Commercial Loan Underwriting and Option Valuation

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Abstract. **This article seeks to answer why over a nineteen-year period the debt-coverage ratio for commercial noninsured properties averages 1.29. The article applies the corporate liabilities extension of the Black-Scholes option pricing model to the equity valuation of a real estate project. The regression results of the modified model robustly sustain its usefulness in explaining the derivation of the debt-coverage ratio. The results confirm that commercial mortgage loan underwriters operate with a five-year horizon in creating the equity cushion needed to protect themselves against interest-rate risk.**

Introduction

The closing of a first mortgage commercial loan is the resolution of two competing strategies. The equity owner extracts from the real estate project a high level of capital funding. The lender provides this funding with expectations that a sufficient equity cushion protects him/her against interest-rate risk and vacancy risk during the loan contract period and at the time of refinancing.

The magnitude of this equity cushion expressed as either a debt-coverage ratio or loan-to-value risk measure has been remarkably stable for properties underwritten by the major life insurers from the period 1968 to 1987. The average monthly debt-coverage ratio (DCR) average, using stabilized first-year net operating income, is 1.29 with a variance of only .003.[1] Likewise the loan-to-value (LV) averaged 72.61% with a variance of 6.94% for the same period.[2] This stability is even more remarkable considering that mortgage loan interest rates over the same period averaged 10.67%, with a roller coaster skewed variance of 3.81%.[3]

Possible reasons for using LV and DCR underwriting measures at these levels include blind herd consensus (groupthink), actuarial basis, and/or stochastically-based debt-equity valuation methodology, in which the equity is priced as an option. The purpose of this study is to examine if applying the corporate liabilities extension of the Black-Scholes option valuation technique to real estate projects to derive the debt-coverage ratio protects the underwriter against interest-rate risk.

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Background

The introduction of commercial mortgage-backed securities has renewed interest in actuarial-based default analysis of uninsured commercial real estate mortgages. Unfortunately, while there have been several contemporary attempts to undertake an examination of commercial real estate default mortgage risk, the lack of centrally-based and standardized information sets has impeded these efforts.[4] The last comprehensive surveys of commercial real estate defaults were in the 1940s and were based on the bond-backed commercial real estate funding schemes of the late 1920s and early 1930s. There is currently considerable interest in the pricing performance of GNMA multifamily mortgages as a surrogate for commercial real estate mortgages. However, these GNMA mortgages are not comparable to the uninsured commercial mortgage, since GNMA mortgages are insured in pools, are multifamily, and are processed via standardized regulations.

The now common approximate 70% loan-to-value risk measure and the corresponding debt-coverage ratio of 1.29 for the uninsured mortgages of office buildings, shopping centers and hotels are partially due to the high default experiences of these early real estate bonds. The uninsured mortgage loan and bond default rates were among the highest of any type of fixed-income instruments during the great depression from 1930 to 1936 [7]. The depression folklore, combined with FHA borrowing experience over the years and deterministic appraisal stress tests (there should always be the residual cushion of the land value), have all contributed to fostering the current 70% consensus loan-to-value. However, the common folklore gives few clues as to how underwriters anticipate interest-rate risk.

Alternate Research Methods

Stochastic approaches have been applied extensively to determine the price of assets, such as mortgages and options. The approaches have evolved, generally into two: a) the contingent claims model (Cox, Ingersoll and Ross [4]) and b) option valuation (Black and Scholes [1] and Sprenkle [13]). Both approaches rely on the analogy of the particle diffusion process found in physics in which a financial instrument travels through a time and financial space subject to the forces of changing financial parameters, a significant proxy being interest rates. Brenner [3] reviews their origins and Brennan–Schwartz [2] proves that, with severely limiting assumptions, the consul bond case, the two models have a common form.

