Mortgage Terminations: The Role of Conditional Volatility

Authors

David M. Harrison, Thomas G. Noordewier and K. Ramagopal

Abstract

This article is the winner of the Real Estate Finance manuscript prize (sponsored by Fannie Mae Foundation) presented at the 2001 American Real Estate Society Annual Meeting.

Studies of mortgage termination decisions typically rely on a competing risks framework comparing defaults and prepayments. While useful tools have been developed to approximate the values of these competing default and prepayment options, the available metrics do not adequately account for the role of the conditional volatility of interest rates and housing prices in option valuation. Using a sample of 1,428 mortgage loan payment histories, this study finds that exponential GARCH estimates of the conditional volatility of housing prices and interest rates influence mortgage termination decisions in a predictable manner. Specifically, increased housing price volatility is shown to enhance default option values, while increased interest rate volatility is shown to enhance prepayment option values. Therefore, it would appear that conditional volatility represents a more refined input into the competing risks option framework.

Introduction

The competing options framework offers powerful insights into mortgage default and prepayment decisions. In this framework, the borrower must ultimately choose between continuing to make timely payments of principal and interest, prepaying the loan in full, or defaulting on the obligation. If the market value of a property drops below the outstanding balance of the loan, the borrower may rationally choose to ruthlessly exercise the default (*i.e.*, put) option.[1] This action would preclude the potentially profitable future exercise of the prepayment option. Conversely, if the value of the property rises, the borrower captures the entire gain in the underlying value of the asset.[2] With respect to prepayments, if market wide interest rates fall, the borrower may choose to exercise the prepayment (*i.e.*, call) option and refinance at a lower rate. Again, such action precludes a future default option exercise on the (now) prepaid obligation. On the other hand, if rates rise, the borrower retains the loan at its (now) below-market interest rate.

The competing risks conceptualization of mortgage contracts has resulted in the increasingly accurate pricing of mortgages and management of risks associated with these securities. Both default and prepayment represent termination alternatives that are potentially valuable to borrowers and costly to lenders. Thus, a borrower's true equity position within a property is most accurately viewed as the equity position in the underlying real estate plus two competing options (Hilliard, Kau and Slawson, 1998).

Since Foster and Van Order's (1984) path breaking application of option theory to the mortgage terminations literature, Deng, Quigley and Van Order (1996) and Ambrose and Capone (1998, 2000) have introduced a pair of measures designed to capture the extent to which the default and prepayment options are in the money. These measures, originally labeled PNEQ and PREPAY, are regarded as state-of-the-art with respect to capturing the relative degree of default and prepayment option moneyness, respectively.[3] Despite their usefulness, these measures do not fully account for the role and importance of conditional volatility as indicated by the empirical options literature.[4] Specifically, the PNEQ measure of default option moneyness incorporates a time-varying volatility metric by using the standard error of the housing price index. However, this volatility input is limited insofar as it does not address the fact that housing price volatility changes more rapidly when there are decreases in market values than when there are increases. The PREPAY measure of prepayment option moneyness does not incorporate any volatility metric.

The main contribution of this study is to develop and test more refined volatility inputs into the default and prepayment competing risks framework. Specifically, exponential Garch techniques are employed to model the conditional volatility of housing prices and interest rates. These volatility estimates are then used as inputs, along with other control variables, to model default and prepayment termination decisions. These conditional volatility estimates exhibit a statistically significant relationship to mortgage termination decisions.

The remainder of the study is structured as follows. The next section considers different approaches to obtaining estimates of conditional volatility for both housing prices and interest rates, presents the rationale for the choice of a particular specification and formally articulates the hypotheses. Following this, the data available for analysis and descriptive statistics are presented. Next, the hypotheses are tested by estimating Cox regression models for the timing of defaults and prepayments, and performing a logistic regression to formally model the competing risks aspect of default and prepayment terminations. The findings from the logistic regression are also utilized to conduct a simulation in which the marginal effects and economic significance of the findings are estimated. The final section offers a summary of the current investigation, and suggests future research possibilities.

The Role of Conditional Volatility

To date, the measurement and analysis of the relative moneyness of the embedded default and prepayment options within mortgage contracts have followed either a Black-Scholes or binomial model. In both of these models, the role for volatility of the underlyings in the competing options framework is typically captured by the unconditional variance (*i.e.*, a constant).[5] To the extent that there is an evaluation of the role for changing volatility, it usually involves measuring the Vega of the option.[6]

Recent research on the distribution characteristics of the returns on securities has revealed that conditional volatility is a critical feature of both equity and debt securities.[7] Applying this research to mortgages, there is ample reason to expect the exercise of the embedded (default and prepayment) options to be influenced significantly, ceteris paribus, by the conditional volatility of the two underlyings: interest rates and housing prices. If this is indeed the case, then using a constant variance assumption may well lead to incorrect pricing of the embedded options. This could lead to biases characterized by volatility smiles.

In order to clarify the role of conditional volatility, consider housing prices—one of the underlyings used in the analysis.[8] Begin with an autoregressive AR(1) process describing changes in housing values. In this description, y_t is the change in housing values relative to the last period "$t - 1$,"[9] a_0 and a_1 are constants, and ε_{t-1} represents innovations or shocks:

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_{t-1}. \tag{1}$$

Now, let $\{\hat{\varepsilon}_{t-1}\}$ be the estimated residuals from this form of model. In this setting, the conditional variance of housing values would be described as:

$$Var\{y_{t+1} \mid y_t\} = E_t\,\{[y_t - a_0 - a_1 y_{t-1}]^2\} = E_t\,[\varepsilon^2_{t+1}]. \tag{2}$$

One approach to modeling the volatility of housing values sets $E_t[\varepsilon^2_{t+1}]$ equal to σ^2, in keeping with the assumption of constant variance. However, when the variance is not constant, the conditional volatility of housing values may be modeled as an autoregressive process of order q, AR(q), where the residuals are squared:

$$\hat{\varepsilon}^2_t = \alpha_0 + \alpha_1\hat{\varepsilon}^2_{t-1} + \alpha_2\hat{\varepsilon}^2_{t-2} + \ldots + \alpha_q\hat{\varepsilon}^2_{t-q} + v_t. \tag{3}$$

