A Procedure for Uncovering Acceptable and Nonacceptable Mortgage Applications through Discriminant Analysis Using Ranked Data

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Abstract. **The procedure developed in this paper uses a less biased statistical technique than conventional *discriminant analysis* and parallels the ranking procedure used by loan officers. A variety of univariate and *multivariate* statistical procedures as well as comprehensive validation methods are used to develop a "best" model. The resulting model provides more accurate classification than other studies have shown, without violating federal law regarding discrimination.**

Introduction

According to conventional economic theory, risk-averse investors prefer less risk to more risk for any level of return. This also applies to mortgage lending institutions that invest in projects (residential mortgage loans) anticipating a positive return while accepting a degree of risk. However, most lending institutions are conservative; the disutility of a "bad" investment is greater than the additional utility resulting from an equal "good" investment. Therefore, the lending institutions rank the possible projects with particular attention to default risk.

A residential mortgage application is commonly processed through two stages of evaluation and ranking. The first occurs when the loan officer counsels with each applicant(s) and ranks all applicants from best to worst using factors that the lending institution considers critical to delineate acceptable borrowers from those that should be rejected. The second stage occurs when the credit committee receives the higher ranked applications from the loan officer and conducts its own ranking from best to worst. The higher ranked applications in the final stage are then approved in order until the supply of loanable funds is exhausted. This is especially true during periods when demand for mortgage money is high.

In the ranking process, the lending institution compares the applicants' occupations, tenure in occupation, credit ratings, payments-to-income ratios and other loan and property characteristics separately (i.e., univariate) to arrive at a final ranking of the applications. An ordinal discriminant analysis procedure performs a similar ranking process except the ranked variables are considered together (i.e., multivariate) to arrive at a final ranking of the applications. Nonetheless, both procedures involve a ranking of variables in order to arrive at a final ranking of the application.

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The lender has a need for a model based upon past accepted and rejected applications that can be used to evaluate and delineate the new or marginal applicant, and to assess whether any of the factors used by the loan officer and the credit committee in their rankings would be considered discriminatory under federal law. Discriminant analysis can be used to develop such a model. In addition, relative measures should be used; the variables are ordinal, not cardinal.

Discriminant analysis assumes that the a priori defined groups are normally distributed. If this assumption is not satisfied, certain parts of the analysis may be biased. Typically, logarithmic data transformations are made and used in the discriminant procedure. However, such transformations may affect the interrelationships among the variables. An alternative transformation uses ranks (ordinal data). Discriminant analysis using ordinal data has been shown to perform comparably to conventional discriminant analysis which uses interval (cardinal) data while mitigating the multivariate normality problem.

The hypothesis tested in this paper is that discriminant analysis using ordinal data will produce a better model for judging future applicants than the traditional discriminant analysis using cardinal data consisting of borrower and property characteristics once they have passed through stage two of the ranking process. This paper argues that a better model can be produced since it more closely replicates the loan evaluation process and is statistically more appropriate. The intent of the paper is to provide a less biased procedure for uncovering the critical factors used by a lender to delineate acceptable applications from those that should be rejected.

Review of Literature

Much of the mortgage lending literature contains research that has been conducted with the objective of detecting discriminatory practices within the models used by lenders to evaluate loan applications. The objective of this paper is to derive a model from ordinal rankings that can delineate accepted from rejected applications. Although detection of discrimination is not a primary objective, the factors contained within the model can be examined closely to determine if it is present. A comparison of ordinal discriminant analysis with the cardinal discriminant analysis found in the literature is useful since it illustrates the differences in results.

Traditional demand-site studies have tested for discrimination in lender behavior by deriving relationships between variables measuring location and mortgage-applicant characteristics and variables measuring loan terms (e.g., Benston (1977), Black, Schweitzer, and Mandell (1978), Warner and Ingram (1982)). Demand-side studies may be subject to a universal criticism that mortgage deficiency may be occurring from a lack of local demand rather than a perceived risk by the lender according to Richardson and Gordon (1979). Although the present study follows the more traditional approach by not explicitly estimating a demand function, the data was drawn from a time period characterized by near record mortgage activity in the local market and the nation which would mitigate the insufficient demand argument.

Warner and Ingram (1982) presented a discriminant analysis approach that attempted to assess mortgage markets for evidence of discriminatory lending practices on the part of financial institutions. The purpose of their study was to develop a model that allows both lenders and regulators to assess the equity of lending patterns within a given market. The study compared a model containing discriminatory (race, sex, etc.) variables intended to detect discrimination as well as risk and return variables to a model containing only risk and return variables in order to distinguish between accepted and rejected mortgage loan applications. Since classification results of the two models were comparable, the authors concluded that there was no evidence of discrimination in the mortgage lending patterns of financial institutions.

