Abstract. **This article provides a matrix representation of the adjustment grid estimator. From this representation, one can invoke the Gauss-Markov theorem to examine the efficiency of ordinary least squares (OLS) and the grid estimator that uses OLS estimates of the adjustments (the "plug-in" grid method). In addition, this matrix representation suggests a generalized least squares version of the grid method, labeled herein as the total grid estimator. Based on the empirical experiments, the total grid estimator outperformed the plug-in grid estimator, which in turn outperformed OLS.**

Introduction

Appraisers commonly use the grid adjustment method to value property. They select, weight and adjust comparable properties using their prior information to estimate the value of the subject property. Such prior information may come from experience, a matched pair analysis on other properties or from the use of other estimators. Evidently, this *modus operandi* performs well.[1]

Due to the difficulty of quantifying appraisers' priors, individuals investigating the grid method have usually resorted to ''plugging in'' the estimates from some procedure such as ordinary least squares (OLS) to operationalize the adjustment process.[2] For clarity, I label this the plug-in grid (PG) method which denotes a specific implementation of the usual adjustment grid method, labeled the general grid (GG) method.[3] The PG method predictions have not dominated predictions from OLS computed using the unadjusted data.[4]

Examination of the literature gives rise to some disquieting thoughts. If the PG method outperforms OLS, why use OLS estimates in its construction? If the PG method underperforms OLS, why use it?

This article develops an estimator that fully executes the grid method concept. If one believes that using comparable properties can reduce the prediction errors on subject properties, one should use this information in estimating the parameters. Lower errors should increase estimation accuracy.

The article provides a matrix representation of the GG estimator for an entire sample whereby each subject property is differenced from its comparables. Naturally, most subject properties will also serve as comparables for other subject properties. The

*Department of Finance, Louisiana State University, Baton Rouge, LA 70803 or Kelleypace@compuserve.com.

matrix representation of the GG estimator gives rise to the total grid (TG) estimator, a generalized least squares (GLS) procedure.

The matrix formulation of the GG estimator also enables one to express it as a linear estimator. Hence, by the Gauss-Markov theorem, the PG estimator does not reach full efficiency when the conditions underlying OLS hold. Moreover, if differencing with comparables lowers exsample errors, this implies the existence on non-i.i.d. (independent and identically distributed) errors. For such an error structure OLS, and hence the PG estimator based on OLS, does not reach full efficiency. In contrast, the TG estimator, with its GLS underpinnings, potentially could reach full efficiency.

To illustrate the potential of the TG estimator, I used 442 properties spread over 24 areas in Memphis to estimate: (1) a nonspatial model via OLS; (2) a spatial model (24 area dichotomous variables) via OLS; and (3) an implicitly-spatial model via the TG estimator. As the results show, the TG estimator using a nonspatial model closely approaches the performance of OLS on the spatial model.

As an initial demonstration of the utility of the ideas presented, the spatial specification of the comparables allowed the PG estimator to display a 15% lower average exsample RMSE than OLS. Exsample error resampling trials showed the even greater potential advantages of TG estimation. For example, OLS ignoring the spatial information produced 56.3% higher and the PG method 32.9% higher average RMSE than the TG estimator in the exsample error trials. Only 2% of the OLS and 12.5% of PG method predictions proved superior to the TG method's predictions in terms of median absolute errors over 1000 trials.

In this article, section two develops the theory behind the general, PG and TG estimators; section three provides an empirical illustration of the potential gains; and section four concludes with the key results.

Grid Estimation

This section provides a theoretical development and examination of the various grid estimators. The first part reviews the GG and OLS estimators; the second part provides a matrix formulation of the GG method; the third part develops the TG estimator, and part four discusses the efficiency of the OLS, PG and TG methods.

The GG and OLS Estimators

The class of linear predictors approximates the subject property value y_0, a scalar, by linearly weighting the p characteristics in the p by 1 vector x_0 by their estimated value in the p by 1 vector $\hat{\beta}$ as in Equation (1). Assume x_0, y_0 are in deviations form (have 0 mean) to avoid the complication of an intercept. The estimates $\hat{\beta}$ could simply represent a particular individual's opinion concerning the value of the various property characteristics.

$$\hat{y}_0 = x_0'\hat{\beta}. \tag{1}$$

Suppose one had m "comparable" properties where $m=1, 2, 3 \ldots$. Let x_i denote a p by 1 vector of the i^{th} comparable's characteristics, y_i denote the i^{th} comparable's sales price and c_i denote the weight given to the i^{th} comparable in Equation (2).

