The Functional Relationship and Use of Going-in and Going-out Capitalization Rates

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Abstract. **In performing a Discounted Cash Flow Analysis for an income-producing property, a traditional rule-of-thumb indicates that the going-out capitalization rate should be one-half to one percent higher than the going-in capitalization rate [4, 10]. So far, there has been no theoretical model or empirical evidence to support or to dispute this assertion. This paper develops a model to examine the determinants of the going-out capitalization rate, as well as the relationship between going-in and going-out capitalization rates in a complete market setting.**

The proposed model indicates that the rule-of-thumb can be challenged, and the selection of an appropriate going-out capitalization rate requires a careful examination of the changes in the assumed income-growth rates, changes in the assumed required rates of return, and changes in the assumed property-appreciation rates during and after the projected holding period. The functional relationship between the property-appreciation rate assumption required for Ellwood methods and the going-out capitalization rate assumption required for DCF analysis also is derived.

Introduction

In recent years, Discounted Cash Flow Analysis (DCF) has been introduced as an updated technique for the valuation of income-producing properties. Considerable efforts have been made by academics and practicing experts to either advocate the use of DCF as a better tool in the income approach to value [2, 6], or to prove that traditional Ellwood yield methods are special cases of DCF with differing assumptions about the income-growth patterns [3, 12].

Researchers have proved that variations of the Ellwood formula are just pre-solved solutions of the DCF technique if both the loan-to-value ratio and the reversion price are specified as a function of the unknown property value [1]. Since the Ellwood equations are pre-solved, the property reversion price can be specified as a percentage growth rate of the current value. In the direct application of DCF analysis, the reversion price cannot be specified as a percentage growth of the unknown current property value unless an iteration

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process is used. When performing a DCF analysis to estimate the current property value, market participants usually use a going-out capitalization rate together with a projected end-period cash flow to estimate the reversion value [4, 10].

The use of a going-out capitalization rate to estimate the property reversion value also conforms better to the underlying promise of the DCF analysis than the use of a property-appreciation rate. The underlying premise of the DCF analysis is that the value of a property is the present value of future cash flows. By capitalizing the end-period cash flow at an appropriate rate, the reversion price actually reflects the present value of the remaining future cash flows after the end of the holding period.

Similar types of techniques also have been used extensively in the finance field for stock and bond valuations. A good example would be the supernormal (nonconstant) dividend growth model which extends the Gordon valuation model by allowing for changes in dividend growth rates [7]. This model, similar to the DCF techniques, also requires the estimation of the end-period cash flow by capitalizing the next period dividend payment at an appropriate rate. Thus, the selection of an appropriate going-out capitalization rate seems to be a necessary, meaningful and important step in the use of DCF analysis.

In recent years, researchers have focused their studies on estimates of the cost of capital and equity rates that are used in the DCF analysis [5, 11]. With the same motivation, this paper examines the use and the selection of the going-out capitalization rate. Previous research on the selection of a going-out capitalization rate is extremely limited and is without any theoretical justification. Some researchers [4, 10] indicated that, as a rule-of-thumb, the going-out capitalization rate is about one-half to one percent higher than the going-in capitalization rate.

When applying the going-out capitalization rate to estimate the property reversion value, there are at least three important issues which require further examination. The first issue concerns the determinants of the going-out capitalization rate. So far, there has been no theoretical model to guide the selection of an appropriate rate.

The second issue concerns the relationship between the going-in capitalization rate and the going-out capitalization rate. Economic reasons to support the rule-of-thumb that the going-out capitalization rate should be higher than the going-in capitalization rate have not yet been established.

The last issue concerns the relationship between the going-out capitalization rate used in the DCF analysis and the property appreciation rate assumed in Ellwood methods. Since both techniques are derived from the same present-value model, a functional relationship must exist between these two variables. This issue is important because, frequently, appraisers use both techniques in the same appraisal report and fail to recognize the inconsistency between these two assumptions.

This paper develops a model to address these three issues, with an emphasis on the second concern. Section two reviews the literature, while section three presents our propositions. Section four shows the implications of the propositions derived in section three. The fifth section contains the conclusions.

