

The Optimal Time of Renovating a Mall

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Abstract. This paper presents a maximization model determining the optimal time at which a mall should be renovated. The analysis is constructed on the assumption of a decreasing rental income over time as a mall ages. It is then shown that the optimal renovation period achieves a balance between the marginal cost and benefits of delaying renovation. We show how this balance is affected by changes in the discount rate, net rental incomes and renovation costs. Numerical simulations are used to demonstrate the method of estimating this optimal renovation time and to illustrate the sensitivity of the optimal renovation period to changes in economic factors such as the discount rate, the level and rates of change of renovation costs and rental incomes.

Introduction

Given that market conditions remain unchanged, net rental incomes (which we define as rental incomes net of recurrent operating costs) derived from a mall can be expected to decrease over time for obvious reasons. The original design will become increasingly outdated and unsuited to new activities and the recurrent costs of maintaining the building fabric will increase as the building deteriorates.

In the worst possible scenario, the building structures may deteriorate so far or become so obsolete that complete redevelopment of the site is desirable. Suppose, however, that this scenario does not arise, i.e., that the structures are good enough to accommodate new designs and refurbishment. It is then the case that periodical renovations are desirable but the timing of such renovations must be decided. This involves balancing two competing forces. Renovation should not be delayed too long since net rental incomes receivable from the unrenovated mall will be decreasing and the costs of renovation will be increasing. On the other hand, renovating a mall too frequently may prove to be uneconomical as the period between major renovations is too short for repayment of the capital investment in refurbishment. In this paper we provide an analysis of the *optimal renovation period* for a mall.

Our analysis is related to recent development timing models applied to real estate: see, for example, Arnott and Lewis (1979), and Turnbull (1988a) for general theoretical analyses of residential or urban development timing; Anderson (1986, 1993) and Turnbull (1988b, 1988c) for theoretical models of taxation impacts on development timing decisions; Clark and Reed (1988) for development timing decisions under uncertainty. We also draw upon analyses of the optimal use of renewable natural

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resources: see, for example, Gong (1991), Howe (1979). The distinguishing feature of our analysis is our concern, not so much with the feasibility and timing of the original mall development, but rather with the timing of the subsequent renovation of that development.

It might be thought that the renovation timing problem is easily resolved using standard investment appraisal techniques embodied in life cycle costing (Flanagan and Norman, 1989). In any given period we can identify the costs of renovation and the additional net rental incomes to which renovation will give rise. It might then appear that renovation should take place when the net present value of the additional net rental incomes minus the renovation costs is positive: by application of standard NPV analysis. This approach is, however, too myopic. It does not take account of the possibility that it might be better to delay the renovation. The net present value of renovation at time T may be positive, but this does not tell us whether renovation at time T is better than renovation at some later time $T+s$. Effectively, we must consider a series of competing investment projects: renovation at time $T_t, T_{t+1}, T_{t+2} \dots$

This paper provides such an analysis. We show that the optimal renovation period must achieve a balance between the benefits of delaying renovation—savings on the capital costs of renovation net of the additional costs of renovation as the mall ages, and the costs of delaying renovation—the loss of net rental incomes from continuing with the unrenovated mall.

The Model

The optimal renovation period is derived from the analysis of a series of competing renovation opportunities. We model this series as a continuum rather than as a set of discrete choices in line with the literature on real estate development (see above) since this is both less restrictive and more mathematically tractable.

Suppose it is possible to restore a mall to a 'brand new' condition by means of renovation. That is, the net rental incomes (rental incomes net of recurrent operating costs) receivable from a newly renovated mall are the same as those receivable at the time when the mall is newly built, and that this can be achieved without going through the process of complete redevelopment. Define the following:

$c(T)$ ≡ cost of renovation to restore a particular mall at an age of T to a completely 'brand new' condition (with $dc/dT > 0, d^2c/dT^2 > 0$);

T_c ≡ time to undertake a renovation, during which no incomes are receivable from the operation of the mall;

$f(s)$ ≡ fraction of renovation cost incurred at time $s \in [0, T_c]$ (with

$$\int_0^{T_c} f(s) ds = 1);$$

$y(t)$ = the net rental income receivable at time t (with $dy/dt < 0; d^2y/dt^2 \geq 0$);

r ≡ the discount rate.

The present value at time T of the renovation costs, denoted $C(T)$, is

$$C(T) = c(T) \int_0^{T_c} f(t)e^{-rt} dt. \tag{1}$$

From our definition of $c(T)$, it follows that $C(T)$ is an increasing function of T .

