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# Shopping Centers 


#### Abstract

This article explains why a profit-maximizing developer may include multiple, competing outlets in a shopping center. While competing outlets presumably dissipate potential profits, thereby lowering aggregate rents that the developer can extract, the presence of shopping externalities causes the developer to be interested not just in individual store profits, but also in the traffic they generate throughout the center. And since competition among identical stores increases traffic, it can create an offsetting advantage that favors multiple outlets. The article provides a theoretical analysis of this problem and illustrates its implications for tenant mix by applying the theory to the problem of filling a vacant store. The paper concludes by explicitly relating the analysis to Brueckner's (1993) model of the optimal allocation of space in shopping centers.


## Introduction

A nascent literature on the economics of shopping centers has examined the importance of inter-store externalities, or spillover shopping, on the structure and composition of centers. The seminal theoretical analysis by Brueckner (1993) focused on the question of how a developer should allocate space in a center among stores whose sales are interrelated through this externality. ${ }^{1}$ According to Brueckner's analysis, the developer should allocate space to a particular store until that store's marginal revenue from an additional square foot equals the marginal cost of space minus the marginal increase in sales enjoyed by all other stores due to the spillover effect. Thus, stores that confer large external benefits on other stores in the center should receive more space, all else equal.

Another implication of this shopping externality is that a profit-maximizing developer may find it profitable to include multiple outlets of a given type of store in a center. The data in Exhibit 1, for example, show that for large shopping centers, there are on average multiple outlets for a wide variety of stores. Standard location theory cannot explain this observation because a developer controls entry into the center rather than there being free entry. Combining this with the fact that location in a shopping center confers some spatial monopoly power on stores suggests that the developer can extract

[^0]Exhibit 1
Number of Outlets per Shopping Center by Store Type: 1995

|  | Type of Shopping Center |  |
| :--- | :--- | :--- |
| Tenant Classification | Super-Regional | Regional |
| General merchandise | 2.4 | 0.0 |
| Food | 1.6 | 1.0 |
| Food services | 1.3 | 0.9 |
| Women's clothing | 14.2 | 8.3 |
| Men's clothing | 4.5 | 2.2 |
| Shoes | 11.7 | 5.4 |
| Home appliances | 1.5 | 0.9 |
| Music | 2.3 | 1.5 |
| Book stores | 2.0 | 1.4 |
| Jewelry | 5.9 | 3.9 |
| Eyeglass stores | 2.2 | 1.5 |

Source: Computed from data in The Dollars and Cents of Shopping Centers: 1995, The Urban Land Institute, Washington, DC.
the highest possible rent from a given type of store by allowing it to exploit fully that monopoly power. Multiple outlets, however, would presumably dissipate some of the joint profits, thereby lowering the aggregate rents that the developer could extract. The shopping externality provides an answer to this puzzle by introducing an offsetting effect to the developer's desire to maximize the profit earned by each store individually (which, as noted, favors one outlet of each type). Specifically, the externality causes the developer to be interested not just in an individual store's profits, but also in the "traffic" it generates, since more traffic will produce larger spillover benefits (and hence profits) for other types of stores. Since we know from basic microeconomic theory that monopolists maximize profits by limiting output (or traffic), a single outlet earning monopoly profits will minimize the external benefits for other types of stores. As more outlets enter, however, output will increase due to the greater competition. The optimal number of outlets therefore balances these two effects in terms of their impact on developer profits.

The first section of this paper develops this argument in greater detail. The analysis is distinguished from that in Brueckner by focusing on output as the choice variable for stores, and the number of outlets of a given type as the choice variable for the developer, while holding space per store fixed. In contrast, Brueckner focused on space as the only choice variable. ${ }^{2}$ The emphasis on output seems to be a more natural way of highlighting the trade-off noted above between inter-store competition versus complementarity. Further, a store's choice of output would seem to be a more fundamental determinant of sales than space. After completing the theoretical analysis, we illustrate the results with a numerical example of a common problem facing a shopping center manager: namely, how to fill a vacant store.