Equation (3) is the ARCH specification using squared residuals to describe conditional volatility, where v_t is a white noise process, and the α_i's are constants. A variant of this specification, where the disturbances enter multiplicatively, is the ARCH model proposed by Engle (1982). The conditional variance in such a situation is:

$$E_t(\hat{\varepsilon}^2_{t+1}) = \alpha_0 + \alpha_1\hat{\varepsilon}^2_t + \alpha_2\hat{\varepsilon}^2_{t-1} + \ldots + \alpha_q\hat{\varepsilon}^2_{t+1-q}. \quad (4)$$

For an ARCH characterization of the volatility of housing prices, the residuals can be generated using a variety of techniques. These include standard OLS regression and autoregressive processes.

A more encompassing description of conditional volatility is the GARCH specification of Bollerslev (1986). In its general form, a GARCH model specifies conditional variance as a function of a constant, the preceding period's error and a long-term variance. In its simplest form, one can specify a GARCH(1,1) model as:

$$\sigma^2_{t+1} = \alpha_0 + \alpha_1\varepsilon^2_t + \beta_1\sigma^2_t, \quad (5)$$

where $\beta_1 \geq 0$. Despite the advantages and the flexibility of the GARCH(p,q) specification, it does not capture an extremely important characteristic of security returns—the leverage effect. In the present context, the leverage effect speaks to the fact that the volatility of housing prices changes more rapidly when there are decreases in the market value of the underlying index than when there are increases. Explicitly addressing this issue, Nelson (1991) offers an exponential version of the GARCH model, referred to as EGARCH. In this model, the leverage effect is depicted as a non-symmetric response to shocks. The simple form of the EGARCH specification, EGARCH(1,1) is:

$$\log(\sigma^2_{t+1}) = \alpha_0 + \alpha_1 f(\varepsilon_{t-1}/\sigma_{t-1}) + \beta_1 \log(\sigma^2_{t-1}), \quad (6)$$

where,

$$f(\varepsilon_{t-1}/\sigma_{t-1}) = \theta_1(\varepsilon_{t-1}/\sigma_{t-1}) + [(\varepsilon_{t-1}/\sigma_{t-1}) - E(\varepsilon_{t-1}/\sigma_{t-1})], \quad (7)$$

where θ_1 is a constant. In the formulation for EGARCH(p,q), the function f(.) is the news impact curve relating conditional volatility, $\log(\sigma^2_{\mathrm{t}})$, to news or innovations, $\varepsilon_{\mathrm{t}-1}$.

Due to its ability to pick up the leverage effect, the EGARCH(1,1) specification is theoretically superior to the GARCH(1,1) specification. Further, the EGARCH(1,1) does not require any restrictions on the values of the coefficients.[10] Therefore, in the empirical work that follows, EGARCH(1,1) estimates of the conditional volatility of both housing prices and interest rates are employed. Ex ante, higher levels of housing price volatility are expected to enhance default option value. In turn, the enhanced default option value reduces borrower incentives to terminate the mortgage via exercise of the competing prepayment option. Thus, increased housing price volatility is expected to increase the relative probability of default. Similarly, higher levels of interest rate volatility are expected to enhance the relative probability of prepayment option exercise.

Data

In order to evaluate the empirical significance of the conditional volatility of housing prices and interest rates on default and prepayment probabilities, respectively, a sample of 1,428 residential loans was obtained from the portfolio of a nationwide mortgage lending institution. The sample was constructed using a probability sample of all loans ever encountering a 90-day delinquency and a random sample of never delinquent (at 90 days) accounts.[11] All sample loans were originated between December, 1989 and June, 1991, with subsequent performance information available through July, 1997. In sum, the dataset is characterized by seasoning of approximately 5.5 to 7 years, and includes 889 loans that experienced a 90+ day delinquency (default) and 539 never delinquent accounts.

In order to minimize the potential for model misspecification and omitted-variable bias, a variety of metrics were included to control for each individual borrower's willingness and ability to repay the mortgage obligation. Traditionally, measures used by lenders to evaluate borrower creditworthiness include the loan-to-value ratio, payment-to-income ratios and credit scores. The unique richness of the current dataset allows for the explicit control of the LTV ratio, the borrower's total debt ratio (*i.e.*, back-end ratio) and the borrower's FICO credit score. Furthermore, the data allow for the control of additional aspects of both borrower heterogeneity and the terms of the credit agreement. A complete list of all the variables used in the empirical investigation is included in Exhibit 1, with Exhibit 2 providing descriptive statistics for each attribute. Before proceeding to analyze the data using multivariate Cox and logistic regression techniques, a brief summary of this information is presented.

Option Characteristics

Looking first at the volatility estimates, a high degree of variability surrounding both of these option characteristics is observed. Specifically, the *EGARCH* measure of housing price volatility (*HPI* − *EGARCH*) reveals that average prices varied by over 2% per quarter, while analogous interest rate volatility

Exhibit 1 | Variable Definitions

INTEREST − EGARCH
This variable captures the conditional volatility of the ten-year constant maturity treasury rate. The specification for conditional variance that is estimated is:

$\log(\sigma^2) = \omega + \beta \log(\sigma^2_{t-1}) + \alpha \mid \varepsilon_{t-1} / \sigma_{t-1} \mid + \gamma(\varepsilon_{t-1} / \sigma_{t-1}).$

Since the left hand side of the expression above is the log of the conditional variance, it implies that the leverage effect is exponential. These forecasts of the conditional variance are guaranteed to be non-negative. The hypothesis that $\gamma < 0$ tests for the leverage effect.

HPI − EGARCH
This variable captures the conditional volatility of the current market value of the property, which is estimated using the Office of Federal Housing Enterprise Oversight (OFHEO) state level weighted repeat sales index to capture the change in single-family housing prices. The functional form of the equation estimated is identical to that for *INTEREST − EGARCH*.