However, the two models evolved differently in their application to financial assets and markets. The contingent claims model (CCM) tests the valuation of fixed-income financial assets whose value is a function of exogenous variables, such as interest rates and inflation rates. However, financial instruments, such as bonds or insured mortgages on which this model was derived, are freely traded in efficient or nearly efficient markets. The contingent claims model (CCM) is a free-form valuation model. Under simplifying assumptions, major exogenous variables determine the value of a financial instrument. The major exogenous variables are the change or variance in interest rates (a single rate or term structure factor), and the speed of that change. At this point the contingent claims model almost reduces to a risk/return concept. The higher the variance and speed of change, the higher the risk and therefore, the higher is the required return.

The contingent claims model (CCM) has also been shown to address the task of valuing of assets and debt. Epperson, Kau, Kennan and Muller [6] applied the CCM to determine the cost of insurance for insured mortgages with different loan-to-value ratios. The value of the house was a function of inflation. The value of the debt was a function of interest rates. There was no theoretical or functional relationship between the house price and the debt, because inflation and interest rates were assumed unrealistically to be independent variables.

However, the point of focus of the underwriter is different from the contingent claims analyst. The contingent claims analyst seeks to answer the question: What is the price that I should pay for this financial bond or mortgage given my expectations of interest rates and volatility? In contrast, shaken by the history of bond and mortgage defaults especially at the time of refinancing [8] the bond raters and mortgage underwriters ask: What is my margin of error? Will there be sufficient value to cover the value of the debt during the holding period and at the time of refinancing? Price is not the major consideration here. The major consideration is the long-range projection that the value of the asset remains greater than the value of the debt since there is no insurance to cover the mortgage loan should a default occur. It can be argued that the bond underwriter and mortgage underwriter, in an evolving general equilibrium market, need only be concerned with the price of the debt to determine the acceptable cushion. However, the objective of the underwriting and bond rating methodologies is to provide an estimate of a long-term comfort level for the debt security when future interest rates are unknown [9].

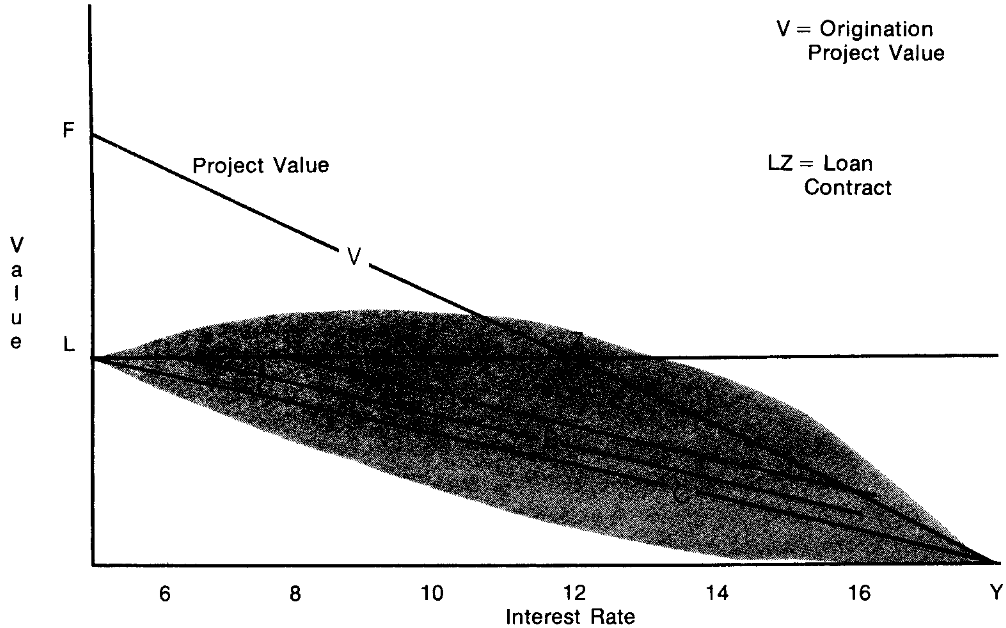
One of the primary theoretical interests of the Black–Scholes option valuation model is to fathom the dependency of the value of one security, an option, which is believed to be a function of another security, the stock. The essential components of this methodology are:

- a major primary instrument such as a stock or firm,
- the secondary dependent instrument such as option, warrant, or debt.

