# Property Taxation of Multifamily Housing: An Empirical Analysis of Vertical and Horizontal Equity

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Abstract

This paper analyzes the uniformity of the property appraisal outcome for multifamily apartment complexes in Phoenix, Arizona. Specifically, it investigates vertical and horizontal equity and equity across assessment methods. Apartment transactions and assessed valuations are examined over a five-year period (1998–2002). Once possible bias in model specification is accounted for, no evidence of vertical inequity in the sample is found. However, there is modest evidence of horizontal inequity: the results suggest that complex size and geographic location are more difficult for the assessor to value uniformly. There is also inequity between small and large properties, as represented by two different valuation methods. Thus, the findings indicate that the income approach is superior to the sales comparison approach for valuing multifamily properties for tax purposes.

## Introduction

The property tax in the United States suffers from an assortment of administrative, economic and political conflicts. These conflicts arise for four principle reasons. First, the property tax is a significant cost of owning real property. Mills (1980) shows that a 2% property tax, based on the asset value of structures and land, is comparable to a 20% annual sales tax. Second, unlike most business taxes, few legal paths exist to sidestep the property tax, leaving few degrees of freedom for tax planners (McLure, 2002). Third, the property tax is a direct tax, but the public benefits received from the tax are often indirect (Palmon and Smith, 1998). Fourth, the public is suspect about the uniformity of the tax (Fisher, 1996). Perhaps the high burden on property owners, the inability to avoid the tax, the misunderstanding of the benefits received from the tax, and concerns about its uniformity partially explain why voters and legislative bodies have tried to limit the property tax by imposing constraints on appraisals, rates, or revenue growth (Advisory Commission on Intergovernmental Relations, 1995). Despite these perceived problems, the property tax remains a major source of revenue for school districts and local governments in the U.S. (Bruce, 2000). Over thirty-five years ago, Netzer (1966) noted that Americans disliked the property tax but used it heavily. He labeled this conflict the *property tax paradox*. This paradox continues today.

Surprisingly, specialists in public finance see advantages to a property tax because it provides fiscal and political autonomy for local governments. In a survey of 1,309 Canadian and U.S. members of the National Tax Association, 93% with training in economics recommended continued use of the property tax as a major source of revenue for local governments (Slemrod, 1995). The property tax provides the potential for increased economic efficiency, because the property tax establishes a ''tax price'' that taxpayers can react to (Oates, 1996). Further, if a property tax is designed and administered correctly, it may provide incentives for the optimal use of land (Brueckner, 1986; and Tideman, Akobundo, Johns and Wutthicharoen, 2002). However, this last advantage highlights the complaints that professionals have with the property tax. If it is not administered properly, the advantage of the property tax is lost, and it may distort land use and create excess burden, especially for mobile capital.

The concern over administration centers on the problem of consistently measuring the annual taxable value or base of individual properties. The starting point for other taxes is based on a flow of value that allows administrators to examine actual economic events—sales or income transactions—to establish the taxable value of the base. The property tax requires a taxable value to be set on a specific date, but few properties sell in any given year, and property tax administrators are left to create hypothetical transactions—what would this property sell for in the open market if it did sell?—to establish the tax base.

The process of estimating the value of an asset that trades infrequently is under continual academic and professional debate. However, many recognize that the process can be quite subjective. Hyman (1998) describes the practice of property assessment by public assessors as an ''art,'' not a science. Kay (1990) suggests that when estimates are used to value a tax base, the possibility of being incorrect is so high that hypothetically determined tax bases may not survive in a global economy where capital is mobile. Concerns about the uniformity of the property tax have been articulated in the literature and the results confirm the belief that the property tax often suffers from inconsistent valuations.

Virtually all the research on the uniformity of the property tax has examined single-family residential homes. In this article, the same methodologies are used to examine equity in the valuation of residential homes to multifamily housing facilities. An important contribution of this paper is to extend an analysis of property tax valuation to other classes of property. Allen (2003), in a carefully designed study, examines the issue of vertical equity of multifamily homes during a one-year period in Broward County, Florida. The study discussed here extends his study in several important areas: (1) the vertical equity of the property tax is examined over a five-year period; (2) horizontal equity in multifamily housing is investigated; and (3) equity across assessment methods is examined.[1] The data used in this study come from the sales of multifamily housing units that occurred over a five-year period in the metropolitan Phoenix, Arizona area (Maricopa County). Unlike most previous studies, this data allows for an investigation of the intertemporal uniformity of the property tax. Generally, equity in property taxation is perceived to exist if there is both vertical and horizontal equity within the tax jurisdiction. Vertical equity results when the assessment ratio (AV/MV)—the ratio of assessed value to market value—is uniform across property value ranges. Horizontal equity results when the assessment ratio is uniform across properties with similar market values. Note: the Maricopa County Assessor's Office uses two different appraisal methods depending on the size of the apartment property. Specifically, a hedonic regression sales comparison approach is used for apartment properties with fewer than thirteen units, and a hedonic regression income approach is used for properties with more than twelve units. This also allows for an examination of equity across assessment methods.

This paper proceeds as follows: The second section briefly reviews the magnitude of multifamily housing units in America and offers some sense of the socioeconomic makeup of apartment dwellers. This section also examines the implications of who actually faces the burden or the incidence of the property tax—the owner of the property or the renter? In the third section, there is a discussion of the unique way that multifamily housing units are valued for property tax purposes in Maricopa County, Arizona and a review of the characteristics of the dataset. In the fourth section, there is a review of both the methodologies that examine vertical and horizontal equity. The fifth section reviews the empirical results. The sixth section presents an investigation of equity across assessment methods. The final section is the conclusion with policy observations for public assessors, owners of multifamily housing units and elected policy makers.

## Multifamily Housing in the United States

Relative to single-family housing, multifamily housing receives modest attention from policy makers. In the U.S., this may be to the detriment of millions of households.[2] Individuals and families who live in rental property represent about 34 million housing units, or about one-third of the population of the U.S. About one-half of the rental units are provided by multifamily complexes—five or more units per building (Schnare, 2001). Thus, about 16.7 million components of the housing stock in the U.S. are of a multifamily nature. The percentage of renters is invariably higher in urban areas than in suburban and rural areas. In some urban areas renters outnumber homeowners. In Los Angeles and New York, over 60% of the housing comes from rental units. In Dallas, Chicago, Houston and San Diego, rental housing provides at least 50% of the total housing stock. In Phoenix, rental housing constitutes about 32% of the total housing.

In addition to the urban nature of multifamily housing, a variety of other socioeconomic markers distinguish renters in the U.S. Compared to homeowners, a higher percentage of renters are poor, single-parent families; come from ethnic and racial groups; and have lower median income levels (Census Bureau, 2001). Because such statistical indicators suggest an economic hurdle must be crossed before an individual or family becomes a homeowner, one might assume that a primary reason to examine the equity of the property tax on multifamily housing is because inequities are passed forward to renters, and renters are often synonymous with poverty. There are of course millions of individuals and families who have the option to live in single-family residential dwellings but choose to live in apartments. Nevertheless, the assumption about poverty and renters is made in a variety of studies on the incidence of the property tax on renters (Chernick and Reschovsky, 1990). It is also the assumption made in the states that grant property tax relief to both homeowners and renters, and is the reason that states that do not have this practice are criticized in the literature (Krueckeberg, 1999).

The assertion that renters pay the property tax however is built on debatable assumptions. Without spiraling into a full discussion on the economic incidence of the property tax, note one contrasting view on this issue. One group of scholars argues that the application of a property tax on economic factors results in the incidence of the tax being placed on the immobile economic factors. This suggests that because renters are mobile, they can avoid a substantial share of the property tax; it is the property owners who face most of the burden of the tax. In other words, if a landlord attempts to raise the rent because of the property tax, renters will consider their options and simply move or they ‘‘vote with their feet.’’ Under this view the property tax is a tax on capital and ultimately the owners of capital (Zodrow, 1986). However, as Mills and Lubuele (1997) note, few believe that the concentration of poor people in the inner city is ‘‘entirely voluntary,’’ and thus the assumption of mobility of the urban rental population faces some policy discount.