The present value of the net rental incomes minus renovation costs derived from the mall over a single renovation period (that is, up to time $t = T + T_c$) is denoted $Y(T)$ and is given by:

$$Y(T) = \int_0^T y(t)e^{-rt} dt - C(T)e^{-rT}. \tag{2}$$

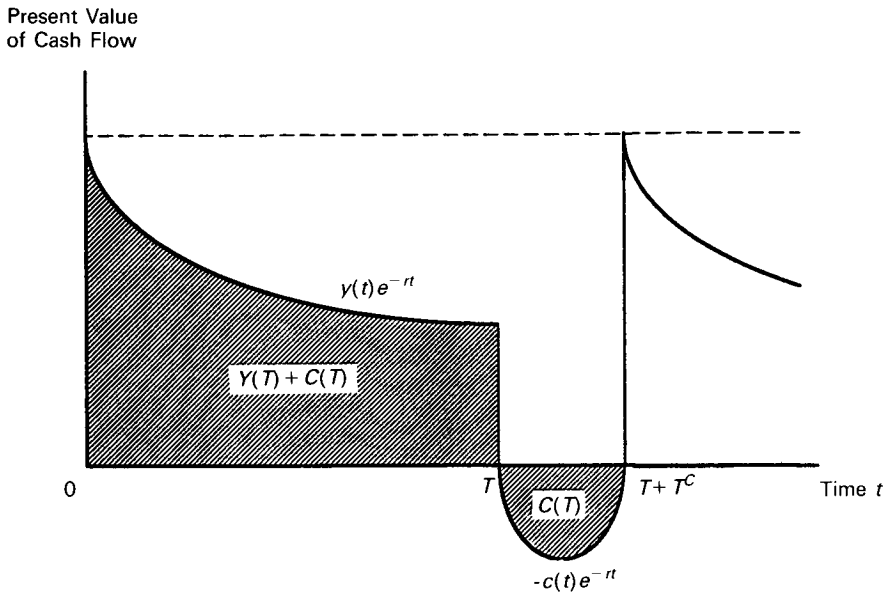
Graphically, $Y(T)$ is represented by the difference between the two shaded areas in Exhibit 1.

Differentiating equation 2 with respect to time, we have

$$Y'(T) = y(T)e^{-rT} - [C'(T) - rC(T)]e^{-rT}. \tag{3}$$

Assuming market efficiency, the capital value of the mall, denoted $V(T)$, can be represented by the sum of the present values of net rental incomes minus renovation costs, $Y(T)$, derived in perpetuity.¹

Exhibit 1 Cash Flow Analysis of a Mall



We assume that the objective function of the owner of the mall is to choose the renovation period to maximize the capital value of the mall, i.e., to choose T to

$$\begin{aligned} \text{Max}_{\{T\}} V(T) &= Y(T) + Y(T)e^{-r(T+T_c)} + Y(T)e^{-2r(T+T_c)} + \dots \\ &= Y(T)[1 + e^{-r(T+T_c)} + e^{-2r(T+T_c)} + \dots] \\ &= \frac{Y(T)}{1 - e^{-r(T+T_c)}}. \end{aligned} \quad (4)$$

The necessary condition for an optimal value of T is:

$$V'(T) = \frac{(1 - e^{-r(T+T_c)})Y'(T) - re^{-r(T+T_c)}Y(T)}{(1 - e^{-r(T+T_c)})^2} = 0. \quad (5)$$

This implies:

$$(1 - e^{-r(T+T_c)})Y'(T) = re^{-r(T+T_c)}Y(T), \quad (6)$$

as $(1 - e^{-r(T+T_c)})^2 > 0$, for any $(T + T_c) > 0$.

Combining equations 3 and 6 to eliminate $Y'(T)$, the optimal renovation period is determined by the condition:

$$y(T^*) + rC(T^*) = rV(T^*)e^{-rT_c} + C'(T^*), \quad (7)$$

where T^* is the optimal renovation period and renovations should take place at T^* , $2T^* + T_c$, $3T^* + 2T_c$,

This condition for optimization is a reasonably standard marginal condition. It implies that renovations should be postponed until the sum of the marginal benefits of delaying renovation [$y(T^*) + rC(T^*)$] equals the sum of the marginal costs of delaying renovation [$rV(T^*)e^{-rT_c} + C'(T^*)$].

We can give a more explicit interpretation of the maximizing condition of equation 7. The marginal benefits of delaying renovation (the left-hand side of equation 7) are $y(T^*)$, the current net rental income at T^* , and $rC(T^*)$, the interest saving on renovation costs that will be achieved by delaying renovation. The marginal costs of delaying renovation (the right-hand side of equation 7), are firstly, $rV(T^*)e^{-rT_c}$ which is the present value of the *implicit rent* derived from the newly renovated mall at time T^* , which would be receivable after the renovation period T_c . This implicit rent is expressed as the interest r on the present value of capital at its maximum, $V(T^*)$, discounted over the time to undertake a renovation T_c . This reflects the additional net income sacrificed by delaying renovation. Secondly, the term $C'(T^*)$ is the marginal increase in renovation costs that will be incurred as the mall 'grows older'.

For $T < T^*$, the sum of marginal benefits (in terms of net rental incomes and interests savings) exceeds the sum of potential marginal costs (as expressed in terms of implicit rents and the extra increase in renovation costs) in delaying renovation. It is therefore better to delay renovation. Note that this is so even if the net present value

of renovation at time T is positive. However, if renovation is delayed beyond T^* (that is, when $T > T^*$), the sum of net rental incomes and interest savings on delayed renovation costs are no longer sufficient to cover potential losses on implicit rents and rises in the costs of renovation.

