Utility Theory and Rent Optimization: Utilizing Cluster Analysis to Segment Rental Markets

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Abstract. The research reported here segments apartment residents and units into homogeneous groups. This segmentation is accomplished via cluster analysis. The purpose of the research is to identify market segments with higher marginal utility preferences for selected project or unit amenities; with the ultimate goal of rental rate optimization. The uniqueness of the research is twofold in that the "tenant response" based hedonic analysis is modeled via cluster analysis instead of the typical regressive framework. The usefulness of the research is based in the presentation of a methodology that should at least be considered as an adjunct technique to the typical regression analysis.

Introduction

Optimizing rental rates in multifamily properties is accepted as one factor in maximizing the value of the investment, value maximation generally being accepted as the primary goal of the business entity. However, rent optimization cannot be accomplished without first ascertaining what factors actually affect rental rates, especially rental rates at the margin, for specific amenities and amenity packages.

The purpose of this paper is to demonstrate how selected tenant demographic profiles and unit characteristic profiles significantly influence the hedonic responses which tenants provided when asked to value specific amenities. Such marketing research techniques, as those applied here, have not been widely used in the multifamily housing industry. In fact, this lack of behavioral-based market research has been mentioned in the literature as an area that needs to be addressed [4]. We will illustrate how the use of market (tenant) surveys and cluster analysis can be of significant benefit to apartment owners, appraisers, and others who are attempting to estimate optimal rent for specific properties with varying amenity packages.

Specifically, we attempted to determine whether statistically significant differences in marginal value responses could be demonstrated utilizing clusters of apartment characteris-

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tics and tenant demographics. While no technique utilized in solo is typically valid or robust enough to accurately predict actual behavior; we propose that our methodology contributes sufficient information in a cost-effective way to be worthy of utilization, as we, as researchers, use a scalpel razor to continuously slice research into ever smaller slices with the hope that each slice will provide at least one new insight.

Review of Related Research

The literature on "hedonics" is generally considered to have begun with Rosen's paper [15], even though Straszheim [19] used the term in an earlier housing study, and in fact, the word hedonic appears in research as early as 1939 [2]. Rosen popularized the term "hedonic" as a result of his presentation where bid and ask curves kissed at the clearing price point. While the theoretical framework for considering individual characteristics of goods that cannot be readily differentiated is considered to have been provided by Lancaster [8], it was Rosen who popularized the framework by recognizing that characteristics have distinct value at the margin because consumers implicitly have a utility function for each characteristic of any product.

Because both apartment units and single-family houses provide essentially the same service (i.e., a permanent place to live), there naturally have evolved studies that considered single-family housing and multifamily housing or both. Because our study concerns multifamily housing, we concentrate the literature review in this area. However, the reader interested in studies related to the single-family housing market will find the following informative: 1) Abelson's noise study in Sydney [1], 2) Martin's study in El Paso [9], 3) Palmquist's seven-city study [11], Smith and Kroll and Smith's studies in Houston and Central Texas [6][7].

As mentioned, there has been limited work focusing specifically on the affect of amenities on rental rates in multifamily housing. Guntermann and Norrbin [3] analyzed apartment rents in the Phoenix area, focusing on physical characteristics, project amenities, and location. They utilized a reduced-form multiple regression to examine information from 104 apartment projects of various unit types. They found that unit size accounted for over 40% of the rental rate variances. Guntermann and Norrbin also found patios, fireplaces, dishwashers, unit age, unit condition, and number of bathrooms to influence rental rates.

