, M)$ ,  $(r, C)$ ,  $(r, F)$ , and  $(r, L)$  spaces, equate, without loss of generality,  $PV^i$  in Equation (1) with zero. Equivalently, given competition among lenders, which implies zero profits, equate the lender's profit in Equation (12) with zero. Then, isolating  $r$  in Equations (1) and (12) yields:

$$r^i = \frac{V(L)(1 + r_f) - P^i(L, F, M, C)X(L)}{[1 - P^i(L, F, M, C)]V(L)} - 1, \quad (14)$$

where  $X = \{\alpha, \beta\}$ .<sup>21</sup>

To simplify the presentation of the analysis, the arguments of the functions are omitted wherever possible in the subsequent equations.

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## Analysis

Consider a mortgage market in which borrowers' default probabilities  $P^h(\cdot)$  and  $P^l(\cdot)$  are unobservable to lenders, where, once again,  $P^h(\cdot) > P^l(\cdot)$  for all  $L, F, M$ , and  $C$ . The role of each of the mortgage attributes in screening borrowers according to their default risk is examined next.<sup>22</sup>

### Constant Payment vs. Graduated Payment Mortgage

**Result 1:** *There exists a separating equilibrium in which the low (high) default risk borrower selects a CPM (GPM) over a GPM (CPM). Furthermore, under the attained equilibrium, the interest rate on the GPM is greater than that on the CPM.*

**Proof:** Following Equations (1) and (12), the separation is attained if the following conditions are satisfied:

*High-Risk Borrower:*

$$\begin{aligned}
 & PV^h(\cdot, C^h, r^h) \\
 &= V(L) \\
 &\quad - \frac{[1 - P^h(L, F, M, C^h)]V(L)(1 + r^h) + P^h(L, F, M, C^h)\alpha}{1 + r_f} \\
 &\geq V(L) \\
 &\quad - \frac{[1 - P^h(L, F, M, C^l)]V(L)(1 + r^l) + P^h(L, F, M, C^l)\alpha}{1 + r_f} \\
 &= PV^h(\cdot, C^l, r^l), \tag{15}
 \end{aligned}$$

*Low-Risk Borrower:*

$$\begin{aligned}
 & PV^l(\cdot, C^l, r^l) \\
 &= V(L) \\
 &\quad - \frac{[1 - P^l(L, F, M, C^l)]V(L)(1 + r^l) + P^l(L, F, M, C^l)\alpha}{1 + r_f} \\
 &> V(L) \\
 &\quad - \frac{[1 - P^l(L, F, M, C^h)]V(L)(1 + r^h) + P^l(L, F, M, C^h)\alpha}{1 + r_f} \\
 &= PV^l(\cdot, C^h, r^h), \tag{16}
 \end{aligned}$$

*Lender:*

$$\begin{aligned}
 & \pi^h(\cdot, C^h, r^h) \\
 &= -V(L) \\
 &\quad + \frac{[1 - P^h(L, F, M, C^h)]V(L)(1 + r^h) + P^h(L, F, M, C^h)\beta}{1 + r_f} \\
 &= -V(L) \\
 &\quad + \frac{[1 - P^l(L, F, M, C^l)]V(L)(1 + r^l) + P^l(L, F, M, C^l)\beta}{1 + r_f} \\
 &= \pi^l(\cdot, C^l, r^l), \tag{17}
 \end{aligned}$$

where  $r^h$  and  $C^h$  ( $r^l$  and  $C^l$ ) are the interest rate and the dominance level of the constant payment feature (as opposed to the graduated payment feature), respectively, on the loan selected by the high-risk (low-risk) borrower.

Note that while the conditions in Equations (15) and (16) assure the self-selection process on the part of the borrowers (each borrower prefers a mortgage loan with different attributes), the condition in Equation (17) guarantees that, on the part of the lender, there is no incentive to deviate from offering these two different loans.

Now, given that  $P^h(\cdot) > P^l(\cdot)$  for all  $L, F, M$ , and  $C$ , then, following Equations (6) and (13), in order for Equation (17) to hold, the combination of  $C^h \geq C^l$  and  $r^h \leq r^l$  may not prevail simultaneously (and, furthermore, the only combinations that may prevail are either  $C^h > C^l$  and  $r^h > r^l$  or  $C^h < C^l$  and  $r^h < r^l$ ). In addition, from Equations (6) and (13), it follows that for the inequality condition in Equation (15) to hold, the combination of  $C^h \geq C^l$  and  $r^h > r^l$  may not prevail simultaneously. However, that both the combination  $C^h \geq C^l$  and  $r^h \leq r^l$  and the combination  $C^h \geq C^l$  and  $r^h > r^l$  may not prevail implies that only  $C^h < C^l$  can sustain. However, if  $C^h < C^l$ , then, following Equations (6), (10), and (13), for Equation (15) and (16) to hold simultaneously,  $r^h > r^l$  must also prevail. Hence,  $C^h < C^l$  and  $r^h > r^l$ .

A graphical presentation of Result 1 is highly explanatory. For that purpose, the borrower's indifference curve and the lender's iso-profit curve in the  $(r, C)$  space is examined by first differentiating  $r^i$ ,  $i = h, l$ , in Equation (14) with respect to  $C$ , which, after reduction, produces:

$$r^i_c = \frac{P^i_c[V(1 + r_f) - X]}{(1 - P^i)^2V} \begin{cases} <0 & \text{if } X = \alpha, \\ >0 & \text{if } X = \beta, \end{cases} \quad (18)$$

where the inequalities follow from Equations (6) and (13).

Further, twice differentiating  $r^i$  in Equation (14) with respect to  $C$  generates, after reduction:

$$r^i_{cc} = \frac{P^i_{cc}[V(1 + r_f) - X](1 - P^i) + 2P^i_c{}^2[V(1 + r_f) - X]}{(1 - P^i)^3V} \begin{cases} <0 & \text{if } X = \alpha, \\ >0 & \text{if } X = \beta, \end{cases} \quad (19)$$

where the inequalities in Equation (19) follow from Equations (6), (10), and (13).