Comparative Statics

The optimal renovation period T^* is determined by three factors that we have treated as exogenous: the discount rate r and the trajectories for net rental incomes $y(t)$ and renovation costs $c(T)$. This section considers how variations in these three factors affect T^* . In order to simplify the analysis without loss of generality, we assume that the time to undertake a renovation T_c is sufficiently short relative to T^* that it can be ignored. Formally we assume $T_c = 0$ from which it follows that $C(T) = c(T)$.

Denote the maximizing condition of equation 7 as $R(T^*)$;

$$R(T^*): y(T^*) + rC(T^*) - rV(T^*) - C'(T^*) = 0. \tag{8}$$

Then the first-order condition for the optimal renovation period can be written:

$$\left. \frac{dV(T)}{dT} \right|_{T=T^*} = \left. \frac{R(T)}{(1 - e^{-rT})} \right|_{T=T^*} = 0.$$

For the second-order condition to be satisfied, we must have:

$$\left. \frac{d^2V(T)}{dT^2} \right|_{T=T^*} = \left. \frac{R'(T^*)}{(1 - e^{-rT^*})} \right|_{T=T^*} < 0 \rightarrow R'(T^*) < 0. \tag{9}$$

Now consider the effect on T^* of a change in the discount rate r . We know that:

$$\left. \frac{dT^*}{dr} = - \frac{\partial R(T) / \partial r}{\partial R(T) / \partial T} \right|_{T=T^*}, \tag{10}$$

and from equation 9 we have:

$$\text{sign} \left(\frac{dT^*}{dr} \right) = \text{sign} \left(\frac{\partial R(T)}{\partial r} \right) \Big|_{T=T^*}. \tag{11}$$

Differentiating equation 8 and reorganizing, we have:

$$\frac{\partial R(T^*)}{\partial r} = C(T^*) - V(T^*)(1 + \mu_r(V(T^*))), \tag{12}$$

where $\mu_r(V(T)) = r/V(T) \cdot \partial V(T)/\partial r$ is the elasticity (sensitivity) of $V(T)$ with respect to the discount rate r .

It is to be expected that the capital value of the mall, $V(T)$, is a declining function of the discount rate r , i.e., that $\partial V(T)/\partial r < 0$, in which case the elasticity $\mu_r(V(T)) < 0$.

Substituting from equation 12 into equation 11 we have:

Proposition 1:

$$\text{Define } |\mu_r(V(T))| \equiv - \frac{\partial r}{V(T)} \cdot \frac{V(T)}{\partial r}.$$

$$(a) \text{ if } |\mu_r(V(T))| \geq \frac{V(T) - C(T)}{V(T)} \Big|_{T=T^*} \text{ then } \frac{dT^*}{dr} > 0;$$

$$(b) \text{ if } |\mu_r(V(T))| < \frac{V(T) - C(T)}{V(T)} \Big|_{T=T^*} \text{ then } \frac{dT^*}{dr} < 0;$$

Proposition 1 has an intuitive appeal. It states that the more elastic (sensitive) is the capital value of the mall to changes in the discount rate the more likely is it that an increase in the discount rate will lengthen the optimal renovation period. An increase in the discount rate increases the marginal benefit of delaying renovation by increasing the interest savings on renovation costs, increases the marginal cost of delaying renovation by increasing the implicit rent from renovation but *reduces* the marginal cost of delaying renovation by decreasing the capital value of the mall. The more elastic is the capital value $V(T)$ with respect to the discount rate the greater is the capital value effect of a discount rate increase relative to the implicit rent effect, as a result of which an increase in the discount rate will discourage frequent renovations.

In evaluating the effect of a change in the net rental income trajectory $y(t)$ on the optimal renovation period we follow Turnbull (1988a).² Define a shift parameter θ_y such that $y(t) \rightarrow y(t; \theta_y)$, $R(T) \rightarrow R(T; \theta_y)$ with:

$$y_{\theta_y}(t; \theta_y) > 0 \text{ and } y_{t\theta_y}(t; \theta_y) \geq 0,$$

where subscripts denote partial differentiation with respect to the appropriate argument.

The parameter θ_y has two effects on $y(t)$: a level effect ($y_{\theta_y}(t; \theta_y) > 0$) and a convexity effect ($y_{t\theta_y}(t; \theta_y)$). When $y_{t\theta_y}(t; \theta_y) = 0$ we have a pure level effect, a parallel shift of $y(t)$ in y - t space, while $y_{t\theta_y}(t; \theta_y) > 0$ reflects an increase and $y_{t\theta_y}(t; \theta_y) < 0$ a decrease in the convexity of $y(t)$.