In the next section we take a first step towards integrating space and output in the model by specifying a demand function that says that the price a store can charge is negatively related to its own output, positively related to the output of other stores through the shopping externality, and also positively related to the amount of space allocated to the store. From this demand curve, we derive Brueckner's sales function, which depends only on the space allocated to all stores in the center. The analysis thus serves as a foundation for Brueckner's basic results regarding the optimal allocation of space to various types of stores. The final section offers concluding remarks.

## Competition versus Complementarity in Output

This section develops a simple model of the optimal number of competing outlets that a profit-maximizing developer should include in a shopping center characterized by inter-store shopping externalities. Shopping externalities arise from the spatial clustering of stores, which allows multi-stop shoppers to economize on transportation costs. Although downtown shopping districts also offer this advantage, shopping center developers can exploit it fully (and thereby make a profit) by optimally planning the selection and physical configuration of stores. ${ }^{3}$ Clearly, including a variety of stores in a center is essential for achieving the maximum extent of this benefit. However, the question we are concerned with in this section is whether it ever makes sense for a developer to include competing stores, or stores selling the identical product, in the center.

In order to focus on this question, we assume that competing stores are perfect substitutes and therefore do not confer external benefits on one another. That is, we assume that shopping spillovers only occur across stores that are either complements or imperfect substitutes. Thus, for example, two shoe stores would not be regarded as competing stores for the purposes of the model if they offered differentiated products. In fact, adding a second shoe store would likely result in some combination of spillover benefits and competition if the two stores stocked some identical and some differentiated products. We abstract somewhat from this case by assuming that a given type of store sells a single good which is either a perfect substitute or a differentiated product from an existing store, but not both.

Brueckner's (1993) analysis of shopping externalities explains why it is optimal for the developer to include complementary outlets in the center. Our object in this section is to show that it may also be optimal for a developer to include multiple outlets selling identical products (e.g., more that one book store), even though there is no complementarity between them. Thus, while our assumption of single-product stores selling either perfect substitutes or differentiated products is somewhat unrealistic, if anything it biases our model away from inclusion of multiple stores selling the same product.

For now, the model focuses exclusively on store output as the choice variable; the amount of space allocated to each store is assumed to be fixed. In the next section we add space as a choice variable.

## Theoretical Analysis

We consider a very simple model in which a shopping center developer chooses the optimal number of outlets, $n$, for a given type of store. As noted, we assume stores sell a single good that defines their type. We assume that the shopping center creates a spatial monopoly for stores of this type, so they face a downward sloping, inverse demand curve of the form:

$$
\begin{equation*}
P=a-b Q, \tag{1}
\end{equation*}
$$

where $P$ is price, $Q$ is aggregate output across the $n$ identical outlets, and $a$ and $b$ are positive constants. ${ }^{4}$ (We assume the demand curve is linear for analytical simplicity.) If we let $q$ be output per outlet, then $Q=n q$. Note that $n$ is constrained to be an integer greater than or equal to one. Although we will focus below on inter-store externalities conferred by this type of store on other types of stores in the center, this store enjoys reciprocal benefits, which we assume are embedded in the parameter $a$. (We examine this aspect of the model in detail in the next section.)

We assume that the shopping center developer is able to extract a rent from each store equal to its expected economic profits. ${ }^{5}$ The rental lease is set up-front, however, so in making their output decisions, we assume stores view the rent $R$ as a fixed cost. Thus, outlet $i(i=1, \ldots, n ; n \geq 1)$ chooses $q_{i}$ to maximize its profit, or net sales:

$$
\begin{equation*}
\pi_{i}=P q_{i}-c q_{i} \tag{2}
\end{equation*}
$$

where $c$ is the (constant) variable cost of output. ${ }^{6}$ Substituting for $P$ from Equation (1) yields:

$$
\begin{equation*}
\pi_{i}=(a-b Q) q_{i}-c q_{i}, \tag{3}
\end{equation*}
$$

It is useful to write aggregate output across the $n$ outlets of this type of store as $Q=$ $\sum_{j \neq i} q_{j}+q_{i}$. Substituting this into Equation (3) yields:

$$
\begin{equation*}
\pi_{i}=\left[a-b\left(\Sigma_{j \neq i} q_{j}+q_{i}\right)\right] q_{i}-c q_{i} . \tag{3'}
\end{equation*}
$$