Unfortunately, while Guntermann and Norrbin's work provided significant insight into coefficients of specific amenities, their analysis was based on actual rental rates, not tenant responses, and with transformed data, the coefficients may have little value—at the margin. It is true that using regression coefficients to measure amenity values can produce results that are useful to appraisers and property owners, but attempting to explain differences in rent, using a regression model, precludes using marginal value responses from tenants as to how they value apartment amenities. This concept of a marginal value for each amenity, or feature, forms the basis for utility theory and might, therefore, be considered as critical to estimating optimum rental rate structure. Therefore, it seems only logical that those who use the product should be queried as to how much, at the margin, they would pay for apartment features and amenities. Guntermann and Norrbin have pointed out that each amenity and feature contributes a different amount toward total rent. While their methodology sought to determine how much each amenity or feature contributed to total rent, our focus is how much, at the margin, each feature or amenity is perceived to be worth to the tenant (i.e., their hedonic price response). Thus, the focus of our methodology is the

marginal value of various amenities to different tenant groups that may be isolated by way of cluster analysis.

It is critical to explicitly state several key points having to do with hedonic research. The first point is that models can be formulated in terms of a bid price framework as we have done here, or the models can involve regressing on price with coefficients in the aggregate having predictive power. The second point is that each of these has a distinct usage. Almost without exception, researchers have reported their results in terms of regression coefficients and simultaneously questioned whether or not the coefficients were a good representation of an amenity's true marginal value. There is no doubt that a regression coefficient of .071 for a fireplace is meaningless for a marginal value estimate in an appraisal (see reference 3, Table 2).

We argue that regression on price may have great predictive power depending upon R^2 , but that to explicitly take the coefficients as marginal values is specious. Simply put, regressing on price is appropriate for predicting price in the aggregate, but not for price at the margin for each amenity. The bid price format as presented here may provide greater benefit because we use non-transformed data and a more robust technique. The robustness of the technique results from freedom of the restrictive assumptions required for regression.

Other studies directly related to this paper are those that focused on the relationship between rental rates and tenant demographics near a major university. Ogur [10] and Jaffe and Bussa [5] found a positive relationship between rent levels and proximity to a university. These earlier findings, as well as the presence of Arizona State University in the market they studied, led Guntermann and Norrbin to test for any differences in their regression models between the university area and other parts of the market examined. They did find that certain variables had larger coefficients than others in the different markets. Their recognition of a separate college student submarket anticipates our research in that college students tend to be demographically clustered in the younger and lowerincome categories. As we have seen, previous studies have considered rents to be largely a function of project characteristics such as unit and project amenities, age of complex, and locational influences. Our study focuses on perceived marginal values of an assortment of these factors. There are also, of course, exogenous factors such as tax law changes that influence what tenants will pay to rent. There is no doubt that even over a limited time horizon, tax law changes affect the supply of, and thus, the price of rental housing. Rose and O'Neil [14] have proposed that the Tax Reform Act of 1986 will result in increased rents.

Utility theory attempts to explain how much, at the margin, a consumer will pay for an additional unit of a particular good or service. An extension of this theory is the search for an assessment of the marginal value of a specific amenity contained within a package of amenities. This is what is typically referred to as "hedonic pricing". There has been very little work done in the area of consumer estimates of the perceived marginal values of specific amenities in housing. This type of research is just emerging in real estate [17][18], and this paper seeks to extend the methodological foundation.

The Data and Model

The data set used consisted of over 400 completed surveys. The sample was selected in such a manner as to be representative of suburban sectors of a larger metropolitan area,

specifically suburban Houston, Texas. The purpose here is to illustrate a particular methodology; therefore, almost any suburban setting would have been appropriate. The survey instruments were administered, in person, to apartment unit dwellers with the goal of determining how much more or less the tenants would pay for their units if they either had or did not have specific amenities. Respondent demographic information was also gathered.

The specific variables of interest are presented in Exhibit 1. As Exhibit 1 illustrates, a wide range of variables were included in the analysis to allow for specification clarity.

The Methodology

The majority of past research in real estate has utilized multiple regression analysis (MRA) to either predict dependent variables or determine hedonic responses using regression coefficients. *Q*-factor analysis, which is a variant of clustering techniques, has also been used, and some work has employed analysis of variance (ANOVA). While MRA minimizes sums of squares to maximize explanation of variance, ANOVA seeks to determine if agglomerated variables' means are significantly "far apart when different groups are compared." Cluster analysis seeks to group observations based on within-group similarity and between-group differences.