Brenner [3] provides an array of both theoretical and empirical studies that link one security to another security, including commodity options, commodity futures, and bank loan portfolios. Black–Scholes [1] called for further explorations into the realm of the valuation relationship of firms and their liabilities. This call for further explorations into firm valuation is the point of departure for this research. The task at hand is to determine whether the Black–Scholes model replicates the decision process of the underwriter in determining the link between the sufficiency of the value of the real estate project, the firm, and the value of mortgage debt during the holding period. The underwriter does not seek precision in a future price, but the comfort of an acceptable latitude in the value created by the income stream. The analyst does so because the uninsured mortgage loan is not freely traded, is generally restrictive in its call or redemption features, and is generally not very liquid. The underwriter assumes that he will be holding the mortgage debt until the time of refinancing; hence there are restrictive prepayment penalties.

The concept of linking the value of two related financial assets has previously been applied in uninsured commercial real estate analysis. Using the Sprenkle warrant pricing model, the predecessor to the Black–Scholes version, Shilton [11, 12] tested the relationship between the value of a real estate project and its debt. In this case, the model was used to illustrate that when net operating incomes are artificially inflated, due to subsidies and tax-exempt uninsured debt, for a subsidized real estate project, there was little likelihood of the project reverting to a value equal that of an unsubsidized project during a normal debt holding period. The results of uninsured portfolios of the various state housing finance agencies confirm this observation.

Exhibit 1
Equity and Debt Pricing



The Loan as a Put Option

A mortgage loan contract is a put option to the lender.[5] At the time of the repayment, what is due to the lender is the outstanding contract amount, i.e., the 'exercise price.' During the holding period, the project value fluctuates because of cap rate changes. The mortgage loan value fluctuates along a pricing path because of interest-rate changes. At the expiration of the loan contract period, the loan is paid off at the contract amount if the project's value is greater than the loan amount. Now, suppose the real estate project was originally worth $1000, and the non-recourse debt contract amount was $800. If the value of the project is reduced to $700, the value of the debt then is reduced to $700. The borrower has recognized a gain of $100 (ignoring tax consequences) because of the default condition. The borrower is the holder of the put. The debt still has value, although the firm's value is below that of the original contract or 'exercise' price. The maximum possible value of the put is the contract loan amount. Theoretically a call option has no maximum value. To transform the put scenario to the equity-mortgage valuations in real estate, we suggest that there is a relationship between cap rates and interest rates. Given an income stream, the higher the interest rate, and therefore the higher the cap rate, the lower the value of the project.

Exhibit 1 depicts the interest rate, the value of the firm and the value of the debt at the initiation of the loan contract. The firm has value *V*, and the debt is value *D*. The equity cushion, the area *VZD*, represents the amount of interest-rate change that the firm can endure until the value of the firm is less than the contract value of the debt. When interest rates rise to a certain level, the firm at point Z is worth the contract debt amount. When interest rates continue to rise above that level, the firm's value then becomes less than the contract debt amount. During the contract life, the value of the debt can travel along many paths, depending upon the volatility of interest rates and the time to maturity. However, no matter what the path is, there is always the confinement imposed by the contract loan amount.

The possible value paths of the debt are similar to those of the value paths of the call or put prices [10]. The longer the time to maturity, the more volatile value path of the debt. The closer to maturity and/or less volatile interest rates, the closer the value path will be to the contract amount. Clearly, in the debt case, the value of the debt during the holding period can exceed the contract amount. Owners of the debt note during the holding period can sell the note (although in a very thinly traded market) and realize gains when interest rates are lower.