We assume Cournot-Nash conjectures on the part of stores. That is, each outlet $i$ chooses its own output, $q_{i}$, taking as given the outputs of all competing outlets (the $q_{j} s, j \neq i$ ). Thus, the first-order condition for $q_{i}$ is:

$$
\begin{equation*}
a-b\left(\Sigma_{j \neq i} q_{j}+q_{i}\right)-b q_{i}-c=0 . \tag{4}
\end{equation*}
$$

In equilibrium, all outlets of a given type are identical. Thus, they all choose the same optimal output, so we set $q_{i}=q_{j}=q^{*}$ in Equation (4) and solve for $q^{*}$ to obtain:

$$
\begin{equation*}
q^{*}=(a-c) / b(n+1) \tag{5}
\end{equation*}
$$

where we assume $a>c$. It follows from Equation (5) that aggregate output is:

$$
\begin{equation*}
Q^{*}=n q^{*}=n(a-c) / b(n+1) \tag{6}
\end{equation*}
$$

Substituting $Q^{*}$ into the demand function in Equation (1) yields the equilibrium price:

$$
\begin{equation*}
P^{*}=a-n(a-c) /(n+1) . \tag{7}
\end{equation*}
$$

Finally, substituting $q^{*}$ and $P^{*}$ into Equation (2) yields per store and aggregate profits, respectively:

$$
\begin{gather*}
\pi^{*}=(1 / b)[(a-c) /(n+1)]^{2},  \tag{8}\\
\Pi \equiv n \pi^{*}=(n / b)[(a-c) /(n+1)]^{2} . \tag{9}
\end{gather*}
$$

The assumption that the developer extracts all economic profits from tenant stores implies that $n R=\Pi$ is the aggregate rents paid by the $n$ competing outlets. It follows from Equation (9) that this quantity is maximized at $n=1$. Specifically, differentiate Equation (9) to obtain:

$$
\begin{equation*}
\partial \Pi / \partial n=\left[(a-c)^{2} / b\right]\left[\left(-n^{2}+1\right) /(n+1)^{2}\right] \tag{10}
\end{equation*}
$$

which is negative for integer values of $n$ greater than one. Thus, writing profit as a function of the number of outlets, $n$, we have $\Pi(1)>\Pi(2)>\ldots$, which leads to the conclusion that, ignoring interstore externalities, the developer should allow entry of only one outlet of a given type of store in order to maximize economic profits. Specifically, suppose initially that there is one store of type $j$ but none of type $k$. If the developer adds a new store, the joint profit from the two stores will necessarily be larger if he adds a store of type $k$ rather than a second outlet of type $j$ since:

$$
\begin{equation*}
\Pi_{j}(1)+\Pi_{k}(1)>\Pi_{j}(2) . \tag{11}
\end{equation*}
$$

Thus, it never pays to duplicate stores. ${ }^{7}$
Now consider the impact of external benefits conferred by a store of type $j$ on other stores in the shopping center. As noted above, these external benefits arise when customers of this store spill over into other stores (i.e., stores selling complementary products), thereby enhancing their sales. Thus, the external benefits are increasing in $Q_{j}^{*}$, the aggregate output (or "traffic") of the type $j$ outlets. To capture this more formally, we write the aggregate external benefits conferred by type $j$ stores on all other (non-identical) stores in the general form $E_{j}\left(Q_{j}^{*}\right)$, where $E_{j}^{\prime}>0 .{ }^{8}$ (This specification is sufficient to demonstrate the desired result; in the next section, we derive $E$ more rigorously beginning with the demand functions of each store.) It follows from Equation (6) that:

$$
\begin{equation*}
\partial E_{j} / \partial n_{j}=E_{j}^{\prime} \cdot\left[\left(a_{j}-c_{j}\right) / b_{j}\left(n_{j}+1\right)^{2}\right]>0 . \tag{12}
\end{equation*}
$$

That is, the external benefits generated by type $j$ stores are increasing in the number of outlets. Thus, if $E_{j}^{\prime}$ is large enough, it will be optimal for the developer to include more than one outlet of store $j$.