Cluster analysis has not been widely used in the real estate literature, and therefore, a brief discussion is in order. The technique is a multivariate approach to identifying similar objects and placing them in groups (clusters). For those unfamiliar with the technique, Romesburg [13] provides a thorough review of the technique, along with criteria as to when a specific option may be appropriate for a particular research problem. As has been pointed out, both cluster analysis and factor analysis are more art than strict scientific discipline. Essentially, cluster analysis is an algorithm which defines subgroups within larger populations, based upon a set of differentiating variables. Ideally, the resulting clusters should be internally homogeneous, but heterogeneous when compared with each other.

While there has not been widespread use of cluster analysis in the real estate literature, the technique is firmly established in marketing and is beginning to receive attention in other business disciplines. Punj and Stewart [12] provide an extensive reference list of previous uses of cluster analysis (CA) in marketing. They also review the various statistical packages available. One ancillary purpose of this article has been to further the use of

Exhibit 1 Amenities Examined

Age of apartment complex
Number of bedrooms per unit
Number of bathrooms per unit
Size of unit in square feet
Rent per unit per month
Age of respondent
Income per month of head of household
Dollar response for fireplaces
Dollar response for washer/dryer connection
Dollar response for washer/dryer

Dollar response for covered parking
Dollar response for price of pool
Dollar response for price of Jacuzzi
Dollar response for price of exercise area
Dollar response for price of open patio
Dollar response for price of covered patio
Dollar response for extra 100 square feet
Dollar response for less 100 square feet
Dollar response for microwave

cluster analysis in the real estate arena. Our approach is not completely at odds with using Chow tests to determine if differences exist between regression coefficients in unique models. We first group, using cluster analysis, and then use Scheffe's Test to see if differences in amenity values exist across clusters.

As already mentioned, various statistical packages available are reviewed by Punj and Stewart. We selected FASTCLUS which is a SAS [16] routine. This algorithm performs a "disjoint cluster analysis on the basis of Euclidean distances." This is a non-hierarchical model, as opposed to hierarchical models (see [13] for a review of each). Several reasons support the use of this specific routine, including its sensitivity to outliers that often form their own cluster or become part of the entropy group, and its K-means-based computational framework that facilitates the testing of differences among clusters. In addition, there was not a priori evidence to believe that the responses consisted of a select hierarchy.

Previous research had demonstrated that single variable market segments do exist. This suggested that combinations of specific variables might have effects upon hedonic responses, i.e., that hedonic responses based on house characteristics or buyer characteristics clusters would be distinctively different. In fact, as reported below, the analysis did reveal distinct clusters and certain hedonic responses varied across these clusters. It should also be noted that our entropy groups proved to be small. This indicates that distinct segments do exist within the larger data set. The small entropy groups also support the conclusion of small numbers of outliers that can also cause bias.

The methodology consisted of three phases, with the first phase involved utilizing the non-hierarchical model to define clusters. The second step involved using one-way ANOVAs to determine whether clustering variable means were statistically significantly different across clusters. High F-ratios resulting from one-way ANOVA of the various clustering variables across clusters should lead to the ex-post conclusion that our ex-ante beliefs were correct. In other words, our hypothesis was that distinct demographic and apartment feature clusters existed and have unique characteristics.

Because of previous research, we had a priori indication that combinations of specific demographic variables might have significant effects upon apartment rents, i.e., that there would be distinctively different market segment clusters. We also anticipated that groups defined by demographic variables would place different values on various amenities. As we will see later, the results do support the notion that there are distinctly varying tenant clusters, and these different groups have different attitudes about the values of amenities. We feel these differences have important implications for planning rental structures based on market demographics. The findings clustered by unit characteristics have similar implications.

Two variables were used to cluster by respondent demographics: age and income. Unit size and monthly rents were used to cluster unit types. Small entropy groups resulted in both instances, and this indicates there are distinct segments within the larger data set.