This study seeks to determine the basis on which the underwriter sets the loan contract line in actual underwriting practice. Is the equity cushion provided by the contract debt insufficient, merely sufficient or excessive, given time horizons and expectations of interest-rate changes and volatility? Can the debt contract line by pushed up? That is, can the debt-coverage ratios be safely decreased and the loan-to-value ratios be safely increased?

The Modified Black–Scholes Valuation Model

As it applies to options, the Black–Scholes valuation model is:

$$V_O = V_S N(d_1) - \frac{V_E}{r^{rt}} N(d_2) \tag{1}$$

where:

V_O = value of the option

V_S = value of the stock

$N(d^*)$ = probability that a deviation less than d will occur in a normal distribution, unit normal.

V_E = exercise price

$$d_1 = \frac{\ln(V_S/V_E) + (r + \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

$$d_2 = \frac{\ln(V_S/V_E) + (r - \tfrac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$$

σ = standard deviation of continuously compounded rate of return on stock

r = riskless rate of interest continuously compounded

t = number of years to expiration date.

A major difference between the CCM model and the Black–Scholes valuation technique is the incorporation of the specific link between the stock and the option in the factors, d_1, and d_2, expressed in the term ln (V_S/V_E). Brenner [3] reviews several possible stochastic distribution variations of this term.

To prepare for the extension of this model to corporate liabilities, recall that the option has value when the stock price is equal to or greater than exercise price. As long as the option contract has value because the stock price is equal to or greater than the exercise price, the holder of the option has a claim against the person who offered the contract. There is value to the option when there is a cushion between the exercise price and the value of the stock. (When the stock is equal to the option exercise price, the option may have value because of transaction cost considerations.)

The option model is now being extended to model liability pricing, i.e., equity-debt valuation. Black–Scholes, in their example, explain that the debt-holders have first right to exercise a claim upon the firm up to the value of expiring debt at the time that corporate debt must be refinanced.[6] The equity-holders only have a claim to the residual. As an example, when there is value to the equity because the firm's value exceeds the contract debt amount the model is transformed:

- the equity, V_{EQ}, of a firm is the 'option,'
- contract value, V_L, of the loan is the 'exercise price,'
- the value of the firm, V_F, is the sum of the value of the equity and the contract value of the loan and is viewed as the 'stock' in the original options model.

At refinancing, if the value of the firm is less than the value of the contract debt amount, the stock of the firm is worthless, as is the case of an option whose exercise price is higher than the stock price.

Within the marketplace, the firm value, the debt value, and therefore the equity value are subject to changes in expectations, interest rates and capitalization rates (as proxies for value of the real estate project).

The option pricing model, transformed to corporate equity-liabilities valuation, becomes the equity-debt pricing model as follows:

$$V_{EQ} = V_F N(d_1) - V_L N(d_2) \tag{2}$$

where:

V_{EQ} = value of equity at time of the underwriting of the mortgage loan,

V_F = value of the project, 'the firm,'

$N(d^*)$ = probability that a deviation less than d will occur in a normal distribution with an expected value of zero and a standard deviation of one,

V_L = value of the debt at time of underwriting,

$$d_1 = \frac{\ln (V_F/V_L) + (r + \frac{1}{2}\sigma^2)t}{\sigma \sqrt{t}}$$

$$d_2 = \frac{\ln (V_F/V_L) + (r - \frac{1}{2}\sigma^2)t}{\sigma \sqrt{t}}$$

σ = standard deviation of mortgage interest rate, the rate of return on the debt, and

t = time, the debt holding period.

However, there are two differences, between the Black–Scholes (B–S) option valuation model and the equity-debt valuation model. In the B–S option model, the option term is discounted because an option does not earn any return during the holding period. However, in the debt-equity model the debt is generating a return and therefore is not discounted. Therefore, the discounting term (e^{rt}) of the B–S option model is omitted. Secondly, the mortgage interest rate is used instead of the riskless rate because it is against this rate the loan has been valued.[7]

Research Design

The hypothesis is that underwriters set their debt-coverage ratio such that the loan-to-value ratio results in a project equity cusion as specified in equation (2). From 1968 to 1987, the changing capitalization rates, mortgage interest rates, debt-cover ratios, and loan-to-values varied the size of the equity cushion.

