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# The Shape of Australian Real Estate Return Distributions and Comparisons to the United States[†]

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***Abstract*. Investment risk models with infinite variance provide a better description of distributions of individual property returns in the Property Council of Australia database from 1985 to 1996 than normally distributed risk models. The shape of the distribution of Australian property returns is virtually indistinguishable from the shape of United States property returns in the NCREIF Property Index for the years 1980 to 1992. Australian real estate investment risk is heteroscedastic, like its U.S. counterpart, but *the characteristic exponent of the investment risk function is constant across time and property type*. It follows that portfolio management and asset diversification techniques that rely upon finite-variance statistics are as ineffectual for the Australian real estate market as they have been found to be for the United States.**

## Introduction

As institutional investors seek real estate investment opportunities among an expanding group of locations both foreign and domestic, it is worthwhile to gain an understanding of the behavioral characteristics of the assets that might be purchased individually or in portfolios. If there are characteristic performance differences among assets in different countries, these differences should be noted in formulating portfolio strategies. If there are similarities among investment characteristics, investors could realize efficiencies by formulating portfolio strategy in the home market and appropriately extending it to foreign soil. This study is a comparative examination of institutional-grade commercial real estate in Australia and the United States.

In an empirical study of disaggregated NCREIF data in the United States, Young and Graff (1995) found that cross-sectional annual returns were not normally distributed during any year between 1978 and 1992. In addition, they found that both the skewness and magnitude of real estate risk changed over time. This study extends that work to the commercial equity real estate returns contained within the database of the Property Council of Australia and compares its results with the earlier results from the NCREIF Property Index.

As in the Young and Graff study, we use the work of McCulloch (1986) to empirically test the shape of real estate return distributions. McCulloch developed a set of simple asymptotically normal estimators for stable Paretian distribution parameters that are

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applicable to both symmetric and asymmetric stable distributions, and that include straightforward formulas for the standard errors of the parameter estimates.

## Stable Distributions

Normal distributions are examples of stable distributions, and are the only examples with finite variance. Other examples of stable distributions are the well-known Cauchy distributions. The normal and Cauchy distributions are the only stable distributions for which probability densities can be expressed in closed form in terms of elementary mathematical functions.

Although most stable distributions and their probability densities cannot be described in closed mathematical form, their characteristic functions can be expressed in closed form. In particular, the log characteristic functions of stable distributions can be written as follows:

$$\psi(t)=\begin{cases} i\delta t-|ct|^{\alpha}\left[1-i\beta\operatorname{sgn}(t)\tan(\pi\alpha/2)\right], & \text{for}\,\alpha\neq 1 \\ i\delta t-|ct|\left[1+i\beta(2/\pi)\operatorname{sgn}(t)\log|t|\right], & \text{for}\,\alpha=1. \end{cases} \tag{1}$$

The four parameters $\alpha$, $\beta$, $c$, and $\delta$ in equation (1) completely characterize the distribution and satisfy the following constraints:

- The **characteristic exponent** $\alpha$ lies in the half-open interval (0,2] and measures the rate at which the tails of the density function decline to zero. The larger the value of the characteristic exponent $\alpha$, the faster the tails shrink toward zero. When $\alpha=2.0$, the distribution is normal.

  A stable distribution with characteristic exponent $\alpha$ has moments of order $<\alpha$, and does not have moments of order $>\alpha$. While the means (first moments) of stable distributions with characteristic exponents $\alpha>1.0$ do exist, variances (second moments) do not exist—i.e., are infinite—for those distributions with characteristic exponents $\alpha<2.0$.
- The **skewness parameter** $\beta$ lies in the closed interval $[-1,1]$, and is a measure of the asymmetry of the distribution.[1] The distribution is symmetric when $\beta=0$.
- The **scale parameter** $c$ lies in the open interval $(0, \infty)$, and is a measure of the spread of the distribution. If $\alpha=2.0$, the scale parameter $c$ is directly proportional to the standard deviation: $c=\sigma/\sqrt{2}$. The scale parameter $c$ is finite for all stable distributions, despite the fact that the standard deviation is infinite for all $\alpha<2.0$. Thus, the scale parameter $c$ can be regarded as a generalization of the standard deviation.
- The **location parameter** $\delta$ may be any real number, and is a rough measure of the midpoint of the distribution. A change in $\delta$ simply shifts the graph of the distribution left or right, hence the term "location."

Under the assumptions that $f$ and $g$ are two random variables, that $f$ and $g$ both have stable distributions, and that the two stable distributions have the same characteristic exponent $\alpha_0$ and the same skewness $\beta_0$, it follows that a simple relation exists between the scale parameter of $f+g$ and the scale parameters of $f$ and $g$, and similarly between the location parameter of $f+g$ and the location parameters of $f$ and $g$:

$$c_{f+g}^{\alpha_0} = c_f^{\alpha_0} + c_g^{\alpha_0} , \tag{2}$$

$$\delta_{f+g} = \delta_f + \delta_g . \tag{3}$$

Equation (2) generalizes the relation between the standard deviation of the sum of two random variables and the individual standard deviations of the two random variables. Equation (3) extends the general principle that the expectation of the sum of two random variables is the sum of the individual expectations.

## Australian Real Estate Performance Data

The Property Council of Australia's Investment Performance Index provides investment performance information on Australian commercial property. As of December 1996, the index comprised 556 office, retail and industrial properties, valued at $A33 billion. The market coverage by area is around 35% of Australia's CBD office stock and in excess of 34% of the total retail stock.

The indices are compiled from information supplied by more than seventy of Australia's largest superannuation funds (pension funds), life insurance companies, and listed property trusts (equivalent to the U.S. REITs).

The Investment Performance Index extends back to December 1984. Until June 1995, the index was calculated on a semiannual basis. In response to the demand for a more frequent information service, the Property Council now produces a quarterly index.

The index was developed with the assistance of the Frank Russell Company and is directly comparable to the NCREIF Index with a slight modification in the denominator of the return formula.[2] Investment returns are calculated using realized income and market values as determined by a registered valuer. The majority of valuations are conducted on an annual basis, although around 25% of the index by market capitalization is revalued on a quarterly basis. Most Australian funds revalue property assets in either June or December. Exhibit 1 presents descriptive characteristics for the Australian Composite Property Index.

The data collection and verification procedures of the Property Council of Australia differ from those of NCREIF, and a few words of explanation are appropriate. Property Council of Australia collects a wide range of information on each property and has put in place an extensive range of validation and consistency checks to ensure the accuracy of results. At each update, checks of property operating and investment performance that indicate outliers trigger a call for verification. If the information is correct, then an explanation is sought from the owner or, in some cases the valuer, for the extreme performance.