After the clusters were identified, the Scheffe's routine of the SAS one-way ANOVA package was used to determine whether differences in hedonic responses existed across the clusters. The Scheffe's Tests were run on all amenities for which data was collected. The Scheffe's Test was used because it is conservative in suggesting statistical significant differences, and thus, we feel confident the differences reported here are real.

Presentation of Findings

The results are presented in summary format in Exhibits 2, 3, 4 and 5. As indicated in Exhibit 2, the variance between clusters in terms of both age and income is quite significant. The Scheffe's Tests of differences in the hedonic responses across the clusters are presented in Exhibit 3. Exhibit 1 lists the amenities included in the Scheffe's Tests. In addition, Exhibit 3 reports *F*-scores and Prob Values in order to allow the reader to assess where the responses across clusters are statistically significantly different.

As mentioned earlier, cluster analysis is certainly not an exact science, and the selection of clustering variables obviously greatly influences the nature of the groups that emerge. We used age and income due to a priori expectation that distinct groups would emerge based on earlier study results. It is worth noting that the number of respondents varied significantly across the clusters. Most notable, the smallest group turned out to be the older, upper-middle-income cluster. If this had not proven to be the case, it would have indicated either an upper-income retirement center contained in the sample, or invalid data. This is due to the fact that when people are in upper-income groups and are older, they typically are living in owner-occupied housing.

The four clusters are shown to be clearly distinct, and with the exception of cluster #2, contained about an equal number of observations. The large sample size precludes the problems associated with potential small sample biases. The standard deviations are also relatively small, particularly in the income groupings, which also suggest that the data set does not contain a large number of spurious responses.

The income groupings ranged from a low of \$500 a month to a high of over \$2,600 per month. This large spread facilitates the clustering and further reinforces the findings. As

Exhibit 2
Summary of Cluster Analysis Results:
Clustered by Respondent Profile

	Response Average & Standard Deviation			
Cluster	Mean Age	Income		
1		'		
(Young, low income)	23.15	500.73		
N = 127	(5.48)	(44.56)		
2				
(Older, upper middle income)	33.43	2,671.45		
N = 44	(9.53)	(239.75)		
3				
(Moderately young, moderate income)	27.23	1,001.00		
N = 119	(8.11)	(112.59)		
4	·			
(Slightly older, middle income)	29.80	1,732.03		
N = 145	(8.19)	(250.14)		
Mean Squares:				
Between Groups	1,600.68	6,440,745.52		
Error	63.16	27,936.64		
Degrees of Freedom	(3,446)	(3,446)		
F	25.34	235.24		
Prob Value	.0001	.0001		

Exhibit 3
Marginal Values of Amenities across Cultures
Clustered by Respondent Profile

<u> </u>	Cluster	Cluster	Cluster	Cluster	F-	Prob.
Amenities	11	2	3	4	Score	Value
Dollar response for price of Jacuzzi	\$ 8.67	\$14.07	\$ 9.26	\$ 8.55	3.15	.025
Dollar response for price of exercise	0.00	0.00	0.50	44.00	4.40	
area	8.66	8.86	8.58	11.06	1.46	.180
Dollar response for price of open patio	8.28	11.58	8.14	10.04	1.29	.276
Dollar response for price of covered						
patio	7.56	14.17	7.71	8.90	6.11	.0005
Dollar response for less 100 square feet	36.02	32.85	36.70	37.90	0.43	.731
Dollar response for						
extra 100 square feet	28.05	29.45	29.04	26.73	0.32	.809
Dollar response for microwave	12.22	10.93	14.45	11.55	1.47	.222
Dollar response for ceiling fans	14.04	15.12	13.73	14.21	0.16	.923
Dollar response for fireplaces	17.03	19.42	17.20	16.85	0.26	.853
Dollar response for washer/dryer			25		5,25	1000
connection	12.31	20.63	18.28	14.41	5.96	.0006
Dollar response for washer/dryer	23.42	26.86	24.63	23.46	0.56	.640
Dollar response for	20.12	20.00	21.00	20.40	0.00	.040
covered parking	8.90	18.18	9.32	12.42	9.90	.0001
Dollar response for price of pool	16.92	19.93	13.51	13.15	2.32	.075
Age of apartment					2.02	
complex	5.97	4.68	4.99	4.53	3.70	.012
Size of unit in square feet	808.98	90.20	752.87	825.83	8.64	.0001

expected, the entire group was a relatively young group because of the previously mentioned tendency for people to purchase homes as they get older. These four distinct clusters are probably not atypical of what might be found in any metropolitan area.

