Discount Points, Effective Yields and Mortgage Prepayments

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Abstract. **Lenders use discount points to vary effective yields from stated contract rates. In the past, this practice resulted from interest-rate regulation. In market, however, there seems to be an absence of economic rationale as to how points are applied in individual cases. This paper provides an economic rationale for the use of points. An option pricing model is used to derive some basic rules for determining the relationship between changes in the contract rate and the payment of points. This explains current practices by lenders in that the option value effect could justify a reduction in the contract rate of well over one-eighth of a percent and shows that lenders are justified in varying the size of the trade-off between points and contract rates based on risk of prepayment. Lenders must charge a premium for providing the prepayment option; however, the payment of points provides a means whereby this call provision can be retained but reduced in value to suit the preferences of individual borrowers.**

Introduction

The use of points in fixed-rate residential mortgage lending is widespread. Lenders reduce the contract rate of the mortgage in return for the payment of points at the origination of the mortgage. However, the magnitude of the reduction in the contract rate per point paid varies from one lender to the next. Furthermore, individual lenders often vary the size of the trade-off over time.

Surprisingly, there is very little formal analysis of the policies that lenders use to determine the points charged to borrowers. In the past, it was often assumed that the use of points was a consequence of interest-rate regulation. Lenders could raise the effective return on a mortgage by demanding the payment of points without raising the contract rate (e.g., see Curley and Guttentag [1974], Colwell, Gunterman, and Sirmans [1979], Zerbst and Brueggeman [1977], and Smith and Sirmans [1984]). While this may explain to some extent the current use of points on FHA and VA mortgages, their current use in conventional mortgage loans is unexplained.[1]

The purpose of this paper is twofold. First, an economic rationale for the use of points is provided in the absence of tax effects, regulatory constraints, and default. Points are a means by which borrowers can lower their effective interest rate, dependent upon their likelihood of prepayment. Second, an option pricing model is used to derive some basic rules for determining the size of the reduction in the contract rate that should be offered in return for the payment of points.

The paper is outlined in the following fashion. Section two presents the economic rationale for the use of points. Section three presents a simple option pricing model for fixed-rate mortgages along with the derivation of a set of basic rules for pricing points.

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Economic Rationale for Discount Points

Fixed-Rate Mortgages (FRMs) are similar to callable bonds. They provide the borrower with an option to call the mortgage at a striking price equal to the outstanding balance of the mortgage. The call option on a mortgage is even more valuable than the call option on a bond, since the striking price for a typical callable bond is set well above the initial market value while the striking price for a mortgage is set at the initial market value. As this section demonstrates, the use of points constitutes a means by which the striking price for the mortgage option may be raised.

Lenders must charge a premium for providing a call option to the borrower. This premium is reflected in a higher required rate of return. In the market for corporate bonds, borrowers are compensated for the higher required rate of return by the value of the option to call the bond at its call price. In the mortgage market, however, the value of the call option for some borrowers may be significantly less than the value of the higher rate of return required by lenders.

A cause of this discrepancy in the value of the call option on an FRM to borrowers and lenders is the relatively high closing costs associated with refinancing a mortgage. A major advantage of the call option on an FRM for borrowers is that it enables them to refinance at a lower rate when interest rates decline. However, in order to refinance, the borrower will incur closing costs on the new loan. Furthermore, these closing costs are typically not received by the original lender. Hence, in many cases the value of the call option on an FRM is less for the borrower who receives it and more for the lender who provides it.

When the call option is worth less to the recipient than to the provider, both parties may benefit by reducing its value or eliminating it altogether. At present, however, virtually all residential FRMs retain the option to prepay the mortgage. One reason for this may be the reduction in default risk that results from retaining an option to prepay. Many mortgage prepayments are in a sense involuntary. For example, the borrower may have to pursue employment in another location or sell the home for other reasons. In the absence of the call option, the payoff on the mortgage may have risen substantially since origination due to a fall in interest rates. The result would be involuntary default, the costs of which would be born in large part by the lender.