Due to the possible bias resulting from the violation of the normality assumption of discriminant analysis, Ingram and Frazier (1982) used discriminant analysis, probit, and logit models in a study of mortgage lending discrimination. Since the choice between logit and probit is a matter of individual preference and no theoretical or experimental basis exists for choosing one approach over the other, the authors compared all three methods. They found that the classification accuracy of the logit model was only marginally higher than the discriminant analysis and probit models (78.18%, 75.90%, and 77.73%, respectively).

None of these studies employing discriminant analysis considered ordinal data or the rank transformation approach. However, due to the problems associated with parametric discriminant analysis assumptions, and the ranking procedure used by lending institutions, such a procedure should be considered.

Rank transformation discriminant analysis also has been used in empirical research. Perry, Cronan and Henderson (1985) and Perry and Cronan (1982) used this procedure to develop more accurate bond rating models. Cronan, Perry and Epley (1983) also used the procedure as an effective data editing methodology.

Methodology

A sample of 750 residential mortgage applications was obtained from the Columbia, South Carolina SMSA.[1] The data set consisted of 250 rejected applications and 500 accepted applications. Variables included risk-return variables and discriminatory (bias) variables. The information for these variables was taken from applications at institutions using Federal Home Loan Mortgage Corporation applications (or reasonable facsimilies thereof). Census tract data related to neighborhood quality was extracted from the *Census of Population and Housing*. Commercial banks and savings and loan associations with headquarters in the Columbia, South Carolina SMSA, and principally lending in the Columbia, South Carolina SMSA, participated. Predictor variables used in the development of the best model included the following risk-return measures: applicant's credit rating, applicant's occupation, applicant's tenure in occupation, loan-to-value ratio, neighborhood crime rate, remaining economic life, applicant's total monthly payment-to-income ratio, lender's yield (APR), and years to maturity (life of loan). Other "bias" variables used included applicant's age, applicant's marital status, applicant's race, applicant's sex, coapplicant's income, coapplicant's tenure in occupation, dwelling age,[2] and neighborhood age. In total, nine risk-return and eight discriminatory (bias) variables were employed. The predictor variables are described in Appendix A, and summary statistics of the data are presented in Appendix B.

The sample was randomly divided into two subsamples — a training sample and a holdout sample. The training sample was used for model development, while the holdout sample was used for validation. Each sample had 375 observations: 250 accepted applications and 125 rejected applications.

Rank Transformation Discriminant Analysis

Discriminant analysis is the classification of an observation X_0, possibly multivariate, into one of several populations $\pi_1, \pi_2, \ldots, \pi_k$ which each have density functions. If these densities can be assumed to be normal with equal convariance matrices, then Fisher's linear discriminant function (LDF) method is used. If the matrices are unequal, a quadratic discriminant function (QDF) is appropriate. These methods (LDF and QDF) assume multivariate normality. Many researchers suggest the use of logarithmic transformations on the original variables so that their distribution function is approximately normal. However, such transformations are not possible when there are negative or zero values in the data. Successful transformation on a variable also requires insight into the true nature of the variable's distribution.

The parametric discriminant function, whether linear or quadratic, also can be contaminated by multivariate outliers. The treatment of outliers on a univariate basis is insufficient. According to Rohlf (1975), in multivariate analysis, an outlier is not just an observation that sticks out on the end. Whatever treatment an outlier receives could bias the classification results.

Conover and Iman (1980) and Moore and Smith (1975) suggest a transformation that applies to all distributions equally well—the rank transformation. Using their terminology, let X_{ij} be the j^{th} observation factor from population i, $j = 1, 2, \ldots, n$ and $i = 1, 2, \ldots, k$. The p components of X_{ij} are denoted X_{ijm}, $m = 1, 2, \ldots, p$. The rank transformation method involves ranking the m^{th} component of all observations X_{ij} from the smallest, with rank 1, to largest, with rank $N = n_1 + n_2 + \ldots + n_k$. Each component, $m = 1$ to $m = p$, is ranked separately. Simply stated, each variable value of the multivariate sample is replaced by its rank from 1 to n of all the groups combined. Sample means and covariance matrices are computed on the ranks and the traditional LDF and QDF are used, hence the rank linear discriminant function (RLDF) and the rank quadratic discriminant function (RQDF). The rank transformation tends to minimize the outlier contamination problem and the nonnormality problem caused by outliers. No knowledge of outliers or distribution form is necessary.[3]

Simulation studies by Conover and Iman (1980) have shown that, when the population is normal, very little is lost in two-group discriminant analysis by using RLDF and RQDF. When the populations are nonnormal, however, the RLDF and the RQDF methods are consistently better than the LDF or the QDF. Hettmansperger and McKean (1978) have also shown that rank procedures do not lose power or efficiency compared to interval-based procedures and that the linear model based on ranks follows a natural extension.