We can apply the same analysis as for a change in the discount rate:

$$\frac{\partial T^*}{\partial \theta_y} = - \frac{\partial R(T; \theta_y) / \partial \theta_y}{\partial R(T; \theta_y) / \partial T} \Big|_{T=T^*} \quad (13)$$

and from equation 9 we have:

$$\text{sign} \left(\frac{\partial T^*}{\partial \theta_y} \right) = \text{sign} \left(\frac{\partial R(T; \theta_y)}{\partial \theta_y} \right) . \tag{14}$$

From equations 2, 4, 7 and 8:

$$\frac{\partial R(T; \theta_y)}{\partial \theta_y} = y_{\theta_y}(T; \theta_y) - \frac{r \int_0^T y_{\theta_y}(t; \theta_y) e^{-rt} dt}{(1 - e^{-rT})} , \tag{15}$$

from which we have:

Proposition 2:

(a) A change in the level of the net rental income trajectory will have no effect on the optimal renovation period:

(i) if $y_{\theta_y}(t; \theta_y) = 0$ then $\frac{\partial R(T; \theta_y)}{\partial \theta_y} = 0$ and $\frac{\partial T^*}{\partial \theta_y} = 0$.

(b) A change in the convexity of the net rental income trajectory will change the optimal renovation period:

(ii) if $y_{\theta_y}(t; \theta_y) < 0$ then $\frac{\partial R(T; \theta_y)}{\partial \theta_y} < 0$ and $\frac{\partial T^*}{\partial \theta_y} < 0$.

(iii) if $y_{\theta_y}(t; \theta_y) > 0$ then $\frac{\partial R(T; \theta_y)}{\partial \theta_y} > 0$ and $\frac{\partial T^*}{\partial \theta_y} > 0$.

In interpreting Proposition 2 it is perhaps easiest to consider the impact on the optimal renovation period of a reduction in net rental incomes, in which case $\theta_y < 0$.

Proposition 2(a) tells us that a reduction in the *level* of net rental incomes changes the marginal costs and marginal benefits of delaying renovation by exactly the same amount and so leaves the optimal renovation period unaltered.³ The optimal renovation period is affected by a reduction in the net rental income trajectory only if there is a change in the *convexity* of that trajectory: Proposition 2(b). $\theta_y < 0$ and $y_{\theta_y}(t; \theta_y) < 0$ imply that the reduction in the net rental income trajectory has reduced the rate of decrease of net rental incomes. This raises the marginal benefits of delaying renovation by more than it raises the marginal costs of delaying renovation, and increases the optimal renovation period. By contrast, $\theta_y < 0$ and $y_{\theta_y}(t; \theta_y) > 0$ imply that the reduction in the net rental income trajectory has increased the rate of decrease of net rental incomes. This raises the marginal benefits of delaying renovation by less than it raises the marginal costs of delaying renovation, and decreases the optimal renovation period.

Now consider the effect of a change in the renovation cost trajectory $C(T)$ (or $c(T)$). Define a shift parameter θ_c such that $C(T) \rightarrow C(T; \theta_c)$, $R(T) \rightarrow R(T; \theta_c)$ with:

$$C_{\alpha}(T; \theta_c) > 0 \text{ and } C_{T\alpha}(T; \theta_c) \geq 0.$$

$C_{T\alpha}(T; \theta_c) > 0$ reflects an increase and $C_{T\alpha}(T; \theta_c) < 0$ a decrease in the convexity of $C(T)$. From equations 2, 4, 7 and 8:

$$\begin{aligned} \frac{\partial R(T; \theta_c)}{\partial \theta_c} &= C_{\alpha}(T; \theta_c) + \frac{rC_{\alpha}(T; \theta_c)e^{-rT}}{(1 - e^{-rT})} - C_{T\alpha}(T; \theta_c) \\ &= \frac{rC_{\alpha}(T; \theta_c)}{(1 - e^{-rT})} - C_{T\alpha}(T; \theta_c). \end{aligned} \quad (16)$$

Equations 13 and 14 hold with the substitution of θ_c for θ_y , and it immediately follows that:

Proposition 3:

- (a) an increase in the renovation cost trajectory $C(T)$ will decrease the optimal renovation period if there is an increase in the convexity of

$$C(T) \text{ such that } C_{T\alpha}(T; \theta_c) > \frac{rC_{\alpha}(T; \theta_c)}{(1 - e^{-rT})};$$

- (b) if there is an increase in the renovation cost trajectory $C(T)$ such that $C_{T\alpha}(T; \theta_c) < rC_{\alpha}(T; \theta_c)/(1 - e^{-rT})$, there will be an increase in the optimal renovation period.

Proposition 3 has an appealing intuitive interpretation. An increase in the renovation cost trajectory increases the marginal benefits and reduces the marginal costs of delaying renovation through its impact on $C(T)$ and $V(T)$ respectively (recall equation 7), while it increases the marginal costs of delaying renovation through its impact on $C'(T)$ and then only if $C_{T\alpha}(T; \theta_c) > 0$, i.e., only if there is an increase the convexity of the renovation cost trajectory. For the increase in renovation costs to lead to a decrease in the optimal renovation period the convexity effect has to be "strong". In other words, for an increase in renovation costs to lead to a reduction in the optimal renovation period it has to be associated with a substantial increase in the rate at which renovation costs escalate with the age of the mall.