Specifically, the developer will choose $n_{j}$ to maximize total revenue from type $j$ stores, which is given by the aggregate rents of these stores plus the spillover sales revenue, or $\Pi_{j}\left(n_{j}\right)+E_{j}\left(n_{j}\right)$. The optimal $n_{j}$ thus solves the first-order condition:

$$
\begin{equation*}
\partial \Pi_{j} / \partial n_{j}+\partial E_{j} / \partial n_{j} \leq 0 \tag{13}
\end{equation*}
$$

We showed that the first term is negative for $n_{j}>1$. Thus, for it to be optimal to include more than one outlet of type $j$, the second term, which captures the marginal spillover benefit, must be large enough to offset the decline in aggregate rents resulting from competition by the additional $j$ outlets. Ultimately, this is an empirical matter that depends on the characteristics of individual stores.

Anikeeff (1996) surveys previous attempts to measure the degree of spillover shopping, or "retail compatibility," across different types of non-anchor stores. The most systematic attempt, by Nelson (1958), classified stores into five categories according to the percentage of customers who visit a given pair of stores. Exhibit 2 lists Nelson's categories (column 1) and the percentages of customer "interchange" that define each (column 2). More recently, Eppli and Shilling (1993) used new data and improved methods to re-estimate the degree of retail compatibility for a sample of stores in fifty-four regional shopping centers in the United States. Column 3 of Exhibit 2 shows the percentages that they assigned to Nelson's categories. Generally, the percentages are higher, indicating greater compatibility. This result is probably due in part to the change in methodology and in part to changes in shopping patterns. Whatever the numbers, these studies reveal that careful selection of a shopping center's tenant mix is crucial in achieving maximum profitability.

## An Illustration: Filling a Vacant Store

The preceding theory suggests that the optimal number of stores of a given type in a shopping center balances two offsetting effects. On the one hand, an additional outlet of a given type results in lower aggregate rent due to greater competition, but on the other, the resulting increase in traffic flow in the center raises the sales (and hence profits) of all other stores. In this section, we apply the insights from this theory to examine a problem that landlords routinely face, namely, how to fill a vacancy in the

Exhibit 2
Categories for the Degree of Retail Compatibility

|  | Percentage of Customer Interchange |  |
| :--- | :---: | :---: |
| Category | Nelson (1958) | Eppli and Shilling (1993) |
| Highly compatible | $10-20$ | $>30$ |
| Moderately compatible | $5-10$ | $10-30$ |
| Slightly compatible | $1-5$ | $5-10$ |
| Incompatible | 0 | 0 |
| Deleterious | $<0$ | $<0$ |

center. In particular, we ask whether the landlord should add an additional outlet of a store that is already present in the center (a competing store) or a new type of store.

Suppose that there is a single vacancy in the center which the landlord can fill with one of two potential tenants, $A$ or $B$. Further, suppose that there already exists an outlet of type $A$ in the center, but there is no outlet of type $B$. Thus, if $A$ is chosen, it would compete with the existing store, thereby lowering net sales revenue and hence rents. Traffic throughout the center, however, would be higher, thereby leading to greater sales (and rents) in non-competing stores. If instead $B$ is chosen, it would operate as a monopoly, generating higher net sales revenue compared to store $A$, but less traffic (all else equal). The landlord's choice between these two options depends on a comparison of these offsetting effects.

To be more specific, suppose that if the landlord chooses store $A$, the yearly profit from the center as a whole would be:

$$
\begin{equation*}
\Pi_{A}(2)+\Sigma \Pi(A) \tag{14}
\end{equation*}
$$

where the first term is the combined profits from the two competing type $A$ stores, and the second term is aggregate profits for all other stores in the center (i.e., all stores except the existing type $A$ store and the new store). The fact that $\Pi_{A}(2)<$ $\Pi_{A}(1)$ reflects the lost profits from competition, but $\Sigma \Pi(A)$ captures the increased sales in all other stores due to greater traffic. ${ }^{9}$