Validation

The derived discriminant functions were validated using the jackknife (Lachenbruch) classification rate, an internal classification (resubstitution) rate, and a holdout classification rate. The jackknife procedure systematically withholds each observation and develops the discriminant function on the remaining observations. That function is then used to classify the withheld observation. The percent correctly classified is the jackknife rate. The internal classification rate is the percent correctly classified when all observations are used to develop the function. These same observations are then used to test the function. The holdout classification rate is determined by classifying the holdout sample using the model derived from the training sample.

Results and Analysis

The BMDP7M program computes jackknife classification rates and performs a forward stepping procedure whereby all variables are free to enter and a "best" linear model can be determined. The variables in the best (highest jackknife rate) linear models, internal classification rate, jackknife classification rate, and holdout classification rate are presented in Exhibit 1; comparable values are also shown for the full seventeen-variable model.[4]

Exhibit 1 reveals that the Wilks' λ and approximate F-values (which determine the ability of the models to discriminate between groups) are significant at the .05 level or less. The average classification rate for the "best" model is 82.39%. The average classification rate for

Exhibit 1
Discriminant Analysis Models and Classification Rates

	Full Model	Best Model
	ATO	APR
	CTO	REL
	AR*	ACR
	AS*	TMPAIR
	AMS*	LVR
	APR	
	YM	
	REL	
	DA*	
	ACR	
	TMPAIR	
	LVR	
	CI*	
	AO	
	AA*	
	CR	
	NA*	
Wilks' λ	0.5482	0.5877
Approximate F	17.306	51.778
Degrees of Freedom	17; 357	5; 369
Classification Rates		
Internal	83.2%	82.4%
Jackknife	80.0%	82.1%
Holdout	81.60%	82.67%
	$x = 81.60\%$	$x = 82.39\%$

*Discriminatory variables

the full seventeen-variable model is 81.6%. This difference (0.79%) is small, however, it should be noted that only five variables (none of which are discriminatory) are in the "best" model as compared to seventeen variables in the full model.

Using the same data set, Warner and Ingram (1982) reported a jackknife (Lachenbruch) rate of 75.09% for the full seventeen-variable model. Their study, however, did not use ordinal data as did the current study. It appears, therefore, that ordinal data results in a higher classification rate with fewer variables.

The best model contained four variables (APR, ACR, TMPAIR, and LVR) that are commonly used in lending practice to judge an applicant's creditworthiness, and one property characteristic variable (REL) that is commonly used in appraisal practice. The latter variable is estimated by the lending institution's appraiser after personally inspecting the property's quality of construction and level of maintenance. Properties that were built with high construction standards and reflect a high degree of owner maintenance typically represent good properties for a loan. Good construction and a high degree of maintenance imply longetivity and little physical deterioration, and in the case of resales, a high degree of pride of ownership. Also, it is important to note that the next variable that would enter the model is NA, neighborhood age, which reemphasizes the importance of the housing quality and neighborhood. A loan officer can reduce the risk of a bad loan by considering properties that are well constructed with good maintenance in a neighborhood that maintains desirable property characteristics.

These results also are consistent with Black, Schweitzer, and Mandell (1978) who found the downpayment, interest rate, and age of the house to be statistically significant in the

Exhibit 2
Classification Matrices
Full Model

Internal Classification

	Total	Reject	Accept	Classification Rate
Reject	125	80	45	64.0%
Accept	250	18	232	92.8%
Total	375	98	277	83.2%

Jackknifed Classification

	Total	Reject	Accept	Classification Rate
Reject	125	74	51	59.2%
Accept	250	24	226	90.4%
Total	375	98	277	80.4%

Holdout Classification

	Total	Reject	Accept	Classification Rate
Reject	125	82	43	65.6%
Accept	250	26	224	89.6%
Total	375	108	267	81.6%

loan decision. The results are different, however, in that they found self-employment and race to be significant while this study did not. Also, they did not consider an economic life property characteristic which was included and found to be part of the best model in this study.

The results of this study are an improvement upon the Ingram and Frazier (1982) approach in that they did not find a "best" model to explain their data. They found the applicant's debt-to-income ratio (TMPAIR) to be significant under discriminant analysis, less significant under probit, and insignificant under logit. They found the applicant's sex, age of the subject property, and the neighborhood age to be significant under discriminant analysis. This study extends their conclusions by deriving one definitive best model that includes only five variables from the data that can be used by a loan officer.