In short, this prior research suggests that improperly imposed property taxes may distort investment and cause inefficient decisions in the rental housing market.

## Empirical Data and Maricopa County Assessment Practices

The data used in this study consist of apartment transactions that occurred in Maricopa County, Arizona, from 1998 through 2002. The data set includes 946 transactions researched by the CoStar Group, Inc.[3] CoStar investigates apartment transactions exceeding $250,000 by physically inspecting each property and confirming the particulars of the transaction with the relevant parties, including buyer, seller and broker. CoStar also reports their evaluation of the overall condition or quality of each property. In addition, representatives of CoStar obtain the assessed value of the property at the time of sale, as reported by the Maricopa County Assessor.

To determine the assessed value of apartment properties, the County Assessor’s office employs a mass appraisal process that uses a hedonic regression model to estimate the value of apartment properties. For the period of study, the County Assessor was revaluing property on an annual basis, although a lag of approximately eighteen months existed between the tax notice date and the age of the transaction and rental data used to estimate value. More recently, the Assessor has moved to a two-year rotation schedule.[4] Besides the frequency of reappraisal, several unique aspects of the approaches are used in Maricopa County to value apartment buildings. Using a hedonic model to value residential properties is common; however, using a hedonic model to estimate value for a multifamily property is less common.[5] For smaller properties, those with fewer than thirteen units, the hedonic regression model incorporates independent variables that reflect physical, location and transaction-related attributes, while the dependent variable is the sales price.

For larger properties, those with more than thirteen units, the Assessor uses independent variables similar to the ones used for smaller facilities, but uses rental income as the dependent variable. The contemporary rental data of apartment properties in Maricopa County is obtained from a nongovernmental data provider. Using rental income as the dependent variable allows the Assessor to estimate the potential gross income for each property. Again this is a unique approach that allows mass appraisal of multifamily housing.[6] At least one International Association of Assessing Officers (IAAO) text makes mention of estimating the value of apartment buildings based on income; however, this method is rarely used (Gloudemans, 1999). Smith and Islam (1998) report limited applications of this approach in the applied appraisal literature; Isakson (2001) notes that this approach has been used on specific properties during appraisal appeals. Because property taxes in the U.S. are based on capital value, one additional step is needed to transform rental income into capital value by applying a gross income multiplier to the estimated rent. A hedonic regression model is also used to estimate a gross income multiplier for each property. When the estimated gross income multiplier is applied to the estimated potential gross income, a capital value estimate is generated for each property.[7]

Arizona property tax law requires that the assessor estimate an assessed value that reflects *full cash*, or market value [Arizona Revised Statutes: 42-11054(B)]. As is common in many states, a legislative modification allows county assessors to adjust the full cash value estimate by a factor of 0.82. This allowance is intended to reduce the assessed value for mass appraisal error, time to sell property, financing costs and personal property. One other likely effect of this policy is to provide the assessor and the assessment process with a ‘‘cushion of error.’’ Goolsby (1997) and Allen and Dare (2002) suggest that assessing officials often underassess properties to minimize tax appeals; however, property tax equity can be maintained as long as all properties in a jurisdiction have a uniform assessment ratio.[8]

Exhibit 1 reports summary statistics on the continuous variables. The mean sales price is approximately $3.2 million; however, the minimum and maximum of $250,000 and $54,000,000, respectively, illustrate the large variation in property

**Exhibit 1** | Descriptive Statistics
Phoenix Apartment Transactions (1998–2002)

| | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|
| Sales Price | 3,214,500 | 6,375,500 | 250,000 | 54,000,000 |
| Assessed Value | 2,291,900 | 4,669,500 | 80,000 | 42,152,500 |
| Number of Units | 69 | 103 | 5 | 768 |
| Building Age (Yrs.) | 27.96 | 13.30 | 1.0 | 109.0 |

*Note:* Values are for all properties; 946 total observations.

values. The typical transaction is for a complex with sixty-nine units and a mean building age of twenty-eight years (standard deviation of thirteen years). The mean assessed value is approximately $2.3 million or about 29% lower than the mean sales price.

Exhibit 2 presents the frequency of the binary variables and reports that nearly 75% of the complexes are in ‘‘average’’ condition, whereas only 8% were considered better than average and 17% worse than average. The Central Phoenix

**Exhibit 2** | Frequency of Binary Variables

| Variable | Mean | Observations |
|---|---|---|
| Total Observations | | 946 |
| Condition | | |
| Better than average | 0.079 | 75 |
| Average | 0.752 | 712 |
| Worse than average | 0.168 | 159 |
| Geographic Areas | | |
| Central Phoenix | 0.653 | 618 |
| Tempe | 0.062 | 59 |
| East Valley | 0.128 | 122 |
| North & West Cites | 0.084 | 79 |
| Scottsdale & Cave Creek | 0.071 | 68 |
| Yearly Time Periods | | |
| 1998 | 0.188 | 178 |
| 1999 | 0.200 | 190 |
| 2000 | 0.192 | 182 |
| 2001 | 0.211 | 200 |
| 2002 | 0.207 | 196 |

area dominates the sample with 65% of the transactions. The East Valley is second with about 13%. North and West Cities, Scottsdale/Cave Creek and Tempe are close third, fourth and fifth, with 8%, 7% and 6%, respectively. The transactions are smoothly distributed across time, with transactions ranging from 178 to 200 in each year.

Panels A and B in Exhibit 3 separate the data by appraisal method. Panel A includes 217 transactions of smaller properties (fewer than thirteen units) composing 23% of the data set. For these smaller properties, the assessed value was obtained by using a mass appraisal hedonic regression sales comparison approach. The mean assessed value of $237,000 is approximately 32% lower than the mean sales price of the sample. Panel B includes 729 transactions of larger properties (more than thirteen units), composing 77% of the sample. A mass appraisal hedonic regression income approach is used to estimate the assessed value for each of these properties. The mean assessed value of $2,903,000 is approximately 29% lower than the mean sales price. The statistics for the entire sample, Exhibit 1, are similar to Panel B because of the significant weight of the larger properties in the complete sample. Two preliminary observations result from the descriptive statistics of Exhibits 1 and 3. First, it is evident that the legislative intent to reduce the assessed values of apartment properties in Maricopa County is working. The assessed values are significantly lower than the respective sales prices. Second, the slightly larger spread between assessed values and sales prices for smaller properties may suggest that the mass appraisal sales comparison approach is less robust compared to the income approach for estimating the assessed value of apartment properties.

Exhibit 4 provides the mean sales price and assessed value for all properties for each of the five years. This exhibit shows that the mean sales price increased in each year of the first fours years; however, a noticeable decline is evident in the fifth year, 2002. In addition, the spread between the assessed value and the sales price was greater in 2002 than in any other previous year. Using averages to

**Exhibit 3** | Appraisal Method

| | Mean | Std. Dev. | Min. | Max. |
|---|---|---|---|---|
| Panel A: Smaller Properties (Fewer than 13 Units; 217 Observations) | | | | |
| Sales Price | 348,631 | 80,050 | 250,000 | 632,500 |
| Assessed Value | 237,073 | 75,117 | 80,096 | 492,500 |
| Panel B: Larger Properties (More than 12 Units; 729 Observations) | | | | |
| Sales Price | 4,067,514 | 7,041,585 | 252,200 | 54,000,000 |
| Assessed Value | 2,903,541 | 5,164,187 | 122,766 | 42,152,500 |