While our analysis abstracts from uncertainty, we can use our comparative static results to comment upon the effect of uncertainty with respect, for example, to the net rental income trajectory. Assume that net rental incomes are subject to uncertainty such that they are expected to be low, $y_l(t)$ with probability p and high, $y_h(t)$ with probability $1 - p$. Expected net rental incomes are $y_e(t) = py_l(t) + (1 - p)y_h(t)$. Our analysis goes through unaltered substituting $y_e(t)$ for $y(t)$.

Now consider the effect of an increase in the probability p of low net rental incomes (perhaps as a result of an increase in the probability of entry of a competitor mall). Proposition 2 indicates that this will affect the optimal renovation period only if it changes the rate of decrease in expected net rental incomes, which requires that the low net rental income trajectory $y_l(t)$ falls at a different rate from the high net rental income trajectory $y_h(t)$. Specifically, an increase in p will reduce the optimal renovation period if the low net rental income trajectory $y_l(t)$ falls faster than the high net rental income trajectory $y_h(t)$.

A Numerical Example

In this section we illustrate the theoretical analysis by means of a numerical simulation.

The practical determination of T^* , the optimal renovation period, depends primarily on our knowledge of the discount rate r , the net rental income trajectory $y(t)$ and the renovation cost $C(T)$. Explicit functional forms for $y(t)$ and $C(T)$ can potentially be estimated from the analysis of empirical data. In the absence of such data we present an analysis using functional forms that we consider to be “reasonable”. We maintain the assumption that the time to undertake a renovation T_c is sufficiently short relative to T^* that it can be ignored.

The simplest approach is to assume net rental incomes $y(t)$ decline at a constant rate while renovation costs $C(T)$ increase at another. This is, we assume

$$y(t) = ab^t \quad 1 > b > 0, \tag{17}$$

and

$$C(T) = hk^T \quad k > 1, \tag{18}$$

where a is the initial net rental income and h is the renovation cost at time $t=0$.

From these definitions of y and C , we have

$$Y(T) = \int_0^T y(t)e^{-rt} dt - hk^T e^{-rT} = \frac{a(1 - b^T e^{-rT})}{r - \ln b} - C(T)e^{-rT} \tag{19}$$

and

$$C'(T) = hk^T \ln k. \tag{20}$$

Substituting into the maximizing condition of equation 8 we have:

$$R(T^*): ab^{T^*} + rhk^{T^*} - r \frac{a(1 - b^{T^*} e^{-rT^*})}{r - \ln b} - hk^{T^*} e^{-rT^*} - hk^{T^*} \ln k = 0. \tag{21}$$

Numerical simulation of equation 21 allows us to calculate the optimal renovation period T^* and to analyze its sensitivity to the parameters a , b , h , k and r .

Suppose a newly renovated mall yields a net rental of \$500,000 per annum, which declines continuously at a rate of 5% per annum. Assume that the initial renovation cost is \$300,000 which increases continuously at a rate of 5% per annum. That is, $a = \$500,000$; $b = .95$; $h = \$300,000$; and $k = 1.05$. Suppose further that the prevailing discount rate r is 6% per annum. Then the optimal renovation period can be estimated by conventional methods of valuation using a spreadsheet program as shown in Exhibit 2.

In this table, all the figures are computed according to the formulae presented above. Column [12] represents the computation of equation 21. In this column, net marginal benefits are positive in years 1 to 5 and are negative from year 6 onwards. The optimal value of T^* is 5.638 years. It should also be noted that the capitalized value of the mall (column 9) is maximized at this point, at $V(T^*) = \$6.314$ million, which is of course, the objective of our analysis. Under market clearing conditions, this $V(T^*)$ also represents the outright capital value of the mall offered by the market.

We use the numerical analysis program MATLAB to simulate the impact on T^* of changes in the parameters a , b , h , k and r . The results are presented in Exhibits 3, 4 and 5.

The first parameters chosen for analysis are h/a (the ratio of renovation cost to initial rent) and the discount rate r . We focus on the ratio h/a because equation 21 indicates that the value of T^* is a function of h/a , as can be seen by dividing both sides of equation 21 by a , giving

$$b^{T^*} + r \frac{h}{a} k^{T^*} - r \frac{(1 - b^{T^*} e^{-rT^*})}{1 - e^{-rT^*}} - \frac{h}{a} k^{T^*} \ln k = 0. \quad (22)$$

Exhibit 3 presents values of T^* for discount rates ranging from 1 to 10% and h/a ratios .5 to 1.4. In relating these figures to Propositions 1-3, consider first the effect of an increase in the discount rate. The elasticity of $V(T)$ with respect to r for our numerical example is $\mu_r(V(T^*)) = .963$ while $(V(T^*) - C(T^*)) / V(T^*) = .937$, the conditions of Proposition 1(a) are satisfied, and an increase in the discount rate increases the optimal renovation period.