Finally, twice differentiating  $r^i$  in Equation (14) with respect to  $C$  and  $P$  yields, after reduction:



$$r_{CP}^i = \frac{2P_c^i[V(1 + r_f) - X]}{(1 - P^i)^3V} \begin{cases} <0 & \text{if } X = \alpha, \\ >0 & \text{if } X = \beta, \end{cases} \quad (20)$$

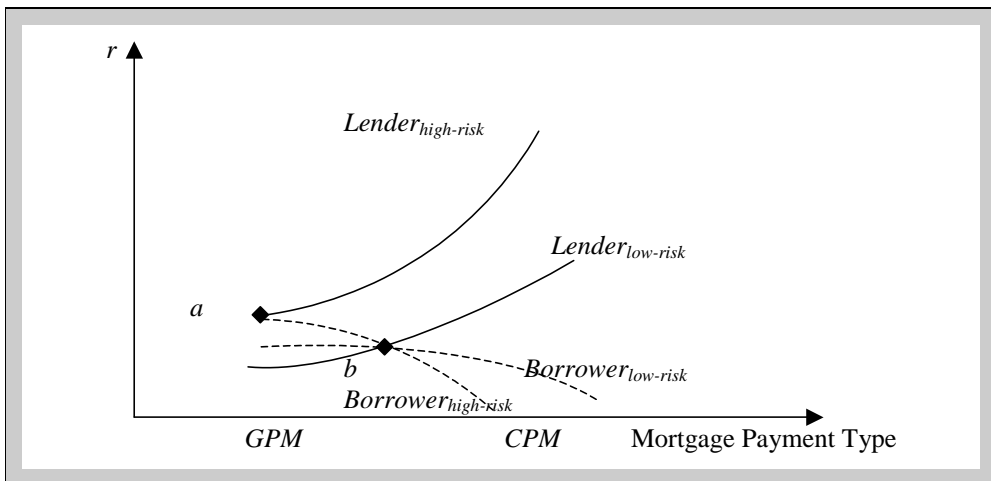
where the inequalities in Equation (20) follow from Equations (6) and (13).

Exhibit 1 demonstrates the separation. Intuitively, by replacing a GPM with a CPM, one substitutes relatively lower initial payments with higher ones and relatively higher later payments with lower ones. However, provided that liquidity is generally more binding in the early years of the loan than in subsequent periods [which is, in fact, the rationale for assuming that  $P_c^i > 0$ ; see the inequality condition in Equation (6)], the default probability rises as one shifts from GPM toward CPM.<sup>23</sup>

Further, to maintain indifference when switching from CPM to GPM, the borrower is willing to increase the interest rate on the loan because a lower expected cost of default is associated with the latter. Moreover, due to their higher default probability, high-risk borrowers, compared to the low-risk ones, are willing to accept a greater interest rate increase for any marginal shift toward a GPM.

Equivalently, for maintaining profit-indifference when shifting from CPM to GPM, the lender decreases the required interest rate on the loan for the equivalent reason; namely, lower expected cost of default is associated with the latter. Here, the lender's iso-profit curve, associated with the high-risk borrower, is more sensitive

**Exhibit 1** | Separating Equilibrium Where the Low- (High-) Risk Borrower Selects a Constant (Graduated) Payment Mortgage



(than that corresponding to the low-risk type) to the change from CPM to GPM because greater expected default cost is avoided.

Note that the effect of the loan interest rate on the probability of default is ignored. While it is a simplifying framework, it emphasizes the effect of the single examined parameter (constant vs. graduated payment in this case) in equilibrium.<sup>24</sup>

Finally, notice that while substitution from GPM to CPM, *ceteris paribus*, yields greater default risk and thus a higher requested interest rate from all borrowers; in equilibrium, it is the high-risk (low-risk) type borrower who selects the GPM (CPM). One can also see from Exhibit 1 that the interest rate on the CPM is lower than that of the GPM in equilibrium. Hence, given the lender's zero profit condition imposed on Equation (12), the CPM is altogether less likely to experience default in equilibrium.

### Constant Payment vs. Price-Level-Adjusted Mortgage

In markets where inflation prevails and, moreover, where the unexpected component of inflation is dominant, one may find a non-negligible market share of the price-level-adjusted mortgage (PLAM).<sup>25</sup> Unlike the constant (*nominal*) payment mortgage (CPM), the PLAM is a constant *real* payment mortgage, which essentially, resolves the “tilt effect” accompanying the CPM in an inflationary environment.<sup>26</sup>

Since income is generally positively correlated with inflation, the PLAM considerably decreases the default probability caused by liquidity crunch because of the better match that it offers between required mortgage payments and borrower's income. Conceptually, it is thus similar to the GPM discussed previously: by replacing a CPM with a PLAM, one substitutes relatively higher early payments with lower ones and relatively lower late payments with higher ones; however, the latter occurs when liquidity is expected to become less binding.

The analysis of the separation by CPM and PLAM is therefore essentially similar to that by CPM and GPM. Formally, provided the assumptions in Equations (6) and (10), the steps presented in the inequalities in Equations (15)–(20) can be reproduced, thus:

**Result 2:** *There exists a separating equilibrium in which the low (high) default risk borrower selects a CPM (PLAM) over a PLAM (CPM).*

See the proof in the Appendix.<sup>27</sup> Substituting the GPM–CPM continuum with the PLAM–CPM continuum on the X-axis in Exhibit 1 depicts the attained separating equilibrium.<sup>28</sup>

Note, once again, the attained phenomenon under which the self-selection and the zero-profit condition imposed on lenders induce the PLAM to exhibit greater

default risk in equilibrium. This occurs despite the greater default risk associated with the CPM, *ceteris paribus*.

### Short vs. Long Maturity

Given the assumptions in Equations (5) and (9), the steps presented in the inequality conditions in Equations (15)–(20) can be repeated—this time with respect to  $M$ —to derive the following:<sup>29</sup>

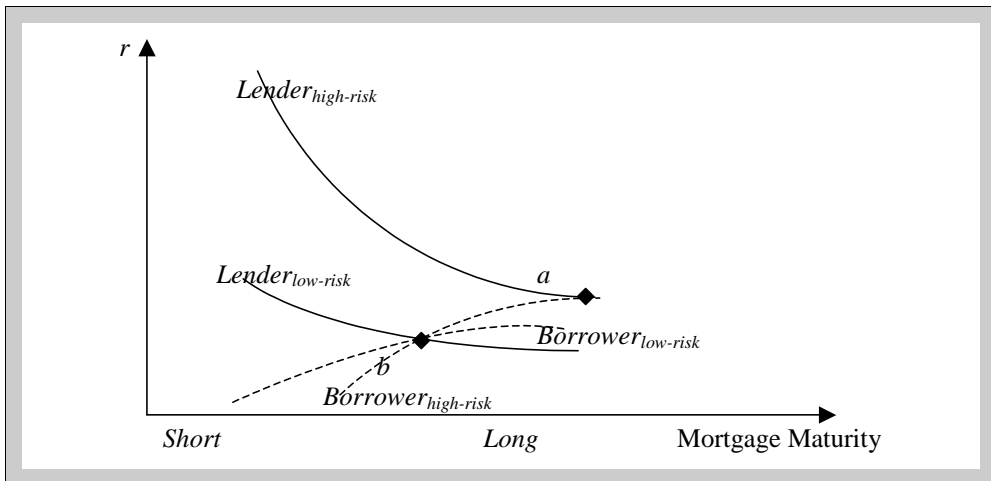
**Result 3:** *There exists a separating equilibrium in which the low (high) default risk borrower selects a short (long) over a long (short) maturity loan. Furthermore, under the attained equilibrium, the interest rate on the longer maturity loan is greater than that on the shorter maturity one.*

See the proof in the Appendix. The attained separating equilibrium is depicted in Exhibit 2. The slopes of the curves follow from inequality conditions in Equations (5), (9), and (13).<sup>30</sup>

It follows that when shifting from short to long maturity along the lender’s iso-profit curve, default risk and, thereby the interest rate, drop (see Exhibit 2); however, the lender offers in equilibrium the higher rate to the longer maturity loan. This stems from the fact that the low-risk borrower chooses the shorter maturity mortgage due to the relative smaller associated cost.<sup>31</sup>

Finally, as in the CPM case, because of both the higher interest rate associated with longer maturity loans under self-selection and the zero-profit condition

**Exhibit 2** | Separating Equilibrium Where the Low- (High-) Risk Borrower Selects a Shorter (Longer) Maturity Mortgage



imposed on the lender, the equilibrium default rate is eventually higher on the longer maturity contract.