**Exhibit 4** | All Properties (Yearly Data)

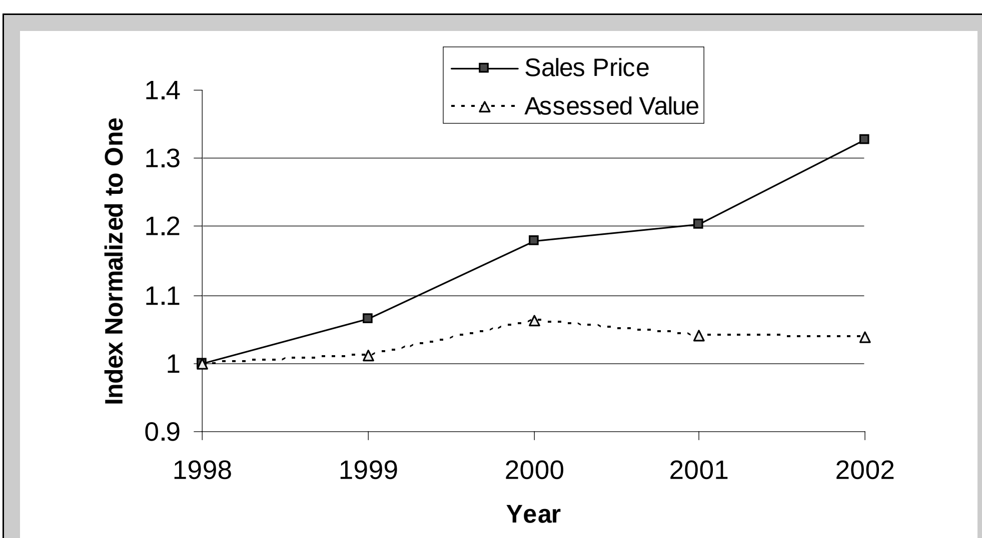
| | 1998 | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|---|
| Sales Price (Mean) | 2,474,682 | 2,893,361 | 3,763,137 | 3,915,966 | 2,972,216 |
| Assessed Value (Mean) | 1,820,217 | 2,021,673 | 2,879,510 | 2,813,805 | 1,903,969 |
| AV % Difference from SP | 26.45% | 30.13% | 23.48% | 28.15% | 35.94% |
| Number of Observations | 178 | 190 | 182 | 200 | 196 |

*Note:* The data were obtained from the CoStar Group, Inc. (formerly Comps InfoSystems, Inc.). Employees at the CoStar Group physically inspect each property and confirm transaction details with relevant parties. The Maricopa County Assessor's Office uses a sales comparison approach to estimate the assessed value of all apartments with fewer than 13 units, and a hedonic income approach to estimate the assessed value of apartments with more than 12 units.

compare the intertemporal nature of sales prices (assessed values) has inherent problems. Specifically, change in prices (assessed values) may either result from true price (assessed value) changes or from the quality differences in transacting properties from one sample period to another. Constant quality hedonic indices rectify this problem. Specifically, the natural log of sales prices (assessed values) is regressed on a vector of independent property characteristic and annual binary time variables. The index is constructed by taking the antilogarithms of the coefficients from the time variables and normalizing to unity. This process holds property characteristics constant, allowing for a clear indication of the intertemporal trend in sales price (assessed value) (Berndt, 1991). Using the data and variables from Exhibits 1–4 in the regression analysis, constant quality indices are generated for both sales price and assessed value. The results from this analysis are illustrated in Exhibit 5.

Assessed values have experienced only a moderate increase of about 3%–5% over the entire five-year period; however, apartment prices have increased by over 30% during the same period. It is clear that assessed values have not kept pace with this robust market, because the difference in assessed values and sales prices increases in each year of the study.

**Exhibit 5** | Sales Price and Assessed Value Indices




*Note:* Berndt (1991) provides an index construction technique that allows for a better understanding of the intertemporal change in sales prices (assessed values). First, a log linear model is employed that regresses sales price (assessed value) on a vector of property characteristic and yearly binary time variables. Second, the index is constructed by taking the antilogarithms of the time variables and normalizing to unity. This process holds property characteristics constant, allowing for a clear indication of the intertemporal trend in sales price (assessed value).

## Examination of Vertical and Horizontal Equity

Two standards of equity are used to monitor the performance of the property tax.[9] The first standard is vertical equity, present when the *AV/SP* ratio is uniform over differing price ranges of the same type of property (Sirmans, Diskin and Friday, 1995). When analysts discover that low-value properties are assessed at a ratio that is higher (lower) than that of higher valued properties, they label the outcome as regressive (progressive) taxation. The second standard is horizontal equity and is based on the public finance concept of equal treatment of equals (Atkinson and Stiglitz 1980; and Musgrave, 1990). In the case of the property tax, this means that properties with similar market value are treated uniformly or are appraised at the same percentage of market value (IAAO 1978; and Allen and Dare, 2002).

### *Vertical Equity Tests*

The measures used to determine vertical equity began with straightforward comparisons of the *AV/SP* ratios between properties controlling for type and value (Oldman and Arron, 1965). Use of this simple comparison has continued in some studies primarily because of the ease at which analysts can explain their results to the general public and elected policy makers (Pearson, 1979). Subsequent work measuring vertical equity has leveraged off variations of the Paglin and Fogarty (1972) model, namely $AV = a_o + a_1 SP$, where $a_o$ and $a_1$ are regression coefficients.[10] Cheng (1974) recommends using the log of *AV* and *SP* to account for nonlinear relationships and to provide for the interpretation of the coefficients as elasticities. The IAAO (1978) posits that the appropriate measure of vertical equity is the *AV/SP* ratio regressed on sales price. This functional form allows for an intuitive explanation: vertical equity exists if there is no correlation between the *AV/SP* ratio and sales price. Kochin and Parks (1982) suggest that these former models are biased toward regressivity due to an "errors in variables" problem because sales price is used as a proxy for market value. Kochin and Parks propose that logging and reversing the dependent and independent variables ($\ln SP = a_o + a_1 \ln AV$) will correct the problem. Bell (1984) argues that the Kochin and Parks model is flawed because it assumes market error but does not allow for subjective assessment error. He defends the traditional approach, but shows that nonlinear inequity is better captured by including a quadratic term ($AV = a_o + a_1 SP + a_2 SP^2$).

Clapp (1990) argues that the Bell (1984) model will be biased toward regressivity and the Kochin and Parks (1982) model will be biased toward progressivity. Clapp (1990) recognizes that market value and assessed value are interdependent because taxes are capitalized negatively into market value; and he proposes a simultaneous equations model to address this issue. Specifically, he introduces an instrumental variable that accounts for the high correlation between market value and assessed value but is uncorrelated with the error terms in the Kochin and Parks model.

Birch, Sunderman and Hamilton (1990) and Sunderman, Birch, Cannaday and Hamilton (1990) show that under some situations the relationship between *AV* and *SP* may be *S*-shaped, and have recommended the use of spline regression models. They base their arguments on the observation that improvements to properties are valued using a cost approach and that the cost approach does not respond well to determining land values or accurately calculating depreciation on high- or low-value properties.[11]

Sirmans, Diskin and Friday (1995) and Smith (2000) examine vertical equity in single-family residences by effectively employing all of the above models in their analysis; however, their results are inconclusive regarding the "best" model. Sirmans et al. do, however, suggest that the Clapp (1990) model provides an effective alternative to the problems encountered with many of the previous models. Benson and Schwartz (1997) reject the models that assume sales price is a function of assessed value by arguing that it is difficult to defend the notion that government officials are better at developing estimates of value than the market. This latter argument has merit; however, only the Clapp (1990) model explicitly recognizes that assessed value feeds back into market value; therefore, a simultaneous equations approach might provide more reliable results. After considering the merits of each technique, four models were selected for the following reasons. First, the IAAO (1978) model is used because of its widespread use and direct interpretation of the results. Second, the Cheng (1974) and Bell (1984) models are used that allow for nonlinearity between *AV* and *SP* and provide intuitive interpretation. Third, the Clapp (1990) model is used because there is convincing evidence that assessed value feeds back on sales price, causing an "errors in variables" problem. Each of these models, as well as related hypotheses, is outlined below:

IAAO (1978): $$\frac{AV}{SP} = a_0 + a_1 SP. \quad (1)$$

The null hypothesis is $H_o{:}a_1 = 0$. The IAAO model offers an intuitive interpretation. If the *AV/SP* ratio is correlated with sales prices, then vertical inequity exists. This correlation is manifest in the coefficient. A negative (positive) coefficient suggests a negative (positive) relationship between *AV/SP* and sale price, consistent with regressive (progressive) vertical inequity.