$$\begin{aligned} y_{comps}&=(y_1, y_2, \ldots, y_m)' \quad (m \text{ by } 1) \\ X_{comps}&=(x_1, x_2, \ldots, x_m)' \quad (m \text{ by } p) \\ c_0&=(c_1, c_2, \ldots, c_m)' \quad (m \text{ by } 1) \\ 1&=c_0'[1]=c_1+c_2+\ldots+c_m. \end{aligned} \tag{2}$$

An alternative linear predictor, the GG method, uses $\tilde{y}_0$ to approximate the subject property value y_0 as a function of the weighted average of the m comparable properties prices, $\bar{y}_{comps}=c_0'y_{comps}$, and characteristics, $\bar{x}_{comps}=(c_0'X_{comps})'$, a p by 1 vector.

$$\tilde{y}_0=\bar{y}_{comps}+(x_0-\bar{x}_{comps})'\hat{\beta}=x_0'\hat{\beta}+c_0'(y_{comps}-X_{comps}\hat{\beta}). \tag{3}$$

Thus, the GG method involves the prediction of y_0 coupled with an adjustment based upon a weighted average of the prediction errors on comparable properties.

A Matrix Formulation of the GG Estimator

Equation (3) specifies the GG prediction for a particular property. Could one express the GG method predictions for all the sample properties in matrix form? Generalizing Equation (3) yields Equation (4),

$$\tilde{Y}=X\hat{\beta}+C(Y-X\hat{\beta}), \tag{4}$$

where C represents an n by n comparable weighting matrix with 0s on the diagonal (the subject property cannot predict itself) and whose row elements (weights) sum to 1 (the equivalent of the weights summing to 1 in Equation (2)). Assume the data Y, X are in deviation form (subtraction of respective means). Let Y denote an n by 1 vector of dependent variable observations with a 0 mean and X an n by p matrix of nonconstant independent variables with 0 means. As well-known, using data in deviation form does not affect the slope coefficients of linear regression models such as OLS or GLS. One can always recover the same intercept after estimating the slope coefficients.[5] Thus, $p+1$ equals k, the total number of variables.

$$\text{(a) } diag(C)=\underset{(n \text{ by } 1)}{[0]} \qquad \text{(b) } \underset{(n \text{ by } n)}{C}\ \underset{(n \text{ by } 1)}{[1]}=\underset{(n \text{ by } 1)}{[1]}. \tag{5}$$

The non-zero entries on the i^{th} row of C represent the comparables for the i^{th} subject property. For example, the i^{th} row of C might appear thus,

$$C_{i,1:n}=\left(\frac{1}{m_i}, 0, \frac{1}{m_i}, 0, 0, \frac{1}{m_i}, \frac{1}{m_i}, 0, \frac{1}{m_i}, 0, \cdots, 0\right)$$

where m_i, the number of comparables for the i^{th} property, equals 5. Multiplying the 1 by n row vector $C_{i,1:n}$ by a n by 1 column vector [1] would yield 1 as assumed in Equation (5). This represents the equiweighted case where each of the five comparables has a weight of 1/5. The next section provides a more extensive example of the structure of C for the empirical example.

Note, the specification of C takes into account spatial effects. Rather than modeling the neighborhood effects in X, one can difference away a large portion of these by the use of C. Differencing out the spatial effects represents a viable alternative to modeling how these affect the mean. This parallels time series—one can use time as an independent variable or one can model the dependency in the error term. Modern time series generally concentrates on the latter rather than the former procedure.

The TG Estimator

The matrix formulation of the GG estimator suggests one might find a better estimate of β than one from, for example, OLS. Writing the matrix formulation of the GG estimator in Equation (4) as a function of the parameters β and a random error term yields Equation (6).[6]

$$Y = X\beta + C(Y - X\beta) + \varepsilon. \tag{6}$$

Suppose ε possesses an independent, 0 mean error structure as specified by Equation (7).

$$\text{(a) } E(\varepsilon) = 0 \qquad \text{(b) } E(\varepsilon\varepsilon') = \sigma^2 I. \tag{7}$$

One can rearrange Equation (6) to form Equation (8), where the dependent and independent variables appear on opposite sides.