Now consider an increase in the parameter a (a decrease in h/a). Simple calculus shows that:

$$y_f(t;a) = \ln b \cdot ab' < 0 \text{ and } y_{fa}(t;a) = \ln b \cdot b' < 0 \text{ since } b < 1.$$

The conditions of Proposition 2(b)(ii) are satisfied and an increase in a decreases the optimal renovation period.

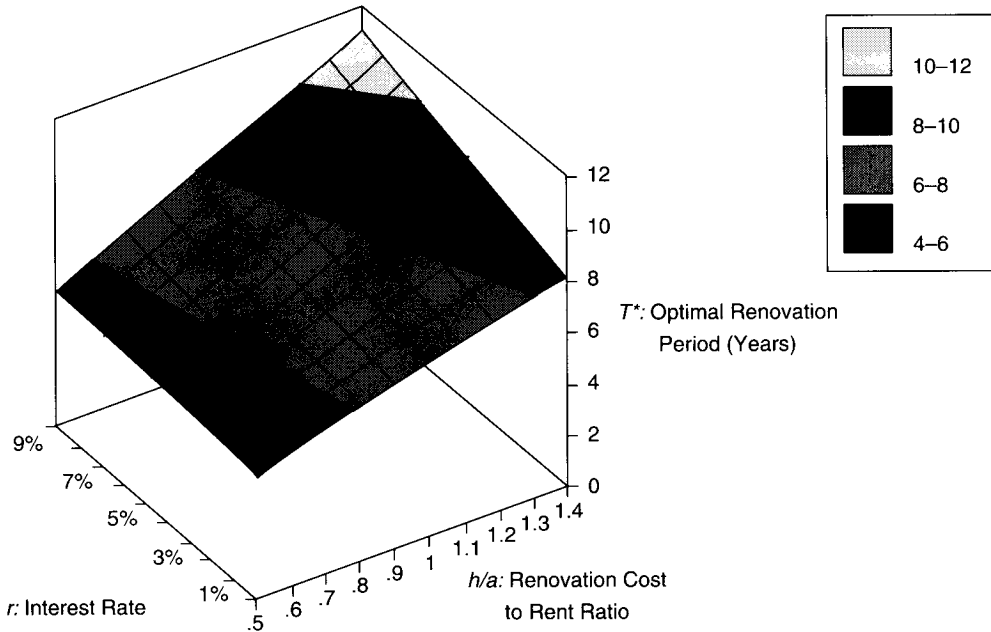
Finally, consider an increase in h (an increase in h/a). Proposition 3 can be simplified by the approximation:

$$r / (1 - e^{-rT}) \approx 1/T,$$

Exhibit 2
Computation of T^* : The Optimal Renovation Period

Ref	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]
Age (in Yrs) After Renovation	$Y(T) = ab^T$	$C(T) = hk^T$	$C(T) = hk^T \ln k$ Change in Renovation Costs	Interest Rate r	$e^{-(rT)}$	Integral of $[1] \times [5]$ $a[1 - b^T Te^{-(rT)}] / (r - \ln b)$	Net Present Value over Renv. Cycle $Y(T)$	$1/[1 - e^{-(rT)}]$	Capitalised Value of the Mall $V(T)$	Marginal Benefits $Y(T) + rC(T)$	Marginal Loss $rV(T) + C(T)$	Net Marginal Benefits
1	\$475,000	\$315,000	\$15,369	6.00%	.941764534	\$473,181	\$176,525	17.17166637	\$3,031,227	\$483,900	\$197,243	\$296,657
2	\$451,250	\$330,750	\$16,137	6.00%	.866920437	\$896,524	\$603,175	8.843330934	\$5,334,080	\$471,095	\$336,182	\$134,913
3	\$428,688	\$347,288	\$16,944	6.00%	.835270211	\$1,275,280	\$985,201	6.070547462	\$5,980,709	\$449,525	\$375,787	\$73,738
4	\$407,253	\$364,652	\$17,791	6.00%	.786627861	\$1,614,143	\$1,327,298	4.686647493	\$6,220,579	\$429,132	\$391,026	\$38,106
5	\$386,890	\$382,884	\$18,681	6.00%	.740818221	\$1,917,317	\$1,633,669	3.858295914	\$6,303,178	\$409,864	\$396,872	\$12,992
5.63848	\$374,425	\$395,000	\$19,272	6.00%	.712975087	\$2,093,964	\$1,812,340	3.48401813	\$6,314,224	\$398,126	\$398,126	\$0 $T^* = 5.63848$
6	\$367,546	\$402,029	\$19,615	6.00%	.697676326	\$2,188,559	\$1,908,073	3.307713177	\$6,311,357	\$391,668	\$398,296	(\$6,629)
7	\$349,169	\$422,130	\$20,596	6.00%	.65704682	\$2,431,232	\$2,153,873	2.915849911	\$6,280,370	\$374,496	\$397,418	(\$22,922)
8	\$331,710	\$443,237	\$21,626	6.00%	.618763392	\$2,648,347	\$2,374,079	2.623180571	\$6,227,638	\$358,304	\$395,284	(\$36,979)
9	\$315,125	\$465,398	\$22,707	6.00%	.582748252	\$2,842,594	\$2,571,384	2.396634659	\$6,162,667	\$343,049	\$392,467	(\$49,418)
10	\$295,368	\$488,668	\$23,842	6.00%	.548811636	\$3,016,382	\$2,748,195	2.216369215	\$6,091,015	\$328,689	\$389,303	(\$60,615)