Cheng (1974): $$\ln AV = a_0 + a_1 \ln SP. \quad (2)$$

The null hypothesis is $H_o{:}a_1 = 1$. Vertical equity exists if the coefficient $a_1$ is equal to one; however, if the coefficient is less (greater) than one, regressive (progressive) property tax inequity is evident.

$$\text{Bell (1984): } AV = a_0 + a_1 SP + a_2 SP^2. \quad (3)$$

This model explicitly accounts for the nonlinearity in $AV/SP$. The null hypothesis for vertical equity is $a_0 = a_2 = 0$; therefore, the variables of interest are the intercept term and the coefficient for the quadratic term. If the quadratic term is insignificant, then a linear model is satisfactory and regressive (progressive) inequity is evident with a positive (negative) and significant intercept term. If the quadratic and intercept terms are positive and negative respectively, then progressive inequity exists at an accelerating rate. If the quadratic and intercept terms are positive, then offsetting progressivity exists. If the quadratic and intercept terms are negative, then offsetting regressivity exists. If the quadratic and intercept terms are negative and positive respectively, then regressive inequity exists at an accelerating rate. If neither the quadratic term nor the intercept terms are significant, then no inequity exists.

$$\text{Clapp (1990): } \ln SP = a_0 + a_1 \ln AV,$$
$$\text{where } \ln AV = \beta_o + \beta_1 \ln Z \quad (4)$$

$Z$ is an instrumental variable defined as follows: $Z = -1$ if $AV$ and $SP$ rank in the bottom one third of the data; $Z = +1$ if $AV$ and $SP$ rank in the top one third of the data; and $Z = 0$ otherwise. The null hypothesis is $H_o: a_1 = 1$. Vertical equity exists if the coefficient $a_1$ is equal to one; however, if the coefficient is greater (less) than one, regressive (progressive) property tax inequity is evident.

In addition to the parametric models, a nonparametric technique is used to examine vertical equity: the Spearman Rank test. This test examines the correlation between the rank order of the $AV/SP$ ratio and sales price. The null hypothesis for this test assumes no correlation between the $AV/SP$ ratio and the sales price. A significant negative (positive) correlation suggests a regressive (progressive) property tax inequity.[12]

## Vertical Equity Empirical Results

The first stage of the investigation examines assessment ratios ($AV/SP$) over different price ranges and year of sale. Exhibit 6 provides the results of these calculations.

The sales price levels were derived by arraying the data by sales price in ascending order and then separating the data into five quintiles, resulting in approximately 189 observations in each quintile. An examination of the ratios finds that in the

**Exhibit 6** | Assessment Ratios (AV / SP) by Year of Sale and Sales Price Range

| | | 1998 | | 1999 | | 2000 | | 2001 | | 2002 | | All | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Level | Sales Price Range | *n* | *AV / SP* | *n* | *AV / SP* | *n* | *AV / SP* | *n* | *AV / SP* | *n* | *AV / SP* | *n* | *AV / SP* |
| 1 | 250,000–384,000 | 41 | 0.86 | 38 | 0.84 | 36 | 0.70 | 49 | 0.65 | 25 | 0.55 | 189 | 0.73 |
| 2 | 385,000–637,250 | 40 | 0.91 | 50 | 0.76 | 35 | 0.70 | 30 | 0.74 | 34 | 0.61 | 189 | 0.75 |
| 3 | 640,000–1,230,000 | 40 | 0.80 | 29 | 0.70 | 41 | 0.71 | 41 | 0.66 | 39 | 0.65 | 190 | 0.70 |
| 4 | 1,235,000–3,625,000 | 26 | 0.71 | 34 | 0.71 | 34 | 0.70 | 33 | 0.70 | 62 | 0.62 | 189 | 0.68 |
| 5 | 3,644,000–54,000,000 | 31 | 0.73 | 39 | 0.72 | 36 | 0.76 | 47 | 0.73 | 36 | 0.66 | 189 | 0.72 |
| | Total / Average | 178 | 0.80 | 220 | 0.75 | 182 | 0.71 | 200 | 0.70 | 196 | 0.62 | 946 | 0.72 |

early years, 1998 and 1999, the ratios generally declined as the sales price increased; suggesting regressive vertical inequity. This trend is reversed, the ratios increasing slightly with sales price in 2000 through 2002, suggesting a slight progressive vertical inequity during these periods. When all years are combined and the ratios are examined over the five price ranges, there is a hint of regressivity. A very interesting finding is the monotonic decline in the average ratio from 1998 through 2002 of 0.80 to 0.62. This is a material decline over a five-year period. There are two possible explanations for this finding: first, the assessor may have systematically decreased the assessed valuations with respect to market value, or second and more likely as observed in Exhibit 5, the market values have been increasing leaving the assessed values behind.

The regression results from the IAAO (1978) model are provided in Panel A of Exhibit 7. In this model the *AV/SP* ratio is regressed on sales price (*SP*). The $a_1$ coefficient for years 1998 and 1999 is negative and statistically significant, suggesting regressive vertical inequity. For the remaining years, the coefficient is positive; however, only year 2000 is statistically significant.

Panel B in Exhibit 7 provides the regression results from the Cheng (1974) model on each year of data. For years 1998 and 1999, the coefficient $a_1$ is less than one and is statistically significant, confirming a pattern of regressive valuation in these years. For the remaining years, the $a_1$ coefficient is positive, suggesting progressive vertical inequity; however, only year 2001 is statistically significant. These results are consistent with the findings in Exhibit 6.

The results from the Bell (1984) model are provided in Panel C of Exhibit 7. The coefficient on the $a_2$ variable is positive and significant in 1998, suggesting progressive inequity; however, all subsequent years exhibit significant regressive inequity. This result is consistent with the expectation of Clapp (1990) who claims the Bell model is biased toward regressivity due to the feedback problem between assessed value and sales price. After correcting for this problem by using a simultaneous equations technique, the Clapp (1990) model finds no significant evidence of vertical inequity (see Panel D of Exhibit 7). Although not significant, the $a_1$ coefficients for each year are consistent with the IAAO and Cheng models.

After examining the individual years, the results of each model were estimated over the entire five years by including binary variables for 1999 through 2002, the omitted variable being 1998. The results of this analysis are provided in Exhibit 8. The results are consistent with expectation: specifically, in the yearly analysis there is evidence of both regressive and progressive vertical inequity in the IAAO and Cheng models. However, taken together, the effects are offsetting suggesting that in the aggregate there is no significant vertical inequity ($a_1$ is not significant). For the Bell (1984) model, regressive vertical inequity is found in four of the five years, therefore, regressive inequity is found when all years are examined together ($a_2$ is significant). After correcting for the errors in variables problem, the Clapp (1990) model finds no inequity when considering the five years together.

**Exhibit 7** | Vertical Equity Analysis

| Year of Sale | $a_0$ | *t*-Stat | $a_1$ | *t*-Stat | Adj. $R^2$ |
|---|---|---|---|---|---|
| Panel A: $AV/SP = a_o + a_1 SP$ (IAAO, 1978) | | | | | |
| 1998 | 0.8342 | 38.54 | −9.257E-9 | −2.20* | 0.0212 |
| 1999 | 0.7625 | 55.02 | −5.054E-9 | −2.20* | 0.0200 |
| 2000 | 0.7003 | 51.30 | 3.321E-9 | 2.09* | 0.0182 |
| 2001 | 0.6855 | 43.45 | 1.701E-9 | 0.94 | 0.0006 |
| 2002 | 0.6151 | 48.00 | 1.862E-9 | 0.93 | 0.0007 |

Panel B: $\ln AV = a_o + a_1 \ln SP$ (Cheng, 1974)

| Year of Sale | $a_0$ | *t*-Stat | $a_1$ | *t*-Stat | Adj. $R^2$ |
|---|---|---|---|---|---|
| 1998 | 0.7216 | 2.55 | 0.9288 | −3.48* | 0.9213 |
| 1999 | 0.2300 | 1.19 | 0.9607 | −2.84* | 0.9621 |
| 2000 | −0.7324 | −3.36 | 1.0257 | 1.67 | 0.9606 |
| 2001 | −0.8175 | −4.28 | 1.0295 | 2.17* | 0.9667 |
| 2002 | −0.8641 | −3.27 | 1.0248 | 1.33 | 0.9394 |