$$(I - C)Y = (I - C)X\beta + \varepsilon. \tag{8}$$

Redefining the independent and dependent variables in Equation (8) leads to Equation (9),

$$\begin{aligned} Y_c &= X_c\beta + \varepsilon \\ Y_c &= (I - C)Y = WY \\ X_c &= (I - C)X = WX, \end{aligned} \tag{9}$$

where $(I - C) \equiv W$, a weight matrix with 1s on the diagonal. Provided X_c is nonsingular, one could estimate an OLS regression on the redefined variables in Equation (9).[7]

$$\tilde{\tilde{\beta}} = (X_c'X_c)^{-1}X_c'Y_c. \tag{10}$$

As well-known, Equation (10) represents a GLS estimator.[8] Since $\tilde{\tilde{\beta}}$ totally executes the ideas of comparable weighting and differencing used by the GG estimator, I call $\tilde{\tilde{\beta}}$ the TG estimator.

Assuming the TG estimate exists, one could predict Y via Equation (11).

$$\tilde{\tilde{Y}}=X\tilde{\tilde{\beta}}+C(Y-X\tilde{\tilde{\beta}}). \tag{11}$$

Furthermore, Equaiton (11) leads to the TG estimated errors in Equation (12).

$$\tilde{\tilde{e}}=Y-\tilde{\tilde{Y}}=Y_c-X_c\tilde{\tilde{\beta}}=W(Y-X\tilde{\tilde{\beta}})=Y-X\tilde{\tilde{\beta}}-C(Y-X\tilde{\tilde{\beta}}). \tag{12}$$

Analogous to Equations (11) and (3), one could compute the TG exsample predicted price for a particular property by Equation (13).

$$\tilde{\tilde{y}}_0=x_0\tilde{\tilde{\beta}}+c_0'(y_{comps}-X_{comps}\tilde{\tilde{\beta}}), \tag{13}$$

where c_0 represents the m by 1 column vector of comparable weights for the subject property.[9] In this case, the m comparable observations (y_{comps}, X_{comps}) could have been used in the formation of $\tilde{\tilde{\beta}}$ (*i.e.*, particularly rows of Y or X) or could represent exsample observations. The latter seems closer in spirit to the way appraisers operate and so I use this in computing the exsample errors in the empirical example.

Efficiency of the Grid and OLS Estimators

Can one compare the GG, PG, TG and OLS estimators? Suppose Equation (14) describes the model generating the data where ε has the 0 mean, i.i.d. properties given by Equation (7).

$$(\Theta Y)=(\Theta X)\beta+\varepsilon. \tag{14}$$

By the Gauss-Markov theorem, if Θ equals I, OLS is the best linear unbiased estimator (BLUE). In repeated sampling, none of the grid estimators could outperform OLS under the assumed conditions.[10] However, if Θ does not equal I, insert OLS just using X, Y is not BLUE. By the Gauss-Markov theorem, GLS is BLUE given knowledge of Θ. If $\Theta=W$, the TG estimator is BLUE.

This exposes the inefficiency of the PG estimator. If OLS assumptions such as Equation (7) prevail, the PG estimator predictions are inefficient. If differencing subject properties using comparable properties induces behavior described by Equation (7), the PG estimator predictions are inefficient since it uses the OLS estimate (nonoptimal for $\Theta \neq I$).[11]

In summary, the PG method possesses a fundamental illogic. If it outperforms OLS, why use OLS estimates in its construction? If the conditions underlying OLS exist, why use the PG estimator? The TG estimator, with its GLS underpinning, avoids this dilemma.

Naturally, estimates of β produced by other estimators for the PG do not attain full efficiency. Hence, the criticism could apply to other estimators as well. However, when estimates of β come from prior information, as they do with appraisers using the GG method, the optimality results from the Gauss-Markov theorem do not apply. Correct prior information can allow the appropriate estimators to outperform OLS or GLS.

An Empirical Example

This section provides an empirical illustration of the gains in prediction accuracy obtainable through the use of the TG method. The first section presents the data; the second section discusses the models; the third section specifies the interactions among the subject and comparable properties; and the fouth section performs the sample estimation; and the last section examines the exsample performance of the TG, PG and OLS methods.

Data

The sample data are from the Memphis Multiple Listing Service's (MLS) *Multiple Listing Book* published by the Memphis Board of Realtors for January 1987. The actual transactions price are from the cumulative index of sold properties. Characteristics data on each of the selected properties came either from this index or from the original listing description. The sample contains observations on 442 single-family dwellings sold within the previous six-month period with complete information on each variable. Stratified random sampling, whereby the proportion of properties in the sample from the 24 different city areas matched the population proportion in these areas, was used to insure a truly representative sample of the population of sold properties. As a result, the sample means of both the dependent and independent variables closely match their population counterparts.