Once the information has been verified and the indices calculated, the Index Technical Committee (ITC) meets to sign off on the nineteen official indices. The ITC, comprised of specialists in index construction, asset consulting, property portfolio management and valuation, plays a vital role in ensuring the integrity and quality of the Property Council of Australia index.

At every update, the ITC receives a set of papers for each of the nineteen indices. The papers include three key tables:

**Exhibit 1**
**Property Council of Australia Composite Property Database Sample Characteristics**

| Year | No. of Properties | Total Value of Index in $A | Average Value in $A |
|---|---|---|---|
| 1984 | 193 | 4,856,613,601 | 25,163,801 |
| 1985 | 236 | 6,795,192,496 | 28,793,189 |
| 1986 | 312 | 9,473,005,639 | 30,362,198 |
| 1987 | 346 | 13,178,090,931 | 38,086,968 |
| 1988 | 408 | 20,208,483,668 | 49,530,597 |
| 1989 | 481 | 27,013,041,087 | 56,160,169 |
| 1990 | 537 | 28,987,375,052 | 53,980,214 |
| 1991 | 555 | 27,907,770,437 | 50,284,271 |
| 1992 | 568 | 26,247,960,998 | 46,211,199 |
| 1993 | 548 | 23,906,768,030 | 43,625,489 |
| 1994 | 562 | 28,746,051,355 | 51,149,558 |
| 1995 | 583 | 32,739,843,806 | 56,157,537 |
| 1996 | 556 | 33,226,777,308 | 59,760,391 |

*Note*: For this study only properties having a full year of performance data were included. Thus the number of properties shown in Exhibit 4 is fewer than shown in Exhibit 1.

- the index results;
- the index sample size—by number of properties, capitalization, total net rentable area, percent of the total market area represented by the index, and average size of each property in the index; and
- the capital returns of each property in the index over the quarter, semiannual and one-year period.

In addition, the ITC is given a summary sheet that provides an explanation for each property that has recorded an extreme movement in capital return while masking identification of the individual property. An extreme movement is defined as a capital return 2.5 standard deviations from the mean or a wide variation in performance between periods. The ITC is asked to review each explanation and to make a decision as to whether or not the property should be removed from the index.

The decision to remove a property from the index is not taken lightly. The ITC considers the magnitude of the movement, the impact on the overall index, and the validity of the explanation. Since its inception, only six properties have been removed from the index database.

To permit comparisons of distributions by property type, the most commonly reported property types have been included: office, retail, and industrial.[3]

Before beginning the data analysis, each annual sample return $r_t$ in the Property Council of Australia database has been replaced with its continuously compounded equivalent, $ln(1+r_t)$. This follows the long-standing preference for continuously compounded returns in studies of the shape of investment risk.

## A Simple Real Estate Return Model

Our real estate market model assumes that differences in reported returns aggregated by property type account for all the differences in expected individual property returns. More precisely, we assume that the observed annual total return on each property $p$ during the calendar year $t$ is of the following form:

$$r_t(p)=\mu_t(h(p))+\varepsilon_t(p), \tag{4}$$

where $h(\ )$is the property type, $\mu_t(\ )$is the expected total return during year $t$ as a function of property type, and $\varepsilon_t(p)$ is a stable (possibly infinite-variance) random variable. In addition, we assume that, for each $t \geq 1985$, the $\varepsilon_t(\ )$ are independent identically distributed random variables with characteristic exponent $\alpha_t > 1.0$ and zero mean, and that $\varepsilon_{t_1}(p_i)$ and $\varepsilon_{t_2}(p_j)$ are independent for all $t_1 \neq t_2$ and all $i$ and $j$.

Under these assumptions, the random variable $\varepsilon_t(p)$ corresponds to the asset-specific investment risk of property $p$ during period $t$, while the systematic and market-sector real estate risk is described by the function $\mu_t(h(\ ))$.

This model implies that two properties of the same type have: (1) the same expected return, and (2) the same investment risk distribution. At first glance this seems quite different from stock market return/risk analysis. Studies of stock market return series suggest that the common stocks of two corporations engaged in the same general economic activities can display very different return/risk profiles. However, the two corporations are also likely to have very distinct capital structures, which in turn implies that the two common stock issues represent very different economic slices of the same kind of economic pie. Because corporate capital structures can be very complex, it is impractical to remove the effects of leverage from common stock returns on a routine basis. Nonetheless, it is reasonable to expect that deleveraged ex ante common stock returns and investment risk functions would be virtually identical for corporations engaged in the same types of business activities.

Standard operating procedure in empirical real estate research has been to assume the normal probability distribution of asset-specific risk as an act of faith, and then apply statistical techniques to obtain descriptions of systematic and market-sector risk. By contrast, our tests will examine asset-specific investment risk with the objectives of (1) confirming or rejecting real-world applicability of the model, and (2) obtaining additional information about the likely shape of real estate investment risk. In particular, the focus of this investigation is the test of a model for the distributional form of $\varepsilon_t(\ )$. We do not propose a time-series model for $\mu_t(h(p))$ (i.e., no model for systematic or market-sector risk).[4]

## Tests and Results

Exhibit 2 shows the distribution of continuously compounded annual total returns for the years 1985–1996 in the aggregate. Superimposed upon the sample histogram is the normal density. The sample density function is more peaked near the mean than the corresponding normal density, has weaker shoulders and fatter tails (i.e., is leptokurtic), and is negatively skewed. This distinction is more clear in Exhibit 3, which graphs the difference between the sample density and the normal density. Although not shown, the distribution of each property type exhibits the same leptokurtic pattern.