Panel C: $AV = a_o + a_1 SP + a_2 SP^2$ (Bell, 1984)

| Year of Sale | $a_0$ | *t*-Stat | $a_1$ | *t*-Stat | $a_2$ | *t*-Stat | $R^2$ |
|---|---|---|---|---|---|---|---|
| 1998 | 102080 | 1.40 | 0.6600 | 20.56 | 3.21E-9 | 2.12* | 0.9486 |
| 1999 | −209058 | −2.34* | 0.9274 | 27.44 | −1.24E-8 | −9.33* | 0.9284 |
| 2000 | −241092 | −1.79 | 0.8944 | 21.85 | −3.33E-9 | −2.88* | 0.9449 |
| 2001 | −79039 | −0.61 | 0.7863 | 23.74 | −2.44E-9 | −2.89* | 0.9300 |
| 2002 | −247499 | −3.21* | 0.8040 | 32.70 | −5.84E-9 | −8.32* | 0.9474 |

Panel D: $\ln SP = a_o + a_1 \ln AV$ where $\ln AV = b_o + b_1 \ln Z$ (Clapp, 1990)

| Year of Sale | $a_0$ | *t*-Stat | $a_1$ | *t*-Stat | Adj. $R^2$ |
|---|---|---|---|---|---|
| 1998 | −0.1472 | −0.22 | 1.0301 | 0.61 | 0.7140 |
| 1999 | −0.0690 | −0.12 | 1.0284 | 0.68 | 0.7663 |
| 2000 | 0.8682 | 1.48 | 0.9636 | −0.85 | 0.7387 |
| 2001 | 1.0737 | 2.15 | 0.9508 | −1.35 | 0.7749 |
| 2002 | 1.2129 | 2.51 | 0.9486 | −1.45 | 0.7861 |

*Notes:* In Panel A, if the coefficient $b_1$ is 0, there is no vertical property tax inequity. If the coefficient is less (greater) than zero, there is regressive (progressive) property tax inequity.
In Panel B, if $a_1$ is equal to one, no vertical tax inequity exists; however, a coefficient less (greater) than one indicates a regressive (progressive) property tax inequity. The null hypothesis for the *t*-Statistic is $H_O: a_1 = 1$.

**Exhibit 7** | (continued)

Vertical Equity Analysis

In Panel C, if the quadratic term is insignificant then a linear model is satisfactory and a regressive (progressive) inequity is evident with a positive (negative) and significant intercept term. If the quadratic and intercept terms are positive and negative respectively then progressive inequity exists at an accelerating rate. If the quadratic and intercept terms are positive then offsetting progressivity exists. If the quadratic and intercept terms are negative then offsetting regressivity exists. If the quadratic and intercept terms are negative and positive respectively then regressive inequity exists at an accelerating rate. If neither the quadratic term nor the intercept terms are significant then no inequity exists. The null hypothesis for vertical equity is $a_0 = a_2 = 0$.
In Panel D, if $a_1$ is equal to one, no vertical tax inequity exists; however, a coefficient less (greater) than one indicates a progressive (regressive) property tax inequity. The null hypothesis for the t-Statistic is $H_O: a_1 = 1$
*Significant at the 0.05 level.

**Exhibit 8** | Models with Time Variables

| Variable | $AV/SP = a_0 + a_1SP + time$ (IAAO, 1978) | $\ln AV = a_0 + a_1 \ln SP + time$ (Cheng, 1974) | $AV = a_0 + a_1SP + a_2SP^2 + time$ (Bell, 1984) | $\ln SP = a_0 + a_1 \ln AV + time$ (Clapp, 1990) |
|---|---|---|---|---|
| $a_0$ | 0.8107 | −0.2192 | −69465 | 0.5126 |
| | (55.39) | (−2.11) | (−0.71) | (2.06) |
| $a_1$ | −2.04E-10 | −0.9969 | 0.7951 | 0.9822 |
| | (−0.02) | (−0.41) | (52.49) | (−0.98) |
| $a_2$ | — | — | −2.95E-9 | — |
| | — | — | (−6.68*) | — |
| 1999 | −0.0635 | −0.0567 | −101606 | 0.0721 |
| | (−3.16) | (−1.92) | (−0.78) | (1.12) |
| 2000 | −0.0987 | −0.1095 | 173961 | 0.1039 |
| | (−4.85) | (−3.66) | (1.32) | (1.60) |
| 2001 | −0.1193 | −0.1421 | −5837 | 0.2072 |
| | (−6.00) | (−4.86) | (−0.05) | (3.26) |
| 2002 | −0.1908 | −0.2526 | −269341 | 0.1356 |
| | (−9.57) | (−8.59) | (−2.08) | (2.12) |
| Adjusted $R^2$ | 0.0914 | 0.9512 | 0.9283 | 0.7597 |

*Note:* The Clapp model includes the instrumental variable computation for ln*AV*. Values in parentheses are *t*-Stats. The number of observations for all models is 946.
*Significant at 0.05 level.

As another analysis of vertical equity, the Spearman Rank nonparametric test was conducted. The null hypothesis assumes no correlation between the assessment ratio (*AV/SP*) and the sales price. As provided in Exhibit 9, the Spearman Rank Correlation coefficients for 1998 and 1999 are −0.266 and −0.213 respectively, while for 2000 through 2002 the coefficients are 0.097, 0.208 and 0.182 respectively. All the coefficients are significant at the 0.05 level. These findings are most similar to those provided by the Cheng model, but are also consistent with the IAAO model, namely that regressive vertical inequity is evident in the early years of the sample and that progressive vertical inequity is found but is more modest in the latter years. The Spearman Rank test finds no vertical inequity over the five years collectively.

To conclude, the results from the Bell model appear to be biased toward regressivity as expected; therefore, little weight is given to these results. The results from the IAAO, Cheng and nonparametric Spearman Rank test suggest regressive inequity is evident in the early years, with a shift towards progressive vertical inequity in the latter years. All of these former models however suffer from the feedback problem noted by Clapp (1990). After correcting for this problem, there is no significant evidence of vertical inequity. In addition, when all models were examined over the entire five years, only the Bell model finds evidence of vertical inequity. In summary, there is a hint of vertical inequity in some years (specifically regressive inequity); however, the overall analysis finds no significant vertical inequity in the Maricopa County apartment market.

## *Horizontal Equity Tests*

In this case of horizontal equity in multifamily housing, the distinction is not based on equity across a value range, but equity within a value range. In other words, properties with similar value should be treated uniformly or appraised at

**Exhibit 9** | Spearman Rank Nonparametric Test

| Year of Sale | Correlation Coefficient |
|---|---|
| 1998 | −0.266* |
| 1999 | −0.213* |
| 2000 | 0.097* |
| 2001 | 0.208* |
| 2002 | 0.182* |
| Aggregate | 0.031 |

*Note:*
* Significant at the 0.05 level.

the same percentage of market value (IAAO, 1978; and Allen and Dare, 2002). Relevant studies of horizontal equity in ad valorem property taxation include Berry and Bednarz (1975), Haurin (1988), Goolsby (1997) and Allen and Dare (2002) who examine single-family residential properties; DeCesare and Ruddock (1998) who examine apartments in Brazil; and Kowalski and Colwell (1986) who examine industrial properties. After reviewing the tests of horizontal equity in the prior studies, the methodologies used by Berry and Bednarz and Goolsby, as well as Allen and Dare (2002), are considered most applicable for the data in this study. Berry and Bednarz regress the (*AV/SP*) ratio on a vector of independent property characteristic and location variables. Goolsby adds a log linear specification to the model as follows:

$$\ln\left(\frac{AV}{SP}\right) = a_0 + a_i \ln(X) + e, \tag{5}$$

where $X$ is a vector of independent property characteristic and location variables and $a_i$ is a vector of coefficients to be estimated. For horizontal equity, the null hypothesis is $H_o : a_i = 0$, suggesting that for properties of similar value, no property characteristic or location variable has significant influence on the assessment ratio. A significant coefficient on an independent variable infers horizontal inequity. For instance, if a geographic binary variable is positive (negative) and significant, then the assessment ratio is higher (lower) for properties in that geographic area compared with similar valued properties in other geographic areas, thus leading to inequity for these properties.