Models and Variables

AREAID varies between 1 to 24 and represents districts within Memphis. I formed 24 dichotomous or dummy variables based upon *AREAID*. *CENTRAL AIR-CONDITIONING*, *WINDOW AIR-CONDITIONING*, *FIREPLACE*, *POOL*, *MASONRY EXTERIOR* and *SIDING* (aluminum or vinyl) are also dichotomous variables with 1 representing the presence of the characteristic. *KITCHEN AREA* and *OTHER AREA* (nonkitchen area) added together equal total area. *LOT AREA* denotes lot size in 1000 square foot units. *BATHS* denotes the number of bathrooms. To simplify the problem of singularity with the TG estimator when an intercept exists, I expressed the data in deviations form (subtracted away the means of the dependent and independent variables). This well-known procedure does not affect the slope coefficients of the OLS based models, the implicit intercept value, nor the degrees-of-freedom. Hence, β_1 appears implicitly in each of the following models. In the results, I will make reference to the following models.

Common Model: β_1*CAR PORT SPACES*$+\beta_2$*GARAGE SPACES*$+\beta_3$*CENTRAL AIR-CONDITIONING*$+\beta_4$*WINDOW AIR-CONDITIONING*$+\beta_5$*AGE*$+\beta_6$*OTHER AREA*$+$

β_7*KITCHEN AREA*$+\beta_8$*BATHS*$+\beta_9$*LOT AREA*$+\beta_{10}$*FIREPLACE*$+\beta_{11}$*POOL*$+$
β_{12}*MASONRY EXTERIOR*$+\beta_{13}$*SIDING*

Nonspatial Model: $\ln(PRICE)=\text{Common Model}+\varepsilon_{ns}$
Spatial Model: $\ln(PRICE)=\text{Common Model}+\alpha_{2-24}AREAID+\varepsilon_s$
Implicitly Spatial Model: $\ln(PRICE)=\text{Common Model}+\varepsilon_{is}$

The use of variables partitioning the property into its components results in a design with relatively low amounts of multicollinearity. Specifically, the condition number for the spatial design, which has the most variables, is 23.3. This constitutes borderline or moderate multicollinearity according to Belsley, Kuh and Welch (1980, p. 153). See Pace and Gilley (1993) or Gilley and Pace (1998) for a discussion of multicollinearity and suggested ways of ameliorating its impact on statistical valuation models. Also, see Pace (1998) for an extensive discussion of the variables in this model and their specification.

Specification of the Weighting Matrix W

The most important part of TG estimation lies in the specification of the weighting matrix W. For OLS, the equivalent is the identity matrix, I. Like I, the diagonal elements of $W=1$. Hence, each subject property (dependent and independent variables) receives equal weighting. In this problem, nonsubject properties sharing the same *AREAID* code are comparable. Hence, those with different *AREAID* codes have $W_{ij}=0$. For the comparable properties, the off-diagonal elements (W_{ij}) equal $-1/m_i$ where m_i represents the number of comparable properties. Hence, m_i equals the number of properties sharing the same *AREAID* code minus 1 (for the subject property). Alternatively, m_i represents the number of elements in the i^{th} row of W not equal to 0 or 1. This means the weighting matrix subtracts from the subject property dependent and independent variables the average of the comparable properties dependent and independent variables. For these data, W is an n by n symmetric, matrix.

As a small example, suppose $n=4$ and properties 1,3 and 4 share the same *AREAID* code while property 2 has no comparables.

$$C=\begin{matrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \end{matrix},\ W=(I-C)=\begin{matrix} 1 & 0 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} & 1 \end{matrix}$$

When multiplying WX or WY, W subtracts the average of the comparable properties from each subject property. Note, observation 2 does not have any comparables and so W would simply select the second row in the original data matrix.

As a further example, suppose $n=6$ and properties 1 and 2 share a common *AREAID* code while properties 3, 4, 5 and 6 also share a common *AREAID* code.

$$C=\begin{matrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \end{matrix}, \qquad W=(I-C)=\begin{matrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & 1 & -\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} \\ 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} & 1 \end{matrix}$$

When multiplying *WX* or *WY*, *W* subtracts row 2 from row 1 for the first observation. For the second observation, *W* subtracts row 1 from row 2. One can interpret the actions of *W* in row (or column 6) as subtracting the average of the comparable properties 3, 4 and 5 from property 6. One can interpret the actions of *W* in row (or column 4) as subtracting the average of the comparable properties 3, 5 and 6 from property 4. Naturally, this happens for both the dependent and independent variables. Thus, *W* simultaneously differences every observation with those sharing a common *AREAID* code.