**Exhibit 2**
**Distribution of Log Annual Total Return Residuals**
**Property Council of Australia Database**
**All Properties, 1985 to 1996**

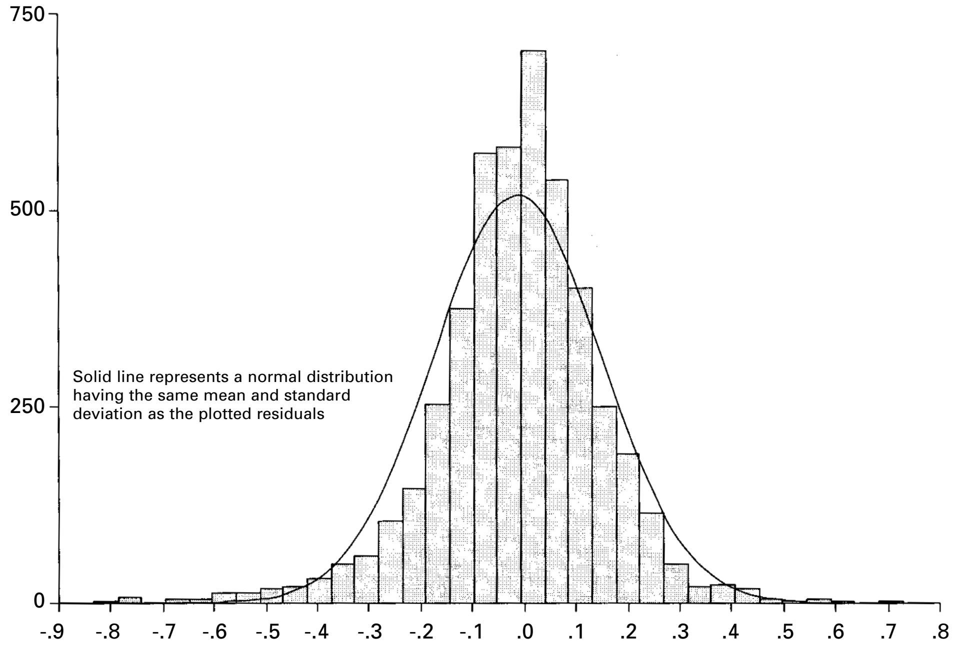
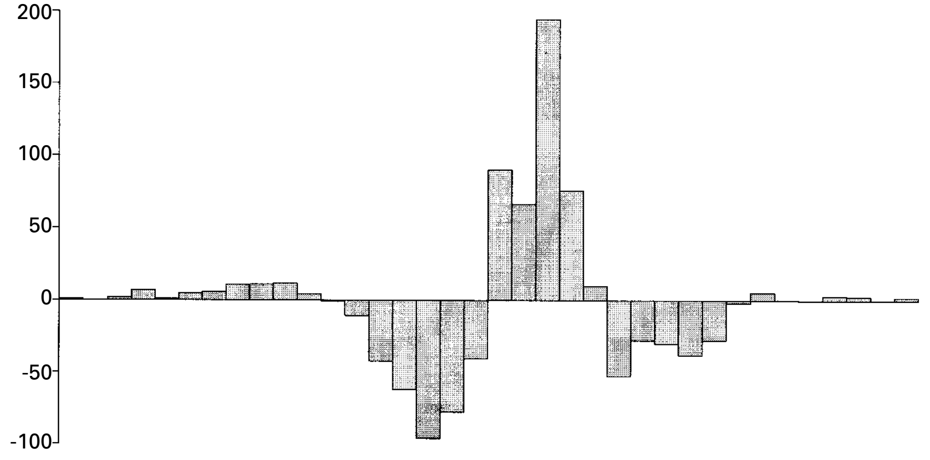



**Exhibit 3**
**Difference in Frequency between Log Annual Total Return Residuals**
**and Corresponding Normal Distribution**
**Property Council of Australia Database**
**All Properties, 1985 to 1996**



Before fitting stable distributions to the sample data, we correct for possible extraneous data dispersion due to changing expected return by reducing each annual return by the corresponding sample mean for that calendar year and property type. The means are shown in Exhibit 4 for purposes of completeness, but are not needed in the subsequent discussion.

We use McCulloch's methodology to fit a stable distribution to each set of residuals decomposed by calendar year and property type. To test whether the parameters varied during the sample period, we also aggregate residuals across calendar years and property types, respectively, and estimate stable parameters for the aggregated sets. These results are tabulated in Exhibit 4 and are displayed graphically together with one and two standard deviation error bands in Exhibits 5, 6, 7, and 8 for the parameters $\alpha$, $\beta$, and $c$ ($\delta$ is irrelevant because the location parameter is an estimator for the mean, and we adjust for the effect of varying means).

Among the characteristic exponents $\alpha_t$ estimated by calendar year and property type, 53% (19 of 36) are distinct statistically from 2.0—the characteristic exponent of the normal distribution—with 95% confidence, and 31% (11 of 36) are distinct from 2.0 with 99% confidence. Among residuals aggregated across property type (the first panel of Exhibit 4), all but two of the twelve sample characteristic exponents $\alpha_t$ are distinct from 2.0 with 95% confidence.

Exhibit 5 displays the sample characteristic exponents $\alpha_t$ of the aggregated residuals. Despite the year-by-year volatility in sample $\alpha_t$ values, after allowing for the width of the error brackets it appears possible in every case that $\alpha_t$ could be time-invariant. Exhibit 6 suggests that $\alpha_t$ is also constant across property type for Australian data, as well as for U.S. NCREIF data.

Exhibit 4 shows that skewness was statistically greater than zero in 1985 and 1986, and statistically less than zero in 1995 and 1996. Thus, the annual variation in skewness during the sample period is larger than can be accounted for by sample noise. Exhibit 7 shows the skewness results together with error brackets in graphical form.

Exhibit 8 shows clearly that the scale parameter $c$ is not time-invariant in the aggregate or by property type. Since $c$ is the stable infinite-variance measure of risk, this means that asset-specific risk is heteroscedastic.

Although Exhibits 4 through 6 suggest that $\alpha_t$ was time-invariant during the test period, we test this proposition rigorously in the next section.

## Test for Time-Invariant $\alpha$

Because all twelve sample estimators for $\alpha_t$ are asymptotically normal, the proposition that the true values are all equal (i.e., that $\alpha_t$ is time-invariant) can be tested by using the fact that, when it is true,

$$\sum w_i (x_i - \overline{x})^2$$

is distributed as $\chi^2$ on eleven degrees of freedom, where each weight $w_i$ is given by the reciprocal of the asymptotic variance of $x_i$, and $\overline{x}$ is the weighted average of $x_i$ (weighted by the $w_i$). [5]

The last column of Exhibit 9 shows the year-by-year $\chi^2$ components for the sample characteristic exponents with the total for the twelve-year period at the bottom of the