Literature Review

Lusht [8], in his survey of 640 appraisals, found that the estimate of reversion value in an appraisal report is among the most difficult parts to understand. In the application of DCF analysis, Graham [4] and Martin [10] pointed out that appraisers normally estimate the reversion value by capitalizing the final cash flow, or the cash flow one year beyond the projected holding period, with a selected going-out capitalization rate. This selected rate, normally, is not substantiated by appraisers and could be higher than or equivalent to the going-in capitalization rate.

Graham [4], by using several case studies, suggested that the selected going-out capitalization rate should be the same as the going-in capitalization rate and should capitalize the cash flow one year beyond the projected holding period. Martin [10] suggested that if the property is at stabilized occupancy level or has an especially limited economic life, the going-out capitalization rate should not be lower than the going-in capitalization rate. He further pointed out that sales costs and an irregular expense pattern could lower the going-out capitalization rate. Both researchers agreed that, in the application of a going-out capitalization rate, appraisers normally add one-half to one percentage point to the going-in capitalization rate.

Unfortunately, both authors failed to provide a conceptual model to support their assertions. The functional relationship between the underlying economic forces and the magnitude of the difference between these two capitalization rates, if these two rates should be different, still is not clear. While the capitalization rate is not the focus of his paper, Lusht [9] established a conceptual two-period model that examines the relationship between the reversion value and the change in income-growth rates during and after the projected holding period. Lusht's work provides the first rigorous model that estimates the reversion value in the context of the economic life of the property; this model requires an estimation of the income-growth pattern beyond the projected holding period.

Following the methodology established by Lusht, this paper develops a model to examine the determinants of going-out capitalization rates as well as the relationship between the going-in and going-out capitalization rates in a complete market setting. The proposed model shows that the going-out capitalization rate could be higher, lower or equivalent to the going-in capitalization rate, depending on the assumptions concerning the changes in income-growth rates, changes in required rates of return, and changes in property-appreciation rates during and after the projected holding period.

The Propositions

Under a present-value model setting, we are able to derive the following four propositions regarding the relationship between the going-in and going-out capitalization rates. The details of the model derivation are provided in Appendix 1:

1. *Scenario One*: As long as there are: (i) a constant income-growth rate before and after the projected holding period, (ii) a constant required rate of return before and after the projected holding period, and (iii) a property-appreciation rate which is a function of the income-growth rate, the going-out capitalization rate always should be equal to the going-in capitalization rate.
2. *Scenario Two*: With all conditions identical to the assumptions in Scenario One except that the income-growth rate after the projected holding period is higher than the income-growth rate during the holding period, the going-out capitalization rate always is lower than the going-in capitalization rate. Similarly, if the income-growth rate is lower after the projected holding period, a reverse relationship will hold.

3. *Scenario Three*: With all conditions identical to the assumptions in Scenario One except that the required rate of return after the projected holding period is higher than the required rate of return during the holding period, the going-out capitalization rate always is higher than the going-in capitalization rate. A reverse relationship holds if the required rate of return is lower after the projected holding period.
4. *Scenario Four*: With all conditions identical to the assumptions in Scenario One except that the property-appreciation rate after the projected holding period is higher than the property-appreciation rate during the holding period, the going-out capitalization rate always is lower than the going-in capitalization rate. Of course, if the property-appreciation rate is lower after the projected holding period, a reverse relationship will hold.

To illustrate *Scenario One*, assume

1. Income Growth Rate (g) = 5% per year,
2. Required Rate of Return (y) = 15% per year,
3. Property Appreciation Rate (a) = 5% per year,
4. Current Net Income (I_o) = \$100,000 per year,
5. Project Holding Period (h) = 5 years.

Under these assumptions and applying the Gordon valuation model, the current value of the property is \$1,050,000. Given this property value and the first-year income of \$105,000, the resulting going-in capitalization is 10%. The property value at the end of year 5 (the projected holding period) and the net operating income at end of year 6 are \$1,340,096 and \$134,010, respectively. The resulting going-out capitalization rate at the end of the holding period, again, is 10%. This result supports our first proposition. Table 1 of Appendix 2 contains mathematical calculations of this scenario.