### Fixed- vs. Adjustable-Rate Mortgage

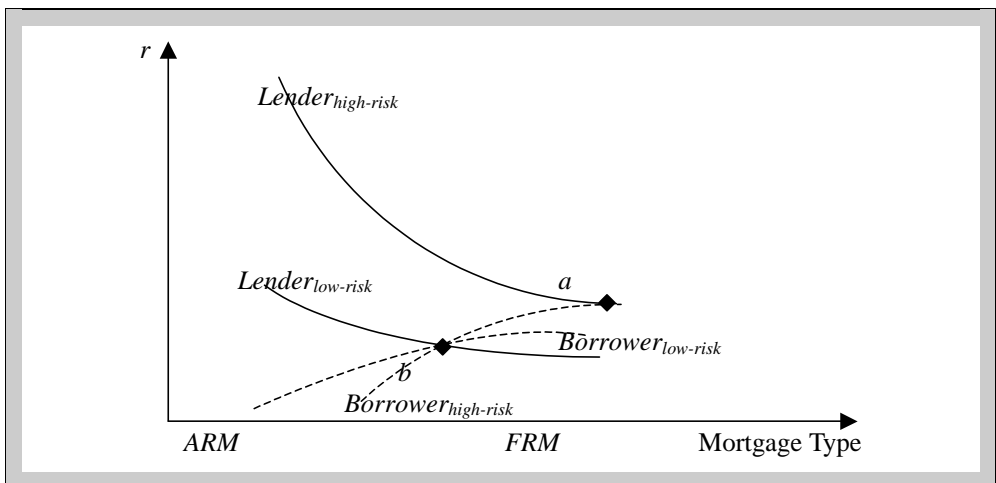
Re-applying the steps shown in the inequality conditions in Equations (15)–(20), this time with respect to  $F$ , produces the following:

**Result 4.** *There exists a separating equilibrium in which the low (high) default risk borrower selects an adjustable- (a fixed-) rate over a fixed- (an adjustable-) rate mortgage. Furthermore, under the attained equilibrium, the interest rate on the fixed-rate mortgage is greater than that on the adjustable-rate one.*

See the proof in the Appendix. The attained separating equilibrium is depicted in Exhibit 3.<sup>32</sup> Intuitively, given the lower default risk associated with the FRM, *ceteris paribus*, the interest rate required by the lender drops as one shifts from an ARM towards an FRM. Equivalently, the interest rate rises along the borrower indifference curve as one shifts from ARM to FRM (in order to maintain indifference, the willingness to pay rises for the safer contract). Moreover, due to its greater default risk, the riskier borrower is more willing to increase the interest rate on the loan in return for the FRM feature than the corresponding low-risk type.<sup>33</sup>

It follows that in equilibrium the interest rate on the FRM is greater than that on the ARM. Furthermore, while the FRM is safer, *ceteris paribus*, it is chosen in equilibrium by the riskier borrower.<sup>34</sup>

**Exhibit 3** | Separating Equilibrium Where the Low- (High-) Risk Borrower Selects an Adjustable (Fixed) Rate Mortgage



### Low vs. High LTV Ratio

The LTV ratio,  $L$ , is yet another mortgage attribute used to screen unobservable default risk.<sup>35</sup> To incorporate the notion of borrowers (lenders) general preference for increasing (constraining) the level of  $L$ , the mortgages offered in the market are posited to comply with the following:<sup>36</sup>

$$\frac{\partial PV^i}{\partial L} > 0 \quad (21)$$

and

$$\frac{\partial \pi^i}{\partial L} < 0.^{37} \quad (22)$$

Thus:

**Result 5.** *There exists a separating equilibrium in which the low (high) default risk borrower selects a relatively low (high) LTV ratio. Furthermore, under the attained equilibrium, the interest rate on the loan rises with the level of the LTV ratio.*

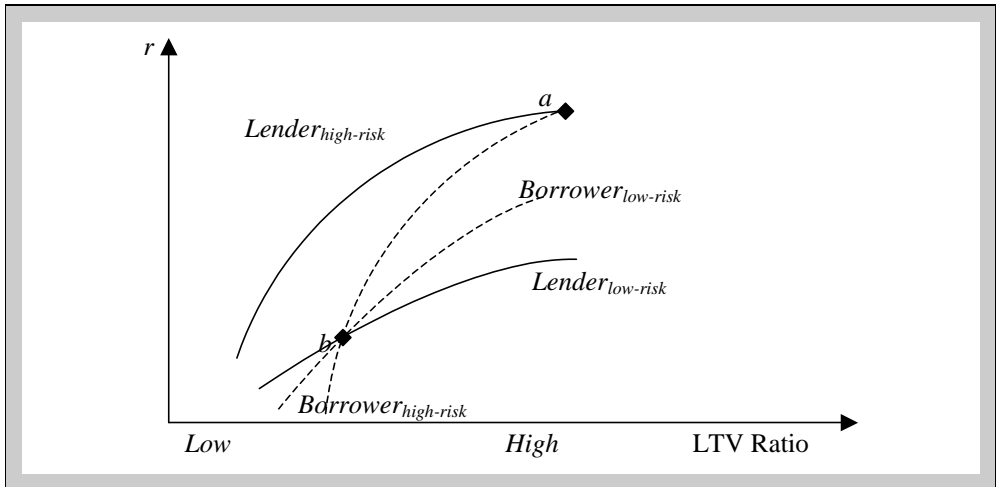
See the proof in the Appendix. The attained separating equilibrium is depicted in Exhibit 4, where the slopes of the curves follow from Equations (2), (13), (21), and (22). Intuitively, because the expected cost of default rises with the LTV ratio, *ceteris paribus*, the lender charges a greater interest rate when the LTV ratio increases in order to maintain a fixed level of expected profit. Furthermore, because of the greater associated default risk, the interest rate for the high-risk type rises faster with LTV along the iso-profit curves.