Exhibit 3
Sensitivity of T^* to h/a : Renovation Cost to Rent Ratio



in which case an increase in h would decrease the optimal renovation period if

$$C_{Th}(T:H) > C_h(T:h)/T|_{T=T^*} \Rightarrow \ln k \cdot k^T > k^T / T|_{T=T^*} \Rightarrow k > 1.194.$$

This condition is not satisfied in our example.

Exhibit 4 illustrates the sensitivity of T^* to b , the rate of decline of net rental incomes: the lower is b the faster is the rate of rental decline. Note that:

$$y_{ib}(t;b) = ab^{t-1}(1 + t \cdot \ln b) > 0 \Rightarrow \ln b > -1/t,$$

which is true for our numerical example in the relevant range of values of t . The conditions of Proposition 2(b)(iii) are satisfied and an increase (decrease) in b increases (decreases) the optimal renovation period.

Exhibit 5 shows that the relationship between T^* and the rate of renovation cost increase k is ambiguous. At low interest rates, T^* decreases as k increases but at high interest rates, T^* initially increases as k increases, before falling as k increases further.

The explanation lies in the condition of Proposition 3. From equation 18 we have:

$$C_{Tk}(T:k) = h \cdot k^{T-1}(1 + T \ln k) \text{ and } C_k(T:k) = T \cdot h k^{T-1},$$

and for an increase in k to lead to an increase in the optimal renovation period we require:

$$1 + T \cdot \ln k > Tr / (-e^{-rT}).$$

Exhibit 4
Sensitivity of T^* to b : The Rate of Net Rental Income Decline

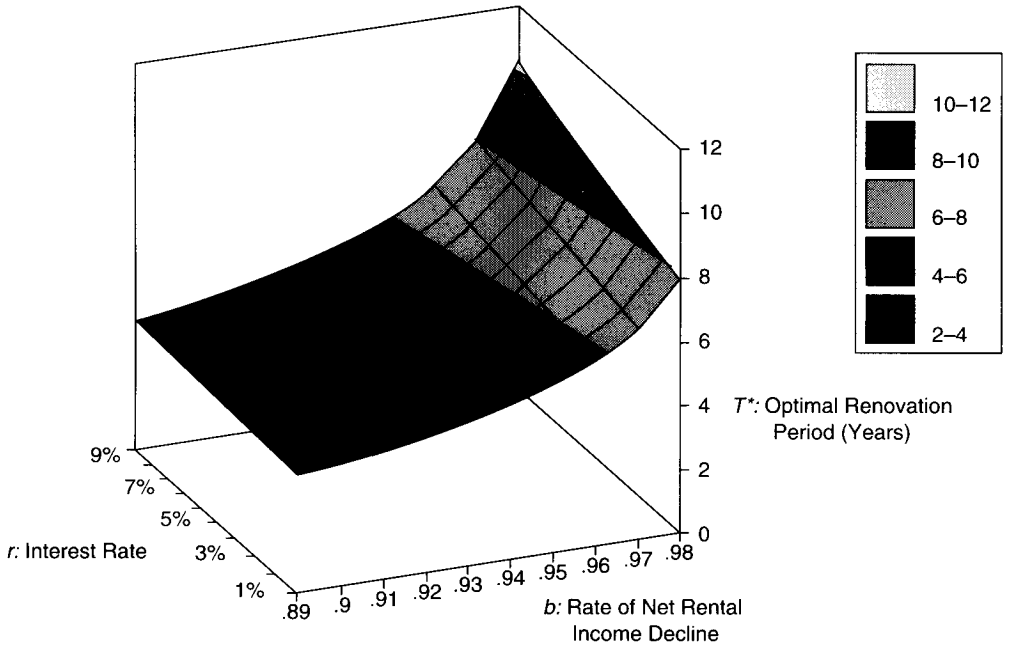
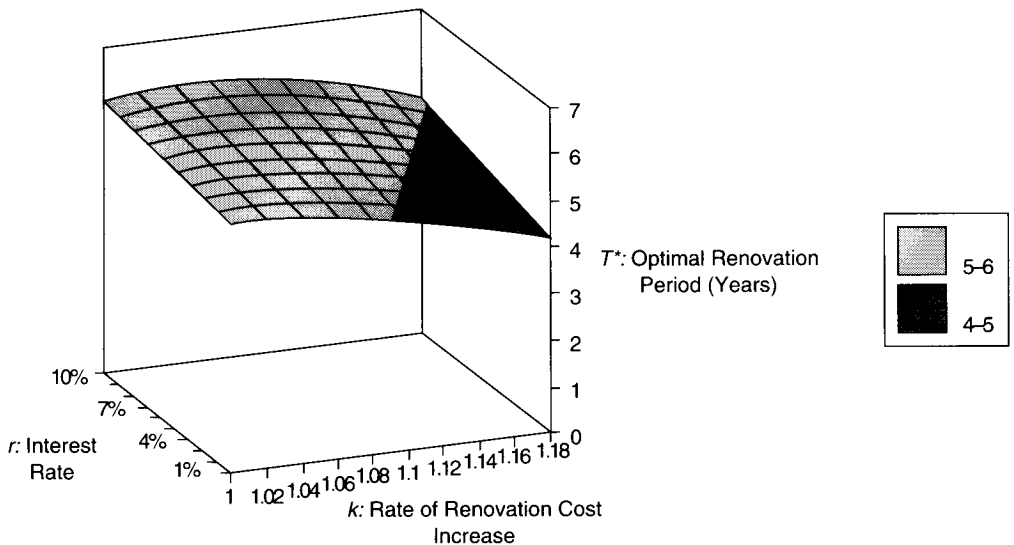