**Exhibit 4**
**Stable Distribution Parameters for the Property Council of Australia Database Log Annual Total Return Residuals, Mean Returns and Number of Properties**

| Year or Period | $\alpha$ | $\beta$ | $c$ | Mean Return | No. of Properties |
|---|---|---|---|---|---|
| All Properties Combined: | | | | | |
| 1996 | 1.575** | -0.495* | 0.064 | 0.067 | 460 |
| 1995 | 1.353** | -0.317* | 0.047 | 0.080 | 468 |
| 1994 | 1.446** | -0.116 | 0.059 | 0.115 | 436 |
| 1993 | 1.652* | -0.848 | 0.091 | -0.008 | 451 |
| 1992 | 1.660* | -0.706 | 0.105 | -0.056 | 455 |
| 1991 | 1.741 | -1.000 | 0.104 | -0.104 | 455 |
| 1990 | 1.591** | -0.433 | 0.074 | 0.074 | 423 |
| 1989 | 1.668* | 0.537 | 0.059 | 0.059 | 382 |
| 1988 | 2.000 | 1.000 | 0.107 | 0.107 | 324 |
| 1987 | 1.641* | 0.927 | 0.081 | 0.081 | 279 |
| 1986 | 1.339** | 0.752** | 0.051 | 0.051 | 258 |
| 1985 | 1.515* | 0.951* | 0.058 | 0.058 | 202 |
| 1985–96 | 1.588** | -0.242** | 0.089 | 0.072 | 4,593 |
| Office Properties: | | | | | |
| 1996 | 1.461** | -0.291 | 0.069 | 0.039 | 234 |
| 1995 | 1.333** | -0.434* | 0.060 | 0.045 | 241 |
| 1994 | 1.324** | -0.297 | 0.066 | 0.085 | 244 |
| 1993 | 2.000 | -1.000 | 0.127 | -0.084 | 263 |
| 1992 | 2.000 | -1.000 | 0.146 | -0.142 | 273 |
| 1991 | 2.000 | -1.000 | 0.142 | -0.159 | 273 |
| 1990 | 1.595* | -0.812 | 0.084 | 0.013 | 254 |
| 1989 | 1.567** | 0.136 | 0.060 | 0.146 | 228 |
| 1988 | 2.000 | 1.000 | 0.112 | 0.266 | 187 |
| 1987 | 1.714 | 1.000 | 0.094 | 0.246 | 158 |
| 1986 | 1.206** | 0.752** | 0.050 | 0.163 | 136 |
| 1985 | 1.553 | 1.000 | 0.072 | 0.153 | 100 |
| 1985–96 | 1.649** | -0.279* | 0.099 | 0.038 | 2,591 |
| Retail Properties: | | | | | |
| 1996 | 1.552* | -0.050 | 0.044 | 0.085 | 117 |
| 1995 | 1.407** | -0.127 | 0.034 | 0.094 | 117 |
| 1994 | 1.602 | 0.194 | 0.038 | 0.145 | 100 |
| 1993 | 2.000 | -1.000 | 0.061 | 0.139 | 98 |
| 1992 | 1.307** | 0.140 | 0.055 | 0.113 | 98 |
| 1991 | 1.380* | -0.815 | 0.058 | 0.046 | 100 |
| 1990 | 1.494* | 0.283 | 0.051 | 0.102 | 94 |
| 1989 | 1.843 | 1.000 | 0.054 | 0.156 | 82 |
| 1988 | 2.000 | 1.000 | 0.096 | 0.219 | 66 |
| 1987 | 1.414* | 0.214 | 0.063 | 0.191 | 56 |
| 1986 | 1.436* | 0.118 | 0.057 | 0.153 | 52 |
| 1985 | 1.548 | 0.986 | 0.037 | 0.160 | 43 |
| 1985–96 | 1.554** | -0.186 | 0.060 | 0.119 | 1,023 |

**Exhibit 4** (continued)

| Year or Period | $\alpha$ | $\beta$ | $c$ | Mean Return | No. of Properties |
|---|---|---|---|---|---|
| Industrial Properties: | | | | | |
| 1996 | 1.382 ** | -0.231 | 0.047 | 0.109 | 109 |
| 1995 | 1.238 ** | 0.347 | 0.028 | 0.142 | 110 |
| 1994 | 1.333 ** | 0.038 | 0.052 | 0.162 | 92 |
| 1993 | 1.872 | -1.000 | 0.072 | 0.053 | 90 |
| 1992 | 1.738 | -1.000 | 0.094 | 0.028 | 84 |
| 1991 | 1.888 | -1.000 | 0.099 | -0.103 | 82 |
| 1990 | 1.801 | -1.000 | 0.068 | 0.019 | 75 |
| 1989 | 2.000 | 1.000 | 0.077 | 0.189 | 72 |
| 1988 | 2.000 | 1.000 | 0.097 | 0.213 | 71 |
| 1987 | 1.437 * | 0.366 | 0.069 | 0.203 | 65 |
| 1986 | 1.219 ** | 0.675 | 0.039 | 0.156 | 70 |
| 1985 | 1.456 * | -0.137 | 0.045 | 0.116 | 59 |
| 1985–96 | 1.635 ** | 0.037 | 0.069 | 0.104 | 979 |

Statistically significant confidence of non-normality ($\alpha \neq 2.0$) or skewness ($\beta \neq 0$): **=99% confidence; *=95% confidence
$\alpha$ is the characteristic exponent, and only equals 2.0 for the normal distribution; $\beta$ is the skewness parameter in the range $-1.0$ to $+1.0$; $c$ is the (positive) scale parameter, which measures the spread of the distribution about $\delta$.

column. The total is 15.82, which is less than the 0.05 significance level of 19.68 for eleven degrees of freedom, and smaller than the 0.10 significance level of 17.28. Thus the twelve year-by-year sample values of $\alpha$ for all properties are consistent with the hypothesis that all twelve samples represent the same true value for $\alpha$. The individual property-type values for $\chi^2$ in the last row of the table imply that the year-by-year retail and industrial property sample values of $\alpha$ are consistent with a single true value for $\alpha$ for each property type. However, the office property sample $\chi^2$ value exceeds the critical value. Thus we reject a single true $\alpha$ for office property at the 0.05 significance level, while accepting it for retail and industrial property.

The $\chi^2$ test can also be used to test whether, for each year during the sample period, the individual property type $\alpha$ estimates are consistent with the hypothesis that the true values of $\alpha$ for the various property types are identical. By computing the weighted average of sample property-type $\alpha$'s for each year, the analog of the $\chi^2$ test described above can be applied to test whether the true values of $\alpha$ for the three property types in a year are identical. This time the critical $\chi^2$ value is 5.99, i.e., the 0.05 significance level of the $\chi^2$ function for two degrees of freedom.