Allen and Dare (2002) propose an alternative method for measuring horizontal inequity as follows:

$$|(AV/MV)_i - \overline{(AV/MV)}| = \beta(X) + e, \tag{6}$$

where in $\overline{(AV/MV)} = \dfrac{\sum (AV/MV)_i}{n}$, $X$ is a vector of independent property characteristic and location variables that may be related to inequity, and $\beta$ is a vector of coefficients to be estimated. The absolute value of the difference between the property's assessment ratio and the mean assessment ratio for the $n$ properties is a measure of an individual property's horizontal inequity. The coefficients in the regression provide insight into the determinants of horizontal inequity for the sample data. For example, a positive (negative) and significant coefficient on a continuous independent variable suggests that an increase in the variable leads to an increase (decrease) in the absolute value of horizontal inequity.

## Horizontal Equity Empirical Results

The two models identified above are applied to the five price level quintiles in Exhibit 6 so as to ensure that properties of similar value are analyzed as required by the definition of horizontal equity. The results of this analysis are shown in Exhibit 10.

Panel A in Exhibit 10 illustrates the results from the Berry and Bednarz (1975) and Goolsby (1997) models. Horizontal inequity is evidenced in price levels 1, 2,

**Exhibit 10** | Horizontal Equity Analysis

| Variable | Price L1 | Price L2 | Price L3 | Price L4 | Price L5 |
|---|---|---|---|---|---|
| Panel A: $(\ln(AV/SP)_i = \beta\ln(X) + e$ (Berry & Bednarz, 1975; Goolsby, 1997) | | | | | |
| Ln Units | 0.2301* | 0.3392* | 0.1895* | 0.0651 | −0.0356 |
| | (3.22) | (6.13) | (3.06) | (1.03) | (−0.87) |
| Ln Age | −0.1507* | 0.0208 | 0.0114 | 0.0118 | 0.0044 |
| | (−3.14) | (0.50) | (0.25) | (0.17) | (0.14) |
| Better Cond. | — | — | 0.4184 | 0.0231 | 0.0608 |
| | — | — | (1.62) | (0.11) | (1.05) |
| Worse Cond. | 0.0617 | −0.0645 | 0.0015 | 0.0068 | 0.2205 |
| | (1.63) | (−1.39) | (0.03) | (0.07) | (1.58) |
| Tempe | −0.0480 | 0.0332 | −0.0591 | 0.0618 | 0.0791 |
| | (−0.56) | (0.25) | (−0.74) | (0.74) | (1.07) |
| East Valley | 0.2402 | 0.2045* | −0.0601 | 0.0930 | 0.1173* |
| | (0.43) | (3.08) | (0.98) | (1.16) | (1.98) |
| NW Cities | −0.0071 | 0.0740 | −0.0179 | −0.0702 | −0.1220 |
| | (−0.09) | (0.97) | (−0.27) | (−0.88) | (−1.74) |
| Scottsdale/CC | 0.2186* | 0.0216 | 0.2310* | 0.2423 | 0.0653 |
| | (3.16) | (0.30) | (2.32) | (1.46) | (1.04) |
| Year 1999 | 0.0165 | −0.1867* | −0.0986 | 0.0635 | −0.0298 |
| | (0.29) | (−3.18) | (−1.58) | (0.75) | (−0.44) |
| Year 2000 | −0.1649* | −0.2223* | −0.0974 | 0.0234 | 0.0618 |
| | (−2.91) | (−3.52) | (−1.68) | (0.28) | (0.89) |
| Year 2001 | −0.2085* | −0.1869* | −0.1607* | 0.0567 | −0.0120 |
| | (−3.85) | (−2.86) | (−2.78) | (0.63) | (−0.18) |
| Year 2002 | −0.3480* | −0.3014* | −0.1718* | −0.0916 | −0.1371* |
| | (−5.23) | (−4.53) | (−2.90) | (−1.12) | (−2.00) |
| Constant | −0.2645 | −1.1694* | −0.9433* | −0.7442* | −0.2138 |
| | (−1.17) | (−5.82) | (−3.53) | (−2.01) | (−0.81) |
| Adj. $R^2$ | 0.342 | 0.284 | 0.088 | 0.012 | 0.047 |

**Exhibit 10** | (continued)

Horizontal Equity Analysis

| Variable | Price L1 | Price L2 | Price L3 | Price L4 | Price L5 |
|---|---|---|---|---|---|
| Panel B: $\lvert(AV/SP)_i - \overline{(AV/SP)}\rvert = \beta(X) + e$ (Allen & Dare, 2002) | | | | | |
| Units | 0.0054 | 0.0112* | 0.0002 | 0.0005 | 0.0001* |
| | (1.51) | (5.11) | (0.18) | (1.37) | (2.06) |
| Age | 0.0007 | 0.0003 | 0.0010 | −0.0015 | 0.0028* |
| | (0.86) | (0.30) | (1.51) | (−1.54) | (2.37) |
| Better Cond. | — | — | 0.1370 | 0.1153 | 0.0161 |
| | — | — | (1.27) | (1.67) | (0.71) |
| Worse Cond. | 0.0078 | −0.0536 | 0.0155 | 0.0511 | −0.0770 |
| | (0.39) | (−1.75) | (0.66) | (1.59) | (−1.30) |
| Tempe | −0.0277 | −0.0080 | 0.0174 | −0.0482 | −0.0100 |
| | (−0.61) | (−0.09) | (0.52) | (−1.67) | (0.32) |
| East Valley | −0.0177 | 0.0758 | −0.0188 | −0.0112 | 0.0246 |
| | (−0.62) | (1.72) | (−0.73) | (−0.40) | (0.98) |
| NWCities | 0.0127 | 0.0373 | −0.0012 | 0.0105 | −0.0096 |
| | (0.32) | (0.74) | (−0.04) | (0.38) | (−0.32) |
| Scottsdale/CC | −0.0230 | −0.0250 | 0.1003* | 0.0415 | −0.0601* |
| | (−0.63) | (−0.53) | (2.40) | (0.72) | (−2.29) |
| Year 1999 | −0.0253 | −0.1266* | −0.0704* | −0.0812* | −0.0467 |
| | (−0.85) | (−3.30) | (−2.68) | (−2.73) | (−1.65) |
| Year 2000 | −0.0618* | −0.0872* | −0.0513* | −0.0599* | −0.0621* |
| | (−2.06) | (−2.11) | (−2.11) | (−2.02) | (−2.13) |
| Year 2001 | −0.0286 | −0.0036 | −0.0616* | −0.0567 | −0.0476 |
| | (−0.99) | (−0.08) | (−2.53) | (−1.82) | (−1.72) |
| Year 2002 | 0.0128 | −0.0483 | −0.0607* | −0.0313 | −0.0196 |
| | (0.36) | (−1.10) | (−2.41) | (−1.10) | (−0.68) |
| Constant | 0.1171 | 0.0407 | 0.1396* | 0.1914* | 0.0983* |
| | (2.32) | (0.72) | (3.61) | (4.67) | (2.63) |
| Adj. $R^2$ | 0.023 | 0.156 | 0.056 | 0.061 | 0.058 |

*Note:* No observations were listed as "better condition" for price levels 1 and 2. The omitted variables include average condition, Central Phoenix, and 1998. Values in parentheses are *t*-Stats.
*Significant at the 0.05 level.

3 and 5. Specifically, the assessment ratio increases with the number of apartment units in price levels 1, 2 and 3. The ratio declines with building age (price level 1). In addition, the ratio is higher in the East Valley (price levels 2 and 5) and in Scottsdale/Cave Creek (price levels 1 and 3) compared with Phoenix. The volatility in the time variables is expected because assessed values are estimated each year.

According to the results provided by the Allen and Dare (2002) model shown in Panel B of Exhibit 10, the absolute value of inequity increased with the number of apartment units (price levels 2 and 5) and building age (price level 5). However, for Scottsdale/Cave Creek the absolute value of inequity increased in price level 3 but decreased in price level 5.