On the borrower's part, however, two forces collide. First, analogous to the lender's consideration, a rise in the LTV ratio increases the probability of default; hence, the borrower is willing to pay a lower interest rate in order to maintain cost-indifference. But, raising LTV incorporates another effect: it provides an opportunity to increase the amount of the loan [a feature that is captured by the function  $V(L)$ ], while the cost of default is bounded at size  $\alpha$ .

It follows that as long as the latter effect sufficiently dominates, then the interest rate rises with LTV along the borrower's indifference curve and, furthermore, that borrowers' indifference curve may be everywhere steeper than the lenders' corresponding iso-profit curve.

Moreover, as the borrower type becomes riskier, the likelihood of loan repayment falls, *ceteris paribus*. Hence, in return for an increase in LTV, the high-risk

**Exhibit 4** | Separating Equilibrium Where the Low- (High-) Risk Borrower Selects a Lower (Higher) LTV Ratio Mortgage



borrower shows a greater willingness to raise the interest rate than does the low-risk type.

Finally, the reason for the difference in the equilibrium level of the requested interest rate in high and low LTV mortgage contracts is twofold: (1) the ex ante greater default risk associated with high LTV, *ceteris paribus*; and (2) the ex post self-selection process according to which the low (high) default risk type borrower chooses a low (high) LTV ratio.<sup>38</sup>

## Conclusion

The understanding of how mortgage markets operate is extended in this study by analyzing the role of various mortgage attributes in screening borrowers' default risk.

Brueckner (2000), Posey and Yavas (2001), and Ben-Shahar and Feldman (2003) show that the high/low LTV ratio continuum, the fixed- versus adjustable-rate (FRM vs. ARM), and the short/long maturity continuum, respectively, function as mortgage attributes that separate high and low default risk borrowers in the presence of asymmetric information. The current study compliments these studies by focusing on additional mortgage attributes and constructing a unified framework to investigate the attained separating equilibria along the lines of Rothschild and Stiglitz (1976) and Spence (1973).

The model shows that low default risk borrowers self-select by choosing low over high LTV ratios, ARM over FRM, and short over long loan maturity. Furthermore, the model extends beyond these mortgage attributes and demonstrates that low default risk borrowers are also screened by their choice of constant payment mortgages (CPM) over graduated payment mortgages (GPM), and CPM over price-level-adjustment mortgages (PLAM).

The analysis thus extends the theoretical literature on mortgage default and generates new empirically testable results. Among others, the derivations predict that, when focusing on default risk, CPM is safer than GPM and PLAM, short is safer than long maturity mortgages, and ARM is safer than FRM.

Yezer, Phillips, and Trost (1994) investigate the simultaneity bias that may arise in empirical estimates of mortgage default. They argue that the separation resulting from asymmetric information must be incorporated into the estimation of the variables explaining default. Consistent with this argument, the analysis presented here predicts, for example, that, while GPMs are, *ceteris paribus*, less risky than CPMs, their risk in equilibrium exceeds that of CPMs due to the self-selection phenomenon. Similarly, while, *ceteris paribus*, longer maturity may inversely affect the default probability, the longer maturity loan may, in fact, become riskier as a result of self-selection.

Hence, the finding, for example, of Epley, Kartono, and Haney (1996) according to which loan maturity is, *ceteris paribus*, positively related to default risk might be found true only as a consequence of equilibrium conditions (i.e., when self-selection is directly addressed in the estimated equations).<sup>39</sup> According to the current study, an equivalent conclusion may hold following the estimation of the relationship between default risk and any one of the examined mortgage attributes (CPM, FRM, and maturity).

## Appendix

**Proof of Result 2:** Following Equations (1) and (12), separation is attained if:

$$\begin{aligned}
 & PV^h(\cdot, C^h, r^h) \\
 &= V(L) - \frac{[1 - P^h(L, F, M, C^h)]V(L)(1 + r^h) + P^h(L, F, M, C^h)\alpha}{1 + r_f} \\
 &\geq V(L) - \frac{[1 - P^h(L, F, M, C^l)]V(L)(1 + r^l) + P^h(L, F, M, C^l)\alpha}{1 + r_f} \\
 &= PV^h(\cdot, C^l, r^l), \tag{A1}
 \end{aligned}$$

$$\begin{aligned}
& PV^l(\cdot, C^l, r^l) \\
&= V(L) - \frac{[1 - P^l(L, F, M, C^l)]V(L)(1 + r^l) + P^l(L, F, M, C^l)\alpha}{1 + r_f} \\
&> V(L) - \frac{[1 - P^l(L, F, M, C^h)]V(L)(1 + r^h) + P^l(L, F, M, C^h)\alpha}{1 + r_f} \\
&= PV^l(\cdot, C^h, r^h), \tag{A2}
\end{aligned}$$

$$\begin{aligned}
& \pi^h(\cdot, C^h, r^h) \\
&= -V(L) + \frac{[1 - P^h(L, F, M, C^h)]V(L)(1 + r^h) + P^h(L, F, M, C^h)\beta}{1 + r_f} \\
&= -V(L) + \frac{[1 - P^l(L, F, M, C^l)]V(L)(1 + r^l) + P^l(L, F, M, C^l)\beta}{1 + r_f} \\
&= \pi^l(\cdot, C^l, r^l), \tag{A3}
\end{aligned}$$

where  $C^h$  and  $r^h$  ( $C^l$  and  $r^l$ ) are the dominance of the constant payment feature (as opposed to the price-level-adjusted feature) and the nominal interest rate (for any given expected inflation rate), respectively, on the loan selected by the high-risk (low-risk) borrower.

Given that  $P^h(\cdot) > P^l(\cdot)$  for all  $L, M, C$ , and  $F$ , it follows from Equations (6) and (13) that, for Equations (A3) to hold, the combination of  $C^h \geq C^l$  and  $r^h \leq r^l$  cannot maintain simultaneously. Also, following Equations (6) and (13), for Equation (A1) to hold, the combination of  $C^h \geq C^l$  and  $r^h > r^l$  cannot prevail simultaneously. Hence,  $C^h < C^l$ . However, following Equations (6), (10), and (13), for Equations (A1) and (A2) to maintain simultaneously,  $r^h > r^l$  must also prevail.