PNEQ
In order to capture the probability that a property has negative equity, *PNEQ* is given by:

$$PNEQ = \Phi\left[\frac{\log(outbal) - \log(mbal)}{\sqrt{\varepsilon^2}}\right],$$

where *mbal* is the current market value of the property estimated using the OFHEO state level weighted repeat sales index to capture change in single-family house prices, *outbal* is the present value of all remaining mortgage payments discounted at the contract interest rate, ε is the standard error of the OFHEO index number estimate and Φ is the cumulative standard normal distribution function.

PREPAY
In order to capture the extent to which the call option is in the money, *PREPAY* is given by:

$$PREPAY = 1 - \frac{pbal}{outbal},$$

where *pbal* is the discounted value of the remaining mortgage payments at the current market rate and *outbal* is the discounted value of the remaining mortgage payments at the contract interest rate (*i.e.*, the unpaid mortgage balance).

FICO
The borrower's credit score calculated using Fair, Isaac and Co.'s proprietary scoring model. These figures typically range from the low 300s into the mid-800s, with higher scores representing better credit characteristics. The scores were generated at the lender's request by one of the three principal credit bureaus: Trans Union, Experian, or Equifax.

LTV
The loan-to-value ratio, based on the lesser of the sales price or appraised value, at the time of loan origination.

ADDINC
A dichotomous variable equal to 1 if the lien contains two or more formally designated borrowers (i.e., at least one co-borrower is formally obligated to satisfy the loan) and the first co-borrower has positive gross income, zero otherwise.

TOTDEBT
A continuous variable equal to monthly principal, interest, taxes, and insurance (PITI) payments plus all other monthly payment obligations relative to total (borrower and co-borrower) gross monthly income.

Exhibit 1 | (continued)

Variable Definitions

LIQUID
A continuous variable equal to the ratio of the sum of the value of all savings, stocks, bonds, etc. held by the borrower relative to total (borrower and co-borrower) gross annual income.

SELFEMP
A dichotomous variable equal to one if the borrower is self-employed, zero otherwise.

SELFLOC
A dichotomous variable equal to one if the borrower is self-employed and their primary work location is within the residence, zero otherwise.

AMORT
A dichotomous variable equal to one if the amortization term of the loan is fifteen years, zero otherwise.

ARM
A dichotomous variable equal to 1 if the mortgage is an adjustable-rate obligation, zero otherwise. Note: within this dataset, all adjustable-rate mortgages are amortized over a thirty-year term.

BALLOON
A dichotomous variable equal to one if the mortgage is a balloon product, zero otherwise. Note: within this dataset, all balloon mortgages are amortized on a 30-year basis

REFINANCE
A dichotomous variable equal to one if the loan is a non-cashout refinancing (defined as a loan for which the value of the ratio of the loan amount to the outstanding lien is less than or equal to 1.06), zero otherwise. Note: no purchase loans are cashout loans in this dataset.

CASHOUTL
A dichotomous variable equal to 1 if the loan is a cashout refinancing where the value of the ratio of the loan amount to the outstanding lien is greater than 1.06, but less than or equal to 1.50, zero otherwise.

CASHOUTH
A dichotomous variable equal to one if the loan is a cashout refinancing where the value of the ratio of the loan amount to the outstanding lien is greater than 1.50, zero otherwise.

JUMBO
A dichotomous variable equal to one if the loan is a jumbo loan, zero otherwise.

ENTRYXXXX
A series of six dichotomous indicator variables identifying the chronological quarter in which each loan was originated. Loans originated during December of 1989 represent the omitted (baseline) case and are characterized by zeros for each indicator variable. All remaining loans assume the value of one for the dichotomous variable corresponding to their quarter of origination and zero for the dichotomous variables corresponding to all other quarters.

TIME
The number of months from loan origination to termination (*i.e.*, prepayment or default).

Exhibit 2 | Variable Means and Standard Deviations

Variable	Mean	Std. Dev.
Option Characteristics		
INTEREST − EGARCH	0.20	0.02
HPI − EGARCH	2.32	1.10
PNEQ	0.35	0.11
PREPAY	−0.15	0.08
Borrower Heterogeneity		
FICO	694	59
LTV	0.74	0.10
ADDINC	0.39	0.49
TOTDEBT	0.30	0.10
LIQUID	0.97	1.72
SELFEMP	0.35	0.48
SELFLOC	0.08	0.27
Loan Characteristics		
AMORT	0.06	0.24
ARM	0.09	0.28
BALLOON	0.08	0.28
REFINANCE	0.07	0.25
CASHOUTL	0.13	0.34
CASHOUTH	0.10	0.30
JUMBO	0.50	0.50
Economic Conditions		
ENTRY90Q1	0.22	0.41
ENTRY90Q2	0.15	0.35
ENTRY90Q3	0.17	0.37
ENTRY90Q4	0.14	0.35
ENTRY91Q1	0.12	0.32
ENTRY91Q2	0.11	0.32

Note: n = 1,428.

(*INTEREST* − *EGARCH*) averaged 20 basis points per month. Given the importance of volatility as demonstrated by the empirical options literature, these estimates suggest that volatility may play a significant role in explaining mortgage termination decisions.

Consistent with the view that default and prepayment are competing options, the models incorporate two variables designed to measure the degree to which each option is in-the-money. Following Deng, Quigley and Van Order (1996) and Ambrose and Capone (1998, 2000), the probability that a mortgaged property investment is in a negative equity situation (*i.e.*, the probability that the option to default is in-the-money, or that the option to ''put'' the loan back to the lender is profitable) is estimated as:

$$PNEQ = \Phi\left[\frac{\log(outbal) - \log(mbal)}{\sqrt{\varepsilon^2}}\right], \quad (8)$$

where *mbal* is the current market value of the property estimated using the Office of Federal Housing Enterprise Oversight (OFHEO) state level weighted repeat sales index to capture changes in single-family house prices, *outbal* is the present value of all remaining mortgage payments discounted at the contract interest rate, ε is the standard error of the OFHEO index number estimate and Φ is the cumulative standard normal distribution function. The positive average value of *PNEQ* in Exhibit 2 indicates that the default option is frequently in-the-money at termination for the loans in this dataset. Ex ante, increases in the probability of negative equity (*PNEQ*) are expected to be associated with enhanced default probabilities.