To illustrate *Scenario Two*, assume

1. Income Growth Rate (g) = 5% per year for the first six years,
2. Income Growth Rate (g_1) = 7% per year thereafter,

with all other assumptions being the same as in Scenario One. Under these assumptions, the current value of the property is \$1,216,566, the first-year income is \$105,000, and the resulting going-in capitalization is 8.63%. The property value at the end of year 5 and the net operating income at end of year 6 are \$1,675,120 and \$134,010, respectively. The resulting going-out capitalization rate at the end of the holding period is 8%. Note that the going-out capitalization rate is lower than the going-in capitalization rate. Table 2 of Appendix 2 contains the mathematical calculations of this scenario.

To demonstrate *Scenario Three*, assume

1. Required Rate of Return (y) = 15% per year for the first five years,
2. Required Rate of Return (y_1) = 17% per year thereafter,

with all other assumptions being the same as in Scenario One. Under these assumptions, the current value of the property is \$938,965, the first-year income is \$105,000, and the resulting going-in capitalization is 11.18%. The property value at the end of year 5 and the net operating income at end of year 6 are \$1,116,746 and \$134,010, respectively. The resulting going-out capitalization rate at the end of the holding period is 12%. Note that

Exhibit 1
Summary Table

Scenario	Income Growth Rate	Required Rate of Return	Property Appreciation Rate	Relationship between Two Cap Rates
One	Same	Same	Same	Same
Two	*G*	Same	Same	*I*> *O*
Three	Same	*G*	Same	*O*> *I*
Four	Same	Same	*G*	*I*> *O*

Notes:
G: If the rate after the holding period is greater than the rate during the holding period.
I: Going-in capitalization rate.
O: Going-out capitalization rate.

the going-out capitalization rate is higher than the going-in capitalization rate. Table 3 of Appendix 2 contains the mathematical calculations of this scenario.

To demonstrate *Scenario Four*, assume

1. Property Appreciation Rate (a) = 5% per year for the first four years,
2. Property Appreciation Rate (a_1) = 6% per year thereafter,

with all other assumptions being the same as in Scenario One. Under these assumptions, the current value of the property is \$1,067,654, the first-year income is \$105,000, and the resulting going-in capitalization is 9.83%. The property value at the end of year 5 and the net operating income at end of year 6 are \$1,375,605 and \$134,010, respectively. The resulting going-out capitalization rate at the end of the holding period is 9.74%. Note that the going-out capitalization rate is lower than the going-in capitalization rate. Table 4 of Appendix 2 contains the mathematical calculations of this scenario.

Exhibit 1 summarizes these four relationships. All other relationships between the going-in and going-out capitalization rates, under different assumptions, can be derived easily by using variations of the model. Finally, we conclude that the going-out capitalization rate should be higher than the going-in capitalization rate if and only if the projected income-growth rate is higher than the projected property-appreciation rate during the projected holding period. The intuitive explanation is simple. At any given time, the property-appreciation rate is dependent upon the growth of future incomes. If the future income-growth rate is lower than the previous income-growth rate, the property should appreciate at a rate lower than the previous income-growth rate. This result also confirms the finding presented in Scenario Two. Appendix 3 contains the mathematical derivation of this relationship.

Implications of the Results

The model shows that, in a complete market setting, the going-in and going-out capitalization rates should be the same if there is no reason to assume that the income-growth rates, required rates of return, or property-appreciation rates are different during and after the projected holding period. The going-out capitalization rate could be higher or lower than the going-in capitalization rate depending on the assumptions made on these parameters. These findings directly contradict the traditional rule-of-thumb that the going-out capitalization rate always should be higher than the going-in capitalization rate.

The going-out capitalization rate could be lower than the going-in capitalization rate if the income-growth rate after the holding period is expected to be higher than the expected income-growth rate during the holding period. This finding is particularly important for an over- or under-supplied market. In an over-supplied market, where the rent is expected to remain constant or to decrease for initial periods, and thereafter to increase at a much faster pace as the inventory is absorbed by the market, the going-out capitalization rate should be lower than the going-in capitalization rate. Since most cities grow in a cyclical pattern, such an income-growth pattern frequently is observed in the real world and can be projected reasonably based on the market supply-and-demand condition.