**Proof of Result 3:** Following Equations (1) and (12), separation is attained if:

$$\begin{aligned}
& PV^h(\cdot, M^h, r^h) \\
&= V(L) - \frac{[1 - P^h(L, F, M^h, C)]V(L)(1 + r^h) + P^h(L, F, M^h, C)\alpha}{1 + r_f} \\
&\geq V(L) - \frac{[1 - P^h(L, F, M^l, C)]V(L)(1 + r^l) + P^h(L, F, M^l, C)\alpha}{1 + r_f} \\
&= PV^h(\cdot, M^l, r^l), \tag{A4}
\end{aligned}$$



$$\begin{aligned}
 & PV^l(\cdot, M^l, r^l) \\
 &= V(L) - \frac{[1 - P^l(L, F, M^l, C)]V(L)(1 + r^l) + P^l(L, F, M^l, C)\alpha}{1 + r_f} \\
 &> V(L) - \frac{[1 - P^l(L, F, M^h, C)]V(L)(1 + r^h) + P^l(L, F, M^h, C)\alpha}{1 + r_f} \\
 &= PV^l(\cdot, M^h, r^h), \tag{A5}
 \end{aligned}$$

$$\begin{aligned}
 & \pi^h(\cdot, M^h, r^h) \\
 &= -V(L) + \frac{[1 - P^h(L, F, M^h, C)]V(L)(1 + r^h) + P^h(L, F, M^h, C)\beta}{1 + r_f} \\
 &= -V(L) + \frac{[1 - P^l(L, F, M^l, C)]V(L)(1 + r^l) + P^l(L, F, M^l, C)\beta}{1 + r_f} \\
 &= \pi^l(\cdot, M^l, r^l), \tag{A6}
 \end{aligned}$$

where  $M^h$  and  $r^h$  ( $M^l$  and  $r^l$ ) are the maturity and the interest rate, respectively, on the loan selected by the high-risk (low-risk) borrower.

Given that  $P^h(\cdot) > P^l(\cdot)$  for all  $L, M, C$ , and  $F$ , then following Equations (5) and (13), for Equation (A6) to hold, the combination of  $M^h \leq M^l$  and  $r^h \leq r^l$  cannot maintain simultaneously. Also, from Equations (5) and (13), for Equation (A4) to hold, the combination of  $M^h \leq M^l$  and  $r^h > r^l$  cannot prevail simultaneously. Hence,  $M^h > M^l$ . However, it follows from Equations (5), (9), and (13) that for Equations (A4) and (A5) to hold simultaneously,  $r^h > r^l$  must also maintain.

**Proof of Result 4:** Following Equations (1) and (12), separation is attained if:

$$\begin{aligned}
 & PV^h(\cdot, F^h, r^h) \\
 &= V(L) - \frac{[1 - P^h(L, F^h, M, C)]V(L)(1 + r^h) + P^h(L, F^h, M, C)\alpha}{1 + r_f} \\
 &\geq V(L) - \frac{[1 - P^h(L, F^l, M, C)]V(L)(1 + r^l) + P^h(L, F^l, M, C)\alpha}{1 + r_f} \\
 &= PV^h(\cdot, F^l, r^l), \tag{A7}
 \end{aligned}$$

$$\begin{aligned}
& PV^l(\cdot, F^l, r^l) \\
&= V(L) - \frac{[1 - P^l(L, F^l, M, C)]V(L)(1 + r^l) + P^h(L, F^l, M, C)\alpha}{1 + r_f} \\
&> V(L) - \frac{[1 - P^l(L, F^h, M, C)]V(L)(1 + r^h) + P^l(L, F^h, M, C)\alpha}{1 + r_f} \\
&= PV^l(\cdot, F^h, r^h), \tag{A8}
\end{aligned}$$

$$\begin{aligned}
& \pi^h(\cdot, F^h, r^h) \\
&= -V(L) - \frac{[1 - P^h(L, F^h, M, C)]V(L)(1 + r^h) + P^h(L, F^h, M, C)\beta}{1 + r_f} \\
&= -V(L) + \frac{[1 - P^l(L, F^l, M, C)]V(L)(1 + r^l) + P^l(L, F^l, M, C)\beta}{1 + r_f} \\
&= \pi^l(\cdot, F^l, r^l), \tag{A9}
\end{aligned}$$

where  $F^h$  and  $r^h$  ( $F^l$  and  $r^l$ ) are the dominance of the fixed-rate attribute (as opposed to the adjustable-rate attribute) and the interest rate, respectively, on the loan selected by the high-risk (low-risk) borrower.

Given that  $P^h(\cdot) > P^l(\cdot)$  for all  $L, F, M$ , and  $C$ , it follows from Equations (4) and (13) that for Equation (A9) to hold, the combination of  $F^h \leq F^l$  and  $r^h \leq r^l$  cannot maintain simultaneously. Also, following Equations (4) and (13), for Equation (A7) to hold, the combination of  $F^h \leq F^l$  and  $r^h > r^l$  cannot maintain simultaneously. Hence,  $F^h > F^l$ . However, it then follows from Equations (4), (11), and (13) that for Equations (A7) and (A8) to hold simultaneously,  $r^h > r^l$  must also maintain.

**Proof of Result 5:** Following Equations (1) and (12), separation is attained if:

$$\begin{aligned}
& PV^h(\cdot, L^h, r^h) \\
&= V(L^h) - \frac{[1 - P^h(L^h, F, M, C)]V(L^h)(1 + r^h) + P^h(L^h, F, M, C)\alpha}{1 + r_f} \\
&\geq V(L^l) - \frac{[1 - P^h(L^l, F, M, C)]V(L^l)(1 + r^l) + P^h(L^l, F, M, C)\alpha}{1 + r_f} \\
&= PV^h(\cdot, L^l, r^l), \tag{A10}
\end{aligned}$$

$$\begin{aligned}
 & PV^l(\cdot, L^l, r^l) \\
 &= V(L^l) - \frac{[1 - P^l(L^l, F, M, C)]V(L^l)(1 + r^l) + P^l(L^l, F, M, C)\alpha}{1 + r_f} \\
 &> V(L^h) - \frac{[1 - P^l(L^h, F, M, C)]V(L^h)(1 + r^h) + P^l(L^h, F, M, C)\alpha}{1 + r_f} \\
 &= PV^l(\cdot, L^h, r^h), \tag{A11} \\
 & \pi^h(\cdot, L^h, r^h) \\
 &= -V(L^h) + \frac{[1 - P^h(L^h, F, M, C)]V(L^h)(1 + r^h) + P^h(L^h, F, M, C)\beta}{1 + r_f} \\
 &= -V(L^l) + \frac{[1 - P^h(L^l, F, M, C)]V(L^l)(1 + r^l) + P^l(L^l, F, M, C)\beta}{1 + r_f} \\
 &= \pi^l(\cdot, L^l, r^l), \tag{A12}
 \end{aligned}$$

where  $L^h$  and  $r^h$  ( $L^l$  and  $r^l$ ) are the LTV and the interest rate, respectively, on the loan selected by the high-risk (low-risk) borrower.