The building age variable is significant in only one price level in each model (price level 1 and 5). Also, East Valley was only significant in the one model (price level 2 panel A). Considering these relatively weak results, these variables do not warrant further scrutiny; however, the results for number of units and Scottsdale/Cave Creek variables do. The coefficients for the number of units variable are significant and positive in multiple price levels in both models. Units can be a proxy for property value; therefore, the results may be capturing evidence of vertical equity as noted in the prior analysis, or the assessor may be having difficulty maintaining stability in the assessment ratio for buildings of similar value, but of different sizes. The Scottsdale/Cave Creek variable is more interesting. Both of these communities are considered more affluent compared with other communities in Maricopa County. Panel A shows that in two of the price levels (1 and 3); the assessment ratio is higher than in Phoenix. In addition, the absolute value of inequity is increasing in price level 3 but decreasing in price level 5. The results suggest that horizontal inequity exists geographically, specifically that Scottsdale/Cave Creek apartments are being overassessed compared with other apartments with similar values.

## Equity Across Assessment Methods

The Maricopa County assessor's office uses a hedonic sales comparison approach to value apartments with less than thirteen units, but a hedonic income approach to value all other apartment properties. This provides a unique opportunity to examine equity across assessment methods. One direct method to examine equity across assessment methods is to determine whether the mean *AV/SP* ratios between the groups are equal. If the means are not statistically different, then the appraisal of both types of properties is uniform. If the population variances of the two groups are equal, then the pooled *t*-Test is appropriate; however, if the variances of the underlying population are not assumed equal, but the populations are normally distributed, the two-sample *t*-Test is appropriate.

The Tiao-Goldberger test for structural change in regression coefficients also provides a direct parametric examination of equity across assessment methods

(Tiao and Goldberger, 1962).[13] In this case, separate regressions using the same functional form are generated on the two property groups, and then the Tiao-Goldberger test determines whether structural stability of an individual coefficient exists between the two regressions. For example, assume that a model is specified as follows: $\ln AV = a_o + a_1 \ln SP$. The coefficient $a_1$ provides the statistical relationship between *AV* and *SP*. If the model were applied to two property groups, then the relationship between *AV* and *SP* (the $a_1$ coefficient) would be similar if equity exists across the groups. The Tiao-Goldberger test allows an explicit test of coefficient equality between regressions at a predetermined confidence level. The null hypothesis is that the coefficients are statistically similar.

The Mann-Whitney test provides a nonparametric test for equity across assessment methods (IAAO, 1978; and Gloudemans, 1999). The null hypothesis is that the two types of multifamily housing are assessed at equal percentages of market value. *AV/SP* ratios from the two groups are pooled and ranked from smallest to largest; then the test determines at a specified confidence level whether the ranks assigned to the two property groups are approximately equal with respect to averages. A *z*-value provides a direct test to determine whether the null hypothesis should be rejected at a specified confidence level. This test assumes that the ratios from each group are normally distributed and have equal variance.

Another frequently used measure that provides insight into equity across two groups, but is not a statistical test, is the Coefficient of Dispersion (COD) (Mehta and Giertz, 1996). The COD measures the degree of dispersion around the median *AV/SP* ratio by calculating the average percentage deviation from the median.[14] One negative aspect of the COD measure is a scale problem. For example, assume a median *AV/SP* ratio for group A is 100, while the ratio for group B is 50. Even

**Exhibit 11** | Means Test of *AV/SP*

| | Pooled *t*-Test | | Two-Sample *t*-Test | |
|---|---|---|---|---|
| Year | *t*-Stat. | *p*-Value | *t*-Stat. | *p*-Value |
| 1998 | −0.3789 | 0.7052 | 0.4669 | 0.6417 |
| 1999 | −0.7415 | 0.4593 | −0.7229 | 0.4724 |
| 2000 | 1.263 | 0.2084 | 1.224 | 0.2260 |
| 2001 | 2.903* | 0.0041 | 3.082* | 0.0026 |
| 2002 | 2.396* | 0.0175 | 2.609* | 0.0112 |
| Aggregate | 2.117* | 0.0345 | 2.186* | 0.0294 |

*Note:*
* Significant at the 0.05 level.

though the medians are very different for each group, the COD measure could be the same for both, depending on the distribution of the *AV/SP* ratios about the median. This scale problem complicates the interpretation of the COD measure (Bowman and Mikesell, 1978).

Exhibit 11 provides the results from the means test. Both tests find that the means are statistically different in years 2001 and 2002 as well as overall (aggregate). These results confirm that there is inequity across the two groups, leading to the supposition that as currently used one valuation method provides stronger results compared with the other.

The Tiao-Goldberger test for structural change between regressions provides an explicit test of equity across assessment methods but also provides for a simultaneous examination of vertical equity. For this test, the sample of transactions is divided between the two groups (small properties sales comparison approach and large properties income approach) and the IAAO, Cheng and Clapp

**Exhibit 12** | Tiao-Goldberger Test for Structural Change

| | $AV/SP = a_0 + a_1SP +$ *time* (IAAO, 1978) | | $\ln AV = a_o + a_1 \ln SP$ + *time* (Cheng, 1974) | | $\ln SP = a_o + a_1 \ln AV$ + *time* (Clapp, 1990) | |
|---|---|---|---|---|---|---|
| Variable | <13 Units Comp. Appr. | ≥13 Units Inc. Appr. | <13 Units Comp. Appr. | ≥13 Units Inc. Appr. | <13 Units Comp. Appr. | ≥13 Units Inc. Appr. |
| $a_0$ | 0.9561 | 0.8089 | 2.3760 | −0.0342 | 4.3342 | 0.3197 |
| | (16.62) | (47.34) | (2.20) | (−0.27) | (7.86) | (1.11) |
| $a_1$ | −3.81E-7 | −4.98E-10 | 0.7958 | 0.9832 | 0.6769 | 0.9973 |
| | (−2.58)* | −0.48 | (−2.41)* | (−1.89) | (−7.27)* | (−0.13) |
| 1999 | −0.0637 | −0.0641 | −0.0755 | −0.0502 | 0.0269 | 0.1084 |
| | (−1.70) | (−2.75) | (−1.32) | (−1.48) | (0.83) | (1.59) |
| 2000 | −0.1432 | −0.0856 | −0.1961 | −0.0814 | 0.0421 | 0.1343 |
| | (−3.76) | (−3.63) | (−3.37) | (−2.38) | (1.29) | (1.95) |
| 2001 | −0.1990 | −0.0890 | −0.2829 | −0.0855 | 0.0800 | 0.1387 |
| | (−5.68) | (−3.77) | (−5.29) | (−2.49) | (2.62) | (2.01) |
| 2002 | −0.2413 | −0.1730 | −0.3505 | −0.2159 | 0.1437 | 0.0339 |
| | (−6.30) | (−7.50) | (−5.98) | (−6.42) | (4.43) | (0.50) |
| Adj. $R^2$ | 0.2284 | 0.0689 | 0.3587 | 0.9438 | 0.5381 | 0.7702 |
| Tiao Statistic | 5.195** | | 4.103** | | 29.93** | |

*Notes:* Values in parentheses are *t*-Stats. Number of observations for Comparable Approach = 217. Number of observations for Income Approach = 729.

*Significant at the 0.05 level.

**Critical $F_{0.05}$ = 3.84.

**Exhibit 13** | Mann-Whitney Test

| Year | Z-Statistic | p-Value |
|---|---|---|
| 1998 | 1.277 | 0.2016 |
| 1999 | −0.493 | 0.6222 |
| 2000 | 1.129 | 0.2585 |
| 2001 | 3.453* | 0.0006 |
| 2002 | 2.609* | 0.0091 |
| Aggregate | 2.026* | 0.0427 |

*Significant at the 0.05 level.

regression models are applied to the two groups.[15] Binary time variables are included to control for the intertemporal change in assessed values and sales price. The coefficient on the $a_1$ variable measures vertical equity in the respective samples. The Tiao-Goldberger test examines the structural stability of individual coefficients between regressions. Specifically, the null hypothesis is that the $a_1$ coefficients are equal between the two samples (large and small properties), testing equity across assessment methods. Exhibit 12 illustrates the results of this analysis. The $a_1$ regression coefficient on all models shows significant regressive vertical inequity for the small property sample. For the large properties, the $a_1$ coefficients are not significant, suggesting no vertical inequity. The Tiao-Goldberger statistic is significant at the 0.05 level for each model, indicating that the $a_1$ coefficient is not uniform or stable between the regressions of the two samples. Therefore, inequity exists across the two assessment methods.