Accepted vs. Rejected Applications

The results reported in Exhibit 1, however, are on an overall basis; an additional analysis of the classification matrices, therefore, is necessary. The classification matrices for the full model and the "best" model are presented in Exhibits 2 and 3.

The classification matrices shown in Exhibits 2 and 3 indicate that, the 91.6% average classification rate for the accepted applications ($x = (90.93 + 92.27)/2$) is higher than the 62.8% average classification rate for the rejected applications ($x = (62.93 + 62.67)/2$). An alternative interpretation is that less than 9% of the accepted applications should have been

Exhibit 3
Classification Matrices
Best Model

Internal Classification

	Total	Reject	Accept	Classification Rate
Reject	125	78	47	62.4%
Accept	250	19	231	92.4%
Total	375	97	278	82.4%

Jackknifed Classification

	Total	Reject	Accept	Classification Rate
Reject	125	78	47	62.4%
Accept	250	20	230	92.0%
Total	375	98	277	82.1%

Holdout Classification

	Total	Reject	Accept	Classification Rate
Reject	125	79	46	63.2%
Accept	250	19	231	92.4%
Total	375	98	277	82.7%

rejected, which suggests no preferential treatment was granted, thus, no discrimination. However, more than 37% of the rejected applications should have been accepted which suggests discriminatory practices possibly existed. A variety of multivariate and univariate procedures, including factor analysis, frequency distributions, and univariate statistics were used to investigate this possibility, but could not resolve the conflict. Other explanations for this conflict and some conclusions are made in the following section.

Summary and Conclusions

The hypothesis examined in this paper was that a best model could be produced that would better explain the factors used by loan officers to delineate acceptable applicants from those that should be rejected. The purpose of the study was to better replicate the ranking conducted by the loan officer and the credit committee prior to approving or rejecting the loan request.

The resulting best model that was derived using discriminant analysis on ordinal data from a set of 750 accepted and rejected loan applications produced a better model with fewer variables and a higher classification rate than Warner and Ingram (1982) and Ingram and Frazier (1982), who used conventional discriminant analysis, logit and probit. The factors contained in the model developed in this paper did not violate any federal laws on discrimination but were discriminatory in the sense that the variables utilized for loan approval or nonapproval were financial in nature.

The best model correctly classified approximately 83% of the loan applications — without the use of any discriminatory variables. This implies that if the best model is used by lenders they should correctly classify a loan application as acceptable or unacceptable 83% of the time—without violating federal law regarding discrimination.

The difference between the "best" model and the full model is only 0.79% (on average); the introduction of the discriminatory variables actually reduces the average classification rate. This agrees with the findings of Warner and Ingram (1982). Analysis of the classification matrices suggests, however, that discrimination may exist, since, of the rejected applications, an average of 37% should have been accepted; only 9% of the accepted applications should have been rejected.

A variety of multivariate and univariate procedures were used to investigate this possibility. The factor analysis showed no major differences in the factor patterns of the subsamples. Analysis of the discriminant analysis probability of misclassification indicated that most misclassifications were marginal ("barely" acceptable). The univariate comparison of frequency distribution of the discriminatory variables indicated no major differences.

This study, used a less biased statistical procedure and comprehensive validation procedures. No evidence of discrimination overall was found, however the classification results suggest that certain discriminatory factors may possibly exist when rejecting a mortgage loan application.

There may be several possible explanations for the conflict; however, two are most plausible. First, the variables used in the study were obtained from the applications. Some of these variables, indeed, represented what they intended to (e.g., APR, REL, YM), but, others were surrogates for risk. The discriminant analysis uses only the variables in the model, whereas, the lender may use additional values on the applications as additional measures of risk.

The second possible explanation is that lenders are inherently conservative. It is their obligation to insure an adequate return at a minimum level of risk. The applications that the discriminant analysis model deemed marginally acceptable were considered too risky by the lender, therefore, giving a lower classification rate and the appearance of discrimination. Even if the exact variables were used by the lender and the model, the weighting could be slightly different and again result in a lower rate.

Discrimination exists when the accept-reject decision is based on age, sex, race, marital status, religion, color, and national origin, *after* considerations for risk. The results of this study suggest that lenders are not discriminating; on the contrary, they are doing as expected—being conservative by rejecting marginal applications.

The discriminant analysis procedure developed in this study is superior to conventional discriminant analysis since it mitigates the multivariate normality problem. It is also intuitively appealing since it parallels the ranking procedure used by lenders and gives high classification results.