The results contained in Exhibit 3 revealed that certain clusters have more price sensitivity or elasticity than others. This increased level of price elasticity is not the result of regression coefficients based upon secondary project input factors, but is based upon "stated preferences" of those actually consuming the service stream of benefits, i.e., the apartment dweller. As can be ascertained from an examination of the *F*-scores and Prob Values, some amenity values are quite significantly different across clusters, while others are not.

Exhibit 3 reveals five specific factors where differences exist. The findings can be summarized as follows:

- 1. Older people with higher incomes will pay more for a Jacuzzi.
- 2. Only the oldest, most upper-income group will pay more for a covered patio.
- 3. While differences exist, there is no clear pattern for washer/dryer connections.
- 4. As was the case for covered patios, the oldest and most upper-income group will pay the most for covered parking.
- 5. Size of unit is positively related to income and age.

Exhibits 4 and 5 present the results of the clusters of the sample by apartment size and monthly rent, along with Scheffe's Tests of the hedonic responses. Again, district clusters do exist and in fact there are more variables with higher *F*-scores in Exhibit 5 than Exhibit 3.

The information in Exhibit 5, in fact, indicates that larger dollar amounts can in some cases be charged for selected amenities in certain market segments.

Benefits of Research

This type of research and concomitant modeling can be of substantial benefit to apartment owners and portfolio managers. By gathering demographic profiles of their residents, they can selectively increase rents for those distinct groups (clusters) which, in the local area, have indicated a willingness to pay more for selected amenities. The manager can collect local responses and then selectively identify those clusters relevant to his/her specific complexes.

Exhibit 4
Summary of Cluster Analysis Results:
Clustered by Rent and Size

	Response Average & Standard Deviation			
Cluster	Mean Size	Rent		
1				
(Larger, Expensive Apartments)	705.94	356.11		
N = 63	(58.539)	(52.67)		
2				
(Small, Inexpensive Apartments)	587.24	245.85		
N = 104	(81.413)	(42.83)		
3				
(Large, Expensive Apartments)	1,063.02	456.32		
N = 68	(75.75)	(50.61)		
4	•			
(Larger, Inexpensive Apartments)	868.88	331.18		
N = 211	(74.08)	(63.23)		
Mean Squares:				
Between Groups	3,437,368.39	619,391.15		
Error	5,564.27	3,110.56		
Degrees of Freedom	(3,446)	(3,446)		
F	617.76	199.13		
Prob Value	.0001	.0001		

Exhibit 5

Marginal Values of Amenities across Cultures
Clustered by Rent and Size

Amenities	Cluster 1	Cluster 2	Cluster 3	Cluster 4	F- Score	Prob. Value
Dollar response for Jacuzzi	\$ 9.72	\$ 9.10	\$ 9.15	\$ 9.50	.06	.982
Dollar response for exercise area	9.25	9.08	10.67	9.14	.41	.746
Dollar response for open patio	10.06	9.32	6.36	9.86	1.51	.215
Dollar response for covered patio	9.24	8.17	5.91	9.67	3.09	.027
Dollar response for less 100 square feet	38.42	33.57	41.56	35.48	1.48	.219
Dollar response for extra 100 square feet	34.12	20.94	32.66	28.20	6.34	.0003
Dollar response for microwave	16.68	9.34	16.09	11.37	7.06	.0001
Dollar response for ceiling fans	17.12	10.60	16.19	13.96	5.79	.0007
Dollar response for fireplaces	22.63	9.14	21.67	17.97	11.23	.0001
Dollar response for washer/dryer						
connection	17.07	11.05	18.55	16.28	4.95	.002
Dollar response for washer/dryer	25.08	17.68	30.39	24.43	8.83	.0001
Dollar response for covered parking	10.95	8.62	10.08	12.87	3.49	.016
Dollar response for pool	14.03	13.30	12.80	17.02	1.49	.216
Respondent's Age	28.02	25.71	27.67	28.39	2.36	.071
Household monthly income	1,313.46	1,102.97	1,485.80	1,311.78	4.24	.006