It is possible to argue that as a property grows older the expected income-growth rate decreases so that the going-out capitalization rate should be consistently higher than the going-in capitalization rate. However, the age of property is not the only determinant of the expected income-growth rate. In fact, the current and future market supply-and-demand conditions might play a more important role than the property age in determining the changes in income-growth rates, especially in a market where the current supply and demand conditions are not in equilibrium. A blind application of the rule-of-thumb for the selection of a going-out capitalization rate without a careful examination of the income-growth assumptions made in the appraisal report seems imprudent.

The going-out capitalization rate also is sensitive to changes in expected required rates of return during and after the holding period. This finding is particularly important if a sharply sloped yield curve is observed in the market. In a normal market, where a long-term rate (which is assumed to apply to an investment period that is significantly longer than the projected property holding period) is higher than or equivalent to a short-term rate, the projected going-out capitalization rate should be higher than the going-in capitalization rate. However, if an inverse yield curve is observed in the market, so that the projected long-term rate is lower than the projected short-term rate, the going-out capitalization rate could be lower than the going-in capitalization rate.

Although an upward-sloping yield curve is typical in the market, inverse yield curves also have been observed in the past. The argument, based on the slope of the yield curve, that the going-out capitalization rate always should be higher than the going-in capitalization rate also is not substantiable.

The analysis also indicates that if a property appreciates at a lower rate after the projected holding period, the going-out capitalization rate should be higher than the going-in capitalization rate. It is possible that property appreciates at a much slower rate at the end of its economic life because of the physical or functional depreciation of the building component. However, if a property is expected to have a change of highest and best use and the land component appreciates at a much faster rate than the property as a whole, a higher property-appreciation rate can be expected after the projected holding period. An argument for a consistently higher going-out capitalization rate, based on the assumption that property appreciates at a slower rate after the projected holding period, may be suspect.

Conclusions

In the application of DCF analysis, appraisers routinely select a going-out capitalization rate that is one-half to one percent higher than the going-in capitalization rate. This paper demonstrates that the going-out capitalization rate should be selected based on the assumptions made concerning the changes in income-growth rates, changes in property-appreciation rates, and changes in required rates of return during and after the projected holding period. The assumptions determine whether the appropriate going-out capitalization rate should be set higher, lower or equal to the going-in capitalization rate. This finding is particularly important in an unstable market with over- or under-supply conditions.

In a stable market, and with the assumption of a decreasing income-growth rate due to the increasing property age and/or the assumption of an upward-sloping yield curve, the going-out capitalization rate should be higher than the going-in capitalization rate. However, even under these circumstances, the magnitude of the difference could vary significantly from case to case. If the property age argument is used, the difference could be much larger if the property is forty years old with a ten-year projected holding period than if the property is one year old with a five-year projected holding period. A blind use of the one-half to one percent rule seems unconvincing.

This paper demonstrates that both the direction and the magnitude predicted by the rule-of-thumb can be challenged. The selection of an appropriate going-out capitalization rate requires a careful examination of the economic parameters assumed during and after the projected holding period. Furthermore, appraisers should be aware that there is a definite relationship between the property-appreciation rate assumption used in Ellwood methods and the going-out capitalization rate assumption used in the DCF analysis. When both techniques are used in the same report, a inconsistency between these two assumptions could seriously damage the credibility of that report.

References

[1] Roger E. Cannaday and Peter F. Colwell. Real Estate Valuation Models: Lender and Equity Investor Criteria. *AREUEA Journal* (Summer 1986), 316–37.

[2] Steven J. Decker. Discounted Cash Flow Analysis and The Appraisal Process: The Application of Probable Investment Value. *The Real Estate Appraiser and Analyst* (Winter 1985), 13 18.

[3] Robert A. Gibson. Ellwood Is Discounted Cash Flow Before Taxes. *The Appraisal Journal* (July 1986), 406 15.

[4] Gale Graham. Use 'Equivalency' Keep Your Friendly Lender Solvent. *The Real Estate Appraiser and Analyst* (Winter 1985), 19 22.