Similarly, as modeled by Deng et al. and Ambrose and Capone, the prepayment option is in-the-money when the discounted value of the remaining mortgage payments at the current market rate is greater than the outstanding mortgage balance. Thus, the degree to which the prepayment option is in-the-money may be calculated as:

$$PREPAY = 1 - \frac{pbal}{outbal}, \quad (9)$$

where *pbal* is the discounted value of the remaining mortgage payments at the current market rate, and *outbal* is the discounted value of the remaining mortgage payments at the contract interest rate (*i.e.*, the unpaid mortgage balance). Positive values of *PREPAY* indicate that the market interest rate is higher than the contract rate and prepayment option exercise is not profitable (*i.e.*, the option is out-of-the-money). Conversely, negative values of *PREPAY* indicate that the market interest rate is lower than the contract rate and option exercise is profitable (*i.e.*, the option is in-the-money). In Exhibit 2, the negative average value of *PREPAY* indicates that the prepayment option is typically in-the-money at termination.

Borrower Heterogeneity

Deng, Quigley and Van Order (2000) advocate the options framework as a useful tool for analyzing and explaining the behavior of mortgage termination options.

However, they qualify this by stating “there exists significant heterogeneity among mortgage borrowers, particularly regarding prepayment.” They continue, “ignoring heterogeneity among mortgage borrowers leads to serious errors in estimating the prepayment risk.”

In order to control for borrower heterogeneity, the model specification includes a variety of borrower-specific risk characteristics. The average borrower in the dataset is relatively creditworthy, as evidenced by a *FICO* credit score of nearly 700, a loan-to-value ratio (*LTV*) of only 74% and a total debt ratio (*TOTDEBT*) of approximately 30%. These three metrics indicate that the average borrower in this dataset easily surpasses traditional standards of creditworthiness on these dimensions.[12] In addition, co-borrower income (*ADDINC*) is available to satisfy the mortgage in the event of short-term cash flow interruptions for 39% of the sample loans, while the typical mortgage within the dataset is secured by liquidatable assets (*LIQUID*) equal to nearly one year’s gross income. Liquidatable assets, like additional income, provide security to the lender in the event a borrower encounters unexpected disturbances in short-term cash flows. Limited employment information was also made available to us, allowing us to control for a borrower who is self-employed (*SELFEMP*), and whether a self-employed borrower works inside or outside their primary residence (*SELFLOC*). Thirty-five percent of the mortgage loans within this sample were made to self-employed borrowers. However, less than 10% of the mortgage loans were made to self-employed borrowers whose primary work location is within their residence.

Loan Characteristics

In order to further minimize model misspecification, the models in this study also control for differential loan attributes. Specifically, explicit controls are established for jumbo loans. In addition, fixed-rate, adjustable-rate and balloon mortgage products are identified. Half of the loans in the dataset exceed secondary market size guidelines for conforming loans, and are thus classified as *JUMBO*. This relatively large fraction of non-conforming loans appears to be driven primarily by the geographic distribution of properties within the dataset. While the sample includes observations from forty states plus the District of Columbia, it is over-represented by loans made in California (32%), New Jersey (18%) and New York (17%). These three areas have all been characterized by traditionally high housing prices and the prevalence of jumbo loans. With respect to ARM and balloon products, approximately 9% of the sample is comprised of adjustable-rate (*ARM*) mortgages, while 8% of the observations represent balloon loans (*BALLOON*). Six percent of the loans are fixed-rate obligations with a 15-year amortization horizon (*AMORT*), while the remaining 77% constitute fixed-rate obligations with a 30-year amortization term. All ARM and balloon products within the dataset are amortized on a 30-year basis.

The dataset also allows for the identification and separation of home purchase and refinance loans. While the clear majority (70%) of the loans represent home purchase loans, 30% of the observations come from refinancings. In a refinancing situation, borrowers often obtain a new loan in an amount equal to the unpaid mortgage balance on their existing obligation. Such a situation is referred to as a ''non-cashout'' refinancing.[13] Alternatively, many times borrowers refinance to extract cash from their equity position in the property and effectively re-lever the investment. Berkovec, Canner, Gabriel and Hannan (1994) present empirical evidence that ''cashout'' refinancings are significantly riskier than their ''non-cashout'' counterparts. In order to control for the possibility of differential risk across loan purposes, the model explicitly identifies and controls for those loans that are non-cashout refinancings (*REFINANCE*), low cash-out refinancings where the new lien is less than or equal to 150% of the previous unpaid mortgage balance (*CASHOUTL*) and high cash-out refinancings where the new lien is greater than 150% of the previous unpaid mortgage balance (*CASHOUTH*). Ex ante, refinance loans are expected to exhibit higher default rates than home purchase loans, with default hazard rates increasing concomitantly with the degree of cash extracted via the refinancing. Within the dataset, 7% of the loans are identified as ''non-cashout'' refinancings, 13% are identified as low ''cashout'' refinancings and 10% are identified as high ''cashout'' refinancings.

Economic Conditions

In order to control for differential economic conditions at the time of loan origination, a series of dummy variables identifying the year and quarter of loan origination is included in the model specification. As shown in Exhibit 2, sample loan originations appear to be fairly evenly distributed across time. Specifically, originations range from a low of 11% for the second quarter of 1991, to a high of 22% for the first quarter of 1990. Nine percent of the loans in the dataset were originated in December, 1989 (captured by the omitted dummy variable).