Given that  $P^h(\cdot) > P^l(\cdot)$  for all  $L, F, M,$  and  $C$ , it follows from Equations (2), (3), (13), and (22) that for Equation (A12) to hold, the combination of  $L^h \geq L^l$  and  $r^h \leq r^l$  cannot maintain simultaneously. Also, note that from Equation (2) it immediately follows that  $PV_{LP}^i$  (the second derivative of  $PV^i$  with respect to  $L$  and  $P$ ) is positive. Furthermore, it is immediate to see from Equation (1) that  $PV_r^i$  (the partial derivative of  $PV^i$  with respect to  $r$ ) is negative and that  $PV_{rP}^i$  (the second derivative of  $PV^i$  with respect to  $r$  and  $P$ ) is positive. Hence,  $PV_r^i < 0, PV_{LP}^i > 0, PV_{rP}^i > 0$  combined with Equations (2) and (3) implies that for Equations (A10) and (A11) to concurrently hold, the combination  $L^h < L^l$  and  $r^h \leq r^l$  cannot maintain simultaneously. Hence,  $r^h > r^l$ . However, the latter combined with Equations (2), (3), and (21) implies that for Equation (A10) to hold,  $L^h > L^l$  must also maintain.

### Endnotes

- <sup>1</sup> Recall that under the mechanics of the PLAM, the periodic ending loan balance is adjusted according to the periodic change in the Consumer Price Index.
- <sup>2</sup> See, for example, Dunn and Spatt (1988), Brueckner (1992), Yang (1992), Brueckner (1994a), Stanton and Wallace (1998), Brueckner (2000), Posey and Yavas (2001), Ben-Shahar and Feldman (2003), and Ben-Shahar (2006).
- <sup>3</sup> Dunn and Spatt (1988) were the first to substantiate the intuition underlying the role of those mortgage characteristics in the context of prepayment.

- <sup>4</sup> Various empirical studies identify different observable borrower characteristics that may ex ante signal default risk. Among those are self-employment, length in job, non-housing wealth, neighborhood rating, number of dependents, and tenure mode (e.g., Vandell and Thibodeau, 1985; Zorn and Lee, 1989; and Cunningham and Capone, 1990). Note, however, that while these observed variables convey information about the borrower's default risk, there is additional information, privately held by the borrower, which may be revealed by the mechanisms discussed in this paper.
- <sup>5</sup> That is, the liquidity constraint arises when the contracted payment is in excess of the borrowers financial capabilities. High (low) probability of default thus implies that the borrower is ex ante more (less) likely to default due to a budget constraint. This motivation is also applied in Posey and Yavas (2001) and summarized in Vandell (1995). Further, VanderHoff (1996) reports that 60% of the payments on the mortgages in his data set are stopped when equity is positive, indicating suboptimal defaults.
- <sup>6</sup> The CPM and FRM features of the mortgages, discussed here, and the graduated payment (GPM) and price-level-adjusted payment (PLAM) features, discussed later, are all considered as continuous variables. For example, a perfect FRM may be viewed as a corner case of the ARM, where the interest rate never adjusts over the life of the loan. The less frequent the adjustment periods and the more constraining the caps and the floors on the ARM, the more the ARM resembles the FRM along the ARM–FRM continuum. An equivalent continuum is assumed for all other mortgage variables. One should note, however, that in some cases it might be difficult to order the loans along the ARM–FRM continuum (e.g., in comparing an ARM with infrequent adjustment periods with an ARM with restricting caps and floors). The analysis is thus restricted to those cases in which the particular mortgage attribute may be viewed along a common continuum.
- <sup>7</sup> The cost represented by  $\alpha$  includes the smaller between the loan balance and the value of the asset, in addition to reputation damages, psychic distress, etc. Furthermore, note that, generally, a greater  $L$  is associated with a greater loan amount [see the inequality condition in Equation (2)] and thus, following the payoff function in Equation (1), when default is triggered by a liquidity crunch, the current cost of default to the borrower rises (even for a non-recourse loan) because of the fact that a greater fraction of the asset value is transferred to the lender (of course, there is also a ceiling on the size of the default cost, namely, when  $L = 1$ ). Yet assuming here that  $\alpha$  increases with  $L$ , while preserving the results, does not introduce any essential intuition and rather complicates the analysis.
- <sup>8</sup> More formally, let  $B$  be the maximum amount to be potentially spent on the asset (given the budget constraint) and let  $Y$  be the borrower's available amount for the purchase of the asset prior to obtaining the loan. Then  $B = Y/(1 - L)$ . Increasing  $L$  therefore raises the level of  $B$ . However, since  $L = V^0/B$ , increasing  $B$  also allows a greater  $V^0$  for any given  $L$ .
- <sup>9</sup> Notice that a greater  $L$  is also associated with a greater probability of ruthless default, since the default option is more likely to be in-the-money.
- <sup>10</sup> See, for example, Brueggeman and Fisher (1997; p. 139) who argue that “when interest rate risk assumed by the borrower increases [...], default risk assumed by the lender increases.”
- <sup>11</sup> In Posey and Yavas (2001) there are two possible sources for default: diminishing income and increased payment in the case of an ARM. Hence, it may be the case that default is experienced with an FRM, while no default would have otherwise occurred

with an ARM. This is when market interest rate and income concurrently fall, in which case one might be able to repay the ARM but not the FRM.

- <sup>12</sup> In fact, while increasing maturity decreases the periodic payment, it also extends the period during which default may arise. The latter might thus increase the default risk. As in Ben-Shahar and Feldman (2003) and consistent with the empirical evidence of Epley, Kartono, and Haney (1996), the former effect is assumed to dominate and that maturity therefore inversely affects the default probability. Furthermore, while the two-period unified framework presented here does not permit capture of the change in maturity, variations in maturity are considered via its essential effect on the probability of default. That is, the effect of the loan maturity on the attained separating equilibrium is examined through its indirect effect on the default probability. As shown later, this also preserves the fundamental result, appearing, for example, in Ben-Shahar and Feldman (2003).
- <sup>13</sup> Also see Brueggeman and Fisher (1997) who argue that the “[GPM reduces] ... the burden faced by the household when meeting mortgage payments from current income in an inflationary environment” (pgs. 115–16) and that “[PLAM] reduce[s] interest rate risk, or the uncertainty of inflation,” which, in turn, decreases the risk of default (p. 131).
- <sup>14</sup> The two-period framework does not permit a perfect description of the real-world difference between CPMs and GPMs and between CPMs and PLAMs, since only one loan repayment exists. Nonetheless, one may suppose, for example, that the first payment on the loan occurs when the loan is originated. This leaves the next (and last) payment to form a payment stream that matches the particular mortgage type. A conceptually similar approach, although applied in a different framework, may be found, for example, in Brueckner (1994a).
- <sup>15</sup> It should be noted that while the derived equilibria depend on these particular second order assumptions, altering the assumptions does not annihilate the equilibria, but might change the specific outcomes.
- <sup>16</sup> This requirement is consistent with, for example, Capozza, Kazarian, and Thomson (1997) who find empirical evidence that default rates rise in a marginally diminishing fashion for higher LTV ratios.
- <sup>17</sup> In fact, if an increase in  $L$  is not accompanied by a rise in the value of the purchased asset, then  $V_{LL} = 0$ . Essentially, the assumption here merely requires that the *marginal increase* in the value of the asset purchased by the borrower (if such an increase ever occurs) does not rise with any increase in the loan-to-value ratio.
- <sup>18</sup> Note that, while an increase in  $M$  is accompanied by a drop in the periodical repayment (thereby decreasing the conditional default probability), the *change* in the repaid amount marginally falls with maturity for any positive interest rate. Hence,  $P_M^i$  is likely to marginally increase with a rise in  $M$ .
- <sup>19</sup> Although the question regarding the sign of  $P_{CC}^i$  and  $P_{FF}^i$  should be resolved empirically, ex ante there is no clear intuition for  $P_{CC}^i$  and  $P_{FF}^i$  to be different from zero. Equilibrium conditions, however, allow these terms to also be positive.
- <sup>20</sup> Recall that the cost of default for the borrower includes, among other things, reputation damages and psychic distress—elements not borne (and, of course, neither received) by the lender. While  $\alpha > \beta$  may potentially facilitate settlements between borrowers and lenders under perfect market conditions, borrower’s propositions for a settlement that postpones the payments due on the loan (when default is triggered by liquidity crunch) may possibly increase the default cost to the lender and might therefore be declined.