The average monthly underwriting data by the American Council of Life Insurance (ACLI) was transformed into average projects based upon a constant unit stream of income. An average monthly project based upon the capitalization rate was formulated and the size of loan calculated by the available debt service and interest rate using an average assumed maturity of twenty-three years. For the November 1968-March 1987 data series, the statistics for each of the financial parameters used in the model were calculated. For each month there was:

- value of the equity at the time of contract signing,
- value of the real estate project derived by capitalizing a unit of net operating income, and
- the value of the mortgage loan and the debt service.

As an example, suppose that the cap rate was .10; the mortgage constant was .095; and the debt-coverage ratio was 1.25. The resulting project value for a given \$100 annual income stream is \$1,000. The amount available for debt service is \$80 that supports a mortgage of \$842. The resulting equity is \$158. Is \$158 sufficient to protect against the interest-rate risk at refinancing? Using the preceding example, the resulting equation to test if the coefficients B_1 and B_2 are *one* is as follows:

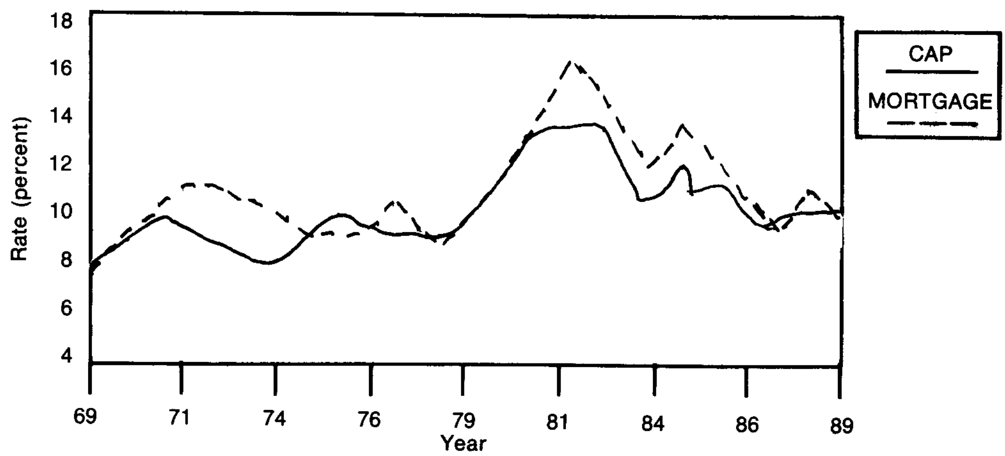
$$158 = B_1 \text{ (1000 * Factor 1)} - B_2\text{(842 * Factor 2)} \quad (2)$$

The underwriting environment varied over the period used in this study. Exhibit 2 suggests that over the period there are three possible financial environments: the pre-1980s positive leverage environment, the early 1980s neutral leverage environment, and the mid-1980s reverse (negative) leverage environment.[8]

There is no theoretical guide as to how the underwriter will assess the interest-rate risk for the changes in these conditions. A dummy variable indicating those periods when there was negative leverage was included in the test. Negative leverage is defined as when the capitalization rate was less than the mortgage interest rate. The reasoning was that the underwriter would increase the debt-coverage ratio to increase the equity cushion.

Now the only parameter left undefined is the expected effective mortgage loan life. While there is a contract maturity of the debt, is it reasonable to assume that all mortgages run to their legal contract limit, especially after pre-penalty payments expire? Determining the appropriate life was elusive. Research has shown that institutional lenders have horizons from five to ten years [14]. While each period had a different group of loan parameters, the average loan contract maturity was slightly over twenty years. However, this average did not reflect the actual holding periods of the loans. Accordingly, four holding periods were

Exhibit 2
Mortgage and Capitalization Rates



tested. Tests were performed with expected mortgage loan lives equal to three, five, seven, and ten years.