Analysis

In order to investigate the importance of conditional volatility to mortgage loan performance, separate Cox regression models for the hazards of default and prepayment were estimated. Exhibit 3 reports the results of the Cox default regression, where defaults are defined as loans that have ever been ninety or more days delinquent.[14] Within the Cox default framework, sample loans that either prepaid in full during the period of study, or were still current as of July, 1997, are treated as censored observations. Consistent with a priori expectations, and as evidenced by the positive coefficient on *PNEQ* ($b = 1.60$), increasing the degree to which the default option is in-the-money increases the probability of default. The corresponding volatility input parameter ($HPI - EGARCH$), while exhibiting the expected positive sign ($b = 0.03$), is statistically insignificant. These results are consistent with the view that *PNEQ* adequately captures the importance of volatility on default option values. As noted, this result is arguably attributable to

Exhibit 3 | Cox Regression Model for the Timing of 90+ Defaults

Variable	Parameter Estimate	Wald Chi-Square
Option Characteristics		
INTEREST − EGARCH	−9.31	8.9***
HPI − EGARCH	0.03	0.3
PNEQ	1.60	4.8**
PREPAY	20.67	274.9***
Borrower Heterogeneity		
FICO	−0.01	149.3***
LTV	3.60	16.5***
ADDINC	−0.77	28.2***
TOTDEBT	0.48	0.7
LIQUID	−0.02	0.2
SELFEMP	0.24	1.8
SELFLOC	0.86	12.0***
Loan Characteristics		
AMORT	−2.17	71.3***
ARM	−3.38	96.9***
BALLOON	0.73	6.5**
REFINANCE	0.47	3.3*
CASHOUTL	1.02	24.3***
CASHOUTH	0.43	3.3*
JUMBO	0.13	0.9
Economic Conditions		
ENTRY90Q1	1.17	24.6***
ENTRY90Q2	1.36	19.3***
ENTRY90Q3	1.04	14.0***
ENTRY90Q4	0.63	3.3*
ENTRY91Q1	0.36	1.1
ENTRY91Q2	−0.29	1.0

Note: n = 1,428. Model χ^2 with 25 DF = 196.9 ($p < 0.01$).
*Statistically significant at the 0.10 level.
**Statistically significant at the 0.05 level.
***Statistically significant at the 0.01 level.

the fact that *PNEQ* explicitly incorporates the standard error of the OFHEO index into its calculation of option moneyness.

With respect to *PREPAY*, the positive coefficient on *PREPAY* (b = 20.67) indicates that as prepayment becomes more profitable, the competing risk of default becomes less likely. As can be seen, not only is the option value variable significant, but so is the corresponding conditional volatility input parameter (*INTEREST − EGARCH*). The negative coefficient (b = −9.31) for this variable is consistent with increasing interest rate volatility enhancing prepayment option value (over and above that captured by the variable *PREPAY*) and thus reducing the probability of the competing hazard of default.

Additional model parameters designed to control for borrower specific risk factors are generally consistent with expectations, as higher *FICO* scores are associated with lower default rates ($b = -0.01$), higher *LTV* ratios are associated with increased default rates ($b = 3.60$), the presence of positive co-borrower income (*ADDINC*) reduces default probabilities ($b = -0.77$), and self-employed individuals working out of their primary residence (*SELFLOC*) are characterized by higher default rates than other borrowers ($b = 0.86$).

The empirical results from the Cox prepayment model are contained in Exhibit 4. This specification treats both loans that have experienced a 90-day delinquency and loans that remain current as of the end of the sample period (July 1997) as censored data. Examining the output, there is strong evidence that conditional volatility influences mortgage termination decisions. With respect to the conditional volatility input metrics, both interest rate and housing price volatility appear significantly related to the prepayment option exercise. Specifically, the positive coefficient on *INTEREST − EGARCH* ($b = 9.61$) suggests that increasing conditional interest rate volatility enhances the prepayment option value and thus increases the probability of prepayment. Conversely, the negative coefficient on *HPI − EGARCH* ($b = -0.14$) suggests that increasing the conditional volatility of housing prices enhances default option value and thus lowers the probability of the competing risk of prepayment. The negative (but insignificant) coefficient ($b = -0.44$) on *PNEQ* is consistent with the notion that the prepayment option exercise becomes less likely as the competing default option becomes more valuable. Surprisingly, *PREPAY* is positive ($b = 5.21$) and significant, indicating that the prepayment option exercise becomes less probable as the option moves deeper into the money. The unexpected sign on *PREPAY* may be due, in part, to multicollinearity between *PREPAY* and its corresponding conditional volatility input parameter, *INTEREST – EGARCH*.[15]

The remaining model parameters in Exhibit 4 are generally consistent with a priori expectations, as borrowers with high *FICO* scores, positive co-borrower income (*ADDINC*) and those taking out *JUMBO* loans are more likely to prepay than their less creditworthy, lower-income and non-*JUMBO* counterparts. Similarly, borrowers with 15-year fixed-rate loans are less likely to exercise the prepayment option than their 30-year counterparts, while *ARM* loan prepayment rates are significantly lower than those observed for fixed-rate instruments. These results are not unexpected as option value should increase with time to maturity, and rate-resetting provisions on ARMs mitigate the need to refinance the lien in order to capture the gains from lower market-wide interest rates. Finally, refinancings where borrowers extract a relatively high degree of equity (*i.e.*, cash) from the outstanding lien are also less likely to prepay than typical home purchase obligations. High cashout refinancings are often observed either because borrowers are cash constrained and need to access all available sources of equity, or because borrowers are consciously choosing to re-lever their investment in hopes of