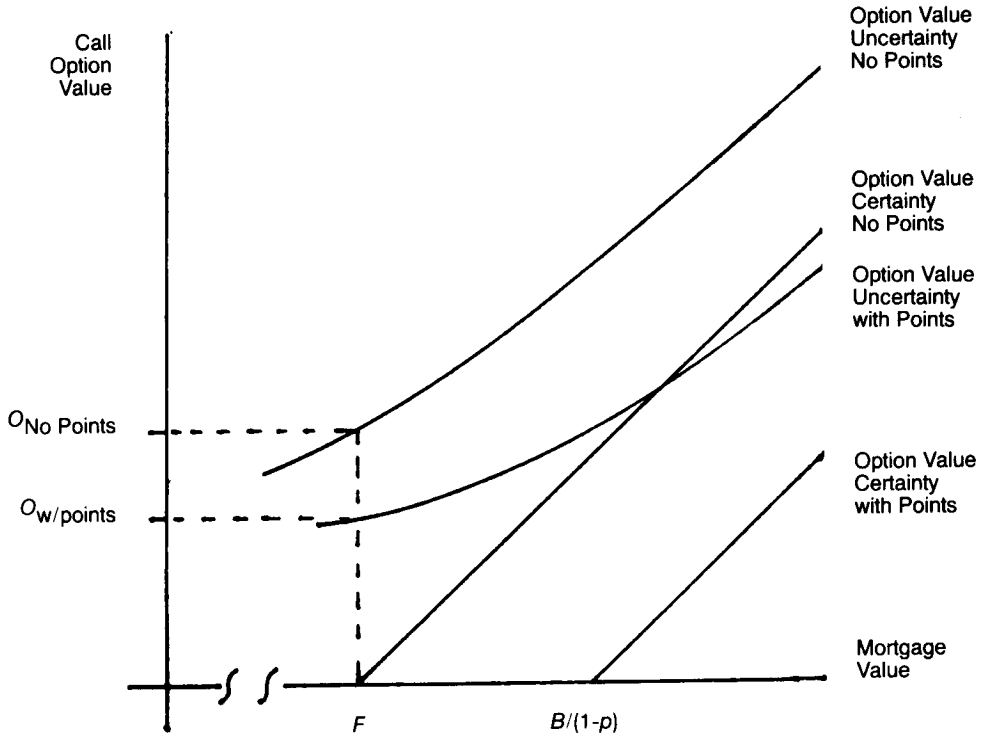
The payment of points constitutes a means by which the call provisions can be retained but reduced in value in order to suit the particular preferences of individual borrowers. In order to demonstrate this, a brief review of some of the basic principles of a call option value is useful. Exhibit 1 is a graph of the value of a call option as a function of the value of the underlying asset. For our purposes the value of the asset (M) is the market value of a bond that is identical to a fixed-rate mortgage except for the absence of an option to prepay at the outstanding balance. This asset value is stochastic and will fluctuate with random movements in interest rates. The value of the call option (C) is the absolute value to the lender of the right to prepay .

The straight line represents the value of the prepayment option under certainty (Oc), i.e., all fluctuations in asset value (M) are known in advance.

$$\begin{aligned} O_c &= M - F, \text{ if } M > F \\ &= O, \text{ if } M < F \end{aligned}$$

This line has a slope of one since the value of the right to prepay at a fixed amount increases one dollar for every dollar increase in the value of the asset. The line intercepts the horizontal

Exhibit 1
Effect of Points on the Value of the Prepayment Option



axis at (*F*) the present value of the asset, which at the time of the mortgage's origination is its face value. Below this value (*F*) the prepayment option has no value, since the borrower has the right not to exercise it.

The curve represents the value of the prepayment option under certainty (O_u). The value of the prepayment option under uncertainty always exceeds its value under certainty. This difference in value is called the premium on the option (O_p),

$$O_p = O_u - O_c.$$

The increase in the value of the option under uncertainty arises, because, when future asset values are uncertain, the potential increase in the value of the option is unlimited whereas the potential decrease is limited since borrowers are not obligated to exercise the option. Since the value of the option is a decreasing function of the exercise price, an obvious method of reducing its value is to increase the exercise price.

The payment of points increases the exercise price of the prepayment option on an FRM. Consider a borrower who wishes to borrow an amount (B) equal to $100,000. In the absence of points, he/she would obtain a mortgage with an original face value (F) equal to $100,000. However, if he/she chooses to pay two points (p), then he/she will need to incur a mortgage with a face value of $102,040.82, i.e.,

$$F = B/(1-p).$$

The face value of the mortgage less the two points ($2,040.82) retained by the lender will still provide the $100,000 to finance the house. Although the borrower has obtained the same amount, the use of points increases the exercise price (F) of the prepayment option. Hence, the value of the prepayment option

$$O \text{ no points} - O \text{ w/points} > 0$$

would be reduced.