[5] Karl L. Guntermann and Richard L. Smith. Derivation of Cost of Capital and Equity Rates from Market Data. *AREUEA Journal* (Summer 1987), 98 109.

[6] Peter F. Korpacz and Mark I. Roth. Changing Emphasis in Appraisal Techniques: The Transition to Discounted Cash Flow. *The Appraisal Journal* (January 1983), 21–44.

[7] Cheng F. Lee and Joseph E. Finnerty. *Corporate Finance: Theory, Method, and Application.* New York: Harcourt Brace Jovanovich, 1990.

[8] Kenneth M. Lusht. The Behavior of Appraisers in Valuing Income Property: A Status Report. *The Real Estate Appraiser and Analyst* (July August 1979), 49 59.

[9] ——. Estimating Reversion Value in Discounted Cash Flow Models. *The Appraisal Journal* (April 1987), 191 97.

[10] Vernon Martin. Nine Abuses Common in Pro Forma Cash Flow Projections. *Real Estate Review* (Fall 1988), 20–25.
[11] C. F. Sirmans and James R. Webb. Expected Equity Returns on Real Estate Financed with Life Insurance Company Loan: 1967–1977. *AREUEA Journal* (Summer 1980), 218–28.
[12] Paul F. Wendt. Ellwood, Inwood, and the Internal Rate of Return. *The Appraisal Journal* (October 1967), 561–74.

Appendix 1
Model Derivation

The model inputs include the following definitions:

1. g_i = 1 + income growth-rate of period i,
2. $G_i = \pi_{t=0}^{i} g_t$ = cumulative income growth-rate at period i,
3. y_i = 1 + required rate of return of period i,
4. $Y_i = \pi_{t=0}^{i} y_t$ = cumulative required rate of return at period i,
5. a_i = 1 + property-appreciation rate of period i,
6. $A_i = \pi_{t=0}^{i} a_t$ = cumulative property-appreciation rate at period i,
7. I_o = current property net operating income,
8. V_o = current property value,
9. $I_i = I_o * G_i$ = property net operating income at the end of period i,
10. $V_i = V_o * A_i$ = property value at the end of period i,
11. h = projected property holding period,
12. m = Economic life of the building,
13. $R_o = I_1 / V_o$ = going-in capitalization rate, and
14. $R_h = I_{h+1} / V_h$ = going-out capitalization rate.

For an income-producing property, the current value of the property is the present value of all the future cash flows, or

$$V_o = \sum_{i=1}^{m} \frac{I_i}{Y_i} + \frac{V_m}{Y_m}$$

$$= \sum_{i=1}^{h} \frac{I_i}{Y_i} + \sum_{i=h+1}^{m} \frac{I_i}{Y_i} + \frac{V_m}{Y_m} \tag{1}$$

In a complete market, the value of a property in any future year should be the present value of the remaining future cash flows after that year, or

$$V_h = \left(\sum_{i=h+1}^{m} \frac{I_i}{Y_i} + \frac{V_m}{Y_m} \right) * Y_h \tag{2}$$

By combining equations (1) and (2), we derive

$$V_o = \sum_{i=1}^{h} \frac{I_i}{Y_i} + \frac{V_h}{Y_h} \tag{3}$$

Note that equation (3) is exactly the format of a DCF analysis and by definition,

$$R_h = \frac{I_{h+1}}{V_h} \quad \text{and}$$

$$R_o = \frac{I_1}{V_o}.$$

By rearranging the terms in equation (3), we obtain

$$V_h = V_o * Y_h - \left(\sum_{i=1}^{h} \frac{I_i}{Y_i} \right) * Y_h \tag{4}$$

From equation (4), the going-out capitalization rate at the end of holding period h, can be rewritten as

$$R_h = \frac{I_{h+1}}{V_h} \tag{5}$$

$$= \frac{I_o * G_h * g_{h+1}}{V_o * Y_h - \sum_{i=1}^{h} \frac{I_o * G_i}{Y_i} * Y_h * \frac{V_o}{V_o}}$$

$$= R_o * \frac{G_h}{Y_h} * \frac{g_{h+1}}{g_1} * \left(\frac{1}{1 - \left\{ \sum_{i=1}^{h} \frac{G_i}{Y_i} \right\} * \frac{I_o}{V_o}} \right) \tag{6}$$