One can define similar schemes for creating *C* or *W* for data with continuous location coordinates such as latitude and longitude. In fact, these techniques work even better for such data.[12] See Pace and Gilley (1997) for examples with complete locational data.

Sample Estimates

Exhibit 1 contains the estimates for the nonspatial, nonintercept variables based upon the entire sample of 442 properties for OLS on the nonspatial model, for OLS on the spatial model and for the TG method on the implicitly spatial model. OLS on the nonspatial model displays two sign violations relative to ex ante expectations and an insignificant coefficient for *Lot Area*.[13] In contrast, both OLS on the spatial model and the TG method on the implicitly spatial model display no sign violations. Moreover, both show a significant coefficient for *Lot Area*. Both OLS on the spatial model and the TG method on the implicitly spatial model exhibit similar coefficients, thus indicating that they effectively use the spatial information.

Both methods greatly outperform OLS on the nonspatial model. OLS on the nonspatial model shows 54.9% higher SSE than the TG method on the implicitly spatial model. In turn, OLS on the spatial model shows 12.9% less SSE than the TG method on the implicitly spatial model. In terms of median absolute error (MAE), OLS on the nonspatial model shows 29.8% higher MAE than the TG method on the implicitly spatial model. In turn, OLS on the spatial model shows 4.8% less MAE than the TG method on the implicitly spatial model.

Exsample Error Performance

To obtain an idea of the exsample error performance from the TG, PG and OLS estimators, I designed a resampling experiment involving 1000 iterations. In each iteration I randomly selected a group of *AREAID* numbers between 1 and 24. As the

Exhibit 1
Spatial OLS (24 Area Variables), Nonspatial OLS and Implicitly Spatial Total Grid Estimates

Independent Variable	Nonspatial OLS	Spatial OLS (24 Areas)	Implicitly Spatial TG
Car Port Spaces	0.0318	0.0427	0.0425
	2.5323	4.0478	4.0858
Garage Spaces	0.0889	0.0658	0.0662
	7.6880	7.0160	7.1204
Central Air-Conditioning	0.3500	0.3264	0.3229
	5.6259	6.4716	6.6985
Window Air-Conditioning	0.2244	0.2006	0.1998
	3.6146	4.0631	4.2394
Age	−0.0016	−0.0037	−0.0032
	−2.1397	−2.9132	−2.6423
Other Area	0.0002	0.0002	0.0002
	17.1959	15.1806	15.4662
Kitchen Area	0.0007	0.0006	0.0006
	5.1195	5.7751	5.8302
Baths	0.0891	0.0787	0.0810
	3.6941	4.1676	4.3814
Lot Area (in 1000 sf)	0.0009	0.0015	0.0015
	1.1239	2.2326	2.2044
Fireplace	0.0798	0.0578	0.0580
	4.2853	3.3818	3.4695
Pool	−0.0040	0.0115	0.0085
	−0.1158	0.4144	0.3066
Masonry Exterior	0.0525	0.0569	0.0580
	2.8609	3.8338	3.9596
Siding	−0.0487	0.0262	0.0243
	−0.9649	0.6575	0.6305
R^2	0.8728	0.9285	0.9179
SSE	9.6833	5.4459	6.2512
SSE Relative to Total Grid	1.5490	0.8712	1.0000
Median Absolute Error (MAE)	0.0909	0.0667	0.0701
MAE Relative to Total Grid	1.2977	0.9520	1.0000
n	442	442	442
k	14	37	37
Degrees-of-Freedom	428	405	405

number of groups and the number of properties within each group varied, the number of sample observations ranged from 28 to 423, with an average of 232. I computed sample estimates using the TG method on the implicitly spatial model and OLS on the nonspatial model. From these I calculated the corresponding exsample errors, as well as the exsample error from the PG method.

Exhibit 2 contains a variety of statistics describing the exsample errors. The first three columns present the level of the statistics and the last two columns express these relative to the TG statistic. OLS displayed 56.3% and the PG estimator 32.9% higher mean RMSE than the TG method. In only 19.6% of the iterations did the PG method yield lower RMSE than the TG method. In only 0.3% of the iterations did OLS on the nonspatial model yield lower RMSE than the TG method.