The resulting twelve $\chi^2$ values are shown in the next-to-last column of Exhibit 9 (the corresponding $\chi^2$ for the data aggregated across the sample period is shown at the bottom of the column). In every case, the observed sample value is not only below the 0.05 significance level, but is also below the 0.10 significance level of 4.61 in all but the year 1992. Thus the observed values are consistent with the conclusion that all thirteen hypotheses are correct.

**Exhibit 5**
**Characteristic Exponent Alpha of Distributions of Log Annual Total Return Residuals Property Council of Australia Database, All Properties**

(bands indicate plus and minus one and two standard deviations)

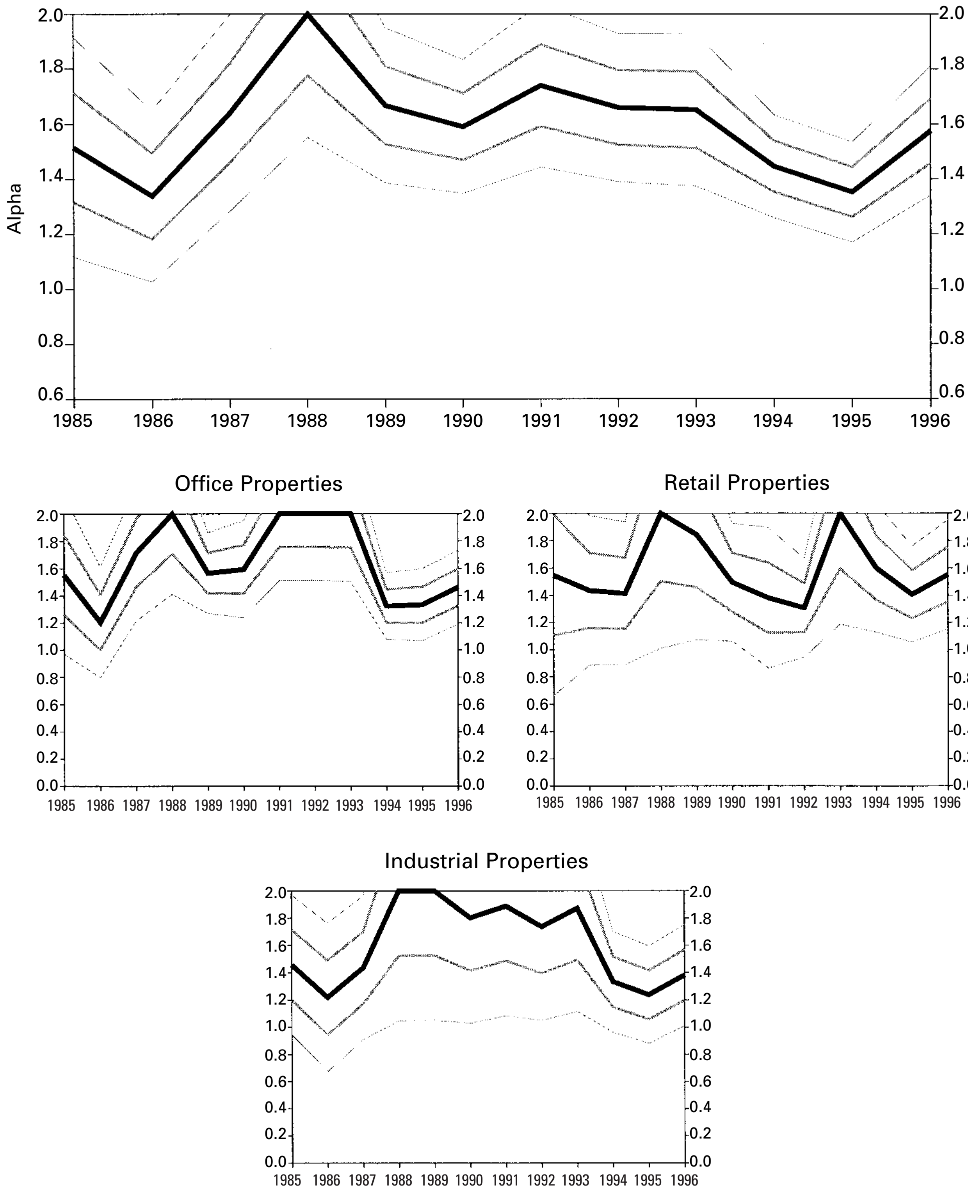
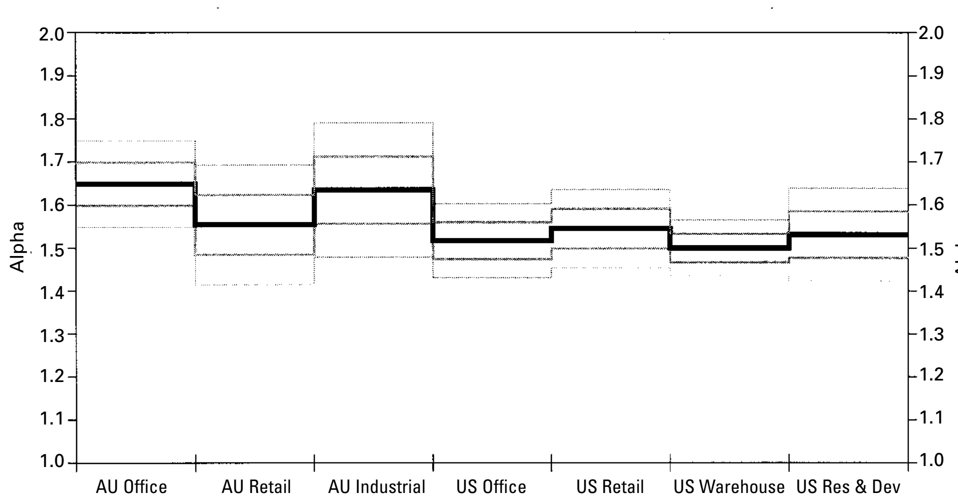
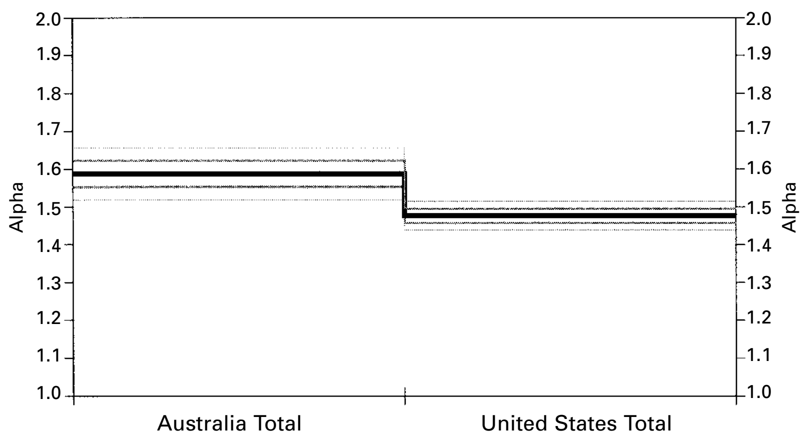