From equation (5), we know that the going-out capitalization rate should be a function of the income-growth rates before and after the holding period, as well as the required rates of return and the property-appreciation rates after the projected holding period of the investment. To simplify the equation, we rewrite equation (6) as

$$R_h = R_o * B \tag{7}$$

B equals to the product of the last three terms on the right-hand side of equation (6), and is a function of the income-growth rates, required rates of return, and property-appreciation rates over the economic life of the property. From equation (7), we know

(1) If $B > 1$, then $R_h > R_o$.
(2) If $B = 1$, then $R_h = R_o$.
(3) If $B < 1$, then $R_h < R_o$.

To simplify the following presentation, the definitions of g, a, and y have been changed to represent the income-growth rate, property-appreciation rate, and required rate of return for each period.

Scenario One

Assume:

(1) $g_i = g$, so that $G_i = (1+g)^i$, for $i = 1$ to m.
(2) $y_i = y$, so that $Y_i = (1+y)^i$, for $i = 1$ to m.
(3) $a_i = g_i = a$, so that $A_i = (1+g)^i = (1+a)^i$, for $i = 1$ to m.

Using these assumptions and equation (7), we obtain

$$B = \frac{(1+g)^h}{(1+y)^h} * \frac{(1+g)}{(1+g)} * \left(\frac{1}{1 - \{ \sum_{i=1}^{h} \frac{(1+g)^i}{(1+y)^i} \} * \frac{I_o}{V_o}} \right)$$

$$= \frac{1 - \{ \sum_{i=1}^{h} \frac{(1+g)^i}{(1+y)^i} \} * \frac{y-g}{(1+g)}}{1 - \{ \sum_{i=1}^{h} \frac{(1+g)^i}{(1+y)^i} \} * \frac{I_o}{V_o}} \quad (8)$$

Note that the current value of the property, under this set of assumptions, is

$$V_o = \sum_{i=1}^{m} \frac{I_o * (1+g)^i}{(1+y)^i} + \frac{V_o * (1+g)^m}{(1+y)^m}$$

$$= \frac{I_o * (1+g)}{y-g} = V* \quad (9)$$

From equations (8) and (9), we know that $B = 1$ and $R_h = R_o$. As long as there are a constant income-growth rate, a constant required rate of return, and a property-appreciation rate that is equal to the income-growth rate, the going-out capitalization rate always should be equal to the going-in capitalization rate. To simplify the following derivations, we denote the V_o derived from equation (9) as $V*$.

Scenario Two

Assume:

(1) $g_i = g$, so that $G_i = (1+g)^i$, for $i = 1$ to $h+1$, $g_i = g_1$, for $i = h+2$ to m, and $g <> g_1$.
(2) $y_i = y$, so that $Y_i = (1+y)^i$, for $i = 1$ to m.
(3) $A_i = (1+k)^i$, for $i = 1$ to $h+1$, $k_1 > k$ if $g_1 > g$, $A_i = (1+k)^{(h+1)} * (1+k_1)^{(i-h-1)}$, for $i = h+2$ to m.

The current value of the property, under this set of assumptions, is

$$V_o=\sum_{i=1}^{h+1}\frac{I_o*(1+g)^i}{(1+y)^i}+\sum_{i=h+2}^{m}\frac{I_o*(1+g)^{(h+1)}*\{(1+g)+(g_1-g)\}^{(i-h-1)}}{(1+y)^i}$$
$$+\frac{V_o*(1+k)^{(h+1)}*(1+k_1)^{(m-h-1)}}{(1+y)^m} \qquad (10)$$

If $g_1>g$, then $V_o>V^*$ and

$$V_o>\frac{I_o*(1+g)}{y-g}$$

From equations (8), (9) and (10), we know that $B<1$ and $R_h<R_o$ if $g_1>g$. With all assumptions identical to the assumptions in Scenario One except that the income-growth rate after the projected holding period is higher than the income-growth rate during the holding period, the going-out capitalization rate always is lower than the going-in capitalization rate.