It is not clear whether the underwriting process places the stress for default on the original contract value or the reduced debt value, due to amortization, at the end of the holding period. If the original contract value is significant, the underwriters should be concerned about possible unexpected refinancing during the holding period and the subsequent expected refinancing. If the reduced amortized loan is significant, the underwriters are probably more concerned about the refinancing at the end of the likely holding period. For all holding periods, the original contract value of the debt and the reduced amortized values were tested as variable V_L.

Results

Of the four regression estimates using different holding periods, the results clearly suggest that mortgage underwriters for uninsured commercial loans apply a five-year holding period interest-rate risk test. The stress is applied to the beginning mortgage loan amount, not the ending reduced mortgage. The statistical significance and the coefficients being close to one for the firm value, V_F, and exercise price of the loan, V_L, point to the five-year holding period results as a confirmation that this risk valuation method applies. The signs are correct for all but the ten-year results. The results of the ordinary least square regression analysis are shown in Exhibit 3.

The residual plot confirms that the data is independently distributed because there is little serial correlation as attested to by the Durbin-Watson statistic of 1.73, significant at the .05 level. The terms approach normal distribution. The skewness and kurtosis for the two independent variables, V_F, and V_L, averaged .30. The significance of the dummy variable for negative leverage confirms that underwriters are concerned about the possible stress of a

Exhibit 3
Debt Holding Period Results

	Constant	V_F	V_E	Dummy	R^2	D-W
Three-year						
F = 38.30*	131.37	.27	−.19	32.99	.58	1.30
	(5.11)*	(6.57)*	(2.72)*	(5.36)*		
Five-year						
F = 42232.60*	−1.129	1.05	−1.10	1.3	.99	1.76*
	(.82)	(316.80)*	(245.76)*	(3.97)*		
Seven-Year						
F = 37.48*	141.50	.27	−.21	31.39	.58	1.29
	(5.51)*	(6.81)*	(3.185)*	(5.01)*		
Ten-Year						
F = 19.97*	155.19	.08	.03	40.93	.46	1.09
	(5.52)*	(2.52)*	(.49)	(6.06)*		

t-statistics in parentheses
*Significant at *alpha* = .05
Data covers the monthly averages, n = 222.
These results are based on computations in which the starting loan contract amount was used for the value of the debt V_L. There was little difference in the results for the three-year, seven-year and ten-year when the ending period value of the debt was used for V_L instead of the starting value.

high interest-rate environment, but the magnitude of the value, 1.3, was not great, and for the five-year holding period, the leverage variable nearly equals the value of the constant term. One explanation for the low value is that negative leverage occurs in higher interest-rate and capitalization-rate environments. In that type of high rate market, there is less likelihood that rates will go higher and that the project value will go lower.

The coefficients for the firm value and the loan exercise price are slightly greater than one for the five-year holding period and result in equity values lower than they would be for a virtually risk-free scenario. This slight deviation from *one* helps explain the underwriter's tolerance to accept some uncertainty. There is a stochastic relationship between the value of the coefficients, and the debt-coverage ratios. If the coefficients were *one*, the model would signify that the underwriter wanted to anticipate every interest-rate scenario during the debt holding period. As an example, suppose the cap rate was 8.5% and the constant were 8%. Coefficients of *one* would result in a debt-coverage ratio of 2.06 that captures over 99.9% of the interest-rate changes. But in the normal business context, a DCR of 2.06 is unacceptable. In part, the bidding process among underwriters determines those who are willing to accept greater levels of risk. The underwriter shaves the risk measure and accepts slightly greater uncertainty to stay competitive. The slightly greater-than-one coefficient, with the mean debt-coverage ratio of 1.29, represents a tolerance level that covers 97% of the interest-rate changes. A slight increase in the regression coefficients substantially lowers the debt-coverage ratio and mirrors the large incremental value changes that occur in the tails of a normal distribution.