Exhibit 4 | Cox Regression Model for the Timing of Prepayments

Variable	Parameter Estimate	Wald Chi-Square
Option Characteristics		
INTEREST − EGARCH	9.61	25.3***
HPI − EGARCH	−0.14	7.4***
PNEQ	−0.44	0.6
PREPAY	5.21	29.1***
Borrower Heterogeneity		
FICO	0.005	17.6***
LTV	−0.11	0.1
ADDINC	0.21	5.2*
TOTDEBT	−0.09	0.0
LIQUID	0.02	0.5
SELFEMP	0.02	0.0
SELFLOC	0.00	0.0
Loan Characteristics		
AMORT	−0.73	14.5***
ARM	−1.85	31.5***
BALLOON	0.08	0.1
REFINANCE	−0.02	0.0
CASHOUTL	0.22	1.8
CASHOUTH	−0.31	3.1*
JUMBO	0.74	48.9***
Economic Conditions		
ENTRY90Q1	0.29	2.6
ENTRY90Q2	1.10	35.0***
ENTRY90Q3	1.00	30.9***
ENTRY90Q4	1.25	43.9***
ENTRY91Q1	0.93	20.3***
ENTRY91Q2	0.81	13.9***

Note: $n = 1{,}428$. Model χ^2 with 25 DF = 548.3 ($p < .01$).
*Statistically significant at the 0.10 level.
**Statistically significant at the 0.05 level.
***Statistically significant at the 0.01 level.

maximizing the value of their portfolios. Borrowers in both situations could rationally be expected to exhibit lower prepayment propensities.

While the Cox regressions for the timing of both defaults and prepayments provide valuable insight into the determinants of mortgage terminations, they do not provide a truly competing risks framework for the analysis of such decisions. In order to explicitly address this shortcoming of the previous analysis, the investigation is extended to a truly competing risks framework where default and prepayment are modeled as alternative mortgage outcomes.[16]

Conceptually, mortgage market outcomes may be thought of as a combination of two separate and distinct stages. A borrower first decides whether to terminate the obligation and then, conditional on termination, chooses the type of termination (*i.e.*, default or prepayment) to pursue. In order to operationalize this process, a generic Cox model of the time to mortgage termination is estimated. Stage one treats both defaults and prepayments as a single termination event, with current loans (as of July 1997) representing the censored data category. As one might expect, given the competing risks nature of prepayment and default, this (unreported) analysis of the timing of mortgage termination decisions is relatively uninformative.

Stage two examines only those (1,335) loans that experienced a termination event. A standard binary logistic model is then used to evaluate the determinants of the type of mortgage termination event observed (*i.e.*, default or prepayment). It is important to note in this context that the probability of default is estimated relative to prepayment, since current loans are excluded from the analysis. Exhibit 5 presents the results of this analysis and provides further evidence in support of the importance of both the competing risks optionality framework and the role of conditional volatility in mortgage termination decisions. Specifically, the positive coefficient on *PNEQ* ($b = 2.48$) indicates that default probabilities rise as the likelihood that a borrower is in a negative equity situation increases. Similarly, the (insignificant) positive coefficient on *PREPAY* ($b = 2.30$) is consistent with the notion that default becomes more likely as the competing option of prepayment becomes less valuable. Turning to the conditional volatility input parameters, the empirical evidence is consistent with the hypotheses that volatility enhances the probability of the termination option exercise. For example, the positive coefficient on housing price volatility ($b = 0.23$ for $HPI - EGARCH$) suggests that greater variability in housing prices increases the value of the default option embedded within the mortgage contract. Similarly, the (insignificant) negative coefficient on interest rate volatility ($b = -2.12$) is consistent with the view that increased interest rate volatility enhances prepayment option value, and thus reduces the probability of default relative to prepayment.

Examining the remaining results for both borrower and loan characteristics generally confirms the importance of these attributes, as documented elsewhere in the literature. For example, more creditworthy borrowers (as measured by higher *FICO* scores) are less likely to default ($b = -0.02$), while high *LTV* loans are more likely to default ($b = 6.18$). Similarly, the presence of positive co-borrower income (*ADDINC*) reduces the relative likelihood of default (b = -0.54), as do reductions in a borrower's total debt burden ($b = 1.78$ for *TOTDEBT*). As expected, self-employed borrowers, particularly those whose primary work location is within their personal residence, are considerably more likely to default than their externally employed counterparts ($b = 0.34$ for *SELFEMP*, and b = 1.10 for *SELFLOC*).

Turning to loan characteristics, *ARM* loans ($b = 0.85$) and refinance loans ($b = 0.84$ for *REFINANCE*, $b = 1.12$ for *CASHOUTL*, and $b = 1.24$ for *CASHOUTH*)

Exhibit 5 | Logistic Model Comparing Defaults (*i.e.*, ever 90+ loans) and Prepayments (relative to prepayments)

Variable	Parameter Estimate	Chi-Square
Option Characteristics		
INTEREST − EGARCH	−2.12	0.2
HPI − EGARCH	0.23	6.4**
PNEQ	2.48	4.7**
PREPAY	2.30	2.1
Borrower Heterogeneity		
FICO	−0.02	67.3***
LTV	6.18	22.1***
ADDINC	−0.54	10.6***
TOTDEBT	1.78	4.2**
LIQUID	−0.03	0.4
SELFEMP	0.34	3.0*
SELFLOC	1.10	11.8***
Loan Characteristics		
AMORT	−0.33	1.0
ARM	0.85	6.5**
BALLOON	0.30	0.8
REFINANCE	0.84	8.8***
CASHOUTL	1.12	17.5***
CASHOUTH	1.24	13.5***
JUMBO	−0.41	3.5*
Economic Conditions		
ENTRY90Q1	0.22	0.3
ENTRY90Q2	−1.06	4.5**
ENTRY90Q3	−1.28	10.4***
ENTRY90Q4	−2.13	27.4***
ENTRY91Q1	−1.63	17.3***
ENTRY91Q2	−3.14	42.2***
Intercept and Time to Termination		
INTERCEPT	9.15	16.3***
TIME	−0.09	127.8***

Note: $n = 1{,}335$. Model χ^2 with 25 DF $= 228.4$ ($p < .01$).
*Statistically significant at the 0.10 level.
**Statistically significant at the 0.05 level.
***Statistically significant at the 0.01 level.

are found to be somewhat more default prone than fixed-rate and home purchase loans, while *JUMBO* loans ($b = -0.41$) are found to be somewhat less likely to default. The positive coefficient on *BALLOON* loans ($b = 0.30$) and the negative coefficient on the 15-year amortization flag ($AMORT = -0.33$) are both consistent with the results from the previously reported Cox default regression. However, they now both fail to attain statistical significance at conventionally accepted levels. Finally, the negative coefficient on the *TIME* variable ($b = -0.09$) confirms that the longer a borrower continues to make timely payments of scheduled principal and interest, the less likely the loan is to default. In sum, the results confirm the importance of traditional measures of borrower creditworthiness (*e.g.*, FICO scores, LTV ratios, etc.) in mortgage termination decisions, provide further evidence of the applicability of the competing risks framework to such events, and refine traditional measures of prepayment and default option values to account for the role of conditional volatility.