Exhibit 13 provides the results from the Mann-Whitney nonparametric test. The null hypothesis assumes that large and small apartment complexes are appraised

**Exhibit 14** | Coefficient of Dispersion Measure by Year and Valuation Method

| Year | <13 Units Comp. Appr. | >12 Units Inc. Appr. | All Properties |
|---|---|---|---|
| 1998 | 17.26 | 25.23 | 23.50 |
| 1999 | 18.34 | 16.91 | 17.26 |
| 2000 | 19.59 | 17.35 | 17.78 |
| 2001 | 21.36 | 19.23 | 20.54 |
| 2002 | 21.13 | 19.95 | 20.79 |
| Aggregate | 22.72 | 20.51 | 21.09 |

at the same percentage of market value. The null is rejected at the 0.05 level for years 2001 and 2002 as well as the aggregate. This result compares with the means tests in Exhibit 11 and solidifies the robustness of the results. The COD by year and valuation method is provided in Exhibit 14. Although the COD measure is not a direct statistical test of equity, it provides insight into the uniformity of the *AV/SP* ratio intertemporally and between apartment groups. One obvious observation from Exhibit 14 is that the reported COD is larger in the small property sample in each year except 1998. This suggests less uniformity intertemporally when the sales comparison approach is used for small properties compared with the income approach that is used for large properties. Consistent with the results found by the Tiao-Goldberger test, this finding provides added support for the conclusion that inequity exists between the samples of small and large properties, which are distinguished by different valuation methods.

## Conclusion and Policy Implications

This article has examined property tax uniformity in the multifamily housing market in Phoenix, Arizona (Maricopa County). It differs from earlier studies because it considers multiple years of data and examines both vertical and horizontal equity issues as well as equity across assessment methods. It also uses both parametric and nonparametric measures of uniformity and applies the Tiao-Goldberger test to examine equity across assessment methods. The results offer mixed evidence of vertical and horizontal inequity in multifamily housing in Maricopa County from 1998 to 2002. With some models there is modest evidence of vertical inequity in the early years of the study. However, extending the analysis to account for the errors in variables problem raised by Clapp (1990), no significant vertical inequity is found in any of the individual years. In addition, when all years are examined collectively, no significant vertical inequity is found; therefore, with respect to vertical inequity for the period 1998 to 2002, the assessor's outcomes are reasonable.

In the examination of horizontal equity within the various price levels, only limited evidence of horizontal inequity is found. In the cases where horizontal inequity is found, it is correlated with the number of units in a complex or geographic location. The results suggest that apartment size and some geographic locations are more difficult for the assessor to value uniformly. The issues appear of a nature that the assessor could make the needed adjustments to improve the outcome.

With respect to the concerns around assessment methods, the analysis reveals that controlling for vertical equity using the sales comparison hedonic model is more difficult than using the income hedonic model. In addition, the parametric and nonparametric tests indicate that there is inequity across the two assessment methods.

In the context of the overall analysis, some limited policy conclusions are offered. As noted in the introduction, the property tax enjoys very little "good press." The complaints may or may not be valid, but for apartment complexes within Maricopa County there is little reason to complain about the uniformity of the property tax. This is an important finding. Assessors are not immune to substantial external and internal political pressure. Property tax assessors are elected officials who often have to manage their offices under significant budget constraints, and as noted, the challenge to value property accurately is substantial. The evidence suggests that in spite of the external influences and the technical complexities of appraisal, the assessor's office for the most part is meeting the challenge to value multifamily property uniformly. This finding is especially noteworthy with respect to the issue of vertical equity in multifamily housing. It is clear that large property owners have greater resources to investigate property tax inequities and appeal property taxes, and that they are more politically connected. This implies that assessing officers may thus be motivated to implement systems that mitigate negative responses from large property owners; however, no significant evidence of differential treatment of properties over different price ranges was found. Naturally this is just one study of a limited geographic area, but it offers hope for policy makers concerned about the ability to achieve uniform property tax outcomes within a single class of property.

## Endnotes

1. Allen (2003) notes that it is difficult to generalize from his study. The limit on generalizations from such studies is an underlying characteristic of any analysis of the property tax and real estate market. The present study when coupled with the Allen study, and anticipated other studies, may begin to help researchers and policy makers understand the implications of the property tax on the multifamily housing market.
2. There is a long series of policies intended to promote the private ownership of housing, away from rental housing (Segal and Sullivan, 1998).
3. CoStar Group, Inc. (formerly Comps InfoSytems, Inc.) investigates and compiles real estate transaction data in many cities in the U.S., including Phoenix, Arizona. CoStar verifies the accuracy and legal nature of the reported sales. In doing so they do not report non-arms length sales or sales of personal property. Summaries of the transactions are provided to interested parties on a subscription basis. We thank Craig Farrington for his generous assistance with the data.
4. Revaluation of property for property tax purposes on a cycle of every two years is an aggressive and commendable schedule.
5. The use of hedonic models to estimate the value of multi-family housing is found in Frew and Jud (2003).
6. Estimating property values based on rental income is a common practice in countries with colonial ties to the British Empire. Referred to as the rating system, it is still used in Hong Kong and Singapore. The tax base is the rental income, and the tax rates are much higher than the tax rates on capital value (Bahl and Linn, 1995).
7. A similar approach has been reported for Stockholm, Sweden (Janssen and Söderberg, 2000).
8. If the assessment ratio is not uniform across properties, then the deliberate under assessment makes it difficult for property owners to seek relief, since an adjustment to property tax is only allowed if the assessed value exceeds market value.

[9] Both standards are based on examining the relationship between the assessed value (AV) of a property and the market value of a property. Analysts generally substitute the sales price (SP) of a property as a proxy for market value (Sirmans, Diskin and Friday, 1995).

[10] Within the Paglin and Fogarty (1972) model, vertical equity in property taxation exists if the intercept term $a_0$ is zero; however, if the intercept is greater (less) than zero, regressive (progressive) vertical tax inequity exists.

[11] The cost approach is not used in Maricopa County to assess apartment buildings, therefore, the Birch, Sunderman and Hamilton (1990) and Sunderman, Birch, Cannaday and Hamilton (1990) models are not applicable in this analysis.

[12] For large samples exceeding fifty, the critical value for the Spearman Rank Test is approximated as $|r_s| \geq 1.96/\sqrt{n-1}$.

[13] Tiao and Goldberger (1962) provides a test of individual parameter equality between regressions. Some of the more recent applications of this test include Slade (2001), Wolverton, Hardin and Cheng (1999), Allen, Springer and Waller (1995) and Michaels and Smith (1990). The *F*-Statistic is as follows:

$$F^{TG} = \frac{\sum_{j=1}^{L} \frac{(\hat{b}_{ij} - \bar{b}_i)^2}{P_{ij}}}{\sum_{j=1}^{L} SSR_j} \times \frac{\sum_{j=1}^{L}(T_j - K_j)}{(L-1)}, \text{ where } \bar{b} = \frac{\sum_{j=1}^{L} \frac{\hat{b}_{ij}}{P_{ij}}}{\sum_{j=1}^{L} \frac{1}{P_{ij}}},$$

and where:

$L$ is the number of models;
$\hat{b}_{ji}$ is the OLS estimates of the *i*th parameter in the *j*th independent model;
$P_{ij}$ is the diagonal element for the *i*th parameter of $(X'X)_j^{-1}$;
$SSR_j$ is the sum of squared residuals for the jth model;
$T_j$ is the number of observations used to estimate the *j*th model; and
$K_j$ is the number of parameters in the *j*th model.

This statistic is distributed as a central *F* distribution.

[14] The COD is found using the following formula:

$$\text{COD} = \frac{\frac{1}{N}\sum_{i=1}^{N}\left|R_i - R^{med}\right|}{R^{med}} x100,$$

where $N$ is the number of properties, $R_i$ is the assessed value-to-price ratio (*AV/SP*) for property *i* and $R^{med}$ is the median of these ratios.

[15] Because of the functional form of the Bell (1984) model, a singularity problem occurs in the calculation of the Tiao-Goldberger statistic; therefore, the Tiao-statistic is not calculated for this model.

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