In contrast, if the landlord chooses the type $B$ store, aggregate profits would be:

$$
\begin{equation*}
\Pi_{A}(1)+\Pi_{B}(1)+\Sigma \Pi(B), \tag{15}
\end{equation*}
$$

where the first term is the profit from the existing type $A$ store, the second term is the profit from the new type $B$ store, both operating as monopolists, and the third term is the aggregate profits from all other stores when the vacancy is filled with a type $B$ store. The net yearly gain in profits from adding a type $A$ store rather than a type $B$ store is therefore given by the difference between Equations (14) and (15), or by:

$$
\begin{equation*}
\Delta=\left[\Pi_{A}(2)-\Pi_{A}(1)-\Pi_{B}(1)\right]+[\Sigma \Pi(A)-\Sigma \Pi(B)] . \tag{16}
\end{equation*}
$$

Note that the first term in square brackets is negative by Equation (11), indicating that, in the absence of interstore externalities, it is always better to have two different types of stores acting as monopolists than to have two stores of the same type competing against one another. The second bracketed term captures the externality, or spillover effect of the new store on the aggregate rents of all other stores. In order for store $A$ to be chosen, the additional revenue from greater traffic generated by competition between the two type $A$ stores must be larger than the additional revenue from traffic generated by individual stores of type $A$ and $B$, each acting as a monopolist (i.e., $\Sigma \Pi(A)-\Sigma \Pi(B)$ must be positive), and this effect must be enough to offset the first term so that $\Delta>0$. (Note that the positivity of $\Sigma \Pi(A)-\Sigma \Pi(B)$ does not imply
that store $B$ confers no external benefits on other stores; it merely says that these benefits are less than those conferred by a second store $A$.)

To illustrate the preceding analysis, consider the following numerical example. Suppose that the developer of a shopping center has one vacant store containing 2,500 square feet (s.f.) of leasable area, and there are two tenants interested in the space: a book store $(A)$ and a toy store $(B)$. Suppose that there is already a 3,000 s.f. book store in the center but no toy store.

The developer has the results of a research study estimating the increased traffic flow from various types of stores. It shows that the second book store would increase traffic by an average of 15 people/day, and the toy store would increase traffic by 8 people/day. The study also indicates that each spillover customer on average spends $\$ 50$ elsewhere in the center. Finally, the study projects that the book store's net sales would be $\$ 240$ per s.f. per year but it would reduce the existing book store's profits by $20 \%$, whereas the toy store's net sales would be $\$ 195$ per s.f. per year. Net sales at the existing book store prior to filling the vacancy are $\$ 260$ per s.f. per year. ${ }^{10}$

Given this information, if the book store is added, the combined profits from the two book stores would be:

$$
(\$ 240 / \text { s.f. } * 2,500 \text { s.f. })+[(\$ 260 / \text { s.f. } * 3,000 \text { s.f. })(1-.2)]=\$ 1,224,000
$$

and the aggregate gain in profits to all other stores would be (15 people/day * 365 * $\$ 50 /$ person $)=\$ 273,750$. Given Equation (13), the increase in profits from the center as a whole would be $\$ 1,224,000+\$ 273,750=\$ 1,497,750$.

If instead the landlord chose the toy store, the existing book store would earn profits of $(\$ 260 /$ s.f. $* 3,000$ s.f. $)=\$ 780,000$, the new toy store would earn profits of $(\$ 195 /$ s.f. $* 2,500$ s.f. $)=\$ 487,500$, and all other stores in the center would earn additional profits of ( 8 people/day $* \$ 50 /$ person $* 365$ ) $=\$ 146,000$. Thus, the overall gain in profits would be $\$ 780,000+\$ 487,500+\$ 146,000=\$ 1,413,500$. The yearly net gain from the book store as compared to the toy store is therefore $\$ 1,497,750-$ $\$ 1,413,500=\$ 84,250$, making the book store the preferred choice.

Of course, this outcome depends on the particular numbers in the example. Exhibits $3-5$ therefore examine the sensitivity of the results to changes in three key parameters: the percentage loss in business to the existing book store from the new book store (Exhibit 3), the traffic generated by the two stores (Exhibit 4) and the average spending per day by spillover customers (Exhibit 5). (This example is indicated in each illustration by the dashed lines.) Exhibit 3 shows that, as the new book store causes a greater loss in profits to the existing book store (all else equal), adding another book store becomes less desirable. Exhibit 4 shows the impact of varying the traffic generated by the stores. The fact that the curves are upward sloping indicates that an increase in traffic from either store increases aggregate profits to the center. The curve for the toy store is everywhere above that of the book store, reflecting the competitive loss to the existing book store. Nevertheless, the book store is still preferred if the

Exhibit 3
Effects of Varying Competitive Loss from Additional Store

combined book stores generate enough additional traffic (as is the case in the above example). Finally, Exhibit 5 shows the effect of varying the amount of spending by spillover customers. It shows that gross profits are increasing in customer spending, though at different rates for the two stores. Thus, as spending increases, a switch point from the toy store to the book store occurs at the intersection of the two curves.