- <sup>21</sup> Note that when  $X = \alpha$ , then Equation (14) becomes a representative of the borrower's indifference curves. When  $X = \beta$ , however, Equation (14) turns into a representative of the lender's iso-profit curves.
- <sup>22</sup> Consistent with other studies in this area (see Brueckner, 1992; Posey and Yavas, 2001; and others), the analysis here separately considers each attribute of the loan. Examining the joint variation of two loan characteristics (e.g., Ben-Shahar and Feldman, 2003; and Ben-Shahar, 2004) is more complex and is beyond the scope of this unified framework analysis.
- <sup>23</sup> It is well documented in the real estate finance literature that the capability to repay the debt rises over time. In fact, among other things, the original motivation for marketing GPMs, and later PLAMs, is to provide a better match between periodic payments and the borrower's growing repayment capability over time (for more on this subject see, for example, Peek and Wilcox, 1991; and Brueggeman and Fisher, 1997).
- <sup>24</sup> In fact, if  $P^i(\cdot)$  is monotonically increasing in  $r$ , modeling the interest rate as a factor in the default probability function merely strengthens the effect of the constant payment shown in the analysis. Nevertheless, for simplicity, this effect will be ignored in the analysis to come. Also, for the analysis of the effect of the interest rate on the probability of default, see, for example, Jaffee and Russell (1976).
- <sup>25</sup> As noted by Peek and Wilcox (1991), the PLAM may be found in the mortgage market of Canada, Australia, Brazil, Colombia, Paraguay, Peru, Finland, Mexico, Argentina, Chile, Ecuador, Ghana, Turkey, and Israel.
- <sup>26</sup> For more on the "tilt effect," see, for example, Brueggeman and Fisher (1997).
- <sup>27</sup> Due to the fact that the proofs of all the results in the article ultimately follow similar conceptual argumentation, the proofs of Results 2–5 are presented in the Appendix.
- <sup>28</sup> Note that the inequality conditions in Equations (18)–(20) also apply to the CPM–PLAM continuum and therefore Exhibit 1 (with the appropriate adjustment of the definition of the X-axis) also corresponds to the CPM–PLAM case. Also, note that the separation with CPMs and PLAMs is more likely to persist in markets where, on one hand, general inflation adjustments—and specifically—PLAMs have become an inseparable part of the economic environment; and on the other hand, actual and expected inflation either are in the process of being or have been virtually defeated. Otherwise, CPMs are extremely risky because of considerable inflation uncertainty.
- <sup>29</sup> Note that the only difference here is that the focus is on  $P_M^i$ , while in the context of Result 1 the focus is on  $P_C^i$ .
- <sup>30</sup> A similar result, namely, that default risk is separated by the loan maturity attribute is shown in a multi-period framework in Ben-Shahar and Feldman (2003).
- <sup>31</sup> As discussed in the closing section, this imposes restrictions when one attempts to empirically estimate the correlation between mortgage maturity and default probabilities due to simultaneity effects.
- <sup>32</sup> Equivalently to the previous analysis, the slopes of the curves in Exhibit 3 follow from the inequality conditions in Equations (4), (11), and (13).
- <sup>33</sup> Put differently, ARMs impose more of the interest rate risk on the borrower and, thereby, increase the likelihood of default. However, this argument more meaningfully applies, ex ante, to the high-risk type borrower, who is thus willing to increase the interest rate more drastically in return for a more dominant FRM feature in the mortgage loan.
- <sup>34</sup> The projected difference in the interest rates conforms to the well-documented phenomenon according to which the equilibrium interest rate on the FRM is greater

than that on the ARM. See further intuition, motivated by prepayment risk considerations, in Dunn and Spatt (1988). Also, the ambiguous outcome regarding the equilibrium default rates is consistent with that of Posey and Yavas (2001).

- <sup>35</sup> Brueckner (2000) shows in detail how LTV ratios are distributed between high and low default risk borrowers. His motivation, however, for observing different behavior on the part of borrowers is their distinct default costs. Further, he assumes that default is driven by ruthless discretion, while here it is assumed that it is exogenous liquidity crunch that induces default. Yet the attained separation here is in line with that derived by Brueckner (2000). Also see Harrison, Noordewier, and Yavas (2004) and Ben-Shahar (2004) for possible variations in the separation by LTV.
- <sup>36</sup> Unlike the variables examined in the previous results, the variable  $L$  is an argument of  $P^i(\cdot)$  and  $V(\cdot)$  [see Equation (1)]. This requires the additional assumptions in Equations (21) and (22).
- <sup>37</sup> In fact, following Equations (1), (12), and (13), for the inequality conditions in Equations (21) and (22) to hold, it is sufficient that  $\alpha < \frac{V_L[1 + r_f - (1 - P^i)(1 + r)] + P_L^i V(1 + r)}{P_L^i}$ . One can see that this assumption is more likely to maintain if  $P^i(\cdot)$  increases with  $L$  more slowly than  $V(\cdot)$  does. The latter is consistent, for example, with the evidence of Capozza, Kazarian, and Thomson (1997).
- <sup>38</sup> Empirical evidence that reinforces the result includes, among others, Jackson and Kasserian (1980), Campbell and Dietrich (1983), Foster and Van Order (1985), Quigley and Van Order (1991), Kau, Keenan, and Kim (1993), Quigley and Van Order (1995), Deng, Quigley, and Van Order (1996), Capozza, Kazarian, and Thomson (1997), Deng (1997), Yang, Buist, and Megbolugbe (1998), Kau and Keenan (1999), and VanOrder and Zorn (2000).
- <sup>39</sup> On this matter, see also Yezer, Phillips, and Trost (1994) and Brueckner (1994b).