In addition, OLS displayed 36.3% and the PG estimator 12.5% higher average MAE than the TG method. In only 12.0% of the iterations did the PG method yield lower MAE than the TG method. In only 2.0% of the iterations did OLS on the nonspatial model yield lower MAE than the TG method.

As the difference between the median and mean RMSE figures shows, OLS and the PG method possess very long tails in their distribution relative to the TG method. The 99th percentile RMSE figures show this very clearly. For some small proportion of the iterations, OLS and the PG method (which uses OLS) perform very poorly relative to the TG method.

The reason for this lies in the nature of the experiment. While the experiment randomly picks areas (blocks of properties), this does not imply uniform coverage over the entire area. For example, the algorithm may randomly (albeit rarely) pick all

Exhibit 2
Resampled Exsample Error Statistics for the Total Grid, Plug-in Grid and OLS Estimators Across 1000 Iterations

Exsample Error Statistics	TG	PG	OLS	PG/TG	OLS/TG
Mean RMSE	0.14	0.18	0.21	1.33	1.56
Proportion of Iterations with Relative RMSE<1				0.20	<0.01
Min. RMSE	0.09	0.09	0.11	1.04	1.23
First Quartile RMSE	0.12	0.13	0.16	1.08	1.30
Median RMSE	0.13	0.14	0.17	1.06	1.29
Third Quartile RMSE	0.15	0.16	0.20	1.07	1.34
90th Percentile RMSE	0.16	0.18	0.24	1.08	1.43
95th Percentile RMSE	0.18	0.20	0.28	1.12	1.53
99th Percentile RMSE	0.21	1.51	1.49	7.07	7.00
Max. RMSE	0.30	2.00	1.93	6.67	6.41
Mean of Median|e|	0.08	0.09	0.11	1.13	1.36
Proportion of Iterations with Relative Median|e|<1				0.12	0.02

of the areas on one side of the city. In this case, OLS applied to the nonspatial model treats the other side of the urban area the same as the sample side. Hence, its estimates may prove quite poor. The PG method improves on these errors but inherits some of the OLS problems. The TG estimator, by differencing out the spatial element, arrives at better estimates. This experimental design exposes the weakness of the nonspatial model estimated via OLS, and by extension the PG method, when spatial information matters.

Conclusion

Attempts to mimic the appraiser's grid estimator have usually taken the OLS coefficient estimates from another sample to use in the comparable adjustment process. As the results of this article illustrate, these methods can improve over OLS. For example, OLS on the nonspatial model in the empirical illustration displayed 17.6% higher average RMSE in the exsample error trials than did the PG estimator.

However, the PG estimator employing OLS based adjustments seems somewhat incomplete. If the PG method outperforms OLS, why use OLS estimates in its construction? This article developed the TG estimator which uses the comparables in the actual coefficient estimation. The resulting GLS estimator outperforms OLS on the nonspatial model as well as the PG estimator using OLS based adjustments. For example, the PG estimator displayed 36.3% higher average RMSE in the exsample error trials than did the TG estimator. In only 12.5% of the exsample error trials did the PG method outperform the TG method in terms of MAE.

The grid estimator in the hands of appraisers works well. Essentially, their ability to inspect the subject and comparable properties allows them to difference away much of the prediction error (*i.e.*, accurately estimate C or W). In the past, the data collection, management and analysis problems more or less enforced the sole use of the grid method where judgment played the dominant part in the adjustment process. However, with the advent of geographic information systems, it should not prove difficult to formulate and compute TG estimates. TG estimates could help hone appraisers' priors based upon experience and other analyses.

From an academic standpoint, the use of a TG framework could aid in the development and testing of rules for comparable weighting and selection (*i.e.*, elements of C or W). For example, one could examine various metrics, spatial orderings or rules mimicking appraiser behavior. Modeling appraiser behavior could also prove important in making progress towards automating appraisals. If nothing else, an automated appraisal system should incorporate appraiser behavior as a control to use in comparing predictions from other estimators. More likely, understanding appraiser behavior would lead to improved predictions, especially when combined with more formal statistical techniques.