While the results of our research found ony a few instances where cluster differences existed, these small numbers can have a significant impact on rental rate maximization. For instance, if a particular portfolio manager can identify, via the type of research methodology presented here, even 10% or 15% of the units he/she manages where rents can be increased by \$20 per month, and if that manager has 5,000 units under his/her control, this would represent an increase of between \$120,000 and \$180,000 per year in gross rental income.

The illustration provided in Exhibit 6 is presented to demonstrate the potentially dramatic effects of implementing the techniques presented in this paper. As is obvious, the benefits of identifying groups that are willing to pay marginally higher rents for selected amenities not previously provided by the complex can be quite significant.

The illustration in Exhibit 6 reveals that in a typical apartment complex of 200 units, the addition of a restricted-use Jacuzzi and covers for fifty parking places can have a dramatic

Exhibit 6 Sample Cost/Benefit Analysis Jacuzzi/Covered Parking Installation 200-Unit Complex

Additional Income Stream: From Jacuzzi	\$ 1,000 ¹
From covered parking	1,000 ²
Total per month	\$ 2,000 × 12
Total per year	\$ 24,000 × 10 ³
Total value added	\$240,000
Less Cost Estimates: For Jacuzzi For covered parking	\$ 5,000 25,000
Total cost	\$ (30,000)
Total Net Present Value	\$210,000

¹Assumes that 100 tenants can be identified who will pay an additional \$10 per month if a Jacuzzi is installed.

effect on both net operating income and property value. The sample complex is assumed to have been lacking in these features prior to the implementation of the research. It is further assumed the amenities are installed subsequent to the research, and Exhibit 6 reports the resulting changes in income and value.

These increases are certainly worth achieving. As it often happens in research, we are investigating small changes in small segments of the market. When this type of analysis is applied to real estate, these small segments can represent large dollar amounts. More importantly, the increases are not just in gross income, but also in net operating income. As demonstrated, this type of research can be of special benefit because each \$100,000 in extra net operating income represents \$1,000,000 in value at a 10% capitalization rate.

It is evident that even if the research and analysis had a cost of \$10,000, the potential benefits would result in a positive net present value. The benefit of using cluster analysis in conjunction with traditional surveys of consumers of rental housing is that distinct submarkets can be defined. Defining such submarkets insures that the hedonic responses being examined are from a distinct, homogeneous group. Such groups can then be specifically targeted.

As this was an illustration of the use of cluster analysis, relatively few demographic variables were used. In applying the technique to other markets, other variables the analyst feels are relevant may be added. Additionally, while we have segmented across certain demographic profiles, the market analyst in each local area should effect numerous profile combinations until those which will be of greatest benefit in that particular geographic area have been identified.

²Assumes that 50 tenants can be identified who will pay an additional \$20 per month if a covered parking stall is provided.

³Assumed capitalization rate of 10%. A variance from this assumed rate, for local market differences, would not significantly alter the decision process.

Conclusions

The research reported here utilizes cluster analysis to segment apartment dwellers into specific demographic profiles, with the objective of identifying those with higher marginal utility preferences for selected project or unit amenities. The purpose of the paper was to illustrate how an analyst might identify those market segments, via hedonic responses, that were willing to pay more or less for selected amenities—with the object of rental rate optimization.

The research revealed that distinct clusters do exist and that in certain cases, the "willingness to pay," i.e., the price elasticity, is different across clusters. This identification of market segments with different pricing elasticity can allow the portfolio manager to optimize rental rate structures in multifamily properties, thus, maximizing the value of the entire portfolio.

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