## Integrating Output and Space in the Model

The theoretical analysis in the previous section focused on competition and complementarity across stores in their choices of output, holding fixed the amount of


space per store. As noted in the introduction, however, this focus on output is in contrast to Brueckner's (1993) original theoretical treatment of the shopping externality, which modeled the externality solely as a function of the amount of space allocated to different stores. Specifically, Brueckner specified a net sales function for each store $i$ that depended on the space allocated to each of the $m$ types of stores in the center (assuming one outlet of each type):

$$
\begin{equation*}
R_{i}=R_{i}\left(S_{1}, S_{2}, \ldots, S_{m}\right) . \tag{17}
\end{equation*}
$$

Brueckner assumed that sales were increasing in a store's own-space and nondecreasing in the space allocated to all other types of stores. That is,

$$
\begin{equation*}
\partial R_{i} / \partial S_{i}>0, \partial R_{i} / \partial S_{j} \geq 0, i=1, \ldots, m, j \neq 1, \tag{18}
\end{equation*}
$$

where the cross-store effects, $\partial R_{i} / \partial S_{j} \geq 0$, captured the shopping externality.
In this section, we extend the model from the previous section to include both output and space. Our objective in doing this is to derive Brueckner's sales function in Equation (18) from profit maximization by individual stores based on an underlying demand function that includes cross-store output effects and own-space effects. Specifically, we write the inverse demand function in Equation (1) for a type $i$ store more explicitly as:

$$
\begin{equation*}
P_{i}=\alpha_{i} S_{i}+\Sigma_{j \neq i} \beta_{j} Q_{j}-b_{i} Q_{i}, \tag{19}
\end{equation*}
$$

where $S_{i}$ is the floor space allocated to store $i$, and $Q_{j}$ is the aggregate output of stores of type $j, j \neq i$. Thus, as above, the price and output of a given type of store are inversely related, as indicated by the term $-b_{i} Q_{i}$. In addition, price is assumed to be positively related to the output of all other types of stores (captured by the term
$\sum_{j \neq i} \beta_{j} Q_{j}$ ), and positively related to the space allocated to store $i$ (the term $\alpha_{i} S_{i}$ ). (Note that the parameter $a$ in Equation (1) is comprised of the first two terms on the righthand side of Equation (19); that is, $a_{i}=\alpha_{i} S_{i}+\Sigma_{j \neq i} \beta_{j} Q_{j}$. $)^{11}$

The inclusion of a store's own space as a positive shift factor in Equation (19) presumably reflects the fact that a larger store will accommodate more customers at a given point in time, thereby shifting out the store's demand curve. ${ }^{12}$ There seems no reason to believe, however, that increasing the space of other stores will directly affect store i's demand curve, so the $S_{j} s$ are not included in Equation (19). However, as we shall show, the $S_{j} s$ will enter store $i$ 's sales function indirectly through the $Q_{j} s$, which will result in a net sales function for store $i$ that depends on the space allocated to each store. In order to simplify the analysis in this section, we will assume (in contrast to the previous section) that there is only one store of each type acting as a monopolist. ${ }^{13}$ In addition, we will assume below that there are only two types of stores (i.e., $m=2$ ). This will be sufficient to illustrate the results while keeping the model tractable.

We assume, as in the previous section, that each store maximizes its profit by choosing its output, $Q$, taking as given the output of other stores, the amount of space allocated to it in the center, and the rent for that space. Thus, each store treats the first two terms on the right-hand side of Equation (19) as parameters. ${ }^{14}$ As a result, the optimal output and net sales revenue for each store are given by Equations (6) and (9), respectively, with $n_{i}=1$ and $a_{i}=\alpha_{i} S_{i}+\sum_{j \neq i} \beta_{j} Q_{j}$. Note, however, that these are not reduced form expressions, given that $a_{i}$ depends on the $Q_{j} s$ through the demand curve in Equation (19). In other words, the $Q_{i} s$ must be determined simultaneously in equilibrium given the cross-store effects.