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## References

- Ben-Shahar, D. Default, Credit Scoring, and Loan-To-Value: A Theoretical Analysis Under Competitive and Oligopolistic Mortgage Markets. Working paper, Technicon-Israel Institute of Technology, 2006.
- Ben-Shahar, D. and D. Feldman. Signaling-Screening Equilibrium in the Mortgage Market. *Journal of Real Estate Finance and Economics*, 2003, 26, 157–78.
- Brueckner, J. K., Borrower Mobility, Self-Selection, and the Relative Prices of Fixed- and Adjustable-Rate Mortgages. *Journal of Financial Intermediation*, 1992, 2:401–21.
- , Borrower Mobility, Adverse Selection, and Mortgage Points. *Journal of Financial Intermediation*, 1994a, 3, 416–41.
- , Unobservable Default Propensities, Optimal Leverage and Empirical Default Models: Comments On “Bias Estimates of Discrimination and Default in Mortgage Lending: The Effects of Simultaneity and Self Selection.” *Journal of Real Estate Finance and Economics*, 1994b, 9, 217–22.
- , Mortgage Default with Asymmetric Information. *Journal of Real Estate Finance and Economics*, 2000, 20, 251–74.
- Brueggeman, W. B. and J. D. Fisher. *Real Estate Finance and Investments*. Irwin, 10th edition, 1997.

- Campbell, T. S. and K. J. Dietrich. The Determinants of Default on Conventional Residential Mortgages. *Journal of Finance*, 1983, 38, 1569–81.
- Capozza, D. R., D. Kazarian, and T. A. Thomson. Mortgage Default in Local Markets. *Real Estate Economics*, 1997, 25:637–55.
- Cunningham D. and C. Capone. The Relative Termination Experience of Adjustable to Fixed-Rate Mortgages. *Journal of Finance*, 1990, 45, 1687–1703.
- Deng, Y., Mortgage Termination: An Empirical Hazard Model with a Stochastic Term Structure. *Journal of Real Estate Finance and Economics*, 1997, 14, 309–31.
- Deng, Y., J. M. Quigley, and R. Van Order. Mortgage Default and Low Downpayment Loans: The Costs of Public Subsidy. *Regional Science and Urban Economics*, 1996, 26, 263–85.
- Dunn, K. B. and C. S. Spatt. Private Information and Incentives: Implications for Mortgage Contract Terms and Pricing. *Journal of Real Estate Finance and Economics*, 1988, 1, 47–60.
- Epley, D. R., L. Kartono, and R. Haney. Borrower Risk Signaling Using Loan-To-Value Ratios. *Journal of Real Estate Research*, 1996, 11, 71–86.
- Foster C. and R. Van Order. FHA Terminations: A Prelude to Rational Mortgage Pricing. *American Real Estate and Urban Economics Association*, 1985, 13, 273–91.
- Harrison, D. M., T. G. Noordewier, and A. Yavas. Do Riskier Borrowers Borrow More? *Real Estate Economics*, 2004, 32, 385–411.
- Jackson, J. R. and D. L. Kasserman. Default Risk on Home Mortgage Loans: A Test of Competing Hypotheses. *Journal of Risk and Insurance*, 1980. 47, 678–90.
- Jaffee, D. M. and T. Russell. Imperfect Information, Uncertainty, and Credit Rationing. *Quarterly Journal of Economics*, 1976, 90, 651–66.
- Kau, J. B. and D. C. Keenan. Patterns of Rational Default. *Regional Science and Urban Economics*, 1999, 29, 765–85.
- Kau, J. B., D. C. Keenan, and T. Kim. Transaction Costs, Suboptimal Termination and Default Probability. *American Real estate and Urban Economics Association Journal*, 1993, 21, 247–63.
- Peek, J. and J. A. Wilcox. A Real, Affordable Mortgage. *New England Economic Review*, 1991, Jan/Feb, 51–66.
- Posey, L. and A. Yavas. Adjustable and Fixed Mortgages as a Screening Mechanism for Default Risk. *Journal of Urban Economics*, 2001, 49, 54–79.
- Quigley, J. M. and R. Van Order. Defaults on Mortgage Obligations and Capital Requirements for U.S. Saving Institutions. *Journal of Public Economics*, 1991, 44, 353–69.
- Quigley, J. M. and R. Van Order. Explicit Tests of Contingent Claims Models of Mortgage Default. *Journal of Real Estate Finance and Economics*, 1995, 11, 99–117.
- Rothschild, M. and J. E. Stiglitz. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics*, 1976, 90, 629–50.
- Spence, A. M., 1973. Job market signaling. *Quarterly Journal of Economics*, 1973, 87: 355–374.
- Stanton, R., and N. Wallace. Mortgage Choice: What's the Point? *Real Estate Economics*, 26, 173–205.



- Van Order, R. and P. Zorn. Income, Location and Default: Some Implications for Community Lending. *Real Estate Economics*, 2000, 28, 385–404.
- Vandell, K. D. Default Risk under Alternative Mortgage Instruments. *Journal of Finance*, 1978, 33, 1279–96.
- , How Ruthless is Mortgage Default? A Review and Synthesis of the Evidence. *Journal of Housing Research*, 1995, 6, 245–64.
- Vandell, K. D. and T. Thibodeau. Estimation of Mortgage Defaults Using Disaggregate Loan History Data. *Journal of the American Real Estate and Urban Economics Association*, 1985, 13, 292–316.
- VanderHoff, J. Adjustable and Fixed Rate Mortgage Termination, Option Values and Local Market Conditions: An Empirical Analysis. *Real Estate Economics*, 1996, 24, 379–406.
- Von Furstenberg, G. M. Default Risk on FHA-Insured Home Mortgages as a Function of the Terms of Financing: A Quantitative Analysis. *Journal of Finance*, 1969, 24, 459–77.
- Yang, T. L. T. Self-Selection in the Fixed-Rate Mortgage Market. *Journal of the American Real Estate and Urban Economics Association*, 1992, 20, 359–91.
- Yang, T. T., H. Buist, and I. F. Megbolugbe. An Analysis of the Ex Ante Probabilities of Mortgage Prepayment and Default. *Real Estate Economics*, 1998, 26, 651–76.
- Yezer, A. M. J., R. F. Phillips, and R. P. Trost. Bias in Estimates of Discrimination and Default in Mortgage Lending: The Effects of Simultaneity and Self-Selection. *Journal of Real Estate Finance and Economics*, 1994, 9, 197–215.
- Zorn, P. and M. Lea. Mortgage Borrower Repayment Behavior: A Microeconomic Analysis with Canadian Adjustable Rate Mortgage Data. *Journal of the American Real Estate and Urban Economics Association*, 1989, 17, 118–36.
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