Marginal Effects

In order to obtain some economic intuition, a simulation is conducted to estimate the marginal effects of selected independent variables on the probability of default. Utilizing the parameter estimates obtained from the logistic model, the simulation estimates the default probability for a typical borrower, and then calculates changes in this probability associated with changes in individual risk characteristics. The estimated marginal effects reported in Exhibit 6 provide additional insight into the competing risks nature of default and prepayment terminations.[17]

In the simulation, the base case borrower is assumed to have a 30-year fixed-rate, fully amortizing loan, a *FICO* score of 710, a *LTV* of 70%, a total debt ratio (*TOTDEBT*) of 30%, and liquidatable assets (*LIQUID*) equivalent to one year's gross income. The origination date is assumed to be the first quarter of 1990. The remaining continuous variables in the model are set approximately at their mean values (*e.g.*, *HPI* − *EGARCH* is assigned the value 2.3), while the remaining dichotomous variables are set equal to 0 (*e.g.*, *REFINANCE* is assigned the value 0). Default rates for the continuous independent variables are calculated by displacing each characteristic by the displayed amount (roughly one standard deviation in either direction at the extremes). In the case of the dichotomous variables, default probabilities are calculated for each variable based on the presence or absence of the characteristic. Marginal effects are estimated as of the origination date.

As reported in Exhibit 6, a reduction (increase) in housing price volatility (*HPI* − *EGARCH*) of 1.2 percentage points (from 2.3% to 1.1%) produces over a one-half percentage point reduction (increase) in the probability of default (from 6.58% to 5.96%). Changes in the other option characteristic, interest rate volatility (*INTEREST* − *EGARCH*), produce less than one-tenth of a percentage point

Exhibit 6 | Simulated Marginal Effect of Selected Independent Variables on Default Probability

Variable	Estimated Default Probability (%)	Change in Default (%)
INTEREST − EGARCH		
0.18	6.67	0.09
0.19	6.63	0.05
0.20	6.58	0.00
0.21	6.53	−0.05
0.22	6.49	−0.09
HPI − EGARCH		
1.1	5.96	−0.62
1.7	6.28	−0.30
2.3	6.58	0.00
2.9	6.87	0.29
3.5	7.15	0.57
FICO		
530	9.75	3.17
590	9.45	2.87
650	8.58	2.00
710	6.58	0.00
770	3.72	−2.86
ARM		
Yes	8.13	1.55
No	6.58	0.00
LTV		
60%	5.12	−1.46
65%	5.87	−0.71
70%	6.58	0.00
75%	7.22	0.64
80%	7.78	1.20
PNEQ		
25%	6.01	−0.57
30%	6.30	−0.28
35%	6.58	0.00
40%	6.85	0.27
45%	7.10	0.52
PREPAY		
−0.25	6.06	−0.52
−0.20	6.32	−0.26
−0.15	6.58	0.00
−0.10	6.83	0.25
−0.05	7.07	0.49

Note: Exhibit 6 simulates the change in default probability associated with changes in selected risk characteristics. The baseline borrower is assumed to have a 30-year fixed-rate loan, a total debt ratio of 30%, a FICO score of 710, a 70% LTV, an origination date corresponding to the first quarter of 1990 and liquidatable assets amounting to one year's gross income. Marginal effects are simulated as of the origination date. All remaining continuous variables are entered approximately at their mean value, with all remaining dichotomous variables entered as zero.

change in the probability of default over the range of values analyzed. The latter result is not surprising given the insignificance of the parameter estimate in the logistic regression. Variability in a borrower's credit score results in rather large default probability swings. For example, lower credit scores produce a default probability increase of over three percentage points (3.17%) for a 180 point decrease in *FICO* (from 710 to 530). The simulation also quantifies the riskier nature of *ARM* loans relative to fully amortizing, 30-year, fixed-rate obligations, since the estimated default rate is approximately one and one-half percentage points higher for adjustable-rate securities. Changes in the origination *LTV* of ten percentage points in either direction result in default rate changes of between one and one and one-half percentage points. Finally, the marginal effects simulation also shows that changes in the probability of negative equity (*PNEQ*) and the value of the prepayment option (*PREPAY*) produce approximately one-half percentage point changes in default probabilities over the range of values analyzed. These simulation results clearly show that the conditional volatility of housing prices exerts an economically significant influence on the mortgage default decision.

Conclusion

The recent literature on mortgage terminations has incorporated options theory into the analysis of default and prepayment decisions by modeling these alternatives as competing risks. Within the competing risks framework, measures have been created to capture the relative moneyness of these embedded put and call options. Despite their usefulness, traditional measures of default and prepayment option moneyness (*PNEQ* and *PREPAY*) do not adequately account for the role and importance of conditional volatility as suggested in the options literature. Specifically, the probability of negative equity (*PNEQ*) incorporates a conditional (time-varying) volatility metric, but does not address the fact that the volatility of housing prices changes more rapidly when there are decreases in market values than when there are increases (*i.e.*, the leverage effect). The traditional measure of the extent to which the call option is in the money (*PREPAY*) fails to incorporate any volatility parameter. The construction of these variables fails to fully account for the findings of the options literature, which suggest that conditional volatility provides a more accurate input into the pricing of options. The goal of this study has been to bridge the gap between the conceptualization of volatility in the empirical options literature and its implementation within the competing risks mortgage terminations literature.