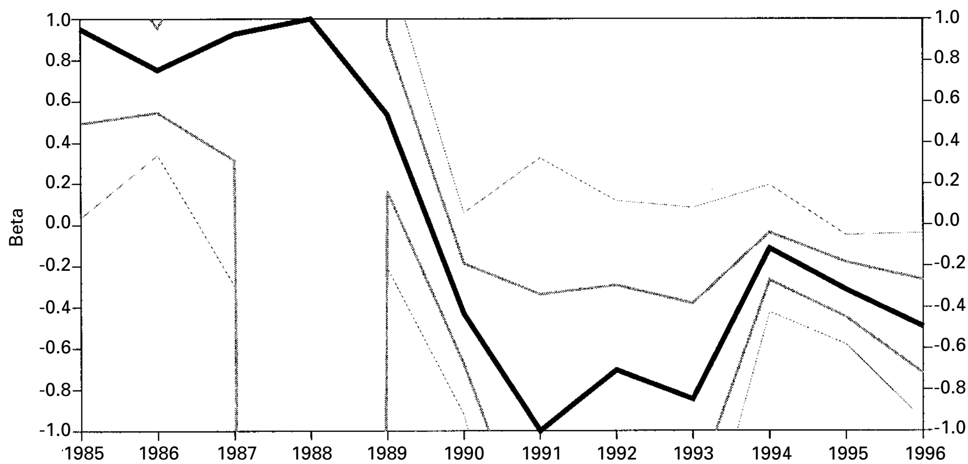
**Exhibit 6**
**Characteristic Exponent Alpha of Distributions of Log Annual Total Return Residuals**
**Australian and United States Data**
**Properties by Type and All Types Combined**
(bands indicate plus and minus one and two standard deviations)




The above analysis implies both that (1) real estate investment risk during the sample period was heteroscedastic, and (2) during virtually all sample subperiods and across property type, stable infinite-variance skewed asset-specific risk functions with a *time-invariant* characteristic exponent of approximately 1.588 modeled the observed distributions of return residuals better than normally distributed risk candidates.

**Exhibit 7**
**Skewness Parameter Beta of Distributions of Log Annual Total Return Residuals**
**Property Council of Australia Database, All Properties**
(bands indicate plus and minus one and two standard deviations)



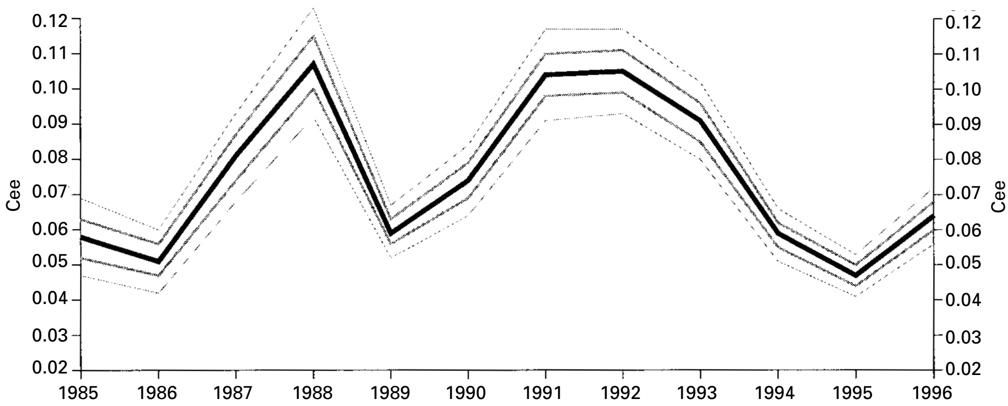

## Comparisons to U.S. Real Estate Return Distributions

The $\chi^2$ test can be employed as in the preceding section to test the proposition that the true values of $\alpha$ for Australian and U.S. property return distributions are equal. As suggested by the two graphs in Exhibit 6, this proposition can be phrased in at least two distinct ways: (1) the three Australian property-type sample values for $\alpha_p$ derived in this article and the four U.S. property-type sample values for $\alpha_p$ derived in Young and Graff (1995) are estimators for a single true value for $\alpha$, and (2) the Australian commercial property sample value for $\alpha$ of 1.588 obtained in this article and the U.S. commercial property sample value for $\alpha$ of 1.477 obtained in Young and Graff (1995) are estimators for a single true value for $\alpha$.

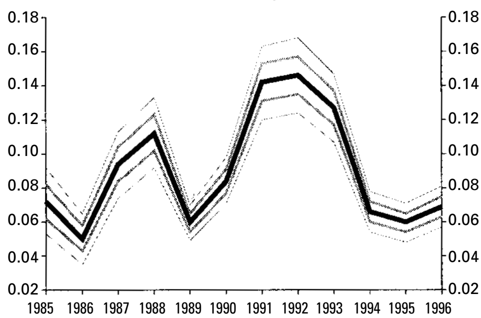
In the case of the first version of the proposition, the critical $\chi^2$ value is 12.59, i.e., the 0.05 significance level of the $\chi^2$ function for six degrees of freedom. The sample $\chi^2$ value obtained from the seven property-type samples for $\alpha_p$ is 8.04, which is below both the critical value for the 0.05 significance level and the critical value of 10.64 for the 0.10 significance level. Thus the seven Australian and U.S. property-type sample values for $\alpha$ are consistent at the 0.10 significance level with the conclusion that there is a single true value for $\alpha$.

For the second version of the proposition the critical $\chi^2$ value is 3.84, i.e., the 0.05 significance level of the $\chi^2$ function for one degree of freedom. The sample $\chi^2$ value obtained from the Australian and U.S. property-type samples for $\alpha$ is 7.89, which is above the critical value of 3.84 for the $\chi^2$ test. Thus the second version of the proposition is rejected at the 0.05 significance level.