Other disciplines have also dealt with errors over space. Agricultural field research examines plots of land with different treatments to discern the effects of more fertilizer, water or sun. Naturally, adjacent plots (comparables) tend to have correlated errors. "Empirical evidence from agricultural field trials indicates that differenced data usually permit the use of a more parsimonious model than undifferenced data."[14] The TG estimator developed herein corresponds to spatial differencing. As in the case of agricultural field research, this differencing can produce a parsimonious yet effective model. The agricultural literature has examined a number of more complicated estimators involving variable amounts of differencing, moving average processes and higher orders of autocorrelation. Some of their experience may prove useful in real estate appraisal.[15]

The emerging field of spatial statistics, coupled with technological advances in geographic information systems, holds forth the promise of both more fully understanding real estate appraisal and improving its accuracy.

Notes

[1]For example, Dotzour (1988) found appraisals displayed about 10% root mean squared error (exsample) using corporate relocation properties. This error rate represents a difficult (but not impossible) target for other techniques to match.

[2]The rôle of judgment and art in appraisal makes it difficult to model how appraisers form and update their priors. For this reason, the academic papers have resorted to using various estimators. While many estimators would work, OLS has obvious theoretical and computational advantages. See Colwell, Cannady and Wu (1983), Lipscomb and Gray (1990) or Vandell (1991) for applications of the PG adjustment method.

Also, prior information enters into the selection and weighting of comparables. In fact, the appraisal profession has partially described the relevant prior information via a number of criteria. For example, part six of *Appraising Residential Properties* (1994) and Chapter 17 of *The Appraisal of Real Estate* (1992) contain discussions relevant to these issues. As *Appraising Residential Properties* (1994, p. 436) states, "... greater reliance is placed on comparables which have been sold most recently, are most similar to the subject, and are subject to the fewer price adjustments." The appraisal texts generally emphasize the rôle of the appraiser's judgment rather than deterministic algorithms, although these also mention the possibility of using regression to arrive at some of the adjustments (*e.g.*, *Appraising Residential Properties*, 1994, p. 425).

The real estate literature generally seeks more deterministic ways of approaching the appraisal problem. For example, Colwell, Cannady and Wu (1983), Isakson (1986), Vandell (1991), Gau, Lai and Wang (1992, 1994) and Green (1994) have proposed various comparable weighting and selection algorithms. These methods typically (1) estimate the distribution of price (β, σ, coupled with the assumption of normality) using an earlier sample of properties; and (2) optimize some objective function (minimum variance or minimum coefficient of variation) with respect to the comparable weights. Because these algorithms rely upon estimates of the distribution of prices, problems in (1) affect the results in (2). The results herein pertain to (1).

[3]The term "plug-in" comes from statistics where it denotes the operation of substituting an estimated quantity for some unknown parameter in a statistic.

[4]For example, in a comparison of OLS and the grid method, Kang and Reichert (1991) found two markets where the PG method improved over OLS and one market where it did not.

[5]Nothing is gained or lost by this procedure. I used the same degrees-of-freedom as if I had estimated an explicit intercept.

[6]This resembles the development of simultaneous autoregressive estimators. See Ripley (1981, pp. 88–98). Pace and Gilley (1998) have estimated a simultaneous autoregression for the data

used in Can (1989, 1992). As in this context, employing spatial information in estimation yields substantial benefits.

[7]The weight matrix, *W*, will often have less than full rank. Although $n \geq \text{rank}(W)$, as long as $\text{rank}(W) \geq k$, WX can have full rank. However, one cannot include an intercept in *X*, as it would turn into a vector of 0s after differencing and make X_c singular.

[8]If one estimated elements of *C*, this would become an estimated GLS estimator (EGLS). At this stage, I assume known elements in *C* or some algorithm which does not involve goodness-of-fit for specifying *C*.

[9]Before this was an *n* by 1 vector since any of the *n* sample properties could serve as a comparable property. For m_0 exsample properties, one only needs m_0 weights.

[10]See Judge et al. (1985) for a more detailed description of the relevant assumptions and for a proof of the Gauss-Markov theorem.

[11]Pace (1996) provided more specific demonstrations for the asymptotic efficiency of the PG estimator relative to OLS when the conditions underlying OLS hold. In such circumstances, the PG estimator with a small number of comparables displays substantially more variation than OLS. For example, with three equi-weighted comparables, the PG estimator has 33% higher variance than OLS.

[12]Having only area dummies available imposes a block diagonal structure on the variance-covariance matrix and thus does not include potentially valuable interblock spatial information. The spatial approach developed here results in estimates close to those from the spatial dummy approach. However, relative to the dummy variable approach, the spatial approach weights less heavily areas with fewer observations.