The Pricing of Discount Points

The previous section has demonstrated that the value of a fixed-rate mortgage is equal to the value of a non-callable bond less the value of the call option. The payment of points has an effect on each of these two components. First, by reducing the net value of the bond, the payment of points increases the effective APR. Second, by reducing the value of the prepayment option, the payment of points lowers the lenders required rate of return. Hence, both effects of points on the value of an FRM result in an increase in the size of the reduction in the contract rate that can be offered in return for the payment of points.

Trade-off of Points and Contract Rates

In order to determine the appropriate trade-off between points and contract rates both the bond effect and the option effect must be estimated. Estimation of the bond effect is relatively simple. Since the actual amount borrowed is the face value of the mortgage less the points paid, the APR will exceed the contract rate. Estimation of the option effect, however, is difficult. The call option on an FRM is a complex form of compound option that features varying exercise prices.[2] It is likely that a closed-form solution for its value does not exist. Nevertheless, a few relatively straightforward propositions about the effect of points on its value can be made.

P1: The value of the option will be reduced by the payment of points, since points have the effect of raising the exercise price. Hence, lenders can accept a greater reduction in contract rate per point paid than implied by the effect on bond value.

P2: The magnitude of the reduction in the option's value will always be less than the amount of the points paid, since the value of a reduction in exercise price under

uncertainty is always less than its value under certainty. Hence, lenders will offer reductions in contract rates per point paid that are less than twice those implied.

P3: The size of the reduction in the option's value per point paid is an increasing function of uncertainty about future mortgage interest rates, since the greater the uncertainty the greater the likelihood of prepayment in order to refinance. Hence, lenders will offer greater reductions in contract rates per point paid during periods of greater uncertainty about the direction of future mortgage rates.

Establishing a Lower Bound

The "pseudo-American call" approach provides a relatively simple means by which a lower bound can be established for the value of the prepayment option.[3] Basically this approach consists of evaluating separate European call options at each payment date over the life of the mortgage, using the Black-Scholes model. Each European option features a maturity (t) equal to the payment date, an exercise price (E) equal to the balance of the mortgage at that payment date, and an asset value (A) equal to the initial face value of the mortgage less the present value of the payments made up to the maturity date of the option:

$$O = A\,N(x) - Ee^{-rt}\,N(x-st^{1/2})$$

where $N(\ )$ is the cumulative normal density and

$$x = [\ln(A/E) + (r + S^2/2)t]/(st^{1/2})$$

and r and s are the risk-free rate and standard deviation of the rate of change in the asset value respectively. The highest of these option values constitutes a lower bound for the value of the prepayment option.

While this procedure may result in an underestimate of the prepayment option's value, the consequences for estimating the change in the value of the option per point paid are much less onerous. The "pseudo-American call" value of the prepayment option is recalculated for each additional point paid. The difference is then reported as the reduction in the value of the prepayment option per point paid. Hence, the underestimates of the two option values tend to cancel out. Nevertheless, the resulting estimated differences must be interpreted with caution, since they only represent differences in lower bounds not differences in actual values, and suggest an area for future research.

Exhibit 2 presents the estimated reductions in the value of the prepayment option that result from the payment of various amounts of points for a thirty-year FRM with a contract rate of 10%. The values are expressed as percentages of the mortgage's value. Because it is impossible to determine accurately the monthly standard deviation of rates of change in mortgage values, the option values were estimated for five possibilities ranging from 1 to 4%.

Exhibit 2
Estimates of the Lower Bound for Reductions in the Value of the Prepayment Option as a Percent of Mortgage Value

Standard Deviation*	Number of Points Paid			
	1	2	3	4
0.01	0.229%	0.378%	0.468%	0.516%
0.015	0.250%	0.459%	0.630%	0.767%
0.02	0.261%	0.497%	0.708%	0.895%
0.03	0.265%	0.519%	0.761%	0.991%
0.04	0.265%	0.524%	0.768%	1.011%

*Std. dev. of monthly rate of change in mortgage value.