Scenario Three

Assume:

(1) $g_i=g$, so that $G_i=(1+g)^i$, for $i=1$ to m.
(2) $y_i=y$, so that $Y_i=(1+y)^i$, for $i=1$ to h, $y_i=y_1$, for $i=h+1$ to m, and $y<>y_1$.
(3) $A_i=(1+k_i)^i$, for $i=1$ to m.

The current value of the property, under this set of assumptions, is

$$V_o=\sum_{i=1}^{h}\frac{I_o*(1+g)^i}{(1+y)^i}+\sum_{i=h+1}^{m}\frac{I_o*(1+g)^i}{(1+y)^h*\{(1+y)+(y_1-y)\}^{(i-h)}}$$
$$+\frac{V_o*(1+k_m)^m}{(1+y)^h*(1+y_1)^{(m-h)}} \qquad (11)$$

If $y_1>y$, then $V_o<V^*$ and

$$V_o<\frac{I_o*(1+g)}{y-g}$$

From equations (8), (9) and (11), we know that $B>1$ and $R_h>R_o$ if $y_1>y$. We demonstrate that, with all assumptions identical to the assumptions in Scenario One except that the required rate of return after the projected holding period is higher than the required rate of return during the holding period, the going-out capitalization rate always is higher than the going-in capitalization rate.

Scenario Four

Assume:

(1) $g_i = g$, so that $G_i = (1+g)^i$, for $i = 1$ to m.
(2) $y_i = y$, so that $Y_i = (1+y)^i$ for $i = 1$ to m.
(3) $a_i = g_i = a$, so that $A_i = (1+a)^i$, for $i = 1$ to $h-1$, $a_i = a_1$, for $i = h$ to m, and $a <> a_1$.

The current value of the property, under this set of assumptions, is

$$V_o = \sum_{i=1}^{m} \frac{I_o * (1+g)^i}{(1+y)^i} + \frac{V_o * (1+a)^{(h-1)} * (1+a_1)^{(m-h+1)}}{(1+y)^m} \tag{12}$$

If $a_1 > a$, then $V_o > V*$ and

$$V_o > \frac{I_o * (1+g)}{y-g}$$

From equations (8), (9) and (12), we know that $B < 1$ and $R_h < R_o$ if $a_1 > a$. A relationship which is similar to the conclusion presented in Scenario One is derived. A higher property appreciation-rate assumption after the projected holding period results in a lower going-out capitalization rate.

Appendix 2
Numerical Examples

Table 1.A: Scenario One

No	Net Income	Reversion Value	Discount Factor	Present Value
0	$100,000	NA	1.00000	NA
1	$105,000	NA	0.86957	$91,304
2	$110,250	NA	0.75614	$83,365
3	$115,763	NA	0.65752	$76,116
4	$121,551	NA	0.57175	$69,497
5	$127,628	$1,340,096*	0.49718	$729,718
6	$134,010	NA	NA	NA
Property Value:				$1,050,000

* = $134,010/(0.15-0.05)

Table 1.B:

No	Property Value	Cap. Rate
0	$1,050,000	10.00%
1	$1,102,500	10.00%
2	$1,157,625	10.00%
3	$1,215,506	10.00%
4	$1,276,282	10.00%
5	$1,340,096	10.00%

Table 2.A: Scenario Two

No	Net Income	Reversion Value	Discount Factor	Present Value
0	$100,000	NA	1.00000	NA
1	$105,000	NA	0.86957	$91,304
2	$110,250	NA	0.75614	$83,365
3	$115,763	NA	0.65752	$76,116
4	$121,551	NA	0.57175	$69,497
5	$127,628	$1,675,120**	0.49718	$896,284
6	$134,010	$1,792,378*	NA	NA
7	$143,390	NA	NA	NA
Property Value:				$1,216,566

* = $143,390/(0.15-0.07)
** = ($1,792,378 + $134,010)/1.15

Table 2.B:

No	Property Value	Cap. Rate
0	$1,216,566	8.63%
1	$1,294,051	8.51%
2	$1,377,909	8.40%
3	$1,468,832	8.27%
4	$1,567,607	8.14%
5	$1,675,120	8.00%
6	$1,792,378	8.00%

Table 1.A calculates the current value of the property. The reversion value at the end of year 5 is estimated by using the Gordon model. The property value of each year in Table 1.B is calculated by compounding the current property value, $1,050,000, at the assumed 5% annual appreciation rate. Note that the resulting property value at the end of year 5 is identical to the reversion value used in Table 1.A. For Table 2.A, the reversion value at the end of year 6 also is estimated by using the Gordon model. The reversion value at the end of year 5 is the present value of the cash flows (net income and reversion value) in year 6, discounted at the required rate of return for one year. This method also is applied to estimate the property value in each year in Table 2.B. Note that the resulting current property value in Table 2.B is identical to the present value of the property estimated in Table 2.A.

Table 3.A: Scenario Three

No	Net Income	Reversion Value	Discount Factor	Present Value
0	$100,000	NA	1.000000	NA
1	$105,000	NA	0.869565	$91,304
2	$110,250	NA	0.756144	$83,365
3	$115,763	NA	0.657516	$76,116
4	$121,551	NA	0.571753	$69,497
5	$127,628	$1,116,746**	0.497177	$618,674
6	$134,010	$1,172,584*	NA	NA
7	$140,710	NA	NA	NA
Property Value:				$938,956

* = $140,710/(0.17–0.05)
** = ($1,172,584 + $134,010)/1.15

Table 3.B:

No	Property Value	Cap. Rate
0	$938,956	11.18%
1	$974,799	11.31%
2	$1,010,769	11.45%
3	$1,046,622	11.61%
4	$1,082,065	11.79%
5	$1,116,746	12.00%
6	$1,172,584	NA

Table 4. A: Scenario Four

No	Net Income	Reversion Value	Discount Factor	Present Value
0	$100,000	NA	1.000000	NA
1	$105,000	NA	0.869565	$91,304
2	$110,250	NA	0.756144	$83,365
3	$115,763	NA	0.657516	$76,116
4	$121,551	NA	0.571753	$69,497
5	$127,628	$1,375,605*	0.497177	$747,373
6	$134,010	NA	NA	NA
Property value;				$1,067,654

* = estimated by using an iteration process

Table 4.B:

No	Property Value	Cap. Rate
0	$1,067,654	9.83%
1	$1,121,037	9.83%
2	$1,177,089	9.83%
3	$1,235,943	9.83%
4	$1,297,741	9.83%
5	$1,375,605	9.74%

The reversion value at the end of year 6 in Table 3.A is estimated by using the Gordon model. The reversion value at the end of year 5 is the present value of the cash flows (net income and reversion value) in year 6, discounted at the required rate of return for one year. This method also is applied to estimate the property value in each year in Table 3.B.

Note that the resulting current property value in Table 3.B is identical to the present value of the property estimated in Table 3.A. Given the assumed property-appreciation and income-growth rates, the reversion value of $1,375,605 in Table 4.A is estimated by using an iteration process. The property value in each year in Table 4.B is estimated by compounding the current value derived in Table 4.A at the assumed appreciation rates. Note that the property value at the end of year 5 is identical to the reversion value derived from the iteration process.

Appendix 3
DCF and Ellwood

Use the same notations as in Appendix 1 and assume:

(1) $g_i = g$, so that $G_i = (1+g)^i$, for $i = 1$ to $h+1$.
(2) $a_i = a$, so that $A_i = (1+a)^i$, for i = 1 to $h+1$.

The reversion value can then be specified as

$$V_h = V_o * A_h = \left(\frac{I_o * (1+g)}{R_o}\right) * A_h = \frac{I_o * G_{h+1}}{R_h} \tag{13}$$

By rearranging the terms in equation (13), we obtain

$$\frac{R_h}{R_o} = \frac{I_o * G_{h+1}}{I_o * (1+g) * A_h} = \frac{G_h}{A_h} \tag{14}$$

Equation (14) demonstrates that a higher going-out capitalization rate implies that the income-growth rate during the holding period is higher than the property-appreciation rate during this period.

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