APPENDIX A

*1.	AA	= applicant's age actual age of head of the household
**2.	ACR	= applicant's credit rating 0 = rating is considered to be neutral or favorable 1 = rating is considered to be a negative indicator
*3.	AMS	= marital status of the applicant 0 = reported as married on the application 1 = unmarried or separated as reported on the application
**4.	AO	= applicant's occupation percent of applicant's income (AI) derived from sales commissions earned by any of the applicant or coapplicant
5.	AOR	= accepted or rejected AOR = 0 for rejected AOR = 1 for accepted
**6.	APR	= lender's yield APR calculated using the requirements of Reg. Z
*7.	AR	= applicant's race 0 = white 1 = non-Caucasian
*8.	AS	= applicant's sex 0 = primary applicant was male 1 = primary applicant was female
**9.	ATO	= applicant's tenure in occupation number of years in current employment
*10.	CI	= coapplicant's income percent of applicant's total annual income made up of the coapplicant's annual income
*11.	CTO	= coapplicant's tenure in occupation number of years in current employment
*12.	DA	= dwelling age number of years that have elapsed since the dwelling was constructed
**13.	LVR	= loan-to-value ratio loan principal requested in the application divided by the appraised value
*14.	NA	= neighborhood age mean age of the homes in the home's census tract
**15.	NCR	= neighborhood crime rate per capita crime rate within the census tract of the home as reported by the Law Enforcement Assistance Administration
**16.	REL	= remaining economic life number of years the home can be used without major rehabilitation as estimated by the appraiser
**17.	TMPAIR	= total monthly payments/applicant's income total monthly payments = monthly mortgage payment including principal, interest, tax escrow, and insurance plus the payments on other existing mortgage debts that existed at the time of the application
**18.	YM	= years to maturity—time period of loan

* discrimination variable
** risk-return variable

APPENDIX B
Predictor Variables and Summary Statistics

Code	Independent Variables	Groupings: Accepted ($n = 500$) Mean (Standard Deviation)	Groupings: Rejected ($n = 250$) Mean (Standard Deviation)
	Risk-Return Variables		
TMPAIR	Total Monthly Payments to Applicant's Income	0.0198 (0.0092)	0.0273 (0.0132)
AO	Applicant's Occupation (Sales Commissions/Total Income)	0.1597 (0.3583)	0.0912 (0.2790)
ATO	Applicant's Tenure in Occupation (Years)	7.0740 (7.5519)	4.6760 (5.7298)
CTO	Coapplicant's Tenure in Occupation (Years)	1.3160 (3.1298)	0.9560 (2.2571)
ACR	Applicant's Credit Rating	0.0040 (0.0532)	0.2640 (0.4417)
LVR	Loan-to-Value Ratio	0.7861 (0.1468)	0.9000 (0.1240)
YM	Years to Maturity	29.2000 (2.7523)	28.4600 (4.1951)
APR	Annual Percentage Rate	0.0958 (0.0040)	0.0923 (0.0029)
REL	Remaining Economic Life	46.5780 (8.0265)	41.7840 (10.6914)
NCR	Neighborhood Crime Rate	3.4080 (5.9847)	3.6280 (5.6903)
	Discriminatory (Bias) Variables		
AS	Applicant's Sex	0.0740 (0.2620)	0.0920 (0.2896)
AR	Applicant's Race	0.0870 (0.2653)	0.1480 (0.3558)
AMS	Applicant's Marital Status	0.2380 (0.3344)	0.2080 (0.4067)
AA	Applicant's Age	36.6600 (7.3198)	34.3560 (5.0783)
CI	Coapplicant's Income	0.0890 (0.1855)	0.0836 (0.1648)
DA	Dwelling Age	10.9360 (14.5788)	14.1240 (17.7097)
NA	Neighborhood Age	12.5465 (6.8862)	14.6568 (6.9825)

Notes

[1]The authors are indebted to Jerry Ingram for sharing his original data set.
[2]In some communities dwelling age could be a proxy for applicant's race. This possibility was investigated by examining the Pearson and Spearman correlation coefficients for dwelling age and race. The respective correlation coefficients are 0.27 and 0.15 for all 750 observations; 0.16 and 0.05 for the 500 accepted applications; 0.38 and 0.28 for the 250 rejected applications. All coefficients were significant at the 0.05 level. Although the coefficients are larger for the rejected applications than the accepted applications, they are not sufficiently large to warrant dwelling age as a proxy for race.
[3]The data set was examined and it was found that several data items had extreme values.
[4]Quadratic models produced similar results.

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