Exhibit 5
Sensitivity of T^* to k : Rate of Renovation Cost Increase



Since $\partial(r/(1 - e^{-rt}))/\partial r$ this condition is more likely to be satisfied at low values of k and/or high values of the discount rate r , as illustrated in Exhibit 5.

The discussion of Proposition 3 indicated that there are two forces relating to $C(T)$ working against each other in determining T^* . On the one hand, $rC(T)$ represents the

benefits in interest savings on capital costs in delaying renovation. This value is high for higher discount rates. On the other hand, $C'(T)$ is the extra cost increase due to waiting, which is an increasing function of k (for $k > 1$). Higher values of k therefore imply extra costs in delaying renovation: the convexity effect to which we have referred.

In our example, at low values of the discount rate r the convexity effect is dominant and increases in k reduce the optimal renovation period. For higher values of the discount rate, however, the interest savings effect is given more weight. As k is increased the interest cost savings from delaying renovation initially outweigh the effects on cost of delay and T^* increases. It is only as k is increased even further that we find the convexity effect dominating once again.

Conclusions

The decision to renovate a mall is not random. It depends on a number of factors including the rate of depreciation of net rental incomes, the level and rate of change of renovation costs, discount rates and changing market conditions. In our model, we have analyzed the interactions of the first three sets of factors in determining the optimal time to renovate. We have shown that this optimal renovation period, T^* , should achieve a balance between the marginal benefits and the marginal costs of delaying renovation. The marginal benefits of delaying renovation are the current net rental income at T^* and the interest saving on renovation costs that will be achieved by delaying. The marginal costs of delaying renovation are the present value of the implicit rent derived from the newly renovated mall plus the marginal increase in renovation costs that will be incurred as the mall 'grows older'.

We have shown that the relationship between the optimal renovation period and the discount rate is determined by the elasticity of the capital value of the mall with respect to the discount rate. The way in which the optimal renovation period is affected by changes in the net rental income and renovation cost trajectories is determined by how the convexity of these trajectories are altered.

Our analysis can be used to identify the likely impact on renovation of changes in market conditions. A change in market conditions that leads to a change in net rental incomes will affect the optimal renovation period only if it alters the rate at which net rental incomes depreciate over time. If, for example, increased competition from new retail developments causes a faster rate of decline in net rental incomes of existing malls, then this will lead to a shortening in the optimal renovation period of the existing malls.

The practical determination of the optimal renovation period is dependent upon data on net rental income depreciation and renovation cost appreciation. Rather than constructing such a data set, we have presented a numerical simulation to illustrate first, the method of calculation and secondly, the probable sensitivity of the renovation period to factors such as the rate of rental decline, the discount rate and the level and rate of change of renovation costs.

A number of extensions to our analysis can be envisaged. We might extend the analysis to cases in which the post-renovation condition of the mall is a choice variable. Moreover, the model developed above can be applied to areas other than retail properties. These are each the subject of further research.

Notes

¹Note that $V(T)$ is defined ignoring the initial construction costs of the mall. This reflects our concern with the renovation of the mall rather than with the feasibility of the initial development, so that we can treat the initial construction costs (which we might denote K_0) as sunk costs. Our subsequent analysis is unaffected if $V(T)$ were to be replaced by $V(T) - K_0$.

²We are grateful to an anonymous referee for this suggestion.

³It should be emphasized that this analysis is concerned solely with renovation of an existing mall. Reduction in the level of net rental incomes may call into question the viability of the mall and require redevelopment of the site, but such a possibility is outside the scope of this paper.

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