In order to illustrate such a solution, we consider the case where $m=2$; that is, where there are only two stores in the center. In that case, we have from (6):

$$
\begin{align*}
& Q_{1}=\left(a_{1}-c_{1}\right) / 2 b_{1},  \tag{20}\\
& Q_{2}=\left(a_{2}-c_{2}\right) / 2 b_{2}, \tag{21}
\end{align*}
$$

where

$$
\begin{align*}
& a_{1}=\alpha_{1} S_{1}+\beta_{2} Q_{2}  \tag{22}\\
& a_{2}=\alpha_{2} S_{2}+\beta_{1} Q_{1} \tag{23}
\end{align*}
$$

Substituting Equation (22) into Equations (20) and (23) into Equation (19) and solving simultaneously yields:

$$
\begin{equation*}
Q_{1}^{*}\left(S_{1}, S_{2}\right)=\left(2 \alpha_{1} b_{2} / \Omega\right) S_{1}+\left(\beta_{2} \alpha_{2} / \Omega\right) S_{2}-\left(2 b_{2} c_{1}+\beta_{2} c_{2}\right) / \Omega, \tag{24}
\end{equation*}
$$

and

$$
\begin{equation*}
Q_{2}^{*}\left(S_{1}, S_{2}\right)=\left(\beta_{1} \alpha_{1} / \Omega\right) S_{1}+\left(2 \alpha_{2} b_{1} / \Omega\right) S_{2}-\left(2 b_{1} c_{2}+\beta_{1} c_{1}\right) / \Omega \tag{25}
\end{equation*}
$$

where $\Omega=4 b_{1} b_{2}-\beta_{1} \beta_{2}$, which is positive assuming $b_{i}>\beta_{i}$ (i.e., own-quantity effects are greater than cross effects). Equations (24) and (25) represent the equilibrium outputs of the two stores as a function of their allocations of space. Differentiating Equations (24) and (25) immediately implies that $\partial Q_{i}^{*} / \partial S_{i}>0$ and $\partial Q_{i}^{*} / \partial S_{j}>0$ for $i, j=1,2$. That is, equilibrium output for both stores is increasing in the space allocated to each store.

Net sales for store $i$ for the case where $n_{i}=1$ are given by Equation (9):

$$
\begin{equation*}
\Pi_{i}=\left(a_{i}-c_{i}\right)^{2} / 4 b_{i}, \quad i=1,2 \tag{26}
\end{equation*}
$$

Substituting for $a_{i}$ using $Q_{i}^{*}\left(S_{1}, S_{2}\right)$ yields:

$$
\begin{equation*}
\Pi_{i}\left(S_{1}, S_{2}\right)=\left[\alpha_{i} S_{i}+\beta_{i} Q_{i}\left(S_{1}, S_{2}\right)-c_{i}\right]^{2} / 4 b_{i}, \quad i=1,2 . \tag{27}
\end{equation*}
$$

Note that this expression corresponds to Brueckner's net sales function $R_{i}\left(S_{1}, S_{2}\right)$ in that it gives store $i$ 's net sales solely as a function of the space allocated to the two stores. It follows from Equations (27), (24) and (25) that:

$$
\begin{equation*}
\partial \Pi_{i} / \partial S_{1}>0 \quad \text { and } \quad \partial \Pi_{i} / \partial S_{2}>0, \quad i=1,2 \tag{28}
\end{equation*}
$$

Thus, net sales are increasing in the space allocated to both stores as conjectured in Equation (18).