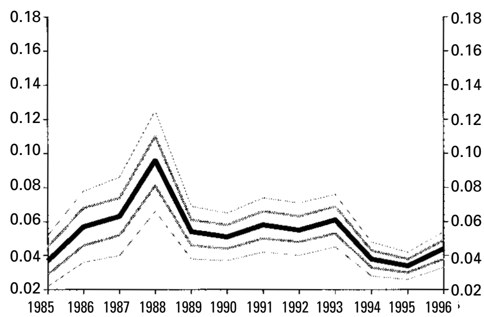
**Exhibit 8**
**Scale Parameter "c" of Distributions of Log Annual Total Return Residuals**
**Property Council of Australia Database, All Properties**
(bands indicate plus and minus one and two standard deviations)




Office Properties




Retail Properties




Industrial Properties

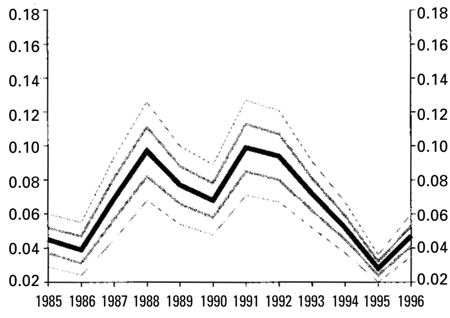


**Exhibit 9**
**Characteristic Exponent $\alpha$ for the Property Council of Australia Database Log Annual Total Return Residual Distributions All Properties, Properties by Type, and Chi-Square Goodness-of-Fit Results**

| Year or Period | All Properties | Office | Retail | Industrial | Annual Property-Type $\chi^2$ | Annual Components of Sample Period $\chi^2$ |
|---|---|---|---|---|---|---|
| 1996 | 1.575 | 1.461 | 1.552 | 1.382 | 0.39 | 0.06 |
| 1995 | 1.353 | 1.333 | 1.407 | 1.238 | 0.46 | 4.49 |
| 1994 | 1.446 | 1.324 | 1.602 | 1.333 | 1.14 | 1.15 |
| 1993 | 1.652 | 2.000 | 2.000 | 1.872 | 0.09 | 0.58 |
| 1992 | 1.660 | 2.000 | 1.307 | 1.738 | 5.45 | 0.72 |
| 1991 | 1.741 | 2.000 | 1.380 | 1.888 | 3.23 | 1.73 |
| 1990 | 1.591 | 1.595 | 1.494 | 1.801 | 0.49 | 0.14 |
| 1989 | 1.668 | 1.567 | 1.843 | 2.000 | 1.10 | 0.75 |
| 1988 | 2.000 | 2.000 | 2.000 | 2.000 | 0.00 | 4.13 |
| 1987 | 1.641 | 1.714 | 1.414 | 1.437 | 0.87 | 0.28 |
| 1986 | 1.339 | 1.206 | 1.436 | 1.219 | 0.50 | 1.77 |
| 1985 | 1.515 | 1.553 | 1.548 | 1.456 | 0.07 | 0.02 |
| 1985–96 | 1.588 | 1.649 | 1.554 | 1.635 | 1.28 | |
| 1985–96 $\chi^2$ | | 22.16 | 5.36 | 9.31 | | 15.82 |

The difference between the $\chi^2$ test outcomes on the two versions of the proposition suggests that the proposition that there is a single true $\alpha$ for all Australian and U.S. real estate return distributions should not be rejected based on results of this study, but that additional evidence is needed in order to accept the proposition.

## Implications for Portfolio Management

In the era of MPT, the central task of portfolio management has been viewed as the optimization of the portfolio return/risk trade-off, subject to portfolio constraints created by investment policy. This involves asset selection and allocation to achieve two independent objectives: (1) minimization of the combined effect of asset-specific risk, and (2) optimization of the trade-off between portfolio return and systematic/sector risk.

The approach to this problem taken in virtually all portfolio research has been: (a) specify the largest tolerable combined asset-specific risk; (b) calculate the minimum number of assets necessary to ensure that the combined effect of asset-specific risk is below the critical threshold; and (c) solve problem (2) under the additional constraint that investment funds be diversified among at least the number of assets determined in (b). In particular, MPT solutions to the management of U.S. stock portfolios take this approach.

To see what would satisfy the additional constraint imposed by (b), it is instructive to make the following simplifying assumptions: all asset-specific risk functions are stable with the same characteristic exponent $\alpha$ and have the same skewness parameter $\beta$, all individual assets have the same level of asset-specific risk (as proxied by the scale parameter $c$ of the distribution for the common asset-specific risk function), and the same percentage of the total portfolio value is invested in each component asset in the optimal portfolio. Then, letting $p$ represent the portfolio and $f$ the common asset-specific risk function equation (2) reduces to:

$$c_p = n^{(1/\alpha)-1} c_f \,. \tag{5}$$

Exhibit 10 shows the impact of varying $\alpha$ upon reduction in asset-specific risk (i.e., diversifiable risk) for various numbers of properties in a portfolio. For any given $\alpha > 1.0$, the reduction in asset-specific risk increases with increasing $n$. As $\alpha$ declines to 1.0 from its upper limit of 2.0, the reduction in asset-specific risk likewise declines for any given $n > 1$.

The sample value $\alpha = 1.588$ implies the following practical estimate for the effect of portfolio diversification on asset-specific risk reduction:

$$c_p \approx n^{-0.370} c_f \,. \tag{5'}$$

A typical Australian listed property trust has ten to fifteen properties.[6] Under the above assumptions, the magnitude of diversifiable risk for such a trust is between 37% and 43% of the magnitude of asset-specific risk for a single-property portfolio. However, if asset-specific risk were normally distributed, diversifiable risk would be between 26% and 32% of the asset-specific risk for a single-property portfolio.