The reductions in the option values as a percent of the mortgage range from a maximum of .265% to a low of .129% (.516/4) per point paid. These values are an increasing function of the variability in mortgage values and a decreasing function of the number of points paid. These values represent the maximum benefit that can be obtained by a borrower who holds the mortgage until maturity. However, if the borrower prepays, his or her net loss may be substantial. Hence, the borrower's choice of whether and how many points to pay will depend on personal circumstances which affect the likelihood of prepayment.

The Price of Points

Because the payment of points reduces the value of the prepayment option, lenders are willing to reduce the required rate of return on the mortgage. This reduction is in addition to the reduction in the contract rate as a result of paying points. The reduction that results from lowering the value of the prepayment option is applied to both the contract rate and the effective APR.

Consider a lender who requires an effective APR of 10.12% on a thirty-year mortgage. The bond effect of the payment of one point would allow the lender to lower the contract rate to 10%, a reduction of 0.12%. If the option effect of the payment of one point increases the value of the mortgage by 0.25% of its value, then the lender would be willing to lower his or her contract rate another 0.03% (= 0.12/4), since the option effect is equal to receipt of another quarter of a point. Hence, the trade-off between the contract rate and the payment of one point would be 0.15% (= .12% + .03%). These values are consistent with the trade-offs that are currently being offered in the residential mortgage market.

One should keep in mind that these values constitute lower bounds, the actual size of the option effect may be substantially greater in some cases. Accurate estimates of the size of the option effect require a well-specified model of the value of the prepayment option. These models are complicated by a variety of factors including term structure effects, amortization, default, and even the actions of misinformed borrowers. Models such as these are beyond the scope of this paper and are currently the focus of research.[4]

Summary and Conclusions

This paper has examined the economic motivations underlying current use of points in the fixed-rate residential mortgage market. FRMs are described as fully amortized bonds on which a call option has been sold to the borrower. The analysis is based upon the separate but additive effects of points on the mortgage's bond value and its option value. Points are shown to have an impact on both the bond and prepayment option component of the FRM. By reducing the amount actually lent, the payment of points raises the effective APR of the mortgage. The magnitude of this increase in effective APR is an increasing function of the contract rate and number of points paid, but a decreasing function of the length of time that the mortgage is held.

By increasing the exercise price of the prepayment option, the payment of points reduces the value of the call provision offered by the lender to the borrower. This in turn lowers the lenders required rate of return on the mortgage and makes possible an additional reduction in the contract rate. Estimates of a lower bound on the value of the prepayment option demonstrates that the magnitude of the reduction in the value of the prepayment option exceeds one-fifth of one percent of the value of the mortgage in most cases. Accurate estimates of the size of the option effect require a well-specified and practical model of the prepayment option's value.

Taken together, the impact of points on both the bond and option value of the FRM explains currently observed practices. The bond value effect would justify a reduction in the contract rate per point paid of slightly less than one-eighth of one percent in most cases. Combined with the option value effect the payment of points could justify a reduction in the contract rate of well over one-eighth of a percent. These results have important implications for both lenders and borrowers. They show that lenders are justified in varying the size of the trade-off between points and contract rates based on perceived risk of prepayment. Borrowers, who for personal reasons are unlikely to prepay, can benefit from the payment of points by an amount equal to the reduction in the value of the prepayment option.

Notes

[1]A recent exception is a working paper by Kau and Keenan [1985]. They attribute the use of points to tax benefits obtained by the lender, which are passed along in part to borrowers.

[2]A compound option is an option on an option. Because an FRM is itself an option, the prepayment option on an FRM is a compound option. See Geske [1979] for a discussion of the valuation of compound options.

[3]The pseudo-American call was first proposed by Black [1975] as a means of approximating lower bounds on options for which early exercise due to cash distribution are likely. The magnitude of monthly payments on a mortgage, which include all interest plus amortization of the principal, makes early exercise very likely. See Jarrow and Rudd [1983] for both proofs and examples.

[4]Among the most sophisticated of the recent models to emerge is a two-state model by Kau, Keenan, Epperson and Muller [1987]. However, simulations of this model are only practical on a supercomputer. For an excellent review of recent work in this area, see Hendershott and Van Order [1987].

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