The driving intuition behind this study draws on research into the distribution characteristics of securities. This research reveals that conditional volatility is a critical feature of both equity and debt security return patterns. Taking this as a starting point, this study estimates the conditional volatility of the underlyings for the two competing options: housing prices and interest rates. In order to estimate the conditional volatility of the underlyings, an exponential GARCH specification is employed. This specification captures the leverage effect that is an important feature of the conditional volatility of the returns of securities.

In the context of a Cox default model, the signs on both the conditional volatility inputs conform to expectations, with the conditional interest volatility coefficient being highly significant. In the Cox prepayment model, both of the conditional volatility inputs are highly significant, and exhibit the expected signs. Finally, in the competing risks logistic model, the HPI conditional volatility input displays both the correct sign and is significant.

A few points in the study are worth highlighting. First, accepting that the competing options framework is a useful paradigm, this study extends the literature by providing a refined input into the empirical operationalization of embedded mortgage options valuation. Additional refinements may further enhance the accuracy with which these embedded options are priced. Second, the results of the investigation may be useful in explaining regional disparities and international differences in mortgage termination behavior. For example, if interest rate or housing price volatility varies across geographic boundaries, the relative propensity of borrowers to default or prepay might likewise vary. Finally, from the perspective of secondary market purchasers of mortgage instruments, the ability to more accurately evaluate and gauge prepayment and default probabilities may well lead to an enhanced ability to manage risk.

Endnotes

[1] Avery, Bostic, Calem and Canner (1996) argue that borrowers may not exercise the default option ruthlessly, but rather suggest that defaults are most likely only when a negative equity situation is accompanied by a ‘‘triggering event’’ such as loss of employment, decline in income, divorce, etc.

[2] While borrowers do capture the entire explicit increase in property values, lenders do benefit from such price appreciation in the form of co-insurance on the debt.

[3] The construction of *PNEQ* and *PREPAY* is detailed below. In more recent work, Deng, Quigley and Van Order (2000) label these metrics Put_Option and Call_Option, respectively.

[4] Consistent with Capozza, Kazarian and Thompson (1998), the term ‘‘unconditional’’ is employed when the volatility is conditioned exclusively on the information set available at origination. Conversely, the term ‘‘conditional’’ is used when volatility is conditioned on information available subsequent to origination.

[5] A powerful hybrid of the binomial pricing method is the model of Hilliard, Kau and Slawson (1996), who use a bivariate binomial lattice to price the competing options embedded in mortgage contracts. Their study uses a simulations methodology to price options. In this setting, the underlying stochastic process defines volatility.

[6] The Vega of the default (prepayment) option is the change in the price of the option relative to the change in the volatility of housing prices (interest rates).

[7] See Kim and Kim (1994). Two additional critical features are skewness and kurtosis.

[8] Conditional volatility estimates for interest rates are derived in an analogous fashion.

[9] The data are first-differenced to ensure stationarity.

[10] In order to ensure stationarity, the restriction on the parameters in the *GARCH* (1,1) is $\alpha_1 + \beta_1 < 1$.

[11] In order to accommodate the probability sampling design employed, all analyses reported below were conducted using the software program SUDAAN, which offers Taylor series linearization to produce correct standard errors.

[12] Traditional lender cut-offs for acceptable borrower credit risk include an LTV of 80% for non-insured loans, a total debt ratio of 31%–33%, and a FICO score of approximately 620.

[13] Closing costs and financing charges associated with refinancing the loan are also frequently rolled into the new mortgage. The underwriting lender for all the loans in the dataset defined non-cashout refinance loans as loans for which the value of the ratio of the loan amount to the outstanding lien is less than or equal to 1.06.

[14] Multiple "default" definitions are possible and have been employed throughout the literature. Consistent with the general consensus of both industry practitioners and the academic community (Ambrose and Capone 1998, 2000), delinquency is viewed as superior to foreclosure as a default metric, since delinquency status is not subject to the differential application of loss mitigation strategies across borrower characteristics or property attributes. Similarly, foreclosure as a measure of default is potentially troublesome due to differential bankruptcy provisions and other jurisdictional variances across geographic regions. For a further discussion of the appropriate definition of bad loans, see Avery, Bostic, Calem and Canner (1996) or Jaske (1997).

[15] The correlation between *PREPAY* and *INTEREST − EGARCH* is 0.21. Estimating the prepayment model and excluding the call option variable *PREPAY* produces a coefficient estimate that is consistent in sign and stronger in both magnitude and significance for the corresponding conditional volatility input parameter *INTEREST − EGARCH*. The conditional volatility of interest rates in the prepayment decision appears robust to the inclusion or exclusion of the *PREPAY* optionality parameter.

[16] Theoretically, the multinomial logit framework is one approach that could be used for estimation purposes, since the dataset includes loans with three distinct, mutually exclusive and exhaustive outcomes (*i.e.*, 889 defaults, 446 prepayments and 93 current loans). However, the distribution of mortgage outcomes across these three classifications leads to convergence and singularity problems when applying the framework to the data, as less than 7% of the sample loans remained current as of July 1997. Therefore, this study employs a two-stage approach to modeling mortgage termination decisions. For a complete discussion of appropriate operationalizations of competing risks models, see Allison (1995).

[17] A potential limitation of the two-stage approach employed in the study, rather than a multinomial logit approach, is that the binary logistic specification involves only defaults versus prepayments. To the extent that current goods are disproportionately more likely to prepay (default) rather than default (prepay) our estimated default probabilities could be overstated (understated) relative to the population of all loan originations.

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The authors thank Peter Christofferson, of the Faculty of Management, McGill University, for his many helpful comments and suggestions on this study.

David M. Harrison, University of Vermont, Burlington, VT 05405-0157 or harrison@bsadpo.emba.uvm.edu.

Thomas G. Noordewier, University of Vermont, Burlington, VT 05405-0157 or tom.noordewier@uvm.edu.

Buchi Ramagopal, University of Vermont, Burlington, VT 05405-0157 or buchi.ramagopal@uvm.edu.