Alternatively, if the question of risk reduction is rephrased to ask the number of assets $n_k$ needed in a portfolio to achieve a reduction of asset-specific risk by a specified factor

**Exhibit 10**
**Risk Reduction for Various $\alpha$ and Number of Assets**

| | Number of Assets | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | 1 | 2 | 4 | 8 | 10 | 20 | 100 |
| 2.00 | 1 | 0.707 | 0.500 | 0.354 | 0.316 | 0.224 | 0.100 |
| 1.90 | 1 | 0.720 | 0.519 | 0.373 | 0.336 | 0.242 | 0.113 |
| 1.80 | 1 | 0.735 | 0.540 | 0.397 | 0.359 | 0.264 | 0.129 |
| 1.70 | 1 | 0.752 | 0.565 | 0.425 | 0.387 | 0.291 | 0.150 |
| 1.60 | 1 | 0.771 | 0.595 | 0.459 | 0.422 | 0.325 | 0.178 |
| 1.50 | 1 | 0.794 | 0.630 | 0.500 | 0.464 | 0.368 | 0.215 |
| 1.40 | 1 | 0.820 | 0.673 | 0.552 | 0.518 | 0.425 | 0.268 |
| 1.30 | 1 | 0.852 | 0.726 | 0.619 | 0.588 | 0.501 | 0.346 |
| 1.20 | 1 | 0.891 | 0.794 | 0.707 | 0.681 | 0.607 | 0.464 |
| 1.10 | 1 | 0.939 | 0.882 | 0.828 | 0.811 | 0.762 | 0.658 |
| 1.00 | 1 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| 0.90 | 1 | 1.080 | 1.167 | 1.260 | 1.292 | 1.395 | 1.668 |

**Exhibit 11**
**Number of Assets Needed for Risk Reduction by the Factor *k***

| | Factor $k$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $\alpha$ | 1 | 2 | 4 | 8 | 10 | 20 | 100 |
| 2.00 | 1 | 4 | 16 | 64 | 100 | 400 | 10,000 |
| 1.90 | 1 | 5 | 19 | 81 | 130 | 558 | 16,682 |
| 1.80 | 1 | 5 | 23 | 108 | 178 | 846 | 31,623 |
| 1.70 | 1 | 6 | 29 | 156 | 269 | 1,445 | 71,969 |
| 1.60 | 1 | 7 | 41 | 256 | 465 | 2,948 | 215,444 |
| 1.50 | 1 | 8 | 64 | 512 | 1,000 | 8,000 | 1,000,000 |
| 1.40 | 1 | 12 | 128 | 1,448 | 3,163 | 35,778 | 10,000,000 |
| 1.30 | 1 | 21 | 407 | 8,192 | 21,545 | 434,307 | $4.6\times10^{8}$ |
| 1.20 | 1 | 64 | 4,096 | 262,144 | 1,000,000 | $6.4\times10^{7}$ | $1.0\times10^{12}$ |

of $k$, then the answer is as follows: $n_k$ is the smallest integer at least as large as $k$ raised to the power 1/0.370. In mathematical notation,

$$n_k = k^{\alpha/(\alpha-1)} + 1 \approx k^{2.70} + 1 \ . \tag{6}$$

This implies that the number of properties in a portfolio needed to achieve, for example, a four-fold reduction in the magnitude of combined asset-specific risk is 43—compared with only 16 properties if asset-specific risk were normally distributed. Similarly, the number of properties in a portfolio needed to achieve a ten-fold reduction in combined asset-specific risk is 502—compared with 100 properties if asset-specific risk were normally distributed. In other words, if purchases are restricted to institutional-grade properties, equally weighted investments in nearly all properties in the Property Council of Australia database would be needed to achieve a ten-fold reduction in the magnitude of combined asset-specific risk.

The effect of varying $\alpha$ upon the portfolio size needed to achieve risk reduction by various specified factors $k$ is shown in Exhibit 11.

## Conclusions

This study is an empirical examination of the shape of cross-sectional returns from Australian equity real estate in the Property Council of Australia database for the years 1985 to 1996.

Interestingly, we find that the shape of the distribution of Australian property returns looks remarkably similar to the shape of the distribution of private property returns in the U.S. In particular, we find that there is a single value for the characteristic exponent of asset-specific risk across both calendar year and property type. A statistical estimate of this common value together with a 95% confidence interval around the value is $1.588\pm0.068$. This is nearly indistinguishable statistically from the range of $1.477\pm0.038$

that describes the shape of U.S. property returns, and is far from the value of 2.0 that represents the characteristic exponent of the normal distribution.

Thus, as is the case with U.S. real estate, investment risk models with infinite variance provide a better description of the distribution of individual Australian equity real estate returns from 1985 to 1996 than normally distributed risk models. In addition, Australian real estate investment risk is heteroscedastic, although the characteristic exponent of investment risk is apparently constant across time and property type. Finally, the true value for the characteristic exponent of Australian real estate investment risk is virtually indistinguishable statistically from the corresponding characteristic exponent for U.S. real estate investment risk.

The low observed value for the characteristic exponent implies that reduction of asset-specific investment risk to levels readily achievable in the stock and bond markets through asset diversification requires a portfolio of far more real estate assets than would be needed for the case of normally distributed risk. In institutional real estate portfolios, substantial risk reduction across multiple risk factors (locational, economic, etc.) could only be achieved by purchasing most of the institutional-grade properties in Australia—a practical impossibility. This implies that institutional real estate portfolio management must be concerned with the asset-specific risk component of each property investment as well as with market/systematic and market-sector risk components.

The fact that real estate investment risk has infinite variance also implies that there is no way to measure codependence among property risk functions with the elementary statistical tools currently available. In particular, sample correlations used in multi-factor MPT real estate risk models do not represent measurements of true risk codependence.

It follows that the current conceptual version of MPT appropriated from stock market analysis is inapplicable to the Australian real estate market.

## Notes

[1]The research literature on stable distributions contains several inconsistencies in the definition of the skewness parameter $\beta$. This difficulty had its origin in the fact that, because of the way early versions of equation (1) were specified, asymmetric stable distributions had skewness that appeared intuitively negative for positive values of $\beta$, and vice versa. McCulloch (1986) discusses this problem in detail.

[2]The formula used to calculate the quarterly time-weighted total returns is: *Total Return*$=(EMV-BMV+PS-CI+NI)\,/\,(BMV-0.5\,PS+0.5\,CI-0.5\,NI)$, where $EMV$ is the quarterly end market value, $BMV$ is the beginning market value for the quarter, $PS$ is partial sales proceeds, $CI$ is capital improvements made in the quarter, and $NI$ is the net property income for the quarter. Unlike the NCREIF formula, the Property Council of Australia formula assumes that net income is received at the midpoint of the quarter rather than monthly.

[3]In comparisons with U.S. real estate return distributions we treat the NCREIF warehouse and research and development classifications as similar to the Property Council of Australia industrial classification.

[4]However, we point out the practical difficulty of testing such a model: in contrast to a test of our model for asset-specific risk, for which we have available nearly 4,600 sample values, any test of a model for systematic or market-sector risk over the same period can call on a database of at most thirty-six sample values (three annual sector means per year times twelve years).

[5]Irwin (1942) and James (1951) present detailed developments of this test in the respective cases of independently distributed normal and asymptotically normal variables.

[6]Australian listed property trusts are similar in nature to U.S. real estate investment trusts.

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