Given Equations (27) and (28), the problem for the developer is to choose $S_{1}$ and $S_{2}$ (for the case of $m=2$ ) to maximize aggregate profits (or rents) for the two stores less the cost of space, denoted $K\left(S_{1}+S_{2}\right)$. The first order conditions for $S_{1}$ and $S_{2}$ are given by

$$
\begin{equation*}
\partial \Pi_{1} / \partial S_{i}+\partial \Pi_{2} / \partial S_{i}-K^{\prime}\left(S_{1}+S_{2}\right)=0, \quad i=1,2, \tag{29}
\end{equation*}
$$

where $\partial \Pi_{i} / \partial S_{i}$ captures the marginal benefit of a store's own allocation of space, the cross derivatives $\partial \Pi_{i} / \partial S_{j}(i \neq j)$ capture the external shopping effects as a function of space, and $K^{\prime}\left(S_{1}+S_{2}\right)$ is the marginal cost of space. This condition says that space should be allocated to a given store up to the point where its marginal sales are equal to the marginal cost of space minus the incremental sales that this store generates for all other stores in the center. Thus, more space should be allocated to those stores that confer greater sales benefits on other stores, all else equal. Again, this is an empirical matter as discussed earlier. The developer fully internalizes these external effects, and therefore makes the optimal allocation of space, given our assumption that the profits of all stores can be extracted in the form of rents. ${ }^{15}$

## Conclusion

This article represents a first effort to integrate the output choice of stores and the allocation of space into the economic model of optimal shopping center design.

Inclusion of output sharpened the model in two ways. First, it provided a more natural way to examine the trade-off between competition and complementarity in the choice of tenant mix by a profit maximizing developer. Second, it allowed us to derive Brueckner's fundamental sales function, which he specified solely as a function space, from profit maximizing behavior by individual stores in the presence of inter-store shopping externalities.

Despite these contributions to the theory, our understanding of the economics of shopping center design is far from complete. This fact, and the increasing importance of shopping centers in the urban landscape, suggests that further research on this topic is warranted.

## Notes

${ }^{1}$ Also see Benjamin, Boyle and Sirmans (1992), Eppli and Shilling (1993, 1995), Miceli and Sirmans (1995) and Anikeeff (1996). Eppli and Shilling (1993) refer to the shopping externality as the "Rule of Retail Compatibility." They provide empirical studies of its magnitude for various combinations of stores.
${ }^{2}$ In an extension, Brueckner also considered a store's choice of "managerial effort." In this article, we do not address this issue or other sorts of agency problems that bear on the choice of tenant mix. For a detailed analysis of these issues, see Miceli and Sirmans (1995).
${ }^{3}$ Variety stores are therefore a small-scale version (and presumably the progenitor) of the shopping center.
${ }^{4}$ The specification of demand in Equation (1) says that, from the perspective of consumers of a given product, price and quantity are inversely related. That is, as the price increases (decreases), the quantity that the store can sell will decrease (increase). Economists refer to this inverse relationship between $P$ and $Q$ as the Law of Demand.
${ }^{5}$ Thus, we follow Brueckner's (1993) scenario of the developer as a perfectly discriminating monopolist.
${ }^{6}$ Since Brueckner (1993) does not explicitly include output in his model, the only cost stores incur in his model is rent, or the cost for space. Thus, his sales function $R$ [see Equation (17)] corresponds to net sales, or $\pi$, in our model.
${ }^{7}$ Note that this is true even if the two j stores make more gross profit than the single $k$ store; that is, even if $\Pi_{j}(2)>\Pi_{k}(1)$.
${ }^{8}$ This assumes that the traffic through store $j$ is proportional (or at least positively related) to its aggregate output.
${ }^{9}$ Thus, $\Sigma \Pi(A)$ would be a function of the degree of retail compatibility between store A and each of the other stores in the center.
${ }^{10}$ Sales figures are based on median sales data for different types of tenants from Dollars and Cents of Shopping Centers, 1995.
${ }^{11}$ For notational simplicity, we do not include an additional constant term in Equation (18).
${ }^{12}$ The fact that more own-space shifts out a store's demand curve is implicit in Brueckner (1993:9).
${ }^{13}$ Thus, $q_{i}=Q_{i}$ for all $i=1, \ldots, m$.
${ }^{14}$ This again reflects Cournot-Nash conjectures, as well as the assumption that the developer dictates a store's allocation of space and its rent. This latter assumption may be unrealistic for the case of large anchor tenants who presumably bargain with the developer for space and rent. ${ }^{15}$ Thus, like a perfectly discriminating monopolist, the developer makes the socially efficient allocation of space.

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