The preceding illustrative values were obtained by iteratively solving the model. By setting the debt-coverage ratios according to debt-equity valuation model, the underwriter processes uninsured commercial mortgage loan to anticipate 97% of possible future financing conditions. This estimate is derived from an analysis of the residual term of the iterations. The coefficients adjust the model from unrealistic certainty to acceptable operating norms of risk management in the uninsured mortgage underwriting process.

Statistical significance as well as interpretive understanding converge on the results for the regression run using the five-year holding period. The results for the other holding periods suggest more crude, ill-fitting models.

Conclusions

The primary importance of these results is evidence that the modified option pricing model (debt-equity model) provides a reasonable rationale for explaining the establishment of debt-coverage underwriting ratios to determine the loan amount for real estate projects. It is an appropriate methodology that confirms the traditional 70% loan-to-value ratio.

The results also confirm that actual holding period expectations for the debt are not synonymous with the contract period. At best the tests indicate that five years is an agreeable likely holding period.

Possible errors in the research design include:

- using the mortgage rate instead of the risk-free rate,
- incorrectly specifying the variance of the interest rate, and
- failure to fully specify the risk differences of the three periods—positive, negative and neutral leverage. Each period should be analyzed with the mean and variance of that interest-rate environment.

Notwithstanding these possible errors, the clear results of the five-year holding period suggests that it is fruitful to apply this risk assessment methodology.

Addendum

Based upon the positive results of this research, the model was run through an iterative process in order to determine the appropriate debt-coverage ratio for an expected holding period using the 1967-1987 period mortgage rate and variance as parameters. The results are as follows.

Exhibit 4
Derived Debt-Coverage Ratios

CAP RATE	DEBT CONSTANT	Five Year Holding Period DCR	Seven Year Holding Period DCR
.08	.07	1.43	1.39
	.08	1.25	1.21
	.09	1.15	1.08
	.10	1.04	1.07
.09	.07	1.61	1.56
	.08	1.41	1.36
	.09	1.22	1.22
	.10	1.12	1.10
	.11	1.05	1.06
.10	.08	1.56	1.52
	.09	1.39	1.35
	.10	1.25	1.21
	.11	1.14	1.05
	.12	1.07	1.08
.11	.09	1.53	1.67
	.10	1.37	1.48
	.11	1.25	1.33
	.12	1.12	1.12
	.13	1.06	1.03

Notes

[1]Data from Table A, "Monthly Totals," Investment Bulletin, American Council of Life Insurance, Washington, D.C.; statistics, n = 222; mean, 1.292; mode, 1.290; variance, .003; minimum, 1.10; maximum, 1.66. The DCR, debt-cover ratio is defined as the seasoned annual net operating income divided by the required annual debt service.

[2]Table A, "Monthly Totals"; statistics, n = 222; mean, 72.613; mode, 73.60; variance, 6.942; minimum, 62.10; maximum, 77.60.

[3]Table A, "Monthly Totals"; statistics, n = 222; mean, 10.674; mode, 9.360; variance, 3.809; minimum, 7.860; maximum, 15.710.

[4]Efforts include the default study group at the Urban Land Institute, the office study program of the Homer Hoyt Institute, and the various proprietary research efforts at the rating agencies and investment houses. See, Leon Shilton, "Commercial Mortgage Loan Underwriting Default Analysis," working paper (for the Urban Land Institute), Real Estate Institute, New York University , 1983; ——, "The Functional Classification of Cities for Investment Analysis," working paper funded by the Homer Hoyt Institute, Fordham Graduate Business School, 1987; ——, "Feasibility Study of a Real Estate Debt Rating System for Moody's," 1986.

[5]A more extensive review of the option and put theory that leads up to the model used in this paper is in the working paper.

[6]Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81: 649-50, 1973.

[7]The selection of rates other than the mortgage rate for use as r in the model invites further research.

[8]Data from Table A, "Monthly Totals," Investment Bulletin, ACLI, various issues.

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