[13]See Pace and Gilley (1993) for a discussion of *ex-ante* expectations in semi-log models.

[14]See Cressie (1993, p. 340).

[15]Actually, the ‘‘plug-in’’ grid method corresponds to a stage in the Papadakis NN estimator (1937). See Cressie (1993, pp. 338–45).

References

Appraisal Institute, *Appraising Residential Properties*, Second edition, Chicago: Appraisal Institute, 1994.

Appraisal Institute, *The Appraisal of Real Estate* Tenth edition, Chicago: Appraisal Institute, 1992.

Belsley, D. A., E. Kuh and R. E. Welch, *Regression Diagnostics: Identifying Influential Data and Source of Collinearity*, New York: John Wiley, 1980.

Can, A., *Modeling Spatial Variation in Housing Price Determinants*, Ph.D. dissertation, Ohio State University: Columbus, OH, 1989.

——, Specification and Estimation of Hedonic Housing Price Models, *Regional Science and Urban Economics*, 1992, 22, 453–74.

Colwell, P. F, R. E. Cannaday and C. Wu, The Analytical Foundations of Adjustment Grid Methods, *Journal of the American Real Estate and Urban Economics Association*, 1983, 11, 11–29.

Cressie, N. A., *Statistics for Spatial Data*, Revised edition, New York: John Wiley, 1993.

Dotzour, M. G., Quantifying Estimation Bias in Residential Appraisal, *Journal of Real Estate Research*, 1988, 1–12.

Gau, G. W., T.-Y. Lai and K. Wang, Optimal Comparable Selection and Weighting in Real Property Valuation: An Extension, *Journal of the American Real Estate and Urban Economics Association*, 1992, 20, 107–24.

——, A Further Discussion of Optimal Comparable Selection and Weighting, and A Response to Green, *Journal of the American Real Estate and Urban Economics Association*, 1994, 22, 655–63.

Gilley, O.W. and R. K. Pace, Improving Hedonic Estimation with an Inequality Restricted Estimator, *Review of Economics and Statistics*, 1995, 77, 609–21.

Green, R. K., Optimal Comparable Weighting and Selection: A Comment, *Journal of the American Real Estate and Urban Economics Association*, 22, 647–54.

Isakson, H. R., The Nearest Neighbors Appraisal Technique: An Alternative to the Adjustment Grid Methods, *Journal of the American Real Estate and Urban Economics Association*, 1986, 14, 274–86.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lutkepohl and T.-C. Lee., The Theory and Practice of Econometrics, Second edition, New York: John Wiley, 1985.

Kang, H.-B. and A. K. Reichert, An Empirical Analysis of Hedonic Regression and Grid-Adjustment Techniques in Real Estate Appraisal, *Journal of the American Real Estate and Urban Economics Association*, 1991, 19, 70–91.

Lipscomb, J. B. and J. B. Gray, An Empirical Investigation of Four Market-Derived Adjustment Methods, *Journal of Real Estate Research*, 1990, 5, 53–66.

Memphis Board of Realtors, *Multiple Listing Book 2*, Memphis, 1987.

Pace, R. K., Relative Efficiencies of the Grid, OLS, and Nearest Neighbor Estimators, *Journal of Real Estate Finance and Economics*, 1996, 13, 203–18.

——, Appraisal Using Generalized Additive Models, *Journal of Real Estate Research*, 15, 77–99.

Pace, R. K. and O.W. Gilley, Improving Prediction and Assessing Specification Quality in Non-Linear Statistical Valuation Models, *Journal of Business and Economics Statistics*, 1993, 301–10.

——, Using the Spatial Configuration of the Data to Improve Estimation, *Journal of the Real Estate Finance and Economics*, 1997, 14, 333–40.

——, Generalizing the OLS and the Grid Estimator, *Real Estate Economics*, 1998, 26, 331–47.

Papadakis, J. S., Methode Statistique Pour Des Experiences Sur Champ, Institut d'Amelioration des Plantes a Thessaloniki, *Bulletin Scientifique*, 1937, 23.

Ripley, B. D., *Spatial Statistics*, New York: John Wiley, 1981.

Vandell, K. D., Optimal Comparable Selection and Weighting in Real Property Valuation, *Journal of the American Real Estate and Urban Economics Association*, 1991, 19, 213–39.

My thanks to Otis Gilley, two anonymous referees and the editors for their valuable comments. I gratefully acknowledge support from the University of Alaska, the University of Connecticut and Louisiana State University.